

Quantum speed limit of Jaynes-Cummings model with detuning for arbitrary initial states

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ABSTRACT

The quantum speed limit (QSL) of the Jaynes-Cummings model with detuning for arbitrary initial states is investigated. We mainly focus on the influences of the detuning, width of Lorentzian spectral density, and coherence of the initial state on the non-Markovian speedup evolution in an open system. It is found that even in the Markovian regime, increasing the detuning parameter leads to quantum speedup. Moreover, we reveal that the QSL has an inverse relation with the population of the initial excited state. Notably, we show that the QSL depends on the quantum coherence of the system's initial state such that the maximal coherent state can saturate its bound.

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1. Introduction

In the study of time evolution of quantum systems, the first question that comes to mind is how fast a quantum process evolves to an orthogonal state? Trying to find the answer of this question is the starting point to understand the concept of quantum speed limit (QSL). Actually, the QSL is the maximum evolution speed of a quantum system. Knowing the QSL of the evolution of a quantum system is useful and has particular importance in various fields of quantum physics, such as quantum communication [1], quantum computation [2], and quantum metrology [3]. In fact, the existence of decoherence makes the attempt to estimate the evolution time of a quantum process to be of key importance in designing quantum control protocols. In order to design quantum control protocols in the implementation of quantum information tasks, estimating the evolution time of a quantum process is of key importance.

How to speedup the evolution of quantum systems is one of the fundamental problems and questions in quantum theory [4–9]. The shortest possible time for a quantum system to transition from an initial state to a final state separated by a predetermined distance

is known as the QSL time [10–19]. Based on theoretical and experimental study models, QSL time has a wide range of operational applications in quantum computation and transportation [20–24]. Besides, the study of QSL time is of key importance in the development of quantum information technology and quantum optimal theory [25–28]. The concept of QSL time has been also used in the study of quantum batteries [29–32].

The first definition for QSL time is provided for the case of closed quantum systems whose their time evolution is described by the unitary time evolution operator. In Ref. [33], Mandelstam and Tamm (MT) show that the minimal time required for a closed quantum system to transform from an initial state to an orthogonal one is described by the following bound

$$\tau \geq \frac{\pi \hbar}{2\Delta E}, \quad (1)$$

where ΔE is the energy variance of the system with $\Delta E^2 = \langle \psi | H^2 | \psi \rangle - \langle \psi | H | \psi \rangle^2$ and H is the Hamiltonian of system. The bound in Eq. (1) is known as MT bound. In addition, Margolus and Levitin (ML) have obtained another bound based on the average energy as [34]

$$\tau \geq \frac{\pi \hbar}{2E}, \quad (2)$$

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where $E = \langle \psi | H | \psi \rangle$ is mean energy. It is also assumed that the energy of the ground state E_0 is zero. The bound in Eq. (2) is known as ML bound. By combining the MT and ML bound, a unified tight bound can be obtained for closed quantum systems as [35]

$$\tau \geq \max\left\{\frac{\pi \hbar}{2\Delta E}, \frac{\pi \hbar}{2E}\right\}. \quad (3)$$

By using various metrics such as trace distance, Bures angle and relative purity, MT and ML bounds have been successfully generalized [36–41]. In addition, for both unitary and non-unitary evolutions, a tight bound for QSL time can be obtained by the distance between the Bloch vectors describing the state of the system [42,43]. The ratio between QSL time and the actual evolution time determines whether is it possible to speedup the dynamical evolution. If this ratio is equal to unity, then the QSL bound is saturated and there is no possibility of speeding up the dynamical evolution. If the ratio of the QSL time to actual evolution time is less than one, then the QSL bound is not saturated and there exist the potential for dynamical evolution speedup.

A real quantum system interacts with its surroundings and the environment disturbs the system as a huge resource of decoherence and dispersion. The study of open quantum systems from various aspects has been the subject of many new researches in the field of quantum information in recent years due to their wide application in quantum information protocols [44–46]. An attractive way of dealing with open quantum systems is through the flow of information between the system and its surroundings. From the insight of memory effects, the evolution of open quantum systems are divided into two categories: Markovian and non-Markovian. Indeed, Markovian process is known as memory-less evolution. In Markovian process the information flow from system to environment monotonically. On the other hand, non-Markovian process is recognized as with memory process where the back-flow of information from the environment to the system will occur. In the non-Markovian process, the back-flow of information is associated with memory effects since the back-flow makes the future states of the system depend on its past states.

In Ref. [36], it is shown that the non-Markovian nature of the process speedup the quantum evolution. Actually, the non-Markovian effects induce the unsaturated QSL bounds for open quantum dynamics. The mechanisms to speedup the quantum evolution through the regulation of non-Markovianity have been studied both theoretically and experimentally [47–52]. Controlling and speeding up due to the non-Markovian feature of the quantum process is completely depend on the environmental parameters. Beside non-Markovian effects, other factors that cause the unsaturated QSL bounds should also be considered [38,53].

In this work, the Jaynes-Cumming model with detuning will be considered. We will study the effects of non-Markovianity and detuning on QSL time. We first determine which environmental parameters with which values cause the non-Markovianity of quantum evolution, and then we study the effect of these parameters on the QSL time. In this study, it is shown that in addition to the non-Markovian effects, the speed of quantum evolution can be increased by adjusting the detuning parameter, even in Markovian evolution. Moreover, we will consider the effects of detuning δ and the width of the Lorentzian spectral density λ on QSL time. Notably, we reveal that in the case of short reservoir correlation time, the non-Markovian effects speedup quantum evolution. We also find that increasing the detuning parameter and deviation from the resonance mode will speedup the quantum process for both Markovian and non-Markovian processes.

The work is organized as follows. In Sec. 2, the considered model is introduced. The non-Markovianity for considered model is studied in Sec. 3. The QSL time for considered model will be

investigated in Sec. 4. Finally, we will summarize the results in Sec. 5.

2. The model

Let us consider a two level system with excited state $|e\rangle$ and ground state $|g\rangle$. The system interacts with an environment consist of the quantized modes of high-Q cavity [54,55]. The model can be characterized by the following Hamiltonian

$$H = \frac{1}{2}\omega_0\sigma_+\sigma_- + \sum_k \omega_k b_k^\dagger b_k + \sum_k (g_k \sigma_+ b_k + g_k^* \sigma_- b_k^\dagger), \quad (4)$$

where ω_0 is transition frequency from excited to ground state, σ_+ and σ_- are rising and lowering operators respectively, ω_k is the frequency of the k th field mode of cavity, b_k and b_k^\dagger are annihilation and creation operators respectively and g_k quantifies the coupling between the system and environment. We consider the case in which there exist a single excitation in the total system, so the initial state of the whole system can be expressed as

$$|\psi(0)\rangle = c_1(0)|e\rangle_S|0\rangle_E + \sum_k c_k(0)|g\rangle_S|1_k\rangle_E, \quad (5)$$

where $|0\rangle_E$ shows the vacuum state of the environment and $|1_k\rangle_E$ is the state of environment with excitation in k th mode. Hence the state of total system at any time t can be written as

$$|\psi(t)\rangle = c_1(t)|e\rangle_S|0\rangle_E + \sum_k c_k(t)|g\rangle_S|1_k\rangle_E. \quad (6)$$

By using the Schrödinger equation, a series of differential equations for probability amplitudes $c_1(t)$ and $c_k(t)$ can be obtained as

$$\dot{c}_1(t) = -i \sum_k g_k \exp[i(\omega_0 - \omega_k)t] c_k(t), \quad (7)$$

$$\dot{c}_k(t) = -i g_k^* \exp[-i(\omega_0 - \omega_k)t] c_1(t). \quad (8)$$

It can be assumed that there are no photons in the initial state of the whole system, which means that probability amplitude $c_k(0)$ is equal to zero. By solving Eq. (8) and substituting the solution into Eq. (7), one can obtain the following integro-differential equation as

$$\dot{c}_1(t) = - \int_0^t dt_1 f(t-t_1) c_1(t_1). \quad (9)$$

In the above equation, $f(t-t_1)$ describes the correlation function, which is related to the spectral density of the environment by the following relation

$$f(t-t_1) = \int d\omega J(\omega) \exp[i(\omega_0 - \omega)(t-t_1)]. \quad (10)$$

So, it can be said that the exact form of the probability amplitude $c_1(t)$ completely depends on the choice of the environment's spectral density. Herein, the Lorentzian spectral density with detuning will be considered as

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma \lambda^2}{(\omega_0 - \omega - \delta)^2 + \lambda^2}, \quad (11)$$

where $\delta = \omega_0 - \omega_c$ is detuning and ω_c is the center frequency of the cavity. Notice that the effective coupling between the qubit and the environment decreases with increasing detuning. In the Lorentzian spectral density, the parameter λ is the spectral width of the environment and is related to the correlation time of the environment as $\tau_E = \lambda^{-1}$. On the other hand, γ is related to the time

scale τ_S , during which the state of the system changes through $\tau_S = \gamma^{-1}$ [44]. By employing the Lorentzian spectral density, the correlation function of the environment $f(t - t_1)$ can be obtained as

$$f(t - t_1) = \frac{1}{2} \gamma \lambda \exp[-(\lambda - i\delta)(t - t_1)]. \quad (12)$$

By substituting the above equation in Eq. (9) and using the Laplace transformation, the integro-differential equation (9) is easily solved and the probability amplitude $c_1(t)$ is obtained as

$$c_1(t) = c_1(0)k(t), \quad (13)$$

where

$$k(t) = e^{-(\lambda - i\delta)t/2} \left[\cosh\left(\frac{\Omega t}{2}\right) - \frac{\lambda - i\delta}{\Omega} \sinh\left(\frac{\Omega t}{2}\right) \right], \quad (14)$$

with $\Omega = \sqrt{(\lambda - i\delta)^2 - 2\gamma\lambda}$. Also, the dynamics of the model can be described by the master equation having the following form

$$\frac{\partial}{\partial t} \rho(t) = \gamma(t) \left(\sigma_\rho(t) \sigma_+ - \frac{1}{2} \{ \sigma_+ \sigma_-, \rho(t) \} \right), \quad (15)$$

where $\gamma(t)$ is the time-dependent decay rate of the model which is defined as

$$\gamma(t) = \text{Re} \left(\frac{2\gamma\lambda \sinh(\Omega t/2)}{\Omega \cosh(\Omega t/2) + (\lambda - i\delta) \sinh(\Omega t/2)} \right), \quad (16)$$

in which $\text{Re}(o)$ is the real part of o . The initial state of the system can be described by $\rho(0) = \sum_{i,j=1}^2 \rho_{ij} |i\rangle\langle j|$. By solving the Eq. (15), the state of the system at time t can be obtained as

$$\rho(t) = \Lambda_t \rho(0) = \begin{pmatrix} \rho_{11} |k(t)|^2 & \rho_{12} k(t) \\ \rho_{21} k^*(t) & 1 - \rho_{11} |k(t)|^2 \end{pmatrix}, \quad (17)$$

where Λ_t is known as quantum dynamical map, which maps initial state at time $t = 0$ to the state at time t . The concept of quantum dynamical map will be described in the following section. Also, the population of excited states at time t can be defined as [52]

$$P_t = \rho_{11} |k(t)|^2. \quad (18)$$

3. Measuring non-Markovianity

In the theory of open quantum systems, the evolution of a quantum system can be described by a quantum dynamical map. The dynamics of an open quantum system can be described by a master equation. Let us suppose that the dynamical map is defined by a master equation in Lindblad form

$$\frac{\partial}{\partial t} \rho(t) = \mathcal{L} \rho(t), \quad (19)$$

where \mathcal{L} is the Lindbladian super-operator given as [56,57]

$$\mathcal{L} \hat{\rho} = -i[\hat{H}, \hat{\rho}] + \sum_i \gamma_i \left[\hat{A}_i \hat{\rho} \hat{A}_i^\dagger - \frac{1}{2} \{ \hat{A}_i^\dagger \hat{A}_i, \hat{\rho} \} \right], \quad (20)$$

where \hat{H} is the Hamiltonian of the considered system, γ_i is decay rate and \hat{A}_i 's are Lindblad operators. If the Lindblad operators and decay rates are time-independent and decay rates have positive value, then Eq. (20) leads to a completely positive trace-preserving (CPTP) map $\Lambda(t, 0) = \exp[\mathcal{L}t]$. For All $t_1, t_2 \geq 0$, the CPTP map satisfies the semi-group property as

$$\Lambda(t_1 + t_2, 0) = \Lambda(t_1, 0) \Lambda(t_2, 0). \quad (21)$$

In this situation, the dynamical CPTP map describes a conventional Markovian process. Of course, it is possible that Hamiltonian \hat{H} , the Lindblad operators \hat{A}_i and decay rates have an explicit dependence on time. In such a situation, Eq. (20) describes a time-dependent Markovian process if the value of decay rate is positive at all times, i.e., $\gamma_i(t) \geq 0$. In the time-dependent scenario, the dynamical map can be written as

$$\Lambda(t, t_0) = T_{\leftarrow} \exp \left[i \int_{t_0}^t \mathcal{L}(s) ds \right], \quad (22)$$

where T_{\leftarrow} is time ordering operator [58]. The Markovian quantum dynamical maps $\Lambda(t, 0)$ have special property that they satisfy the divisibility condition. The divisibility condition is defined in such a way that a CPTP map can be expressed as a composition of two other CPTP maps as

$$\Lambda(t_2, t_0) = \Lambda(t_2, t_1) \Lambda(t_1, t_0). \quad (23)$$

It is important to mention that the value of decay rate $\gamma_i(t)$ may become negative in some time intervals during the evolution. In such a situation, there is an intermediate dynamical map $\Lambda(t_2, t_1)$ which is not CPTP in the interval that $\gamma_i(t)$ is negative. So, the divisibility condition is violated in these intervals [59,60]. It is recalled that what has been introduced as a time-dependent Markovian process is based on the divisibility property. Many criteria for measuring the non-Markovianity of the quantum process are based on the divisibility condition. However, some criteria are based on the back-flow of information from the environment to the system. In this work, we focus on a criterion that is based on the flow of information between the system and the environment. The employed measure is quantified by trace distance of a pair of arbitrary states $\rho_1(t)$ and $\rho_2(t)$, which is given by [59,60]

$$D(\rho_1(t), \rho_2(t)) = \frac{1}{2} \| \rho_1(t) - \rho_2(t) \|. \quad (24)$$

Actually, trace distance is a measure to quantify the distinguishability of a pair of quantum states. So, its changes during evolution can be interpreted as the exchange of information between the system and the environment. If the distinguishability of the two states in Eq. (24) decreases monotonically ($W_t = \partial_t D(\rho_1(t), \rho_2(t)) < 0$), then it can be said that information continuously flows from system to environment, however, the back-flow of information to the system will not occur and the process is Markovian. On the opposite side, $W_t > 0$ means that there exists the back-flow of information from the environment to the system and so the process is non-Markovian. Accordingly, Breuer et al. [61] proposed a non-Markovian measure as

$$\mathcal{N}(\Lambda) = \max_{\rho_1(0), \rho_2(0)} \int_{W_t > 0} W_t dt. \quad (25)$$

Based on Eq. (25), it should be necessary to perform an optimization process on all possible initial states $\rho_1(0)$ and $\rho_2(0)$ to determine the degree of non-Markovianity. In Ref. [61], it is shown that the maximum value of Eq. (25) will be obtained for initial states $\rho_1(0) = |0\rangle\langle 0|$ and $\rho_2(0) = |1\rangle\langle 1|$, by considering a large sample set of pairs of initial states and using strong numerical evidence. By choosing these two initial states, the trace distance in Eq. (24) for the model defined in this work in the previous section is obtained as

$$D(\rho_1(t), \rho_2(t)) = |k(t)|^2, \quad (26)$$

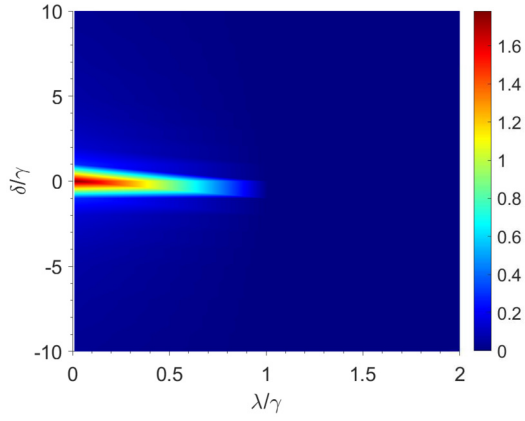


Fig. 1. Non-Markovianity (25) as functions of detuning parameter δ and spectral width of the environment λ .

where $k(t)$ is given by Eq. (14).

To illustrate this result, Fig. 1 shows the non-Markovianity \mathcal{N} as a function of detuning parameter δ and spectral width of the environment λ . From this figure, one can see that in the resonance case when $\delta = 0$, the non-Markovianity has its maximum value for small values of λ . While for the detuning case $\delta \neq 0$, the degree of non-Markovianity decreases with increasing the absolute values of detuning parameter δ . In the following, we will use the results extracted from Fig. 1 to study the QSL time for both Markovian and non-Markovian cases.

4. QSL time

In recent works, various methods have been proposed to study the QSL time. The geometric approach in quantum information theory is usually applied to introduce a criterion for quantifying the QSL time of quantum evolution. In Ref. [37], the authors have provided the QSL time for mixed initial states. Here, we use the method which has introduced by them. In their work, the function of relative purity has been used to define the QSL time, which is given by

$$\Theta(\rho(0), \rho(t)) = \arccos \left(\sqrt{\frac{\text{tr}[\rho(0)\rho(t)]}{\text{tr}[\rho(0)^2]}} \right). \quad (27)$$

Based on the metric introduced in Eq. (27), a comprehensive expression of QSL time for the mixed initial states is obtained as follows

$$\tau_{QSL} = \max \left\{ \frac{1}{\Phi_{\tau}^{op}}, \frac{1}{\Phi_{\tau}^{tr}}, \frac{1}{\Phi_{\tau}^{hs}} \right\} \sin^2[\Theta(\rho(0), \rho(\tau))] \text{tr}[\rho(0)^2], \quad (28)$$

where τ is the actual driving time and

$$\begin{aligned} \Phi_{\tau}^{op} &= \frac{1}{\tau} \int_0^{\tau} dt \|\dot{\rho}(t)\|_{op}, \\ \Phi_{\tau}^{tr} &= \frac{1}{\tau} \int_0^{\tau} dt \|\dot{\rho}(t)\|_{tr}, \\ \Phi_{\tau}^{hs} &= \frac{1}{\tau} \int_0^{\tau} dt \|\dot{\rho}(t)\|_{hs}, \end{aligned} \quad (29)$$

where $\|\dot{\rho}(t)\|_{op} = \lambda_1$ is the operator norm of $\dot{\rho}(t)$ (λ_1 is the largest singular value of $\dot{\rho}(t)$), $\|\dot{\rho}(t)\|_{tr} = \sum_i \lambda_i$ is the trace norm of $\dot{\rho}(t)$

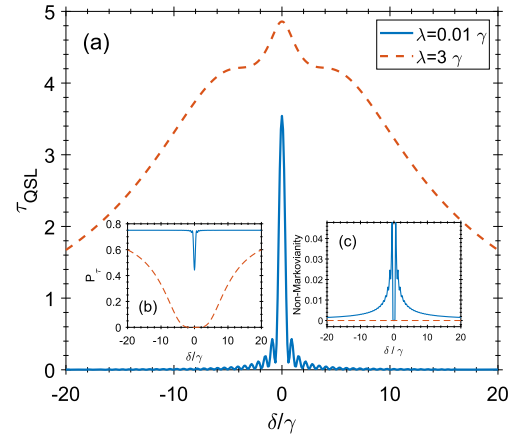


Fig. 2. (a) QSL time (35), (b) population of excited states $P_{\tau} = \rho_{11}|k(\tau)|^2$ (18), and (c) non-Markovianity (25) versus δ/γ for both Markovian ($\lambda = 3\gamma$) and non-Markovian ($\lambda = 0.01\gamma$) regimes with driving time $\tau = 10$. For two plots (a) and (b) we set $r_x = r_y = r_z = 0.5$.

(λ_i 's are the all singular values of $\dot{\rho}(t)$) and $\|\dot{\rho}(t)\|_{hs} = \sqrt{\sum_i \lambda_i^2}$ is the Hilbert-Schmidt norm of $\dot{\rho}(t)$. In Eq. (28), if the denominator of the fraction is Φ_{τ}^{op} and Φ_{τ}^{tr} , we have generalized ML type QSL bound for open quantum systems, while if it is Φ_{τ}^{hs} , we have MT type bound on the QSL time for non-unitary dynamics.

For a matrix like B , the following inequality holds for the norms which are used in Eq. (28)

$$\|B\|_{tr} \geq \|B\|_{hs} \geq \|B\|_{op}. \quad (30)$$

From Eq. (30), one can obtain the following order for denominator in Eq. (28) as

$$\Phi_{\tau}^{op} \leq \Phi_{\tau}^{hs} \leq \Phi_{\tau}^{tr}. \quad (31)$$

It is clear from above equation that ML type bound based on the operator norm is the tightest QSL time bound for non-unitary evolution.

Now, we would like to use this bound to investigate the effects of environmental parameter on QSL time for the model which has described in Sec. 2. According to the fact that the bound can be used for the mixed initial states, let us consider a general two-level system. In the Bloch representation, the general initial two-level state can be written as

$$\rho(0) = \frac{1}{2} \begin{pmatrix} 1 + r_z & r_x - ir_y \\ r_x + ir_y & 1 - r_z \end{pmatrix}, \quad (32)$$

where r_x , r_y and r_z are components of the Bloch vector \mathbf{r} . So, from Eq. (17), the density matrix at time t can be written as

$$\rho(t) = \frac{1}{2} \begin{pmatrix} (1 + r_z)|k(t)|^2 & (r_x - ir_y)k(t) \\ (r_x + ir_y)k^*(t) & 2 - (1 + r_z)|k(t)|^2 \end{pmatrix}. \quad (33)$$

As mentioned before, the ML type bound of QSL time based on the operator norm is the tightest bound, therefore, the QSL time can be considered as

$$\tau_{QSL} = \frac{1}{\Phi_{\tau}^{op}} \sin^2[\Theta(\rho(0), \rho(\tau))] \text{tr}[\rho(0)^2], \quad (34)$$

From Eq. (33) and using Bloch representation, the QSL time is obtained as

$$\tau_{QSL} = \frac{(1 - k(\tau))[r_x^2 + r_y^2 + r_z(1 + r_z)(1 + k(\tau))]}{\frac{1}{\tau} \int_0^{\tau} |k(t)| \sqrt{r_x^2 + r_y^2 + 4k(t)^2(1 + r_z)^2} dt}. \quad (35)$$

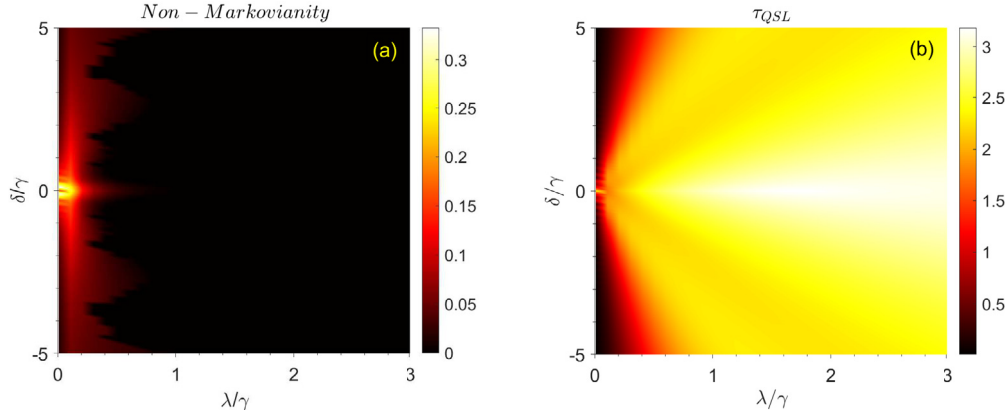


Fig. 3. (a) Non-Markovianity as functions of δ/γ and λ/γ for driving time $\tau = 30$, and (b) QSL time with $r_x = r_y = r_z = 0.5$ for driving time $\tau = 30$.

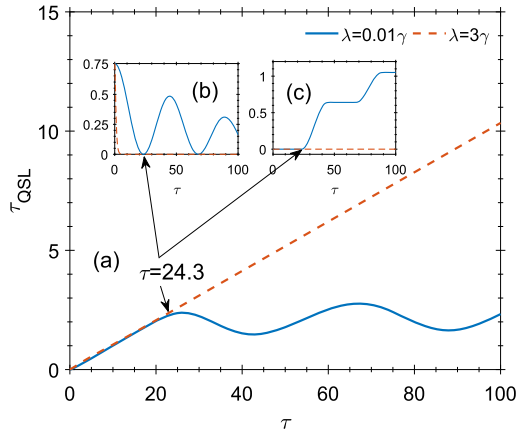


Fig. 4. (a) QSL time, (b) population of excited states, and (c) non-Markovianity versus the actual evolution time τ for both Markovian ($\lambda = 3\gamma$) and non-Markovian ($\lambda = 0.01\gamma$) regimes at resonance case $\delta/\gamma = 0$. For two plots (a) and (b) we set $r_x = r_y = r_z = 0.5$.

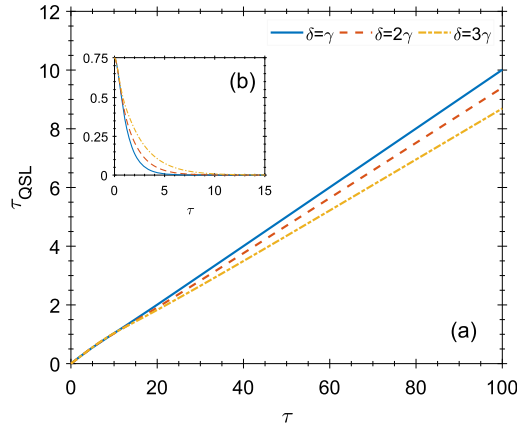


Fig. 5. (a) QSL time and (b) population of excited states in terms of actual evolution time τ for different values of detuning parameter δ in Markovian regime with $\lambda = 3\gamma$ and $r_x = r_y = r_z = 0.5$.

In Fig. 2a, the QSL time is plotted versus detuning parameter δ/γ . Two cases will be considered: Markovian and non-Markovian dynamics. From Fig. 1, one can choose $\lambda = 3\gamma$ and $\lambda = 0.01\gamma$ to have the Markovian and non-Markovian dynamics, respectively. For both Markovian and non-Markovian dynamics, the actual driving time has considered to be $\tau = 10$. It can be seen that the QSL time for non-Markovian dynamics is shorter than Markovian case. Besides, the population of excited states P_τ is plotted versus detuning pa-

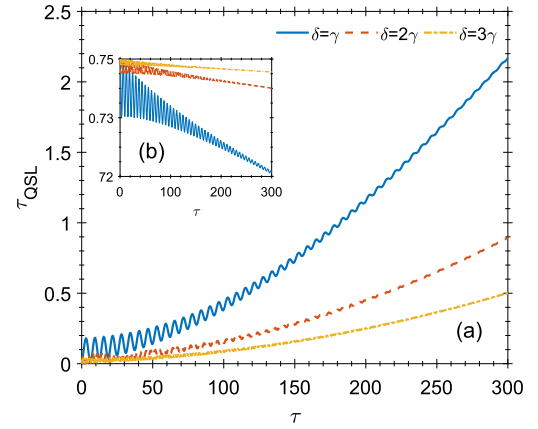


Fig. 6. (a) QSL time and (b) population of excited states in terms of actual evolution time τ for different value of detuning parameter δ in non-Markovian regime with $\lambda = 0.01\gamma$ and $r_x = r_y = r_z = 0.5$.

rameter in Fig. 2b. As can be seen, the population of excited states increases with increasing detuning parameter for both markovian and non-Markovian dynamics. Therefore, it can be concluded that there exists inverse relation between the population of excited states and the QSL time. Moreover, Fig. 2c shows the degree of non-Markovianity during actual evolution time versus detuning parameter. We see for $\lambda = 3\gamma$, the evolution is Markovian while for $\lambda = 0.01\gamma$, we have non-Markovian dynamics. It is also observed that the degree of non-Markovianity decreases with increasing detuning parameter, in agreement with Fig. 1.

In Fig. 3, the QSL time and non-Markovianity are shown as a density plot in terms of λ/γ and δ/γ for a detailed analysis. By comparing the two plots, it can be found that the QSL time in the non-Markovian regime is shorter than that in the Markovian regime, which indicates that the non-Markovian effects can speedup the evolution compared to the Markovian effects.

Now, in Fig. 4a, we illustrate the QSL time as a function of actual driving time τ in both Markovian ($\lambda = 3\gamma$) and non-Markovian ($\lambda = 0.01\gamma$) regimes for $\delta = 0$. As expected, the QSL time for non-Markovian regime is shorter than that of the Markovian regime. Fig. 4b shows the time variation of the population of excited states P_τ for both Markovian and non-Markovian regimes. By comparing Fig. 4a and Fig. 4b, it can be seen that there is an inverse relation between the QSL time and the population of excited states P_τ . In Fig. 4c, the degree of non-Markovianity is plotted in terms of actual evolution time τ . As expected for $\lambda = 3\gamma$, non-Markovianity is zero for all actual time while for $\lambda = 0.01\gamma$ at $\tau = 24.3$, the non-Markovian nature of the evolution is revealed. The interesting point in Fig. 4 is that with the revelation of

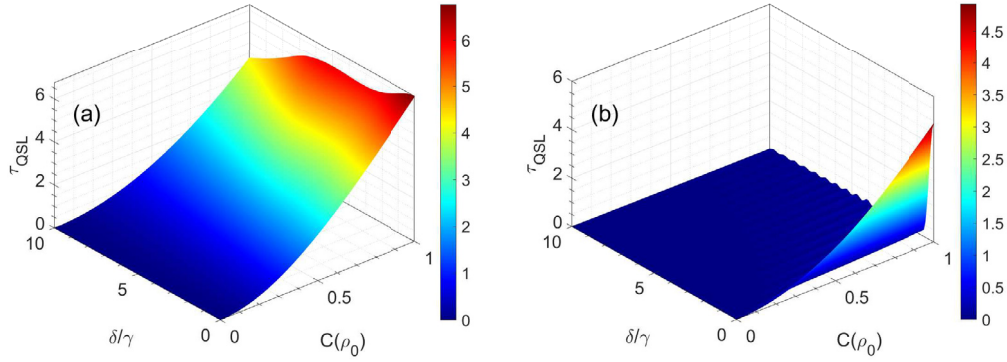


Fig. 7. QSL time as functions of quantum coherence of initial state $C(\rho_0)$ and the detuning parameter δ when population of initial excited state is $P_0 = 1/2$ at $\tau = 10$. (a) Markovian dynamics with $\lambda = 3\gamma$ and (b) non-Markovian dynamics with $\lambda = 0.01\gamma$.

the non-Markovian nature of the evolution at $\tau = 24.3$, the non-monotonic behavior of population of excited states will be started and the QSL time will be shorter than the QSL time in Markovian regime.

In Fig. 5a, the QSL time is plotted versus actual driving time τ for different values of detuning parameter δ in Markovian regime with $\lambda = 3\gamma$. From this plot, it can be seen that the QSL time decreases with increasing detuning parameter δ in Markovian regime. It is an interesting result because in the absence of non-Markovian effects and just by adjusting the detuning parameter δ , the evolution speed has been increased. On the other hand, Fig. 5b represents the population of excited states P_τ versus τ for the same values in Fig. 5a. From Fig. 5b, we see that the population of excited states increases with increasing the detuning parameter.

Fig. 6 is similar to Fig. 5, with the difference that the QSL time and the population of excited states have been shown in a non-Markovian regime. In Fig. 6a, it can be observed that for the non-Markovian regime, we have quantum speedup with increasing the detuning parameter. Here, although the non-Markovian effects themselves are effective for protecting the population of excited states, one can notice that the increase in the detuning parameter δ still plays an important role in protecting the population.

Notably, by using the l_1 -norm of coherence measure $C(\rho) = \sum_{i \neq j} |\langle i | \rho | j \rangle|$ to quantify the quantum coherence [62,63], it can be concluded that the coherence of the initial state ρ_0 (32) is obtained as $C(\rho_0) = \sqrt{r_x^2 + r_y^2}$. Also, the z-component of the Bloch vector r_z can be expressed in terms of the population of initial excited state from Eq. (18), which we denote it by $r_z = 2P_0 - 1$. Considering these cases, Eq. (35) can be rewritten in terms of the coherence of the initial state and the population of initial excited state as

$$\tau_{QSL} = \frac{(1 - k(\tau))[C(\rho_0)^2 + 2P_0(2P_0 - 1)(1 + k(\tau))]}{\frac{1}{\tau} \int_0^\tau |\dot{k}(t)| \sqrt{C(\rho_0)^2 + 16k(t)^2 P_0^2} dt} \quad (36)$$

In Fig. 7, the QSL time has been shown as function of the quantum coherence of initial state $C(\rho_0)$ and detuning parameter for both Markovian and non-Markovian regimes. From Fig. 7a, we see that in the Markovian regime, the QSL time has direct relation with the coherence of initial state such a way that with increasing the coherence of initial state, the QSL time increases. Besides, it is observed that the QSL time decreases with increasing the value of the detuning parameter. In a similar way, we have same interpretation for Fig. 7b with the difference that the dynamics is non-Markovian and the QSL time is shorter than Markovian regime in Fig. 7a.

5. Conclusion and outlook

In this work, the quantum speedup mechanisms involving the regulation of detuning in Jaynes-Cumming model are investigated. Here, the relative purity based quantum speed limit bound is selected to study the quantum speedup in Jaynes-Cumming model with detuning. We have first determined the range of environmental parameters in which the non-Markovian nature of the evolution becomes obvious. It is observed that the non-Markovian effects lead to quantum speedup in both resonance and detuning cases in a certain condition. As the main result of the work, it can be said that even in the Markovian regime, increasing the detuning parameter δ leads to quantum speedup. In other words, even in the absence of memory effects, the evolution speed can be increased with the detuning parameter. It was also observed that the initial state of the system has a direct effect on the quantum speed limit time. Moreover, we have revealed that the quantum speed limit has an inverse relation with the population of the initial excited state. It is also shown that the quantum speed limit time depends on the quantum coherence of the system's initial state such that the maximal coherent state can saturate the quantum speed limit bound. An interesting topic concerns the relationship between the quantum Fisher information [64–66] and quantum speed limit. This issue will be explored in a forthcoming work.

CRedit authorship contribution statement

Yasin Shahri: Writing – review & editing, Writing – original draft, Methodology, Investigation, Conceptualization. **Maryam Hadipour:** Writing – review & editing, Writing – original draft, Supervision, Software, Methodology, Investigation, Conceptualization. **Saeed Haddadi:** Writing – review & editing, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Hazhir Dolatkah:** Writing – review & editing, Methodology, Conceptualization. **Soroush Haseli:** Writing – review & editing, Writing – original draft, Supervision, Software, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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