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Entanglement sensitivity to signal attenuation and amplification

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We analyze general laws of continuous-variable entanglement dynamics during the deterministic attenuation and amplification of the physical signal carrying the entanglement. These processes are inevitably accompanied by noises, so we find fundamental limitations on noise intensities that destroy entanglement of Gaussian and non-Gaussian input states. The phase-insensitive amplification $\Phi_1 \otimes \Phi_2 \otimes \cdots \Phi_N$ with the power gain $\kappa_i \geqslant 2$ (≈ 3 dB, $i = 1, \ldots, N$) is shown to destroy entanglement of any N-mode Gaussian state even in the case of quantum-limited performance. In contrast, we demonstrate non-Gaussian states with the energy of a few photons such that their entanglement survives within a wide range of noises beyond quantum-limited performance for any degree of attenuation or gain. We detect entanglement preservation properties of the channel $\Phi_1 \otimes \Phi_2$, where each mode is deterministically attenuated or amplified. Gaussian states of high energy are shown to be robust to very asymmetric attenuations, whereas non-Gaussian states are at an advantage in the case of symmetric attenuation and general amplification. If $\Phi_1 = \Phi_2$, the total noise should not exceed $\frac{1}{2}\sqrt{\kappa^2 + 1}$ to guarantee entanglement preservation.

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Introduction. Creation, manipulation, and evolution of entangled states are in the basis of many applications including quantum information protocols [1] and interferometry [2]. The physical implementation of such applications raises an important problem of noisy entanglement dynamics and robustness of entangled states [3]. The problem of continuous-variable entanglement dynamics in different physical models of system-environment interactions was considered in the papers [4–17]. The particular results depend on many aspects, namely, the structure of composite system comprising (in)distinguishable particles, the entanglement measure, the noise model, the initial state, the interaction among particles of the system, and the form of external driving. Such a variety of scenarios makes the full characterization of entanglement dynamics hardly possible. Moreover, the extent to which the evolved entanglement remains useful depends on the particular quantum application. However, a general entanglement-assisted application relies on the presence of nonvanishing entanglement, an exceptional quantum property regardless of its magnitude. Thus, the fundamental limitation on the application performance is imposed by those noises that completely destroy the entanglement of an input state.

In this Rapid Communication, we analyze the limiting noises that accompany the physical processes of deterministic signal attenuation and amplification [18]. The former one is a standard model to describe losses in continuous-variable systems [13,14], whereas the latter one is used in so-called quantum cloning machines [19] and other applications [20]. The limiting noises for such operations were found in the one-sided scenario, i.e., for a quantum channel of the form $\Phi_1 \otimes \mathrm{Id}_2$, which transforms any input state into a separable one [21]. Such quantum channels Φ are known as entanglement-breaking (EB) ones [22,23]. However, the attenuation or amplification does not have to be one sided. Our goal is to find parameters of the general channel $\Phi_1 \otimes \Phi_2 \otimes \cdots$ that fundamentally restrict the use of locally attenuated or amplified signals in entanglement-assisted applications.

This problem was partially explored in the paper [24], which announced the existence of non-Gaussian states that are more robust to the action of homogeneous two-mode amplification $\Phi \otimes \Phi$ than Gaussian ones (in contrast to the beforehand opposite conjecture [25–28]). Our results improve those of Ref. [24] and provide evidence that non-Gaussian states of little energy can outperform high-energy Gaussian states also in the case of two-mode attenuation. We extend our results to asymmetric channels and multiple numbers of modes.

Attenuators and amplifiers are distinguished examples of Gaussian channels [29–31] that are usually used to describe the deterministic lossy process and linear amplification of bosonic quantum states. The bosonic quantum state is defined by the density operator ϱ or, equivalently, by the characteristic function $\varphi(\mathbf{z}) = \mathrm{tr}[\varrho W(\mathbf{z})]$, where $W(\mathbf{z}) = \exp[i(q_1 x_1 + p_1 y_1 + \cdots + q_N x_N + p_N y_N)]$ is the Weyl operator, N is the number of modes, the operators q_i and p_j satisfy the canonical commutation relation $[q_i, p_j] = i\delta_{ij}$, and $\mathbf{z} = (x_1, y_1, \ldots, x_N, y_N)^\top$ corresponds to coordinates in the real symplectic space $(\mathbb{R}^{2N}, \mathbf{\Delta})$, with $\mathbf{\Delta}$ being the symplectic form $\mathbf{\Delta} = \bigoplus_{i=1}^{N} \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$.

In terms of the characteristic functions, the Gaussian channel acts as follows:

$$\varphi_{\mathrm{out}}(\mathbf{z}) = \varphi_{\mathrm{in}}(\mathbf{K}\mathbf{z}) \exp\left(-\tfrac{1}{2}\mathbf{z}^\top \mathbf{M}\mathbf{z}\right). \tag{1}$$

The one-mode Gaussian channels are characterized in [31]. Suppose $\mathbf{K} = \sqrt{\kappa}\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ and $\mathbf{M} = \mu\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, then the transformation (1) defines processes of one-mode attenuation ($0 < \kappa < 1$), addition of classical noise ($\kappa = 1$), and amplification

($\kappa > 1$). These processes are fair physical channels (completely positive maps) if the total noise $\mu \geqslant \frac{1}{2}|\kappa - 1|$ [29]. The minimal noise $\mu_{\rm QL} = \frac{1}{2}|\kappa - 1|$ corresponds to a so-called quantum-limited operation, and the quantity $a = \mu - \mu_{\rm QL} \geqslant 0$ is the extra noise. The one-mode channels $\Phi(\kappa,\mu)$ altogether form a set $\mathcal{C}$.

The action of channel $\Phi \in \mathcal{C}$ takes a simple form in the diagonal sum representation $\Phi[X] = \sum_{ij} A_{ij} X A_{ij}^\dagger$, where the explicit form of Kraus operators $\{A_{ij}\}_{i,j=0,1,\ldots}$ in the Fock basis has been found for all κ and a in the seminal paper [32]. To work easily with the coherent states $|\alpha\rangle$, we derive the representation $\Phi[\varrho] = \pi^{-2} \iint d^2\alpha\, d^2\beta\, \tilde{A}_{\alpha\beta} \varrho \tilde{A}_{\alpha\beta}^\dagger$, where

$$\tilde{A}_{\alpha\beta} = \int \frac{d^2\gamma}{\pi\sqrt{\tau}} \exp\left(-\frac{|\alpha|^2 + |\beta|^2 + |\gamma|^2}{2} + \sqrt{1-\eta}\,\alpha\gamma + \frac{1}{2\tau}|\sqrt{\tau - 1}\beta + \sqrt{\eta}\,\gamma|^2\right)\left|\sqrt{\frac{\tau-1}{\tau}}\beta + \sqrt{\frac{\eta}{\tau}}\gamma\right\rangle\langle\gamma|, \tag{2}$$

$$\eta = \frac{\kappa}{\tau}, \qquad \tau = \begin{cases} 1 + a, & 0 < \kappa < 1, \\ \kappa + a, & \kappa > 1. \end{cases} \tag{3}$$

The parameter η defines the attenuation factor of the quantum-limited attenuation $\Phi_{\mathrm{QL}\eta}$ and τ defines the power gain of the quantum-limited amplifier $\Phi_{\mathrm{QL}\tau}$; the concatenation of these channels results in the channel $\Phi(\kappa,\mu)$ given by (1), i.e., $\Phi_{\mathrm{QL}\tau} \circ \Phi_{\mathrm{QL}\eta} = \Phi(\kappa,\mu)$ [32].

Entanglement annihilation. The phenomenon of complete entanglement degradation is known as entanglement annihilation [33] and was analyzed for discrete variable systems in the papers [34–36]. The density operator ϱ acting on $\mathcal{H}^{\otimes N}$ is called mode entangled (separable) if it cannot (can) be represented as a convex sum $\sum_i p_i \varrho_i^{(1)} \otimes \cdots \otimes \varrho_i^{(N)}$, where $p_i \geqslant 0$, $\varrho_i^{(j)} \geqslant 0$. The channel $\Upsilon : \mathcal{T}(\mathcal{H}^{\otimes N}) \mapsto \mathcal{T}(\mathcal{H}^{\otimes N})$ annihilates entanglement of some input state ϱ, if the output state $\Upsilon[\varrho]$ is separable. If this property holds true for all ϱ from some domain $\mathcal{S}$, then Υ is called entanglement annihilating on $\mathcal{S}$.

The channel $\Phi^{\otimes N}$ is of a great interest in quantum communication: The parts of a composite quantum state (encoded in time bins or radiation modes) are sent through the same communication line modeled by the channel Φ [37]. If $\Upsilon = \Phi^{\otimes N}$ is entanglement annihilating on $\mathcal{S}$, then Φ is called N-locally entanglement annihilating (N-LEA$_{\mathcal{S}}$). We will refer to the channel Φ as ∞-LEA if $\Phi \in N$-LEA for all $N = 2,3,\ldots$. If Φ is entanglement breaking, then it is a measure and prepare operation [22,23] that definitely disentangles the part it acts on from all other parts of the multipartite system. The inclusion diagram follows [33]:

$$\text{EB} \subset \infty\text{-LEA}_{\mathcal{S}} \subset \cdots \subset 3\text{-LEA}_{\mathcal{S}} \subset 2\text{-LEA}_{\mathcal{S}}. \tag{4}$$

Further, this relation will be specified for the channels from class $\mathcal{C}$.

Gaussian input states find applications in many quantum information protocols [20]. The characteristic function of a Gaussian state $\varrho \in \mathcal{G}$ reads $\varphi(\mathbf{z}) = \exp(-\frac{1}{2}\mathbf{z}^\top \mathbf{V}\mathbf{z} + i\mathbf{l}^\top\mathbf{z})$, where $\mathbf{V} \geqslant \frac{i}{2}\mathbf{\Delta}$ is the covariance matrix and $\mathbf{l}$ is the vector of average values of q_i, p_i, $i = 1,\ldots,N$. The vector $\mathbf{l}$ is irrelevant for entanglement properties, so we let $\mathbf{l} = 0$.

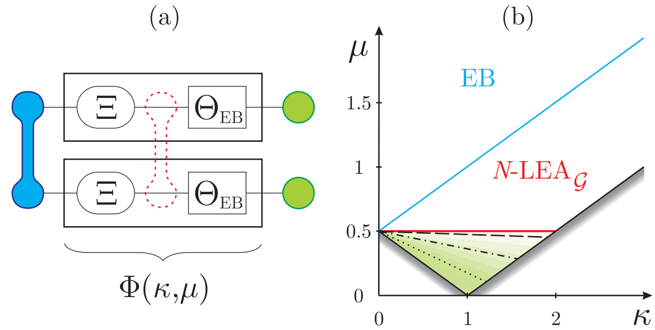

FIG. 1. (Color online) (a) Decomposition of the one-mode channel $\Phi(\kappa,\mu)$ into the scaling map Ξ with $\kappa_\Xi \gg 1$ and $\mu_\Xi = 0$ and the entanglement-breaking map $\Theta_{\rm EB}$. Action of the map $\Xi^{\otimes N}$ on any Gaussian input results in a valid Gaussian state (red dotted line), whose entanglement is then annihilated by the entanglement-breaking maps. (b) Map $\Phi(\kappa,\mu)$ is a valid channel above gray shading. Channel $\Phi(\kappa,\mu)^{\otimes N}$ is entanglement annihilating for points (κ,μ) above the horizontal red solid line for every $N = 2,3,\ldots$. Entanglement of the two-mode squeezed state with energy $\mathcal{E}$ (measured in photons) is preserved by the channel $\Phi(\kappa,\mu)^{\otimes 2}$ below the lines: dotted if $\mathcal{E} = 0.1$, dash-dotted if $\mathcal{E} = 1$, and dashed if $\mathcal{E} = 10$.

Proposition 1. The channel $\Phi(\kappa,\mu) \in \mathcal{C}$ is N-LEA$_{\mathcal{G}}$ for all $N = 2,3,\ldots$ if and only if the total noise level $\mu \geqslant \frac{1}{2}$.

Proof. Let us verify when the channel $\Phi(\kappa,\mu)$ can be represented as a concatenation $\Theta_{\rm EB} \circ \Xi$ of the scaling map Ξ given by formula (1) with $\kappa_\Xi \gg 1$, $\mathbf{M} = \mathbf{0}$, and the entanglement-breaking attenuator $\Theta_{\rm EB}$ with $\kappa_\Theta \ll 1$ [Fig. 1(a)]. The scaling map Ξ is not positive in general but it transforms any Gaussian state into another Gaussian state because the transformed covariance matrix satisfies the condition $\mathbf{V}_{\rm out} = \kappa_\Xi \mathbf{V}_{\rm in} \geqslant \frac{i}{2}\mathbf{\Delta}$. The relation $\Phi(\kappa,\mu) = \Theta_{\rm EB} \circ \Xi$ holds if $\kappa = \kappa_\Xi \kappa_\Theta$ and $\mu = \mu_\Theta \geqslant \frac{1}{2}(1 + \kappa_\Theta)$, the latter inequality being a necessary and sufficient condition for the entanglement-breaking property of Θ [21]. In the limit $\kappa_\Theta \to 0$ and $\kappa_\Xi \to \infty$ with keeping $\kappa_\Xi\kappa_\Theta = \kappa = \text{const}$, we obtain $\mu \geqslant \frac{1}{2}$. Thus, the channel $\Phi(\kappa,\mu)$ is a concatenation of the scaling map (positive on Gaussian inputs) and entanglement-breaking map if $\mu \geqslant \frac{1}{2}$. Those entanglement-breaking maps make the output state separable, which proves sufficiency.

If $\mu < \frac{1}{2}$, then $\Phi(\kappa,\mu)^{\otimes 2}$ preserves entanglement of the two-mode squeezed vacuum state $|\psi\rangle = \sqrt{1 - \tanh^2 r}\sum_{n=0}^{\infty}(\tanh r)^n |n\rangle \otimes |n\rangle$ when $r \to \infty$. This can be checked, e.g., by Simon's criterion [38] applied to the covariance matrix

$$\mathbf{V} = \frac{1}{2}\begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -\sinh 2r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -\sinh 2r & 0 & \cosh 2r \end{pmatrix}. \tag{5}$$

Thus, $\Phi(\kappa,\mu)$ is not 2-LEA$_{\mathcal{G}}$ and, consequently, not N-LEA$_{\mathcal{G}}$. This proves the necessity. ■

We emphasize that Proposition 1 is valid for any $N = 2,3,\ldots$, which complements the previously known result for $N = 2$ [24] and is in agreement with the attenuation experiment of Ref. [14].

$\Phi(\kappa,\mu)$ is entanglement breaking if and only if $a \geqslant \min(\kappa,1)$ [21]. This means that $\mathrm{EB}^{\mathcal{C}} \neq \infty\text{-LEA}_{\mathcal{G}}^{\mathcal{C}}$ and the inclusion diagram (4) takes the following form for $\mathcal{C}$ channels and Gaussian inputs:

$$\mathrm{EB}^{\mathcal{C}} \subsetneq \infty\text{-LEA}_{\mathcal{G}}^{\mathcal{C}} = \cdots = 3\text{-LEA}_{\mathcal{G}}^{\mathcal{C}} = 2\text{-LEA}_{\mathcal{G}}^{\mathcal{C}}. \tag{6}$$

Gaussian state entanglement cannot survive the amplification with $\kappa > 2 \approx 3$ dB. Even if the total noise $\mu < \frac{1}{2}$, the Gaussian state should have enough energy to protect its entanglement from annihilation. The two-mode squeezed vacuum has energy $\mathcal{E} = \cosh 2r - 1$ photons and its entanglement is annihilated by $\Phi(\kappa,\mu)^{\otimes 2}$ unless $\mu < \frac{1}{2}[1 - \kappa + \kappa(\sqrt{\mathcal{E}(2+\mathcal{E})} - \mathcal{E})]$ [see Fig. 1(b)].

The result of Proposition 1 can be extended to the case of nonhomogeneous local channels.

Corollary 1. The channel $\Upsilon = \Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2) \otimes \cdots \otimes \Phi(\kappa_N,\mu_N)$ annihilates entanglement of all Gaussian N-mode states if $\mu_i \geqslant \frac{1}{2}$, $i = 1,\ldots,N$.

Proof. Similarly to the proof of Proposition 1, concatenation of the homogeneous scaling map $\Xi^{\otimes N}$ and a map $\bigotimes_{i=1}^{N} \Theta_{\mathrm{EB}i}$ composed of the individual entanglement-breaking attenuators with $a_{\Theta i} = \kappa_{\Theta i} = \kappa_i/\kappa_{\Xi}$ leads to the map $\Phi(\kappa_1,\frac{1}{2}) \otimes \Phi(\kappa_2,\frac{1}{2}) \otimes \cdots \otimes \Phi(\kappa_N,\frac{1}{2})$ in the limit $\kappa_{\Xi} \to \infty$. The map Υ may be readily obtained by adding classical noise $(\mu_i - \frac{1}{2})$ into the ith mode, $i = 1,\ldots,N$. ■

Corollary 2. Suppose the channel $\Upsilon = \Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2) \otimes \cdots \otimes \Phi(\kappa_N,\mu_N)$ such that $\min_{i=1,\ldots,N-1} \frac{2\mu_i - 1}{\kappa_i} = s \geqslant 0$. The channel Υ annihilates entanglement of all Gaussian N-mode states if $\mu_N \geqslant \frac{1}{2}(1 - s\kappa_N)$.

Proof. If $s \geqslant 1$, then all the channels $\Phi(\kappa_1,\mu_1)$, $\Phi(\kappa_2,\mu_2),\ldots,\Phi(\kappa_{N-1},\mu_{N-1})$ are entanglement breaking and the statement becomes trivial. If $0 < s < 1$, let us represent Υ as a concatenation of the homogeneous scaling map $\Xi^{\otimes N}$ and a map $(\bigotimes_{i=1}^{N-1} \Theta_{\mathrm{EB}i}) \otimes \Theta_{\mathrm{QL}N}$ composed of $N-1$ entanglement-breaking attenuators and a quantum-limited attenuation of the Nth mode [see Fig. 2(a)]. In fact, put $\kappa_{\Xi} = s^{-1} > 1$ and $\kappa_{\Theta i} = s\kappa_i$, $i = 1,\ldots,N-1$, then the relation $a_{\Theta i} = \mu_i - \frac{1}{2}(1 - s\kappa_i) \geqslant \kappa_{\Theta i}$ makes Θ_i entanglement breaking for $i = 1,\ldots,N-1$, which guarantees separability of the output state for all N-mode Gaussian inputs. The application of the quantum-limited attenuator $\Theta_{\mathrm{QL}N}$ with $\kappa_{\Theta N} = s\kappa_N$ results in the noise $\mu_N = \frac{1}{2}(1 - \kappa_{\Theta N}) = \frac{1}{2}(1 - s\kappa_N)$. Greater noises in Nth mode can be realized by adding classical noise. The case $s = 0$ corresponds to the closure of the set of separable states. ■

Corollary 3. The channel $\Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2)$ annihilates entanglement of all two-mode Gaussian states if and only if $\kappa_1\mu_2 + \kappa_2\mu_1 \geqslant (\kappa_1 + \kappa_2)/2$.

Proof. The sufficiency follows from Corollary 2 applied to the case $N = 2$. The necessity follows from Simon's criterion [38] applied to the two-mode squeezed vacuum with the covariance matrix (5), where $r \to \infty$. ■

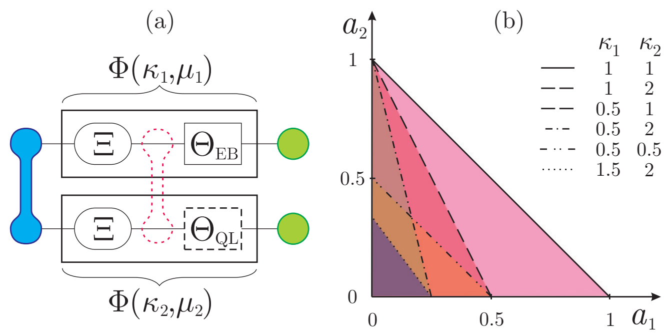

FIG. 2. (Color online) (a) Decomposition of the local channel $\Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2)$. Scaling map $\Xi^{\otimes 2}$ transforms any Gaussian input into a valid quantum state (red dotted line). Channel Θ_{EB} is entanglement breaking, channel Θ_{QL} is quantum limited. (b) Regions of additional noises a_1, a_2 in the channel $\Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2)$, where the entanglement of high energy Gaussian states is preserved.

Corollary 3 is followed by the observation, if $\Phi(\kappa_1,\mu_1)$ is a quantum-limited attenuator, then $\Phi(\kappa_2,\mu_2)$ must be an entanglement-breaking channel to annihilate entanglement of all Gaussian states. This fact is in agreement with the experimental entanglement detection for a two-mode squeezed state, one-half of which is subjected to a near quantum-limited amplification [i.e., $\Phi(\kappa_1,\mu_1) = \mathrm{Id}$ and $\Phi(\kappa_2,\mu_2)$ introduces a low noise] [39]. However, if $\Phi(\kappa_1,\mu_1)$ is a quantum-limited amplifier, that property does not hold anymore. Figure 2(b) illustrates additional noises a_1 and a_2 in the channel $\Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2)$ that can be tolerated by Gaussian entangled states.

Non-Gaussian input states. Further, we demonstrate that the entanglement of some low-energy non-Gaussian states can be more robust to attenuation and amplification than that of Gaussian ones.

Proposition 2. The channel $\Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2)$ is not entanglement annihilating under the following conditions:

(i) $\kappa_1 < 1$, $\kappa_2 < 1$, $a_1 < \frac{\kappa_1(1+a_2)}{2(1+a_2)-\kappa_2}$, and $a_2 < \frac{\kappa_2(1+a_1)}{2(1+a_1)-\kappa_1}$;

(ii) $\kappa_1 < 1$, $\kappa_2 \geqslant 1$, $a_1 < \frac{\kappa_1(\kappa_2+a_2)}{\kappa_2+2a_2}$, and $a_2 < 1 - \kappa_2 \frac{1+a_1-\kappa_1}{2(1+a_1)-\kappa_1}$;

(iii) $\kappa_1 \geqslant 1$, $\kappa_2 \geqslant 1$, $a_1 < 1 - \frac{\kappa_1 a_2}{\kappa_2+2a_2}$, and $a_2 < 1 - \frac{\kappa_2 a_1}{\kappa_1+2a_1}$.

Proof. It suffices to find a two-mode state $|\psi\rangle$ such that $\Phi(\kappa_1,\mu_1) \otimes \Phi(\kappa_2,\mu_2)[|\psi\rangle\langle\psi|]$ is entangled for parameters $\kappa_{1,2}$ and $a_{1,2}$ satisfying (i)–(iii). Let $|\psi\rangle = [2(1 - e^{-|\gamma|^2})]^{-1/2}(|\gamma\rangle|0\rangle - |0\rangle|\gamma\rangle)$, its energy $\mathcal{E} = (1 - e^{-|\gamma|^2})^{-1}|\gamma|^2 \to 1$ when $|\gamma| \to 0$. From a series of powerful entanglement detection techniques [40–46] we choose [46] and modify it to obtain the following witness:

$$W_{\lambda} = \int \frac{d^2\alpha}{\pi} \frac{d^2\beta}{\pi} e^{\lambda(|\alpha|^2+|\beta|^2)} |\alpha\rangle\langle\beta| \otimes |\beta\rangle\langle\alpha|. \tag{7}$$

For all pure factorized states $|\xi\rangle \otimes |\upsilon\rangle$ we have $\mathrm{tr}[W_{\lambda}|\xi\rangle\langle\xi| \otimes |\upsilon\rangle\langle\upsilon|] = |\int \frac{d^2\alpha}{\pi} e^{\lambda|\alpha|^2} \langle\xi|\alpha\rangle\langle\alpha|\upsilon\rangle|^2 \geqslant 0$, whereas $\mathrm{tr}[W_{\lambda}\varrho] < 0$ indicates entanglement of ϱ. If $\lambda > 0$, the operator W_{λ} becomes unbounded but its average with the output state can still be finite and negative (indication of entanglement). A straightforward integration with the Kraus

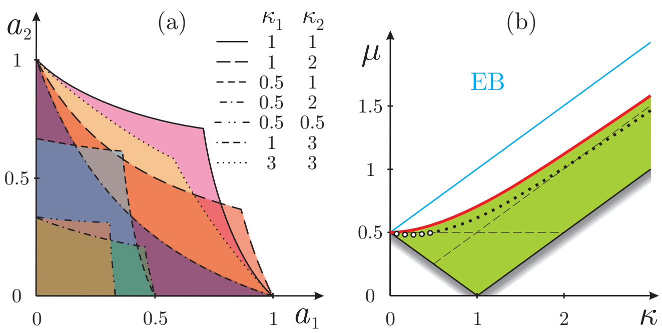

FIG. 3. (Color online) (a) Regions of additional noises a_1,a_2 in the channel $\Phi(\kappa_1,\mu_1)\otimes\Phi(\kappa_2,\mu_2)$, where the entanglement of low-energy state $|\psi\rangle \propto |\gamma\rangle|0\rangle - |0\rangle|\gamma\rangle$ is preserved. (b) Channel $\Phi(\kappa,\mu)^{\otimes 2}$ does not annihilate entanglement of the state $|\psi\rangle \propto |\gamma\rangle|0\rangle - |0\rangle|\gamma\rangle$ for points (κ,μ) below the red solid line (dashed lines are asymptotes). Dotted line is the result of Ref. [24] obtained for the state $\frac{1}{\sqrt{2}}(|n\rangle|0\rangle + |0\rangle|n\rangle)$, $n=5$, and circles represent the points where Gaussian states outperform that result.

operators (2) and the witness operator (7) yields

$$\begin{aligned}
&\mathrm{tr}\,\{W_\lambda\ \Phi(\kappa_1,\mu_1)\otimes\Phi(\kappa_2,\mu_2)[|\psi\rangle\langle\psi|]\}\\
&=\left\{\left(1-e^{-|\gamma|^2}\right)[\tau_1\tau_2(1-\lambda)^2-(\tau_1-1)(\tau_2-1)]\right\}^{-1}\\
&\quad\times\left\{\exp\left[-\frac{\eta_1\tau_1\,[1-\lambda(2-\lambda)\tau_2]}{\tau_1\tau_2(1-\lambda)^2-(\tau_1-1)(\tau_2-1)}\,|\gamma|^2\right]\right.\\
&\quad+\exp\left[-\frac{\eta_2\tau_2\,[1-\lambda(2-\lambda)\tau_1]}{\tau_1\tau_2(1-\lambda)^2-(\tau_1-1)(\tau_2-1)}\,|\gamma|^2\right]\\
&\quad\left.-2\exp\left[-\left(1-\frac{\sqrt{\eta_1\tau_1\eta_2\tau_2}(1-\lambda)}{\tau_1\tau_2(1-\lambda)^2-(\tau_1-1)(\tau_2-1)}\right)|\gamma|^2\right]\right\},
\end{aligned}$$

which is justified for $\lambda<\lambda_0 = 1-\sqrt{(\tau_1-1)(\tau_2-1)/\tau_1\tau_2}$. The average value obtained takes negative values in the widest region of parameters $\eta_{1,2},\tau_{1,2}$ if $|\gamma|\to 0$. In this case, the output state is entangled if there exists a solution λ_* of the inequality $2[\tau_1\tau_2(1-\lambda)^2-(\tau_1-1)(\tau_2-1)-\sqrt{\eta_1\tau_1\eta_2\tau_2}\,(1-\lambda)] < \eta_1\tau_1+\eta_2\tau_2-\lambda(2-\lambda)\tau_1\tau_2(\eta_1+\eta_2)$ such that $\lambda_*<\lambda_0$. The reader will have no difficulty in showing that such a solution λ_* exists if $2-\eta_1-\tau_2(2-\eta_1-\eta_2)>0$ and $2-\eta_2-\tau_1(2-\eta_1-\eta_2)>0$. Substituting expressions (3) for $\eta_{1,2}$ and $\tau_{1,2}$ yields formulas (i)–(iii). ■

The result of Proposition 2 is depicted in Fig. 3(a). If $\Phi(\kappa_1,\mu_1)$ is a quantum-limited attenuator, then the entanglement of the non-Gaussian state $|\psi\rangle \propto |\gamma\rangle|0\rangle - |0\rangle|\gamma\rangle$ is preserved in a narrower range of parameters κ_2,a_2 than for the Gaussian state with large squeezing. Thus, Gaussian state entanglement is favorable for transmission through lossy channel with high asymmetry in noises. On the contrary, if the losses are quite similar, then the non-Gaussian state $|\psi\rangle$ is at an advantage. As far as amplifiers are concerned, non-Gaussian states undoubtedly outperform Gaussian ones. Moreover, if $\Phi(\kappa_1,\mu_1)$ and $\Phi(\kappa_2,\mu_2)$ are amplifiers with $|\kappa_1-\kappa_2|\leqslant 2$ and one of them is quantum limited, then the other has to be entanglement breaking to destroy entanglement of the state $|\psi\rangle$.

Corollary 4. The channel $\Phi(\kappa,\mu)\in\mathcal{C}$ is not N-LEA for any $N=2,3,\dots$ if the total noise level satisfies $\mu<\frac{1}{2}\sqrt{\kappa^2+1}$.

Proof. Applying the result of Proposition 2 for $\kappa_1=\kappa_2=\kappa$ and $a_1=a_2=a$ and solving the corresponding inequality with respect to a gives $a<\frac{1}{2}(\sqrt{\kappa^2+1}-|\kappa-1|)$. The relation $\mu=|\kappa-1|/2+a$ leads to $\mu<\frac{1}{2}\sqrt{\kappa^2+1}$, which indicates when $\Phi(\kappa,\mu)$ is not 2-LEA and, consequently, not N-LEA. ■

If $\mu<\frac{1}{2}\sqrt{\kappa^2+1}$, the state $|\psi\rangle\propto|\gamma\rangle|0\rangle-|0\rangle|\gamma\rangle$ remains entangled irrespective to the value of $\gamma\neq 0$, i.e., for all energies $\mathcal{E}\in(1,+\infty)$. This fact follows from the expression $\mathrm{tr}\{W_\lambda\Phi(\kappa,\mu)^{\otimes 2}[|\psi\rangle\langle\psi|]\}$ containing the difference of two exponents only. Note that Corollary 4 could be proven by considering input states $|\psi_n\rangle = \frac{1}{\sqrt{2}}(|n\rangle|0\rangle - |0\rangle|n\rangle)$ and the witness $\widetilde{W}_\lambda = \sum_{i,j=0}^{\infty}\lambda^{i+j}|i\rangle\langle j|\otimes|j\rangle\langle i|$ when λ tends to $\tau/(\tau-1)$. It is shown in Ref. [24] that the state $\frac{1}{\sqrt{2}}(|n\rangle|0\rangle+|0\rangle|n\rangle)$ outperforms robustness of Gaussian states with respect to homogeneous attenuation and amplification $\Phi(\kappa,\mu)\otimes\Phi(\kappa,\mu)$ for some region of parameters κ and μ, however, there was no evidence that these states outperform Gaussian states for $\kappa<0.43$ even if $n\to\infty$ [see Fig. 3(b)]. We have just used a stricter entanglement detection method and proved the existence of non-Gaussian states with little energy that outperform Gaussian states for all values of $\kappa>0$.

To conclude, we have analyzed the noises that accompany attenuation and amplification and impose fundamental limitations on the performance of entanglement-assisted devices. The important practical conclusion is that Gaussian states of high energy are quite robust to lossy channels with high asymmetry in the noises, whereas non-Gaussian states are more robust in the case of similar attenuations. Gaussian state entanglement cannot withstand amplification with power gain 2 (≈3 dB), whereas non-Gaussian states of small energy can preserve the entanglement for arbitrarily large power gains if the introduced noise is sufficiently small.

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