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Measurement uncertainty and dense coding in a two-qubit system: Combined effects of bosonic reservoir and dipole–dipole interaction

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ABSTRACT

The uncertainty principle imposes a limitation on the measurement accuracy of two incompatible observables and has potential applications in quantum information science. We explore the bipartite entropic uncertainty relation and dense coding for two qubits coupled via dipole–dipole interaction and subject to a bosonic reservoir. It is shown that there exists a trade-off between the uncertainty bound and the dense coding capacity which results in their opposite dynamical behaviors. Moreover, the behaviors of the measurement uncertainty and the dense coding capacity rely crucially on the initial system–reservoir correlation, the strength of the dipole–dipole interaction, and the degree of non-Markovianity, and one can reduce the measurement uncertainty and enhance the dense coding capacity by tuning these parameters to appropriate values.

Introduction

One of the fundamental subjects in quantum mechanics is the uncertainty principle. It reveals that one cannot measure two incompatible observables with arbitrary precision. This principle was introduced by Heisenberg [1] and the first uncertainty relation for two non-commuting observables $\hat{x}$ (position) and $\hat{p}$ (momentum) was presented by Kennard [2]. Then it was generalized to arbitrary two incompatible observables by using the standard deviation of the measurement results [3,4]. An entropic version of the uncertainty relation based on the Shannon entropy was suggested by Deutsch [5] and its lower bound was subsequently improved [6,7]. Recently, Berta et al. [8] used a quantum memory to further reduce the measurement uncertainty of two observables Q and R, and this quantum-memory-assisted entropic uncertain relation reads

$$S(Q|B) + S(R|B) \geq \log_2 \frac{1}{c} + S(A|B), \tag{1}$$

where $S(A|B) = S(\rho_{AB}) - S(\rho_B)$ is the conditional von Neumann entropy of ρ_{AB} with $\rho_B = \mathrm{Tr}_A(\rho_{AB})$ being the reduced state of the memory particle B, $S(Q|B) = S\left(\rho_{QB}\right) - S\left(\rho_B\right)$ is the conditional von Neumann entropy regarding the post-measurement state $\rho_{QB} = \sum_i (|q_i\rangle_A\langle q_i| \otimes \mathbb{1}_B)\rho_{AB}(|q_i\rangle_A\langle q_i| \otimes \mathbb{1}_B)$ with $\mathbb{1}_B$ being the identity operator, and likewise for ρ_{RB}. Besides, $c = \max_{i,j}\{|\langle q_i|r_j\rangle|^2\}$ denotes the maximal overlap between the eigenstates $\{|q_i\rangle\}$ of Q and $\{|r_j\rangle\}$ of R.

To understand the entropic uncertainty relation with quantum memory, one can consider a game between two players Alice and Bob. In this intriguingly game, Bob prepares and shares a two-particle entangled state between himself and Alice. Then, Alice measures one of the two incompatible observables, Q and R, on her state and then she tells Bob about her choice. By using the information received from Alice, Bob can minimize his uncertainty about Alice's measurement outcomes, e.g., he can guess accurately Alice's outcomes when the shared state is maximally entangled [8].

In the literature, the uncertainty relations have been extensively studied and great efforts have been made to improve them as well as to find their connections with quantum correlations [9–37]. Moreover, those bipartite entropic uncertainty relations [38–40] could be

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extended to the tripartite scenario where the two particles serving as the quantum memories are in possession of Bob and Charlie [41,42]. In a recent paper [43], the tripartite uncertainty relation was generalized to the multipartite scenario with quantum memory which covers some well-known uncertainty relations. Moreover, Li and Qiao [44] introduced an optimal measurement strategy based on the conditional majorization to suppress the measurement uncertainty.

In this work, we consider one of the entropic uncertainty relations for a two-qubit system for which its lower bound is tighter than the other bounds, namely [12]

$$S(Q|B)+S(R|B)\geq \log_2\frac{1}{c}+S(A|B)+\max\{0,\delta\}:=U_b, \tag{2}$$

where U_b represents the entropic uncertainty bound (EUB) with $\delta = I(A:B)-I(Q:B)-I(R:B)$, $I(A:B)=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$ is the quantum mutual information of ρ_{AB}, while $I(Q:B)$ is the quantum mutual information of ρ_{QB}, and likewise for $I(R:B)$.

From a practical point of view, many tasks of quantum communication and information processing can be realized by using entanglement [45–47], among which dense coding is a well-known protocol that exploits the entangled states in its realization [48–50]. This significant protocol aims fundamentally to transmit two classical bits of information by sending, via a quantum channel, just a single qubit if the sender (say, Alice) and receiver (say, Bob) share a two-qubit maximally entangled state [51,52]. The transmission process can be done in a perfect way when a noiseless quantum channel between Alice and Bob is available. However, this is not entirely possible in a realistic scenario where the noise is present. Thereby, the amount of information that can be conveyed through a quantum channel is closely related to the quality of the latter. In view of this, the dense coding capacity (DCC) was presented to measure the quantum channel quality, which is quantified by how much classical information might be accessed from a quantum ensemble in which the information is encoded. It is worth mentioning that the optimal amount of information is generally bounded by the Holevo quantity [53]. For a general bipartite state ρ_{AB} shared between Alice and Bob, the DCC reads as

$$\chi = S\left(\bar{\rho}_{AB}\right)-S\left(\rho_{AB}\right), \tag{3}$$

where $S\left(\rho_{AB}\right)$ is the von Neumann entropy of ρ_{AB}, while $S(\bar{\rho}_{AB})$ is the von Neumann entropy of the average state after the dense coding operation. It can be obtained from the probe entangled state ρ_{AB} by subjecting the part received by Alice to a Pauli operation σ_i^A with probability p_i. In the particular situation of equal probability $p_i = 1/4$ ($\forall i$), the average state of the signal ensemble is given by

$$\bar{\rho}_{AB}=\frac{1}{4}\sum_{i=0}^{3}(\sigma_i^A\otimes \mathbb{1}_B)\rho_{AB}(\sigma_i^A\otimes \mathbb{1}_B), \tag{4}$$

where $\sigma_0^A=\mathbb{1}_A$ and $\sigma_{1,2,3}^A$ are the three Pauli operators. The DCC is valid when $\chi>1$, while $\chi=2$ corresponds to the optimal DCC, i.e., Alice can transmit two bits of classical information by sending only one qubit.

When considering an explicit quantum information processing task, although various works have been performed either focused on the influence of the initial system–reservoir correlation or the dipole–dipole interaction on an open quantum system [54–62], there are few studies considering the combined effects of the initial system–reservoir correlation and the dipole–dipole interaction of two qubits [63–65]. But there are situations for which such an effect should be considered. Our main motivation is to study dynamical behaviors of a system consisting of two qubits interact with a structured bosonic reservoir with the addition of the dipole–dipole interaction between them. Specifically, we will explore effects of the initial system–reservoir correlation and the dipole–dipole interaction of two qubits on dynamics of the EUB and DCC in both the Markovian and non-Markovian regimes. Therefore, this paper is prepared and organized as follows. In Section “Theoretical model”, we present the physical model and its solution. The influence of different system parameters on dynamical behaviors of the EUB and DCC are analyzed in Section “Results and discussion”. Finally, a brief conclusion remark is given in Section “Final remarks”.

Theoretical model

The Hamiltonian of two two-level atoms (serving as two qubits) that interact with a bosonic reservoir at zero temperature and with the addition of the dipole–dipole interaction between them can be written as (see Fig. 1)

$$\hat{H}=\omega_0\left(\sigma_+^A\sigma_-^A+\sigma_+^B\sigma_-^B\right)+\sum_k\omega_k b_k^\dagger b_k+\sum_k\left[g_k\left(\sigma_-^A+\sigma_-^B\right)b_k^\dagger+\text{H.c.}\right] + d\left(\sigma_-^A\sigma_+^B+\sigma_+^A\sigma_-^B\right), \tag{5}$$

where $\hbar=1$ is assumed, ω_0 is atomic transition frequency, $\sigma_\pm^j$ ($j=A,B$) are the atomic inversion operators, b_k ($b_k^\dagger$) is the annihilation (creation) operator of the kth mode of the reservoir with frequency ω_k, g_k is the coupling strength of the atoms to the kth mode of the reservoir, and d is the dipole–dipole interaction strength between the two atoms.

Here, let us assume that the initial state has only one excitation in the total system

$$|\psi(0)\rangle = c(0)|\Psi^+\rangle_{AB}|0\rangle_r+\sum_k c_k(0)|gg\rangle_{AB}|1_k\rangle_r, \tag{6}$$

where $|\Psi^+\rangle_{AB}=(|eg\rangle_{AB}+|ge\rangle_{AB})/\sqrt{2}$, the collective state $|0\rangle_r$ represents the vacuum state of the reservoir, and $|1_k\rangle_r$ is the quantum state of the reservoir with one excitation in the kth mode. It should be mentioned that in order to study effect of the initial system–reservoir correlation on dynamics of the EUB and DCC, one can take $c(0)\neq\{0,1\}$, otherwise there will be no initial correlation between the qubits and the reservoir. By defining $|1\rangle_r = \sum_k c_k(0)|1_k\rangle_r/[\sum_k|c_k(0)|^2]^{1/2}$ and considering $|c(0)|^2+\sum_k|c_k(0)|^2=1$, one can further write the initial state as

$$|\psi(0)\rangle = c(0)|\Psi^+\rangle_{AB}|0\rangle_r+\sqrt{1-|c(0)|^2}|gg\rangle_{AB}|1\rangle_r. \tag{7}$$

Now, by using Eq. (5), one can write the state of the whole system at time t as

$$|\psi(t)\rangle = c(t)|\Psi^+\rangle_{AB}|0\rangle_r+\sum_k c_k(t)|gg\rangle_{AB}|1_k\rangle_r, \tag{8}$$

where the equations of motion for $c(t)$ and $c_k(t)$ can be obtained respectively by solving the Schrödinger equation in the interaction picture

$$\begin{aligned} i\dot{c}(t) &= d\,c(t)+\sqrt{2}\sum_k g_k^* e^{i(\omega_0-\omega_k)t}c_k(t),\\ i\dot{c_k}(t) &= \sqrt{2}g_k e^{-i(\omega_0-\omega_k)t}c(t), \end{aligned} \tag{9}$$

then from the second term of Eq. (9) one can derive $c_k(t) = -i\sqrt{2}g_k\int_0^t \mathrm{d}t_1 e^{-i(\omega_0-\omega_k)t_1}c(t_1)$, by inserting of which into the first term, one comes to

$$\dot{c}(t) = -id\,c(t)-i\sqrt{2}\sum_k c_k(0)g_k^* e^{i(\omega_0-\omega_k)t} - 2\sum_k|g_k|^2\int_0^t \mathrm{d}t_1 e^{i(\omega_0-\omega_k)(t-t_1)}c\left(t_1\right), \tag{10}$$

where the initial system–reservoir correlation generates the second term on the right-hand side of Eq. (10). To achieve an analytical solution, let us consider the case that the coefficient $c_k(0)$ is proportionate to the coupling strength g_k, that is, $c_k(0)=\alpha g_k$, which means that the increase of g_k induces the increase of occupied number of the excited state in kth mode at the initial time [63]. In the limit of large number of modes k, one has $\sum_k|g_k|^2\longrightarrow\int \mathrm{d}\omega J(\omega)$, in which $J(\omega)$ is a spectral density function. In the following, we concentrate on the Lorentzian spectral density of the structure

$$J(\omega)=\frac{\gamma_0}{2\pi}\frac{\lambda^2}{\left(\omega-\omega_0\right)^2+\lambda^2}, \tag{11}$$

where ω_0 denotes the center frequency of structured reservoir, γ_0 indicates the decay of excited state of the qubits in the Markovian limit, and

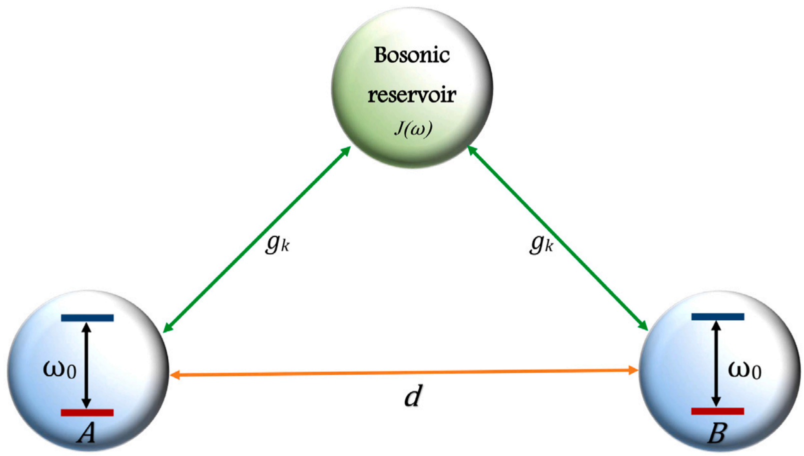

Fig. 1. A schematic of two qubits interacting with a bosonic reservoir with the Lorentzian spectrum $J(\omega)$, where g_k is their coupling strength to the reservoir and d is the dipole–dipole interaction.

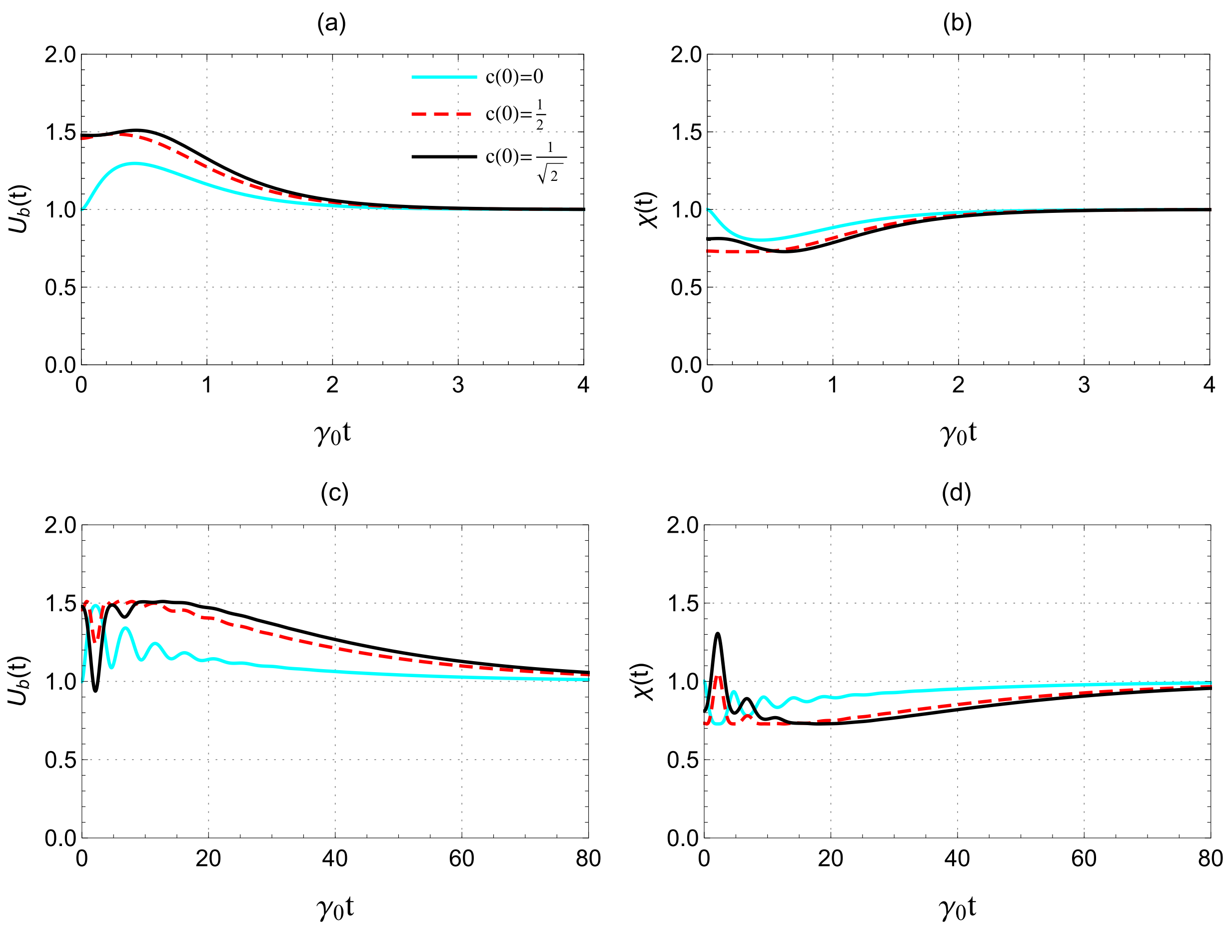

Fig. 2. Time evolution of the EUB and DCC with $d = \gamma_0$ and different $c(0)$. The other parameter is given by $\lambda = 5\gamma_0$ in (a) and (b) and $\lambda = 0.2\gamma_0$ in (c) and (d). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

λ determines the spectral width. In general, there are two dynamical regimes [66,67], namely, the weak-coupling regime ($\lambda > 2\gamma_0$) for which the dynamics is Markovian and the strong-coupling regime ($\lambda < 2\gamma_0$) for which it is non-Markovian.

The reservoir correlation function $f(t) = \frac{1}{2}\gamma_0\lambda e^{-\lambda|t|}$ can be obtained by its definition $f\left(t-t_1\right) = \int \mathrm{d}\omega J(\omega) e^{i(\omega_0-\omega)(t-t_1)}$. Also, regarding $c_k(0) = \alpha g_k$, one arrives at $\sum_k |c_k(0)|^2 = |\alpha|^2 f(0) = \frac{1}{2}\gamma_0\lambda|\alpha|^2$, with $\alpha = \sqrt{2\left(1-|c(0)|^2\right)/\gamma_0\lambda}$. Equipped with these equations, we rewrite Eq. (10) as follows

$$\dot{c}(t) = -id\,c(t) - i\sqrt{2}\alpha f(t) - 2\int_0^t \mathrm{d}t_1 f\left(t-t_1\right) c\left(t_1\right), \tag{12}$$

then by using the Laplace transform method, one obtains [64]

$$c(t) = \frac{e^{-\frac{1}{2}(p+\beta)t}}{2\beta}\left\{-2b(1-e^{\beta t}) + c(0)[p(1-e^{\beta t}) + \beta(1+e^{\beta t})]\right\}, \tag{13}$$

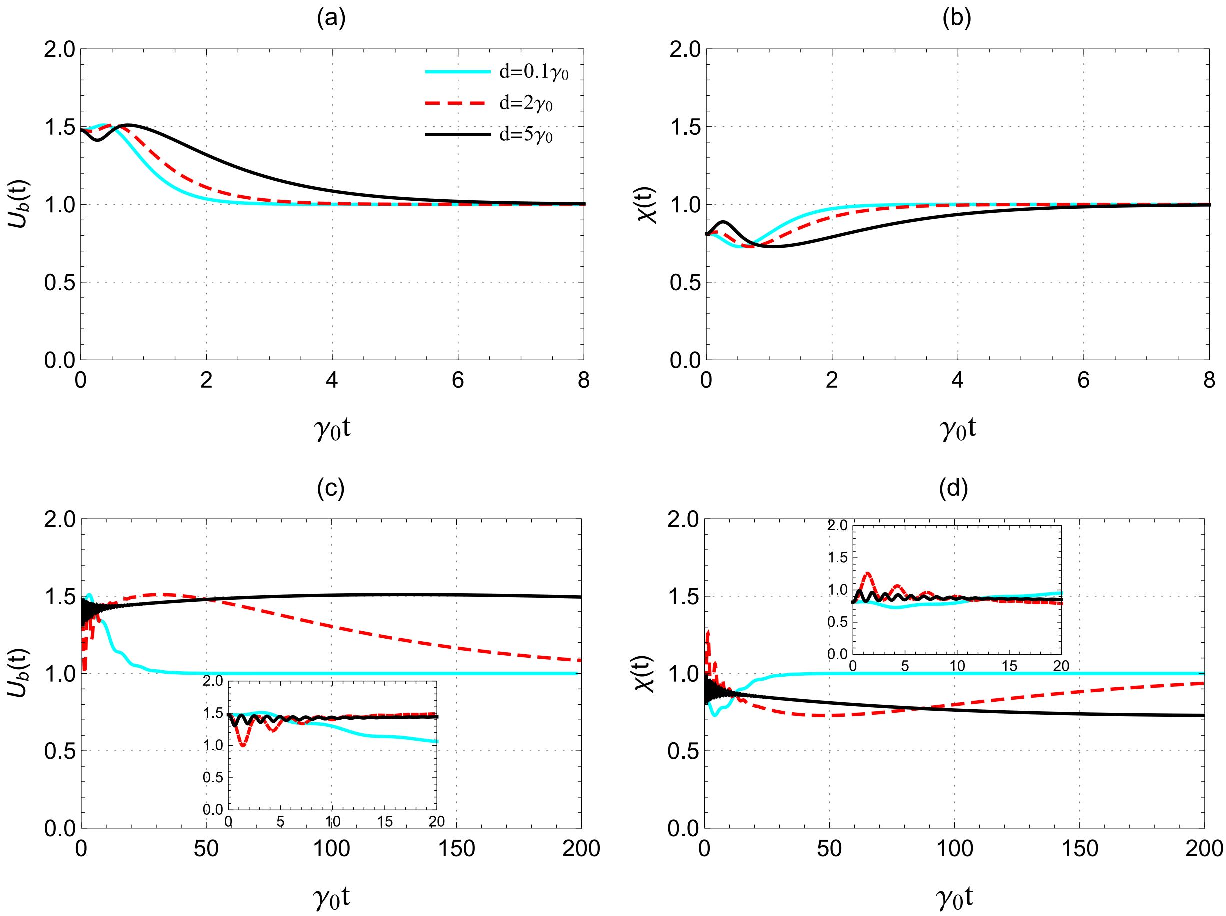

Fig. 3. Time evolution of the EUB and DCC with $c(0) = 1/\sqrt{2}$ and different dipole–dipole interaction d. The other parameter is given by $\lambda = 5\gamma_0$ in (a) and (b) and $\lambda = 0.2\gamma_0$ in (c) and (d).

where $\beta = \sqrt{p^2 - 4\lambda\left(\gamma_0 + id\right)}$, $p = \lambda + id$, and $b = \lambda[c(0) - i\alpha\gamma_0/\sqrt{2}]$. Finally, by denoting $\kappa = |c(t)|$, the time-evolved density matrix of two qubits in the standard basis $\{|gg\rangle, |ge\rangle, |eg\rangle, |ee\rangle\}$ can be obtained analytically as [64]

$$\rho_{AB}(t) = (1-\kappa^2)|gg\rangle\langle gg| + \frac{\kappa^2}{2}\left(|ge\rangle\langle ge| + |eg\rangle\langle eg| + |eg\rangle\langle ge| + |ge\rangle\langle eg|\right). \tag{14}$$

Results and discussion

In the uncertainty game as bring up previously, after preparing the bipartite state, the qubit A is sent to Alice and the memory qubit B to Bob. After receiving qubit A, Alice measures either Q or R and informs Bob about her measurement choice, based on which Bob can reduce his uncertainty regarding Alice's measurement outcomes. In the following, we take $Q = \sigma_1$ and $R = \sigma_3$ as two incompatible observables without loss of generality. Then the conditional von Neumann entropy for the time-evolved state of Eq. (14) can be obtained as

$$S(A|B)_t = H_2(\kappa^2) - H_2(\kappa^2/2), \tag{15}$$

while the mutual information and the Holevo quantities are given by

$$\begin{aligned} I(A:B)_t &= 2H_2(\kappa^2/2) - H_2(\kappa^2),\\ I(\sigma_1:B)_t &= H_2(\kappa^2/2) - H_2(\xi/2),\\ I(\sigma_3:B)_t &= 2H_2(\kappa^2/2) - H_2(\kappa^2) - \kappa^2, \end{aligned} \tag{16}$$

where $\xi = 1 + \sqrt{1 - 2\kappa^2 + 2\kappa^4}$ and $H_2(x) = -x\log_2 x - (1-x)\log_2(1-x)$ denotes the binary Shannon entropy function. So one can obtain the EUB as

$$U_b(t) = 1 + H_2(\kappa^2) - H_2(\kappa^2/2) + \max\{0, \kappa^2 - H_2(\kappa^2/2) + H_2(\xi/2)\}. \tag{17}$$

Moreover, the average state of the signal ensemble is given by

$$\bar{\rho}_{AB}(t) = \frac{1}{4}[(2-\kappa^2)(|gg\rangle\langle gg| + |eg\rangle\langle eg|) + \kappa^2(|ge\rangle\langle ge| + |ee\rangle\langle ee|)], \tag{18}$$

then the DCC defined in Eq. (3) can be obtained analytically as

$$\chi(t) = 1 + H_2(\kappa^2/2) - H_2(\kappa^2), \tag{19}$$

and remarkably, by combining Eqs. (17) and (19) one can obtain a simple trade-off between the EUB and DCC as follows:

$$\chi(t) + U_b(t) = 2 + \max\{0, \kappa^2 - H_2(\kappa^2/2) + H_2(\xi/2)\}. \tag{20}$$

Start from the above equations, one could discuss in detail combined effects of the initial system–reservoir correlation and the dipole–dipole interaction of two qubits on the EUB and DCC.

We begin with considering effects of the initial system–reservoir correlation on the EUB and DCC dynamics of two qubits both in the Markovian and non-Markovian regimes. First, let us consider the case where the strength of the dipole–dipole interaction is fixed ($d = \gamma_0$). As is shown in Fig. 2(a,b), we have plotted the EUB and DCC versus the rescaled time $\gamma_0 t$ for three different values of the initial system–reservoir correlation $c(0)$ in the Markovian regime ($\lambda = 5\gamma_0$). It can be observed that with the increasing evolution time t, the EUB first increases to a peak value and then decays asymptotically to its steady-state value 1 in the infinite-time limit, while the DCC shows an opposite time dependence compared to the EUB, i.e., it first decreases gradually and then turns to be increased asymptotically to the steady-state value 1. In Fig. 2(c,d), we have sketched the time evolution of the EUB and

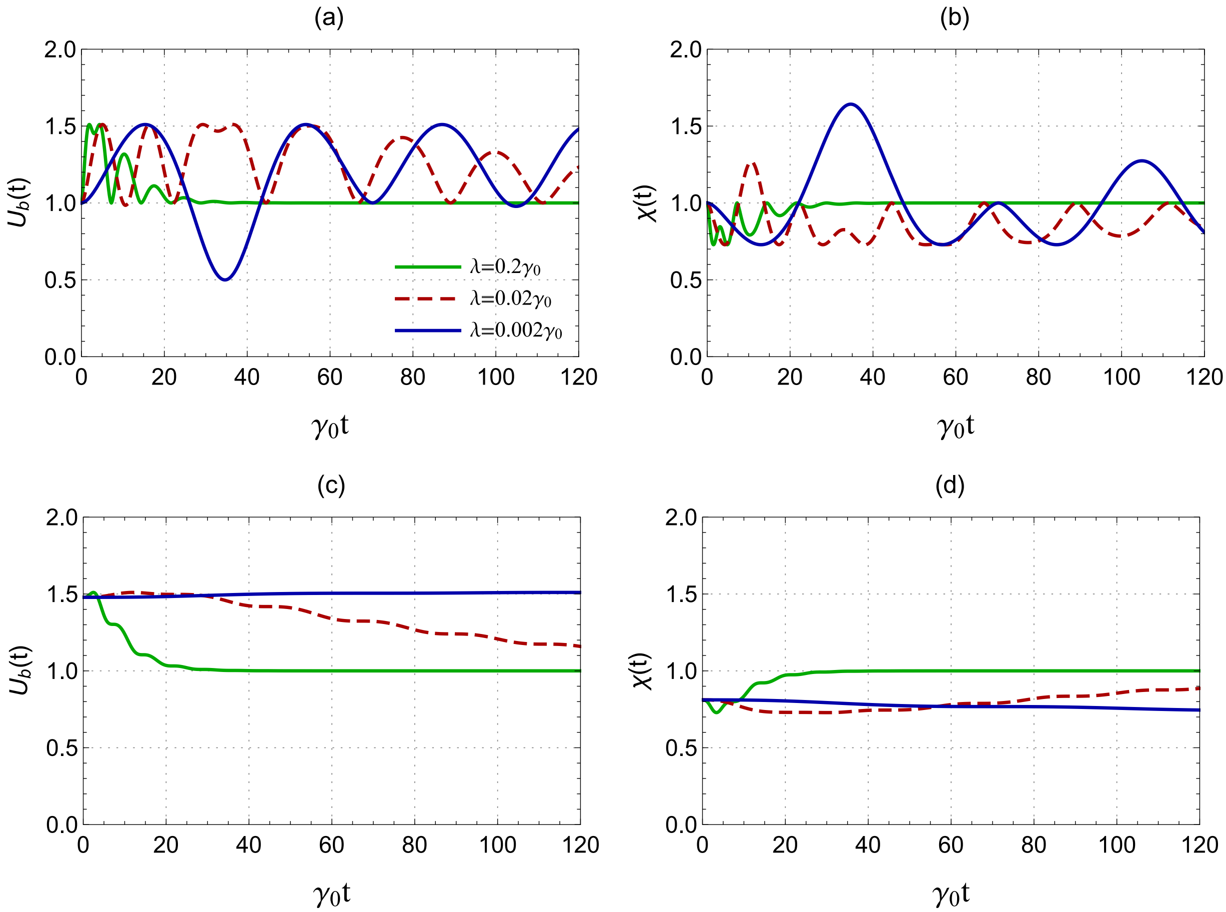

Fig. 4. Time evolution of the EUB and DCC in the non-Markovian regime with $d = 0$ and different λ. The other parameter is given by $c(0) = 0$ in (a) and (b) and $c(0) = 1/\sqrt{2}$ in (c) and (d).

DCC in a non-Markovian regime ($\lambda = 0.2\gamma_0$), from which one can see that the EUB behaves as damped oscillations in the short time region and also tends to the asymptotic value 1 in the infinite-time limit. This indicates that the non-Markovian memory effect dominates time evolution of the EUB only in the relative short-time regions. Similar to the case in the Markovian regime, the DCC in the non-Markovian regime also shows an opposite time dependence compared to the EUB. The underlying physical reason for these damped oscillations of the EUB and DCC is the backflow of information from the reservoir to the two-qubit system. Moreover, by comparing the lines of different colors in Fig. 2, one can observe that in the short-time region, the $c(0)$ dependence of the EUB and DCC are complicated and there are time regions during which the EUB (DCC) is reduced (enhanced) obviously for the non-Markovian case. Apart from the short-time regions, the EUB (DCC) of two qubits without initial correlation with the reservoir is smaller (larger) than that with the initial correlation. Thus, the initial system–reservoir correlation plays a negative role in the long-time region. In general, the EUB is maximum when the DCC is minimum and vice versa, and such a phenomenon is rooted in the trade-off relation given in Eq. (20).

Next, let us turn the topic to the problem of how the dipole–dipole interaction between two qubits affects the measurement uncertainty and the dense coding capacity for the physical model we considered. In Fig. 3, we show the EUB and DCC as a function of the rescaled time $\gamma_0 t$ with $c(0) = 1/\sqrt{2}$ and different strengths of the dipole–dipole interaction d. In the Markovian regime, one can note that the behaviors of the EUB and DCC are different for different d, but with the increase of the evolving time t, they both approach to the asymptotic value of 1. In the non-Markovian regime, both the EUB and the DCC oscillate rapidly in the short-time region, especially for the case of strong dipole–dipole interaction. Moreover, as can be seen from Fig. 3, a combination of the dipole–dipole interaction and non-Markovianity can reduce the EUB and enhance the DCC to some extent in the short-time regions. But such a positive role is weakened with the increasing time and it is reversed in the long-time region as the EUB increases and the DCC decreases with the increasing d.

To see the degree of the non-Markovianity on the EUB and DCC, we show their time dependence with different λ in Fig. 4 for two decoupled qubits (i.e., $d = 0$) which interact with the non-Markovian reservoir. Specifically, we investigate two typical cases: the case of two qubits without initial correlation to the reservoir [Fig. 4(a,b)] and that with the initial correlation to the reservoir [Fig. 4(c,d)]. For $c(0) = 0$ which corresponds to the initial state $|gg\rangle_{AB}|1\rangle_r$ [see Eq. (7)], both the EUB and DCC take the initial value 1. With the increasing time t, as is shown in Fig. 4(a,b), the non-Markovianity induces pseudo-period oscillations of the EUB and DCC. In particular, the stronger the degree of the non-Markovianity (i.e. small λ/γ_0), the more the extent to which the EUB (DCC) is reduced (enhanced). That is to say, the presence of strong non-Markovianity is beneficial for reducing the EUB and enhancing the DCC. On the other hand, it can be observed that all plots fulfill the expected reciprocal behaviors on the dynamics of EUB and DCC. When $c(0) = 1/\sqrt{2}$, the initial value of EUB is close to 1.5 and the DCC is 0.8 as the initial state of Eq. (7) reduces to $(|\Psi^+\rangle_{AB}|0\rangle_r + |gg\rangle_{AB}|1\rangle_r)/\sqrt{2}$. For this case, as is shown in Fig. 4(c,d), the strong non-Markovianity turns to be detrimental as the EUB (DCC) is increased (decreased) with the increase of λ apart from the short-time region. Besides, different from the case of $c(0) = 0$, the EUB and DCC are insensitive to the evolving time and they approach to the steady-state values quickly.

Final remarks

In summary, we have investigated combined effects of the bosonic reservoir with a Lorentzian spectrum and the dipole–dipole interaction between two qubits on the dynamics of the EUB and DCC. We first obtained a trade-off between the EUB and DCC and show through explicit examples that they exhibit opposite dynamical behaviors, and an evident reduce of the EUB and enhancement of the DCC was observed due to the non-Markovian effects of the bosonic reservoir. Moreover, the initial qubit–reservoir correlation characterized by $c(0)$ has a typical influence on the EUB and DCC. For the non-Markovian case, the EUB (DCC) can be reduced (enhanced) to some extent by tuning $c(0)$ to certain finite values in the relative short-time regions. Furthermore, an optimal choice of the dipole–dipole interaction between two qubits is also beneficial to the EUB and DCC in the short-time regions, both for the Markovian and non-Markovian cases. Due to the fundamental significance of uncertainty principle in quantum theory and the practical significance of dense coding in quantum information science, it is vital to seek efficient ways to suppress detrimental effects of noises on quantum channel. The results we obtained in this work provide a potential way to achieve this goal, thereby we think it would be useful for implementing quantum computation [68–72] and improving measurement precision of non-commuting observables in quantum metrology [73–77].

CRediT authorship contribution statement

Saeed Haddadi: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing -original draft, Writing - review & editing. **Ming-Liang Hu:** Conceptualization, Formal analysis, Project administration, Resources, Validation, Visualization, Writing -original draft, Writing - review & editing. **Youssef Khedif:** Conceptualization, Formal analysis, Investigation, Resources, Validation, Visualization, Writing -original draft, Writing - review & editing. **Hazhir Dolatkhah:** Conceptualization, Investigation, Resources, Validation, Writing – original draft. **Mohammad Reza Pourkarimi:** Conceptualization, Investigation, Validation, Writing – original draft. **Mohammed Daoud:** Conceptualization, Validation, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

This work is theoretical research and has no the associated data.

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