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Characterizing tripartite entropic uncertainty under random telegraph noise

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Abstract

The uncertainty principle bounds the measurement precision of two complementary variables and the bipartite scenario of quantum-memory-assisted entropic uncertainty relation (QMA-EUR) has been extensively studied. We investigate the tight uncertainty bound (UB) of a tripartite scenario of QMA-EUR in a system consisting of three qubits coupled either to their independent random telegraph noises (RTNs) or to a common source of RTN. For both the initial GHZ- and W-type states, the results show that the UB always increases monotonically with time in the Markovian regime and oscillates in the non-Markovian regime. Moreover, the UB can be significantly reduced due to the non-Markovian effects in the finite-time region and approaches the same asymptotic values in the infinite-time limit for both Markovian and non-Markovian cases.

Keywords Tripartite uncertainty bound · Open quantum system · Random telegraph noise

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1 Introduction

The uncertainty principle plays a fundamental role in the quantum theory [1]. Apart from the conventional uncertainty relation expressed via the variance of measurement outcomes [2], the uncertainty principle can also be expressed in terms of the Shannon entropy. The most well-known entropic uncertainty relation (EUR) was proposed by Deutsch [3]. Its lower bound was improved by Kraus [4] and later was strictly proven by Maassen and Uffink [5]. However, this EUR holds only for the situation that the observer can access only to the classical information. In a pioneer work, Berta et al. showed that the measurement uncertainty can be further decreased by using the quantum information stored in a memory particle B which is entangled with the particle A to be measured [6]. This leads to a new uncertainty relation known as bipartite quantum-memory-assisted entropic uncertainty relation (QMA-EUR). By denoting X and Z as two incompatible observables, the QMA-EUR can be expressed as [6]

$$S(X|B) + S(Z|B) \geq \log_2 \frac{1}{c} + S(A|B), \tag{1}$$

in which $S(A|B)$ is the conditional von Neumann entropy of ρ_{AB}, $c = \max_{\{i,j\}} |\langle x_i|z_j\rangle|^2$ is the incompatibility of X and Z, with $\{|x_i\rangle\}$ and $\{|z_j\rangle\}$ being the eigenstates of X and Z, respectively. Moreover, $S(\mathcal{O}|B) = S(\rho_{\mathcal{O}B}) - S(\rho_B)$ ($\mathcal{O} = X$ or Z) is the conditional von Neumann entropy of the post-measurement state $\rho_{\mathcal{O}B} = \sum_i (\Pi_A^i \otimes \mathbb{1}_B)\rho_{AB}(\Pi_A^i \otimes \mathbb{1}_B)$ after measuring $\mathcal{O}$ on particle A, where the measurement operator $\Pi_A^i = |x_i\rangle\langle x_i|$ ($|z_i\rangle\langle z_i|$) for $\mathcal{O} = X$ (Z).

In the last decade, generalizing the EUR has been the subject of many researches in the field of quantum information science and a great deal of efforts have been devoted to tightening the lower bound of the QMA-EUR [7–22]. Moreover, the dynamics of the measurement uncertainties have been investigated for various quantum systems [23–46]. On the other hand, the bipartite QMA-EUR can be generalized to the tripartite scenario in which two extra particles B and C are played as the quantum memories [47–51]. The tripartite QMA-EUR could be described by the uncertainty game between three players Alice (A), Bob (B), and Charlie (C). First, a quantum state ρ_{ABC} is shared among them. Then, Alice measures one of two non-commuting observables X and Z on her particle A. If Alice measures X, then it is Bob's task to minimize his uncertainty about X, and if she measures Z, then it would be Charlie's task to minimize his uncertainty about Z. The tripartite QMA-EUR mathematically can be written as $S(X|B) + S(Z|C) \geq \log_2(1/c)$ [6], and recently, some works have been put into improving its bound [50, 51]. In particular, in Ref. [51], a tight uncertainty bound (UB) for the tripartite QMA-EUR was obtained as

$$S(X|B) + S(Z|C) \geq \log_2 \frac{1}{c} + \frac{S(A|B) + S(A|C)}{2} + \max\{0, \delta\}, \tag{2}$$

where

$$\delta = \frac{1}{2}[I(A:B) + I(A:C)] - [I(X:B) + I(Z:C)], \tag{3}$$

with $I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ being the quantum mutual information, $I(X:B) = S(\rho_B) - \sum_i p_i S(\rho_{B|i})$ being the Holevo quantity [52], and likewise for $I(A:C)$ and $I(Z:C)$. Here, $S(\rho_{B|i}) = \mathrm{Tr}_{AC}(\Pi^i_A \rho_{ABC} \Pi^i_A)/p_i$ is the collapsed state of B after Alice measuring X on particle A, and $p_i = \mathrm{Tr}(\Pi^i_A \rho_{ABC} \Pi^i_A)$ is the probability of the outcome i. It has been shown that the UB of Eq. (2) is tighter than the other bounds that have been introduced in the literature [53–55]. For a detailed review on the bipartite and tripartite QMA-EUR as well as their applications, see Refs. [8, 9].

In the real situation, any quantum system will be affected by its surrounding environments, which induces decoherence of the state in most cases [56–58]. Thus, it is crucial to investigate the influence of environmental noises on the QMA-EUR in different physical systems. Although the decohering effects on the QMA-EUR have been widely studied in the bipartite systems [59–67], the tripartite QMA-EUR has been investigated only in few studies [53–55]. Motivated by this fact, we investigate dynamics of the tripartite QMA-EUR in an explicit system consisting of three non-interacting qubits and subject to a classical random telegraph noise (RTN). Indeed, RTN can be caused by charge trapping in the surfaces of thin films [68]. In addition, the experimental investigation of RTN has revealed dephasing influences in the dynamics of open systems, resulting in rapid quantum correlations degradation [69]. In this work, we focus on the dephasing effect of RTN because it is common in many quantum information protocols [70, 71]. Two different initial states are considered, i.e., the three-qubit GHZ- and W-type states. Here, the classical noise is characterized by a stochastic Hamiltonian with a coupling term mimicking the RTN, and the time evolution of the system can be obtained by averaging over the noise [72–76]. Specifically, the three qubits coupled either to their independent RTNs or to a common source of RTN will be considered.

This paper is organized as follows. In Sect. 2, the physical model consisting of three qubits subject to a classical RTN will be explained. In Sect. 3, dynamics of the UB for the tripartite QMA-EUR with two different initial states is analyzed. The results will be summarized in Sect. 4.

2 Solution of the model

In this paper, we consider a physical model consisting of three non-interacting identical qubits subject either to three independent RTNs or to a common source of RTN [72, 73]. For the independent RTNs, each qubit of the system interacts independently with its own environment, whereas for the common RTN, the three qubits are coupled simultaneously to the same environment. A schematic diagram for these configurations is shown in Fig. 1. The system's Hamiltonian can be written as [72, 73]

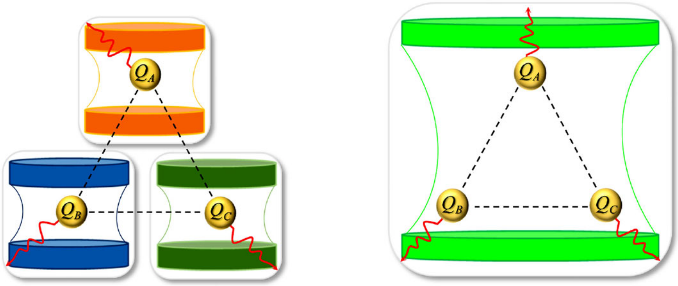

Fig. 1 A schematic diagram of three qubits interacting with independent (left) and common (right) RTN. The black dashed lines represent the entanglement between the qubits and the red wavy lines show the coupling of each qubit to the RTN (Color figure online)

$$H(t) = H_A(t) \otimes \mathbb{1}_{BC} + H_B(t) \otimes \mathbb{1}_{AC} + H_C(t) \otimes \mathbb{1}_{AB}, \tag{4}$$

where $\mathbb{1}_{BC}$ is the identity operator in the Hilbert space of BC, and likewise for $\mathbb{1}_{AC}$ and $\mathbb{1}_{AB}$. Moreover, $H_L(t)$ stands for the single-qubit Hamiltonian for qubit $L \in \{A, B, C\}$, which reads [77, 78]

$$H_L(t) = \varepsilon \mathbb{1}_L + v\eta_L(t)\sigma_L^x, \tag{5}$$

where ε denotes the qubit energy without noise, v is the coupling constant of the qubit L to the RTN, and σ_L^x is the first Pauli operator for the qubit L. Here, the RTN corresponds to a stochastic process characterized by $\eta_L(t)$ describing a coin-flip variable switching randomly between the possible values ± 1 with rate γ.

The time-dependency of $H(t)$ in Eq. (4) results in a stochastic dynamics of the qubits due to the random nature of the telegraph processes. As a consequence, we need to take an average over differential noise configurations to acquire the time-evolved state under the influence of RTN [77–79]. To be explicit, for the given noise configuration $\{\eta\} = \{\eta_A, \eta_B, \eta_C\}$, the evolution of the system's density operator can be expressed as

$$\rho(t) = \langle U(\{\eta\}, t)\rho(0)U^\dagger(\{\eta\}, t)\rangle_{\{\eta\}}, \tag{6}$$

with $\rho(0)$ being the initial state of the system and $U(\{\eta\}, t)$ is the corresponding unitary evolution operator for the given noise configuration $\{\eta\}$. Since there are no direct interactions among the three qubits, $U(\{\eta\}, t)$ can be written as the tensor products of the single-qubit evolution operators, $U(\{\eta\}, t) = U_A(\eta_A, t) \otimes U_B(\eta_B, t) \otimes U_C(\eta_C, t)$. It should be mentioned that $\eta_A \neq \eta_B \neq \eta_C$ corresponds to the independent RTNs and $\eta_A = \eta_B = \eta_C$ corresponds to the common RTN. By setting $\hbar = 1$, $U_L(\eta_L, t)$ can be expressed as [72]

$$U_L(\eta_L, t) = e^{-i\int_0^t H_L(t')\mathrm{d}t'} = e^{-i\varepsilon t}\begin{pmatrix} \cos\phi_L(t) & i\sin\phi_L(t) \\ i\sin\phi_L(t) & \cos\phi_L(t) \end{pmatrix}, \tag{7}$$

where $\phi_L(t) = -v \int_0^t \eta_L(t')\mathrm{d}t'$ is the stochastic phase picked up during the time interval $[0, t]$. Then one can obtain the time-evolved density operator $\rho(t)$ of Eq. (6) by first estimating the averaged terms of the type $\langle[\cos\phi(t)]^m[\sin\phi(t)]^k\rangle$ $(m, k \in \mathbb{N})$, which can be expressed in terms of the average of the characteristic function of the noise phase $\langle e^{in\phi(t)}\rangle$ $(n \in \mathbb{N})$ as [80]

$$\left\langle e^{in\phi(t)}\right\rangle = \langle\cos n\phi(t)\rangle + i\,\langle\sin n\phi(t)\rangle\,, \tag{8}$$

where $\langle\sin n\phi(t)\rangle = 0$ and

$$\langle\cos n\phi(t)\rangle = G_n(t) = e^{-\gamma t}\big[\cosh(\delta_{nv}t) + \tfrac{\gamma}{\delta_{nv}}\sinh(\delta_{nv}t)\big], \tag{9}$$

where $\delta_{nv} = \sqrt{\gamma^2 - (nv)^2}$. Note that when $\gamma > nv$ (weak coupling), $G_n(t)$ decays exponentially with time t and the evolution is Markovian. But when $\gamma < nv$ (strong coupling), $\delta_{nv} = i|\delta_{nv}|$, hence $\cosh(\delta_{nv}t) = \cos(|\delta_{nv}|t)$ and $\sinh(\delta_{nv}t) = i\sin(|\delta_{nv}|t)$, then $G_n(t)$ oscillates with time and the evolution will be non-Markovian [81].

3 Results and discussion

In this section, we explore effects of the RTN on the UB of the tripartite QMA-EUR for the system introduced above. Without loss of generality, we consider two types of initial states, i.e., the three-qubit GHZ- and W-type states. As illustrated in Fig. 1, the dynamics of the UB will be investigated for both the cases of independent and common RTN, and herein, the incompatible observables measured on particle A are taken to be $X = \sigma^x$ and $Z = \sigma^z$.

3.1 The initial GHZ-type state

Let us consider the initial GHZ-type state of the following form

$$\rho_G(0) = r|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| + \frac{1-r}{8}\mathbb{1}_{ABC}, \tag{10}$$

where $|\mathrm{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ is the three-qubit GHZ state, $0 \le r \le 1$ represents the corresponding ratio of $|\mathrm{GHZ}\rangle$ in $\rho_G(0)$, and $\mathbb{1}_{ABC}$ is the identity operator.

For the initial state $\rho_G(0)$, the time-evolved states have been obtained in Ref. [72] when the three qubits are coupled to their independent environments (ie) and common environment (ce) of RTN (see also Appendix A), based on which one can obtain the UB of the tripartite QMA-EUR as follows:

$$U_b\left[\rho_G^{\mathrm{ie}}(t)\right] = 1 + H_2\left(\frac{1 + rG_2^2(t)}{2}\right), \tag{11}$$

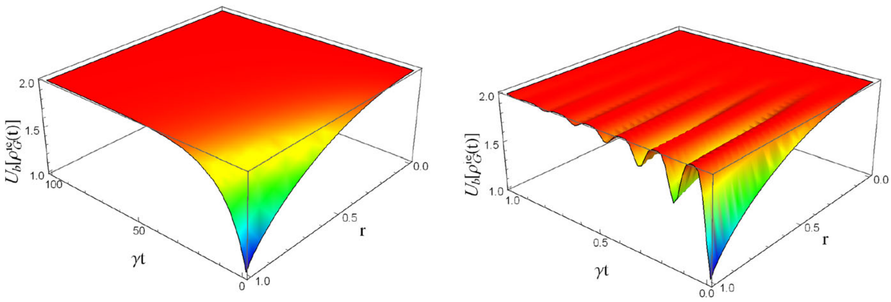

Fig. 2 The UB of the tripartite QMA-EUR versus γt and r when the three qubits are initially prepared in the GHZ-type state $\rho_G(0)$ and are coupled to their independent RTNs, where the parameter is chosen to be $\gamma/v = 10$ (left) and $\gamma/v = 0.1$ (right)

and

$$U_b[\rho_G^{\mathrm{ce}}(t)] = 1 + \frac{1}{2}H_2\left(\frac{1+rG_4(t)}{2}\right) + \frac{1}{2}H_2\left(\frac{1+r}{2}\right) + \max\left\{0, \delta_G^{\mathrm{ce}}(t)\right\}, \quad (12)$$

where $H_2(x) = -x\log_2 x - (1-x)\log_2(1-x)$ denotes the binary Shannon entropy function and

$$\delta_G^{\mathrm{ce}}(t) = H_2\left(\frac{2+r+rG_4(t)}{4}\right) - \frac{1}{2}H_2\left(\frac{1+rG_4(t)}{2}\right) - \frac{1}{2}H_2\left(\frac{1+r}{2}\right). \quad (13)$$

For the case that the three qubits are coupled to their independent RTNs, the UB of the tripartite QMA-EUR (11) as functions of the rescaled time γt and the initial purity r is plotted in Fig. 2. Two different values of γ/v, namely $\gamma/v = 10$ corresponding to the Markovian dynamics and $\gamma/v = 0.1$ corresponding to the non-Markovian dynamics, are considered. For the special case $r = 0$ which corresponds to a maximally mixed state, the UB remains the constant 2 in the whole time region. For $r \neq 0$, as can be seen from Fig. 2, the UB increases monotonically with time in the Markovian regime and approaches the asymptotic value 2 when $\gamma t \to \infty$. In the non-Markovian regime, however, the UB oscillates with time in the finite-time region due to the memory effects caused by the temporal correlations occurring in the time evolution of each qubit. With the increasing time, the memory effects of the RTN will be weakened and its detrimental effects turn to dominate, so the UB still approaches its asymptotic value 2 in the infinite-time limit. This indicates that the memory effects of the RTN play a positive role in reducing the measurement uncertainty in the finite-time region.

When the three qubits are coupled to a common source of RTN, we show in Fig. 3 dependence of the UB (12) on γt and r. For such a case, due to the RTN-mediated indirect interactions among the qubits, the dynamical behaviors of the UB may be complicated. As can be seen from Fig. 3, it still remains the constant 2 when $r = 0$, regardless of the Markovian or non-Markovian nature of the noise. When $r \neq 0$, one can see that in the Markovian regime, the UB shows a similar behavior to that of the independent RTNs, and the only difference is that its asymptotic value in the infinite-time limit decreases monotonically with the increase of r. In the non-Markovian

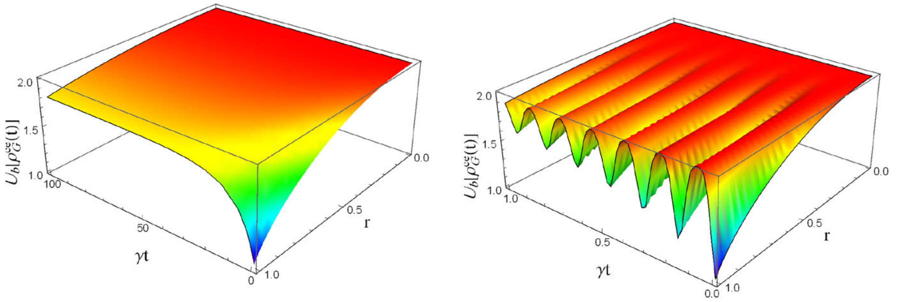

Fig. 3 The UB of the tripartite QMA-EUR versus γt and r when the three qubits are initially prepared in the GHZ-type state $\rho_G(0)$ and are coupled to a common RTN, where the parameter is chosen to be $\gamma/v = 10$ (left) and $\gamma/v = 0.1$ (right)

regime, the UB also oscillates with time and approaches the same asymptotic values as those in the Markovian regime. But the amplitudes of oscillations are more evident than those for the independent RTNs. This shows that for the initial three-qubit GHZ-type state, the common RTN is beneficial for reducing the measurement uncertainty compared to the independent RTNs.

3.2 The initial *W*-type state

In this subsection, we turn to consider the case that the three qubits are prepared initially in the following generalized W-like state

$$\rho_W(0) = r|W\rangle\langle W| + \frac{1-r}{8}\mathbb{1}_{ABC}, \tag{14}$$

where $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$ is the three-qubit W state and $0 \leq r \leq 1$ is the ratio of $|W\rangle$ in $\rho_W(0)$.

For such an initial state, the time-evolved state for both the cases of the independent and common RTN can be found in Ref. [72] (see also Appendix A). Therefore, one can obtain the UB of the tripartite QMA-EUR as

$$\begin{aligned} U_b[\rho_W^{\mathrm{ie}}(t)] =& \frac{3}{2} - \sum_{i=0,1}\left[\frac{3 - rG_2^2(t) + (-1)^i\Theta(t)}{12}\log_2\frac{3 - rG_2^2(t) + (-1)^i\Theta(t)}{12}\right] \\ &+ \frac{1}{2}H_2\left(\frac{3+2r+rG_2^2(t)}{6}\right) - H_2\left(\frac{x(t)}{6}\right) + \max\{0, \delta_W^{\mathrm{ie}}(t)\}, \\ U_b[\rho_W^{\mathrm{ce}}(t)] =& 1 - \frac{1-r}{4}\log_2\frac{1-r}{4} - \frac{3+2r+3rG_4(t)}{12}\log_2\frac{3+2r+3rG_4(t)}{12} \\ &- H_2\left(\frac{x(t)}{6}\right) - \sum_{i=+,-}\left[\frac{y^i(t)}{24}\log_2\frac{y^i(t)}{24}\right] + \max\{0, \delta_W^{\mathrm{ce}}(t)\}, \end{aligned} \tag{15}$$

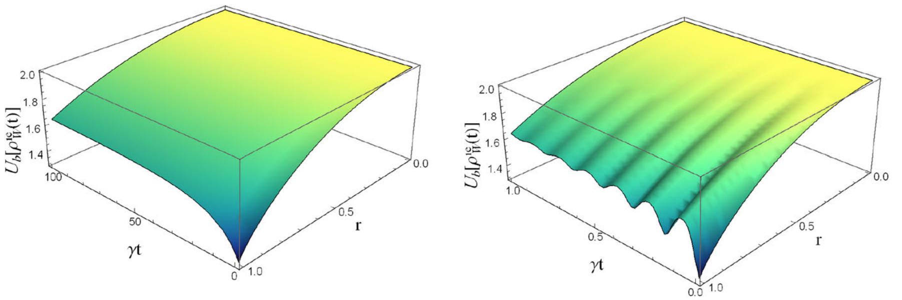

Fig. 4 The UB of the tripartite QMA-EUR versus γt and r when the three qubits are initially prepared in the W-type state $\rho_W(0)$ and are coupled to their independent RTNs, where the parameter is chosen to be $\gamma/v = 10$ (left) and $\gamma/v = 0.1$ (right)

where

$$\begin{aligned}
\delta_W^{\mathrm{ie}}(t) &= H_2\left(\frac{\epsilon(t)}{6}\right) - H_2\left(\frac{x(t)}{6}\right) + \frac{3+2r+3rG_2^2(t)}{12}\log_2\frac{3+2r+3rG_2^2(t)}{12} \\
&\quad + \frac{3-2r-rG_2^2(t)}{12}\log_2\frac{3-2r-rG_2^2(t)}{12} - \frac{3+rG_2^2(t)}{6}\log_2\frac{3+rG_2^2(t)}{12} \\
&\quad + \sum_{i=0,1}\left[\frac{3-rG_2^2(t)+(-1)^i\Theta(t)}{12}\log_2\frac{3-rG_2^2(t)+(-1)^i\Theta(t)}{12}\right. \\
&\quad \left. - \frac{3-rG_2^2(t)+(-1)^i 2rG_2(t)}{12}\log_2\frac{3-rG_2^2(t)+(-1)^i 2rG_2(t)}{12}\right], \\
\delta_W^{\mathrm{ce}}(t) &= \frac{1-r}{4}\log_2\frac{1-r}{4} + H_2\left(\frac{\epsilon(t)}{6}\right) - H_2\left(\frac{x(t)}{6}\right) \\
&\quad + \frac{3+2r+3rG_4(t)}{12}\log_2\frac{3+2r+3rG_4(t)}{12} \\
&\quad - \frac{6-r+3rG_4(t)}{12}\log_2\frac{6-r+3rG_4(t)}{24} \\
&\quad + \sum_{i=+,-}\left[\frac{y^i(t)}{24}\log_2\frac{y^i(t)}{24} - \frac{\pi^i(t)}{24}\log_2\frac{\pi^i(t)}{24}\right],
\end{aligned} \tag{16}$$

and the parameters $\Theta(t)$, $\epsilon(t)$, $x(t)$, $y^{\pm}(t)$, and $\pi^{\pm}(t)$ appeared in the above equations are given by

$$\begin{aligned}
&\Theta(t) = 2r\sqrt{1-G_2^2(t)+G_2^4(t)},\ \epsilon(t) = 3+r\sqrt{4+G_2^2(t)},\ x(t) = 3+rG_2(t), \\
&y^{\pm}(t) = 6+r-3rG_4(t) \pm r\sqrt{9(G_4(t)-1)^2+16G_2^2(t)}, \\
&\pi^{\pm}(t) = 6+r-3rG_4(t) \pm 4rG_2(t).
\end{aligned} \tag{17}$$

To show effects of the independent RTNs on the UB of the QMA-EUR for the initial three-qubit W-type state, we display in Fig. 4 dependence of $U_b[\rho_W^{\mathrm{ie}}(t)]$ on the

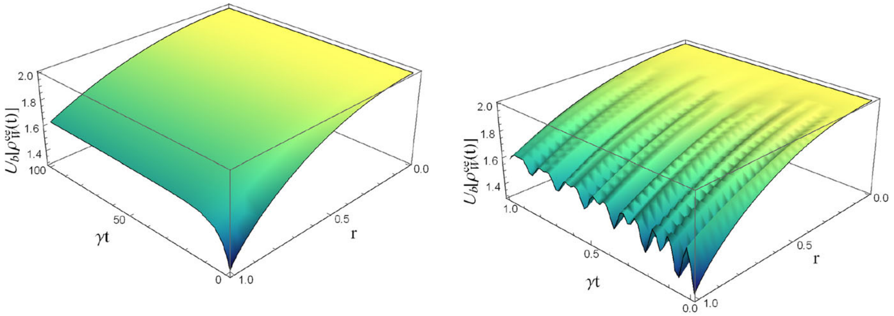

Fig. 5 The UB of the tripartite QMA-EUR versus γt and r when the three qubits are initially prepared in the W-type state $\rho_W(0)$ and are coupled to a common RTN, where the parameter is chosen to be $\gamma/v = 10$ (left) and $\gamma/v = 0.1$ (right)

rescaled time γt and purity r, where the left panel is plotted for the Markovian case and the right panel is plotted for the non-Markovian case. When $r = 0$, the UB always remain the constant 2. When $r \neq 0$, it can be found that for the Markovian case, the UB increases monotonically with the evolving time, and for the non-Markovian case, the memory effects of the RTNs induce oscillations of the UB. By comparing Fig. 4 with Fig. 2, one can also note that $U_b[\rho_W^{\text{ie}}(t)]$ is smaller than $U_b[\rho_G^{\text{ie}}(t)]$. This indicates that the W-type state is beneficial for reducing the measurement uncertainty compared to the GHZ-type state. The physical reason for such an observation may be rooted in the fact that the GHZ state is significantly more fragile under environmental coupling than the W state [82]. In particular, there is no pairwise entanglement in the GHZ state, while there is pairwise entanglement between any two qubits of the W state.

Finally, we see the case that the three qubits are coupled to a common source of RTN. The corresponding dynamical behaviors of the UB of the tripartite QMA-EUR are shown in Fig. 5. In the Markovian regime, one finds again the monotonic increase of the UB with time, while in the non-Markovian regime, the UB also oscillates with the evolving time. But different from the case of the independent RTNs, one can see that in the non-Markovian regime of the common RTN, there exist two different patterns of oscillations for which we term them as the main oscillation and sub-oscillation, that is, there is an extra shallow valley between the two deep valleys.

4 Summary

A number of works have been devoted to exploring quantum correlations for a tripartite system coupled to different sources of environments, and it was found that their dynamical behaviors are strongly dependent on the types of the system-environment coupling [83–96]. As the measurement uncertainty in a QMA-EUR is intimately related to quantum correlations of the system, it is expected that it may also be affected by the system-environment coupling. In this paper, we have investigated a tight UB of the tripartite QMA-EUR in an explicit system consisting of three non-interacting qubits which are coupled either to independent or to a common source of RTN. We have considered two types of initial states, i.e., the GHZ- and W-type states. It was shown that the behaviors of the UB strongly depends on the nature of the system-environment coupling. Specifically, the UB increases monotonically with time in the Markovian regime and oscillates with time in the non-Markovian regime. Moreover, the memory effects of the non-Markovian dynamics are beneficial for reducing the UB in the finite-time region. In the infinite-time region, however, the detrimental effect of the RTN dominates and thus the UB approaches the same asymptotic values, regardless of the Markovian or non-Markovian nature of the system dynamics.

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Author Contributions HD has put forward the main idea and also prepared the draft. SH has performed the calculations and designed the diagrams. HD, SH, and MLH all contributed to the development and completion of the idea, analyzing the results, discussions and writing the manuscript. Thorough checking of the manuscript was done by all authors.

Data availability This paper is a theoretical research and has no associated data.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A: Time-evolved states for the initial GHZ- and W-type states

For the initial GHZ-type state of Eq. (10), the explicit form of the time-evolved state has been obtained in Ref. [72]. We list it here for facilitating discussion of the tripartite QMA-EUR in the main text. First, for the case that the three qubits are coupled to their independent environments (ie) and common environment (ce), one has [72]

$$\rho_G^{\mathrm{ie}}(t) = \begin{bmatrix} \alpha(t) & 0 & 0 & 0 & 0 & 0 & 0 & \sigma(t) \\ 0 & \theta(t) & 0 & 0 & 0 & 0 & \beta(t) & 0 \\ 0 & 0 & \theta(t) & 0 & 0 & \beta(t) & 0 & 0 \\ 0 & 0 & 0 & \theta(t) & \beta(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta(t) & \theta(t) & 0 & 0 & 0 \\ 0 & 0 & \beta(t) & 0 & 0 & \theta(t) & 0 & 0 \\ 0 & \beta(t) & 0 & 0 & 0 & 0 & \theta(t) & 0 \\ \sigma(t) & 0 & 0 & 0 & 0 & 0 & 0 & \alpha(t) \end{bmatrix}, \tag{A1}$$

and

$$\rho_G^{\mathrm{ce}}(t) = \begin{bmatrix} \frac{1}{8}+3\mu(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & \frac{r}{8}+3\mu(t) \\ -\lambda(t) & \frac{1}{8}-\mu(t) & \lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & -\lambda(t) \\ -\lambda(t) & \lambda(t) & \frac{1}{8}-\mu(t) & \lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & -\lambda(t) \\ -\lambda(t) & \lambda(t) & \lambda(t) & \frac{1}{8}-\mu(t) & \lambda(t) & \lambda(t) & \lambda(t) & -\lambda(t) \\ -\lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \frac{1}{8}-\mu(t) & \lambda(t) & \lambda(t) & -\lambda(t) \\ -\lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \frac{1}{8}-\mu(t) & \lambda(t) & -\lambda(t) \\ -\lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \lambda(t) & \frac{1}{8}-\mu(t) & -\lambda(t) \\ \frac{r}{8}+3\mu(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & -\lambda(t) & \frac{1}{8}+3\mu(t) \end{bmatrix}, \tag{A2}$$

where the parameters appeared in the above two equations are given by

$$\begin{aligned} &\alpha(t) = \frac{1}{8}\left[1+3rG_2^2(t)\right],\ \beta(t) = \frac{r}{8}\left[1-G_2^2(t)\right],\ \theta(t) = \frac{1}{8}\left[1-rG_2^2(t)\right], \\ &\sigma(t) = \frac{r}{8}\left[1+3G_2^2(t)\right],\ \mu(t) = \frac{r}{16}\left[1+G_4(t)\right],\ \lambda(t) = \frac{r}{16}\left[1-G_4(t)\right]. \end{aligned} \tag{A3}$$

Similarly, for the three qubits being prepared initially in the W-type state of Eq. (14) and subject to their independent RTNs and common RTN, the time-evolved density matrices of the system can be obtained, respectively, as [72]

$$\rho_W^{\mathrm{ie}}(t) = \begin{bmatrix} \frac{\kappa(t)}{8}+\varsigma & 0 & 0 & \frac{\kappa(t)}{12} & 0 & \frac{\kappa(t)}{12} & \frac{\kappa(t)}{12} & 0 \\ 0 & \frac{\tau(t)}{24}+\varsigma & \frac{\vartheta(t)}{12} & 0 & \frac{\vartheta(t)}{12} & 0 & 0 & \frac{\chi(t)}{12} \\ 0 & \frac{\vartheta(t)}{12} & \frac{\tau(t)}{24}+\varsigma & 0 & \frac{\vartheta(t)}{12} & 0 & 0 & \frac{\chi(t)}{12} \\ \frac{\kappa(t)}{12} & 0 & 0 & \frac{\xi(t)}{24}+\varsigma & 0 & \frac{\psi(t)}{12} & \frac{\psi(t)}{12} & 0 \\ 0 & \frac{\vartheta(t)}{12} & \frac{\vartheta(t)}{12} & 0 & \frac{\tau(t)}{24}+\varsigma & 0 & 0 & \frac{\chi(t)}{12} \\ \frac{\kappa(t)}{12} & 0 & 0 & \frac{\psi(t)}{12} & 0 & \frac{\xi(t)}{24}+\varsigma & \frac{\psi(t)}{12} & 0 \\ \frac{\kappa(t)}{12} & 0 & 0 & \frac{\psi(t)}{12} & 0 & \frac{\psi(t)}{12} & \frac{\xi(t)}{24}+\varsigma & 0 \\ 0 & \frac{\chi(t)}{12} & \frac{\chi(t)}{12} & 0 & \frac{\chi(t)}{12} & 0 & 0 & \frac{\chi(t)}{8}+\varsigma \end{bmatrix}, \tag{A4}$$

and

$$\rho_W^{\mathrm{ce}}(t) = \begin{bmatrix} \frac{1}{8}+\Lambda(t) & 0 & 0 & \Xi(t) & 0 & \Xi(t) & \Xi(t) & 0 \\ 0 & \frac{1}{8}+\Upsilon(t) & \Omega(t) & 0 & \Omega(t) & 0 & 0 & \Gamma(t) \\ 0 & \Omega(t) & \frac{1}{8}+\Upsilon(t) & 0 & \Omega(t) & 0 & 0 & \Gamma(t) \\ \Xi(t) & 0 & 0 & \frac{1}{8}+\Phi(t) & 0 & \Delta(t) & \Delta(t) & 0 \\ 0 & \Omega(t) & \Omega(t) & 0 & \frac{1}{8}+\Upsilon(t) & 0 & 0 & \Gamma(t) \\ \Xi(t) & 0 & 0 & \Delta(t) & 0 & \frac{1}{8}+\Phi(t) & \Delta(t) & 0 \\ \Xi(t) & 0 & 0 & \Delta(t) & 0 & \Delta(t) & \frac{1}{8}+\Phi(t) & 0 \\ 0 & \Gamma(t) & \Gamma(t) & 0 & \Gamma(t) & 0 & 0 & \frac{1}{8}+\Psi(t) \end{bmatrix}, \tag{A5}$$

where the parameters appeared in the above two equations are given by

$$
\begin{aligned}
&\kappa(t) = r\,[1+G_2(t)]^2\,[1-G_2(t)]\,,\ \varsigma = \frac{1}{8}(1-r),\\
&\chi(t) = r\,[1-G_2(t)]^2\,[1+G_2(t)]\,,\\
&\tau(t) = r\,[1+G_2(t)]\left\{[1+G_2(t)]^2 + 2\,[1-G_2(t)]^2\right\},\\
&\vartheta(t) = r\,[1+G_2(t)]\left[1+G_2^2(t)\right],\\
&\xi(t) = r\,[1-G_2(t)]\left\{2\,[1+G_2(t)]^2 + [1-G_2(t)]^2\right\},\\
&\psi(t) = r\,[1-G_2(t)]\left[1+G_2^2(t)\right],\\
&\Lambda(t) = r\left(\frac{1}{16} + \frac{3G_2(t)}{32} - \frac{3G_4(t)}{16} - \frac{3G_6(t)}{32}\right),\\
&\Xi(t) = r\left(\frac{1}{16} + \frac{3G_2(t)}{32} - \frac{G_4(t)}{16} - \frac{3G_6(t)}{32}\right),\\
&\Upsilon(t) = r\left(\frac{-1}{48} + \frac{7G_2(t)}{96} + \frac{G_4(t)}{16} + \frac{3G_6(t)}{32}\right),\\
&\Omega(t) = r\left(\frac{5}{48} + \frac{7G_2(t)}{96} + \frac{G_4(t)}{16} + \frac{3G_6(t)}{32}\right),\\
&\Gamma(t) = r\left(\frac{1}{16} - \frac{3G_2(t)}{32} - \frac{G_4(t)}{16} + \frac{3G_6(t)}{32}\right),\\
&\Phi(t) = r\left(\frac{-1}{48} - \frac{7G_2(t)}{96} + \frac{G_4(t)}{16} - \frac{3G_6(t)}{32}\right),\\
&\Delta(t) = r\left(\frac{5}{48} - \frac{7G_2(t)}{96} + \frac{G_4(t)}{16} - \frac{3G_6(t)}{32}\right),\\
&\Psi(t) = r\left(\frac{1}{16} - \frac{3G_2(t)}{32} - \frac{3G_4(t)}{16} + \frac{3G_6(t)}{32}\right).
\end{aligned}
\tag{A6}
$$

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