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ATOMIC PHYSICS

Quantum Speed Limit Time of Topological Qubits Influenced by the Fermionic and Bosonic Environments

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Abstract
Quantum theory sets a bound on the minimum time required to transform from an initial state to a target state. The bound is known as quantum speed limit time. Quantum speed limit time can be used to determine the rate of quantum evolution for closed and open quantum systems. In the real world, we are dealing with open quantum systems. So, the study of quantum speed limit time for open quantum systems has particular importance. In this work, we consider the topological qubit realized by two Majorana modes. We consider the case in which the topological qubit is influenced by the fermionic and bosonic environment. Fermionic and bosonic environments are assumed to have Ohmic-like spectral density. The quantum speed limit time is investigated for the various environments with different Ohmic parameters. It is observed that for the super-Ohmic environment with increasing Ohmic parameter the quantum speed limit time gradually reaches a constant value and so the speed of evolution reaches a uniform value. It is also observed that the quantum speed limit time reaches zero value by increasing initial time parameter for small value of Ohmic parameter while it reaches constant value for larger Ohmic parameter. The effects of the external magnetic field on the quantum speed limit time are also studied. It is observed that with increasing magnitude of the magnetic field, the quantum speed limit time decreases.

Keywords Quantum speed limit · Topological qubit · Environment

1 Introduction

The minimum time required for the transformation of a quantum system from an initial state to a target state is known as quantum speed limit (QSL) time. It can be said that the QSL time stems from the time–energy uncertainty principle. The maximum speed of quantum evolution can be obtained using QSL time. The QSL time is used in many topics of quantum information theory such as quantum communication [1], investigation of exact bounds in quantum metrology [2], computational bounds of physical systems [3], and quantum optimal control algorithms [4].

For closed quantum systems whose evolution is described using unitary operations, the QSL time is obtained using distance measures such as Bures angle and relative purity [5–12]. Among the most important QSL time bound for closed systems, one can mention two bounds, one is Mandelstam–Tamm (MT) bound [11] and the other is Margolus–Levitin (ML) bound [12]. Given that isolating a quantum system from its surroundings is difficult and almost impossible, and any real quantum system interacts inevitably with its surroundings, the study of open quantum systems is one of the fascinating topics in quantum information theory [13–15]. Therefore, due to the importance of open quantum systems and their role in quantum information theory, the study of QSL time for these systems has been considered in many recent works [16–41]. In general, Mandelstam–Tamm bound and Margolus–Levitin bound are used to describe QSL time in closed and open quantum systems. The generalizations of these two bounds for open

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quantum systems are given in Refs. [7, 8]. In Ref. [24], Deffner et al. present a comprehensive and unified bound for non-Markovian dynamics that includes both MT and ML bounds. In Ref. [28], Zhang et al. provide the QSL time bound for arbitrary initial states. They have shown that the QSL time depends on the quantum coherence of the initial state. Based on the definition they provide, QSL time is the minimum time required for the evolution of an open quantum system from an initial state at time τ to target state at time $\tau + \tau_D$, where τ_D is driving time. In this work, we will consider the QSL time bound which has introduced by Zhang et al. [28].

It has been observed that topological quantum computing is a promising design for the realization of quantum computers with stable qubits [42]. According to recent studies, there exist different and new types of topologically ordered states that are physically achievable, such as topological insulators and superconductors [43–45]. For these systems, some of the excitations are topologically protected, provided that some symmetries, such as time inversion, are maintained. In other words, the local perturbations that maintain these symmetries cannot disentangle the topological excitations. The most interesting of these topological excitations are Majorana modes localized on topological defects, which follow the non-Abelian anyonic statistics [46–48]. The Kitaev 1D spineless p-wave superconductor chain model is the most common model for realizing such Majorana modes. Each onsite fermion can be decomposed into two Majorana modes. By properly adjusting the model, Majorana modes can be dangling at the end of the chain without pairing with other nearby Majorana modes to form common fermions. So, these two separate Majorana modes can create a topological qubit. There exist two meanings to the word topological here: One meaning is that it is composed of Majorana modes that are topological excitations, another implication is that the topological qubit itself is non-local, meaning that the two Majorana states are very separate and therefore cannot be combined into a common fermion. From quantum information insight, the topological qubit is EPR-like, because it encodes quantum state non-locally. Both characteristics explain its resistance to local disturbances. Given that topological excitations are robust against local perturbations, the question that may arise is whether topological qubits are also robust against decoherence when they are considered as an open quantum system coupling to the non-topological environment. The open system setup is more logical and realistic when performing quantum computations. Since quantum information is carried by physical excitations, stability against decoherence indicates stability against local perturbations, but the opposite is not true. Even if local excitations are stable to local perturbations, however, the quantum information carried by the topological qubit may still penetrate the environment. However, since the topological qubit is non-local, its interaction with the environment is quite different from that of conventional fermions, and quantum decoherence behaviors are expected to be unusual. This motivates us, in this work, to examine QSL time for decoherence of topological qubits.

In this paper, the QSL time for the dynamics of a topological qubit realized by two Majorana modes coupled to a fermionic and bosonic Ohmic-like reservoir is discussed in detail.

2 Decoherence of Topological Qubits

A topological qubit composed of two Majorana modes of the one-dimensional Kitaev chain that are spatially separated. These Majorana modes are located at the two ends of a quantum wire and are denoted by γ_1 and γ_2, and the following relations are established for them [49]

$$\gamma_a^{\dagger} = \gamma_a, \quad \left\{\gamma_a, \gamma_b\right\} = 2\delta_{ab}, \tag{1}$$

where $a, b \in \{1, 2\}$. The Majorana modes are influenced by their surroundings in an incoherent way, which causes the decoherence of the topological qubit. The Hamiltonian describing the intended general system is defined as follows

$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_I, \tag{2}$$

where $\hat{H}_S$ represents the Hamiltonian of topological qubit system, $\hat{H}_E$ describes the Hamiltonian of the environment, and $\hat{H}_I$ is the interaction Hamiltonian describes the influence of the environment on topological qubit which reads

$$\hat{H}_I = G_1\gamma_1\hat{Q}_1 + G_2\gamma_2\hat{Q}_2, \tag{3}$$

where $G_{1(2)}$ describes the real coupling constant and $\hat{Q}_{1(2)}$ is the composite operator consists of the electron creation operator $a^{\dagger}$ and annihilation operator a . According to hermeticity condition of interaction Hamiltonian, i.e., $\hat{H}_I^{\dagger} = H_I$ we have

$$\hat{Q}_a^{\dagger} = -\hat{Q}_a. \tag{4}$$

When the system is affected by a fermionic environment, the Majorana modes are located at two ends of a quantum wire which is placed over an s-wave superconductor. Majorana modes are affected by a magnetic field B whose direction is along the quantum wire. Each of the Majorana modes is paired with a metallic nanowire through a tunnel junction with tunneling strength B_i that is adjusted by an external gate voltage. The schematic of this type of interaction is drawn in Fig. 1.

When the system is affected by a bosonic environment, the Majorana modes are placed at two ends of a quantum ring with a space between them. In this case, some

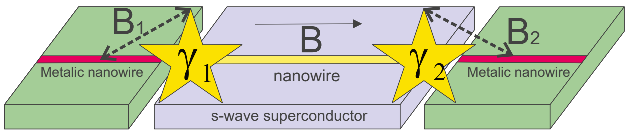

Fig. 1 Schematic representation of topological qubit realized by two Majorana modes γ_1 and γ_2, interacting with a fermionic environment

environmental bosonic operators interact with the two Majorana modes locally. The frequency dependence in the bosonic environment is provided by a magnetic flux Φ passing through the quantum ring. The schematic of this type of interaction is drawn in Fig. 2. In this work, we consider Ohmic-like environmental spectral density $J(\omega) \propto \omega^s$, for both fermionic and bosonic environments. When $s < 1$, $s = 1$ and $s > 1$, we have sub-Ohmic, Ohmic, and super-Ohmic environment, respectively.

Before interaction, the Majorana modes form a topological qubit with states $|0\rangle$ and $|1\rangle$, which are related as follows

$$\frac{1}{2}\big(\gamma_1 - i\gamma_2\big)|0\rangle = |1\rangle, \quad \frac{1}{2}\big(\gamma_1 + i\gamma_2\big)|1\rangle = |0\rangle. \tag{5}$$

Given that γ_a's must satisfy Eq. (1), they can be selected as

$$\gamma_1 = \sigma_1, \quad \gamma_2 = \sigma_2, \quad i\gamma_1\gamma_2 = \sigma_3, \tag{6}$$

where σ_i's are Pauli matrices.

Here, it is assumed that the state of the whole system $S + E$ is product, i.e., $\rho_S(0) \otimes \rho_E$ and $\rho_S(0) = \sum_{i,j=0}^{1} \rho_{ij}|i\rangle\langle j|$ is the initial state of the topological qubit. For the case where the topological qubit is affected by a fermionic environment, the state of the topological qubit at time t is obtained as

$$\rho_S^F(t) = \frac{1}{2}\begin{pmatrix} 1 + \big(2\rho_{00} - 1\big)\alpha^2(t) & 2\rho_{01}\alpha(t) \\ 2\rho_{10}\alpha(t) & 1 + \big(2\rho_{11} - 1\big)\alpha^2(t) \end{pmatrix}, \tag{7}$$

and for the case where the topological qubit is influenced by a bosonic environment, the state of the topological qubit at time t is obtained as

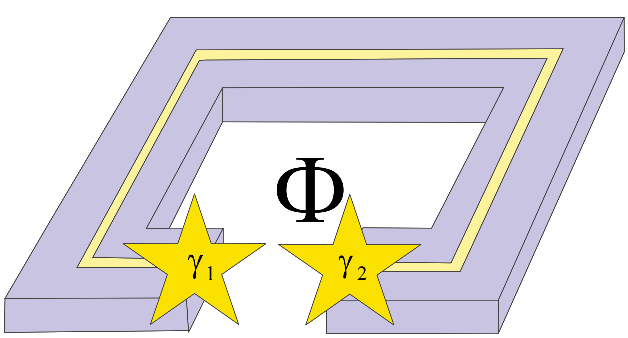

Fig. 2 Schematic representation of topological qubit realized by two Majorana modes γ_1 and γ_2, interacting with a bosonic environment

$$\rho_S^B(t) = \begin{pmatrix} \rho_{00} & \rho_{01}\alpha(t) \\ \rho_{10}\alpha(t) & \rho_{11} \end{pmatrix}, \tag{8}$$

where

$$\alpha(t) = e^{-2B^2|\beta_{F,B}|\mathcal{I}_s(t)}. \tag{9}$$

In relation Eq. (9), the decay parameter $\mathcal{I}_s(t)$ is given by

$$\mathcal{I}_s(t) = \begin{cases} 2\Gamma_0^{s-1}\Gamma\left(\frac{s-1}{2}\right)\left(1 - {}_1F_1\left(\frac{s-1}{2};\frac{1}{2};\frac{-\Gamma_0^2t^2}{4}\right)\right) & s \neq 1 \\ \frac{\Gamma_0^2t^2}{2}\,{}_2F_2\left(\{1,1\};\left\{\frac{3}{2},2\right\};\frac{-\Gamma_0^2t^2}{4}\right) & s = 1, \end{cases} \tag{10}$$

where Γ_0 describes the cutoff frequency of the environment, $\Gamma(z)$ is the Gamma function, and ${}_iF_j$ is the generalized hypergeometric function. β_F and β_B are time-independent coefficients of fermionic and bosonic environments, respectively, and are

$$\beta_F = \frac{-4\pi}{\Gamma\left(\frac{s+1}{0}\right)}\big(\Gamma_0\big)^{-(s+1)} \tag{11}$$

and

$$\beta_B = \begin{cases} -\frac{N_{ss}^2\Gamma(3-\Delta)\epsilon^{2(\Delta-4)}}{4\pi^2\Gamma(\Delta-2)2^2\Delta-5}\sin\pi\Delta & 2 < \Delta \notin \mathbf{N} \\ -\frac{N_{sc}^2\epsilon^{2(\Delta-4)}}{4\pi(\Delta-3)!2^{2\Delta-5}} & 2 \leq \Delta \in \mathbf{N}, \end{cases} \tag{12}$$

where N_{sc} represents the number of degrees of freedom of the dual conformal field theory, ϵ is the UV cutoff of the length, $\Delta = (s+4)/2$ shows the conformal dimension and N stands for the set of natural numbers.

3 Quantum Speed Limit Time

Quantum mechanics sets a bound on the evolution speed of a quantum process for a close or open quantum system. In general, the minimum time required to transform from an initial state to a target state is known as QSL time. Mandelstam and Tamm have introduced the QSL time bound known as (MT) bound for a close quantum system as [11]

$$\tau \geq \tau_{QSL} = \frac{\pi\hbar}{2\Delta\mathcal{E}}, \tag{13}$$

where $\Delta\mathcal{E} = \sqrt{\langle\hat{H}^2\rangle - \langle\hat{H}\rangle^2}$ is the variance of energy of initial state and $\hat{H}$ is the time-independent Hamiltonian describing the evolution of the closed quantum system. Another bound for closed quantum systems has been introduced by Margolus and Levitin [12]. It is known as (ML) bound, which is defined as follows

$$\tau \geq \tau_{QSL} = \frac{\pi\hbar}{2\mathcal{E}}, \tag{14}$$

where $\mathcal{E} = \langle \hat{H} \rangle$. By combining the two ML and MT bounds for the close quantum system, one can obtain a unified bound for QSL time as follows

$$\tau \geq \tau_{QSL} = \max\left\{ \frac{\pi\hbar}{2\Delta E}, \frac{\pi\hbar}{2E} \right\}. \tag{15}$$

In the real world, the interaction of a system with its environment is inevitable, so in practice, the study of open quantum systems is of particular importance. The evolution of an open quantum system is described using a time-dependent master equation as

$$\dot{\rho}_t = \mathcal{L}_t \rho_t, \tag{16}$$

where ρ_t is the state of the open quantum system at time t and $\mathcal{L}_t$ is the positive generator [15]. The main goal here is to find the minimum time required to evolve from an initial state ρ_τ to a target state $\rho_{\tau+\tau_D}$ for an open quantum system, where τ is initial time and τ_D is driving time. This minimal time is called QSL time. To quantify the bound of QSL time one should use an appropriate distance measure. In Refs. [23, 28], the authors use relative purity to quantify the bound for QSL time. A notable feature of the bound defined by them is that their bound can also be used for mixed initial states. The relative purity between the initial state ρ_τ and the target state $\rho_{\tau+\tau_D}$ is defined as follows

$$f\left(\tau + \tau_D\right) = \frac{\mathrm{tr}\left(\rho_\tau \rho_{\tau+\tau_D}\right)}{\mathrm{tr}\left(\rho_\tau^2\right)}. \tag{17}$$

By following the method given in Ref. [28], the ML bound of QSL time can be obtained as

$$\tau \geq \frac{\left|f\left(\tau + \tau_D\right) - 1\right| tr\left(\rho_\tau^2\right)}{\overline{\sum_{i=1}^{n} \kappa_i \varrho_i}}, \tag{18}$$

where κ_i and ϱ_i are the singular value of $\mathcal{L}_t(\rho_t)$ and ρ_τ, respectively, and $\overline{X} = \frac{1}{\tau_D} \int_\tau^{\tau+\tau_D} X dt$. By following a similar method, the MT bound of QSL time for open quantum system can be obtain as

$$\tau \geq \frac{\left|f\left(\tau + \tau_D\right) - 1\right| \mathrm{tr}\left(\rho_\tau^2\right)}{\overline{\sqrt{\sum_{i=1}^{n} \kappa_i^2}}}. \tag{19}$$

By combining ML and MT bounds, a unified bound can be achieved as follows

$$\tau_{(QSL)} = \max\left\{ \frac{1}{\overline{\sum_{i=1}^{n} \kappa_i \varrho_i}}, \frac{1}{\overline{\sqrt{\sum_{i=1}^{n} \kappa_i^2}}} \right\} \times \left|f\left(\tau + \tau_D\right) - 1\right| tr\left(\rho_\tau^2\right). \tag{20}$$

4 Results

In this section, we find the QSL time for the topological qubit when they interact with the bosonic or fermionic environment. Let us consider the initially mixed state for topological qubit as

$$\rho_0 = \frac{1}{2} \begin{pmatrix} 1 + v_z & v_x - iv_y \\ v_x + iv_y & 1 - v_z \end{pmatrix}. \tag{21}$$

When the topological qubit interacts with the fermionic environment its time evolution reads

$$\rho_S^F(t) = \frac{1}{2} \begin{pmatrix} 1 + v'_z & v'_x - iv'_y \\ v'_x + iv'_y & 1 - v'_z \end{pmatrix}. \tag{22}$$

Now we find the singular values of ρ_τ and $\mathcal{L}_t(\rho_t)$. The singular values of ρ_τ are

$$\begin{aligned} \varrho_1 &= \tfrac{1}{2}\left(1 - \sqrt{v'^2_x + v'^2_y + v'^2_z}\right), \\ \varrho_2 &= \tfrac{1}{2}\left(1 + \sqrt{v'^2_x + v'^2_y + v'^2_z}\right), \end{aligned} \tag{23}$$

where $v'_x = \alpha(\tau) v_x$, $v'_y = \alpha(\tau) v_y$ and $v'_z = \alpha(\tau)^2 v_z$. The singular values κ_i of $\mathcal{L}_t(\rho_t)$ are given by

$$\kappa_1 = \kappa_2 = \tfrac{1}{2} |\dot{\alpha}(t) \sqrt{v_x^2 + v_y^2 + 4\alpha(t)^2 v_z^2}|. \tag{24}$$

From Eqs. (23) and (24) we conclude that $\varrho_1 \kappa_1 + \varrho_2 \kappa_2$ is always less than $\sqrt{\kappa_1^2 + \kappa_2^2}$ and so the ML bound on QSL time is tighter than MT bound for open quantum systems.

For the bosonic environment, we follow the calculations as the fermionic environment. We consider the initial state as Eq. (21). So, the state of the topological qubit at time t is given by

$$\rho_S^B(t) = \frac{1}{2} \begin{pmatrix} 1 + v'_z & v'_x - iv'_y \\ v'_x + iv'_y & 1 - v'_z \end{pmatrix}. \tag{25}$$

We calculate the singular values of ρ_τ and $\mathcal{L}_t(\rho_t)$ for the bosonic environment. The singular values of ρ_τ are

$$\begin{aligned} \varrho_1 &= \tfrac{1}{2}\left(1 - \sqrt{v'^2_x + v'^2_y + v'^2_z}\right), \\ \varrho_2 &= \tfrac{1}{2}\left(1 + \sqrt{v'^2_x + v'^2_y + v'^2_z}\right). \end{aligned} \tag{26}$$

where $v'_x = \alpha(\tau) v_x$, $v'_y = \alpha(\tau) v_y$ and $v'_z = v_z$, and the singular values κ_i of $\mathcal{L}_t(\rho_t)$ are

$$\kappa_1 = \kappa_2 = \tfrac{1}{2} |\dot{\alpha}(t) \sqrt{v_x^2 + v_y^2}|, \tag{27}$$

In this work, we consider the maximally coherent initial state with $v_x = v_y = 1/\sqrt{2}$ and $v_z = 0$. So, the QSL time for topological qubit inside both fermionic and bosonic environments is given by

$$\tau_{QSL} = \frac{|\alpha(\tau)^2 - \alpha(\tau)\alpha(\tau+\tau_D)|}{\frac{1}{\tau_D}\int_{\tau}^{\tau+\tau_D} |\dot{\alpha}(t)|}. \tag{28}$$

We see the same relation for the fermionic and the bosonic environments, but from Eqs. (9), (11) and (12) we find different values of $\alpha(t)$ for the bosonic and the fermionic environments.

Figure 3 shows the QSL time as a function of the Ohmic parameter for both bosonic and fermionic environments. For the bosonic environment, the value of QSL time decreases to zero for a small value of s and increases again with increasing s. The situation is slightly different for the fermionic environment. In this case, the QSL time begins from zero for small values of s and increases steadily with increasing s. It is important to note that in any case, the QSL time is less than the driving time τ_D. It is obvious that except for the interval $s < 0.2$ and $s \in [0.2, 1.68]$, that the QSL time of the fermionic case is more than the bosonic environment, the QSL time of the bosonic environment is always more than the QSL time of the fermionic case.

The QSL time versus initial time τ for different values of the Ohmic parameter are shown in Fig. 4 for both fermionic and bosonic environments. Figure 4(a) shows the QSL time as a function of initial time τ for the sub-Ohmic environment. From Fig. 4(a), one can see that for the sub-Ohmic environment the QSL time decreases until it reaches zero for both the fermionic and the bosonic case. Moreover, it is obvious that the QSL time for the bosonic environment is more than the fermionic environment. In Fig. 4(b), the QSL time is plotted for the Ohmic bosonic and fermionic environments. We see that the QSL time of the fermionic environment is more than the QSL time of the bosonic case. In Fig. 4(c) and (d), the QSL time is shown for both fermionic and bosonic super-Ohmic environments. We observe that by increasing s the QSL time increases for both fermionic and bosonic environments. We can see from Fig. 4(d) that for the large values of s, due to the occurrence of coherence trapping [50] the QSL time would be gradually trapped to a fixed value and leads to a uniform evolution speed for the open system.

In Fig. 5, the QSL time is plotted versus the magnetic parameter B for both bosonic and fermionic environments. As we see, the QSL time decreases with increasing the magnetic parameter for sub-Ohmic, Ohmic, and super-Ohmic bosonic and fermionic environments.

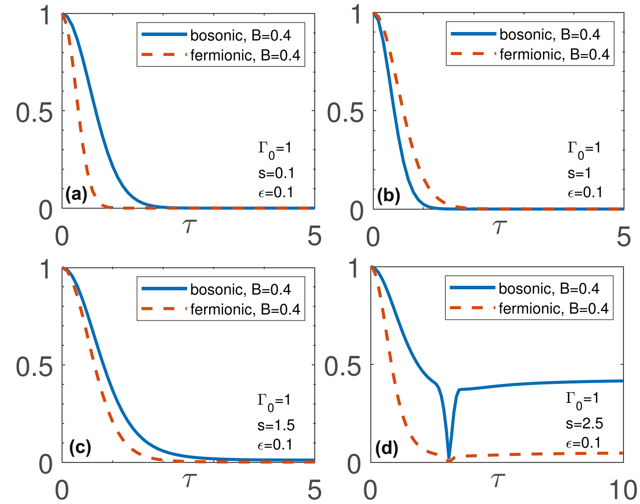

Fig. 4 (Color online) QSL time versus initial time parameter τ for different bosonic and fermionic environments when $\tau_D = 1$ (**a**) s=0.1, (**b**) s=1, (**c**) s=1.5 and (**d**) s=2.5

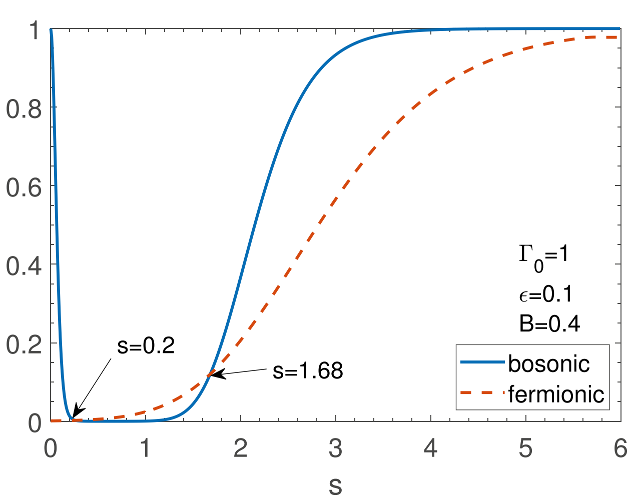

Fig. 3 (Color online) QSL time versus Ohmic parameter s for different bosonic and fermionic environments, B=0.4, $\tau_D = 1$ and $\tau = 1$

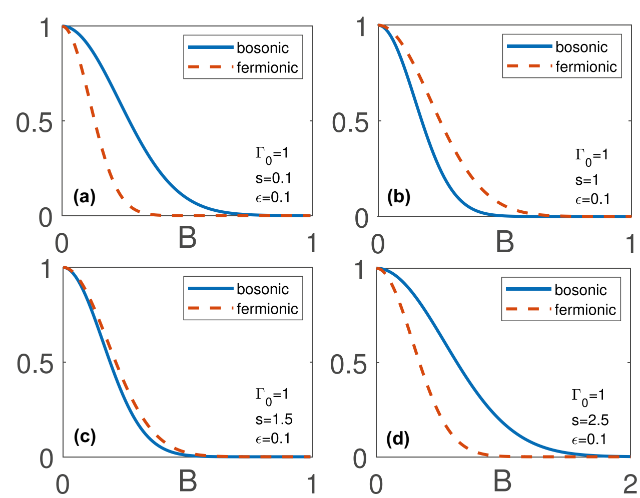

Fig. 5 (Color online) QSL time versus magnetic field parameter B for different bosonic and fermionic environments when $\tau_D = 1$ (**a**) s=0.1, (**b**) s=1, (**c**) s=1.5 and (**d**) s=2.5

5 Conclusion

In this work, we have studied the QSL time for topological qubit when interacts with fermionic and bosonic environments. This model was considered because the topological qubits are physically achievable. We consider the situation in which the fermionic and bosonic environment has the Ohmic-like spectral density. In this work, the effects of environmental parameter such as Ohmic parameter and the magnitude of magnetic field on quantum speed limit time has been studied. About the effect of Ohmic parameter on quantum speed limit time, it was shown that for both fermionic and bosonic environment the quantum speed limit time reaches constant value for larger value of Ohmic parameter. Also, it was shown that the quantum speed limit time decreases by increasing the magnitude of magnetic field for both bosonic and fermionic environments with different Ohmic parameters. It was also shown that the quantum speed limit time reaches zero value for larger value of initial time parameter τ for both sub-Ohmic and Ohmic fermionic and bosonic environment. In other words, the topological qubit open quantum system experience a speeded-up dynamics. For super-Ohmic bosonic and fermionic environment, the quantum speed limit time reaches constant value by increasing initial time parameter τ.

Declarations

Conflicts of Interests The authors have no conflicts of interest to declare. All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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