PHYSICAL REVIEW A **88**, 032316 (2013)

Bipartite entanglement-annihilating maps: Necessary and sufficient conditions

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(Received 27 June 2013; published 16 September 2013)

We fully characterize bipartite entanglement-annihilating (EA) channels that destroy entanglement of any state shared by subsystems and, thus, should be avoided in any entanglement-enabled experiment. Our approach relies on extending the problem to EA positive maps, the cone of which remains invariant under concatenation with partially positive maps. Due to this invariancy, positive EA maps adopt a well characterization and their intersection with completely positive trace-preserving maps results in the set of EA channels. In addition to a general description, we also provide sufficient operational criteria revealing EA channels. They have a clear physical meaning since the processes involved contain stages of classical information transfer for subsystems. We demonstrate the applicability of derived criteria for local and global depolarizing noises, and specify corresponding noise levels beyond which any initial state becomes disentangled after passing the channel. The robustness of some entangled states is discussed.

DOI: 10.1103/PhysRevA.88.032316 PACS number(s): 03.67.Mn, 03.65.Ud, 03.65.Yz

I. INTRODUCTION

Entanglement is a quantum phenomenon with numerous potential quantum-information applications [1,2]. However, the practical realization of such applications is typically faced with various sources of noise, which affect the performance and design of entanglement-enabled technologies. It is of practical interest to understand how entanglement is influenced by any such experimental imperfections. This problem has stimulated considerable research effort, which has introduced the concepts of entanglement sudden death and revival [3–5], entanglement robustness [6–9], and entanglement-breaking [10–15] and entanglement-annihilating [16] processes.

One of the main lessons of entanglement theory [1,2] is that the presence of entanglement is in general extremely difficult to verify. Therefore, some limitations are typically imposed on both initial states and noise models in most of the studies on the dynamics of entanglement [6–9,17–21]. No doubt such analysis is in many cases of great practical relevance; however, the conclusions do not necessarily capture the universal behavior of entanglement. In fact, it is questionable whether some universal entanglement dynamics features do exist. For example, are there processes capable of creating (not decreasing) entanglement regardless of the initial state? Or, on the other hand, are there processes that destroy any entanglement? Is there some equation capturing the dynamics of entanglement?

The first of these questions resulted in considering various aspects of entangling and disentangling capabilities of quantum processes [22,23]. The other two questions were mostly studied for one-side noisy processes $\Phi \otimes \mathrm{Id}$, where the noise Φ acts only on one of the subsystems while the rest of subsystems evolve in a noiseless manner (Id). For such processes, the so-called evolution equation for entanglement has been derived [24–27]. It says that the change of the entanglement due to one-sided noise is quantitatively bounded by its action on the maximally entangled state. Then, all the noises Φ that disentangle the maximally entangled state will also disentangle a given subsystem (under the noise action) from any other subsystem (noiseless) regardless of the initial state of the global system, *ipso facto* forming a class of entanglement-breaking (EB) processes [10–15].

In practice, however, the noise is rarely one sided. This is the reason why the notion of entanglement-annihilating (EA) processes was introduced in Ref. [16]. Formally, the noise (not necessarily one sided or local) is EA if its action disentangles all the subsystems forming the composite system. EA processes acting on a composite system do not necessarily disentangle the system from its surrounding, they only destroy entanglement between subsystems accessible in experiment. For instance, it could happen that the joint action of local noises on individual subsystems constitutes an EA process even if none of the local noises is EB [16,28].

Although EA processes impose fundamental limitations on the performance of entanglement-enabled experiments, they are not explored much. In this paper, we provide explicit characterization of general bipartite EA channels and derive sufficient criteria for their detection. We employ these criteria to specify the maximal noise levels above which no entanglement can be preserved.

II. PRELIMINARIES

The states of a quantum system associated with a d-dimensional Hilbert space $\mathcal{H}_d$ are identified with density operators (positive and unit trace) and form a convex set $\mathcal{S}(\mathcal{H}_d)$. Quantum processes are modeled as channels, i.e., completely positive trace-preserving (CPT) linear maps $\Phi : \mathcal{T}(\mathcal{H}_{\mathrm{in}}) \to \mathcal{T}(\mathcal{H}_{\mathrm{out}})$ on trace-class operators $\mathcal{T}(\mathcal{H}_{\mathrm{in}})$. We say a state of the system S composed of subsystems $\mathscr{A},\mathscr{B},\ldots$ is separable if $\varrho = \sum_j p_j \varrho_j^{\mathscr{A}} \otimes \varrho_j^{\mathscr{B}} \otimes \cdots$, with $\{p_j\}$ being a probability distribution. Otherwise it is called entangled. We say a channel $\Phi^S \equiv \Phi^{\mathscr{A}\mathscr{B}\cdots}$ is EA if $\Phi^S[\varrho^S]$ is separable (with respect to partition $\mathscr{A}|\mathscr{B}|\mathscr{C}|\cdots$) for all input states. Φ^S is EB

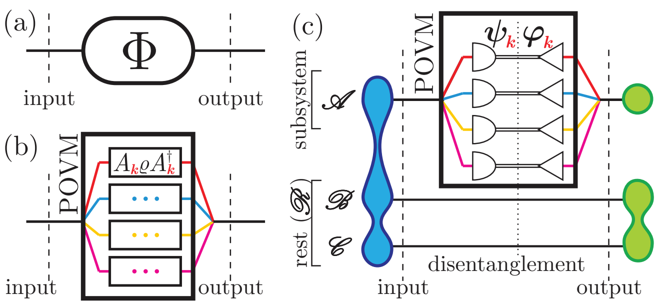

FIG. 1. (Color online) (a) Quantum channel as an input-output device. (b) Physical interpretation of the diagonal-sum representation. (c) Structure of EB channels.

if $(\Phi^S \otimes \mathrm{Id}^E)[\omega^{SE}]$ is separable with respect to partition $S|E$ for all states ω^{SE} of system S and an arbitrary environment E.

Quantum channel Φ can be written in a (nonunique) sum diagonal representation $\Phi[\varrho] = \sum_k A_k \varrho A_k^\dagger$, where Kraus operators A_k satisfy the normalization $\sum_k A_k^\dagger A_k = I_{\rm in}$ (identity operator). Because of that, the channel Φ can be seen as a sum of conditional outputs of a measurement, in which the outcomes k are occurring with probability $p_k = \mathrm{tr}[\varrho A_k^\dagger A_k]$ while the state is undergoing the conditional (post-selected) transformation $\varrho \mapsto p_k^{-1} A_k \varrho A_k^\dagger$ [Figs. 1(a) and 1(b)].

Any linear map $\Phi : \mathcal{T}(\mathcal{H}_{\rm in}^S) \rightarrow \mathcal{T}(\mathcal{H}_{\rm out}^S)$ can be described by a so-called Choi matrix [29,30]

$$\Omega_\Phi^{SS'} := (\Phi^S \otimes \mathrm{Id}^{S'})[|\Psi_+^{SS'}\rangle\langle\Psi_+^{SS'}|], \qquad (1)$$

where $|\Psi_+^{SS'}\rangle = (d^S)^{-1/2} \sum_{i=1}^{d^S} |i \otimes i'\rangle$ is a maximally entangled state shared by system S and its clone S', $\langle i|j\rangle = \langle i'|j'\rangle = \delta_{ij}$. It is well known [29,30] that the map Φ^S is completely positive (CP) if and only if $\Omega_\Phi^{SS'} \geqslant 0$, i.e., $\Omega_\Phi^{SS'} \in \mathcal{S}(\mathcal{H}_{\rm out}^S \otimes \mathcal{H}_{\rm in}^{S'})$. The matrix in Eq. (1) defines the map:

$$\Phi[X] = d^S\, \mathrm{tr}_{S'} \left[\Omega_\Phi^{SS'} \left(I_{\rm out}^S \otimes X^{\rm T} \right) \right], \qquad (2)$$

where $X^{\rm T} = \sum_{i,j} \langle j|X|i\rangle |i'\rangle\langle j'| \in \mathcal{T}(\mathcal{H}_{\rm in}^{S'})$ and $\mathrm{tr}_{S'}$ denotes the partial trace operation.

A general positive linear map Λ that transforms positive operators into positive ones gives rise to the Choi matrix which can be nonpositive in general. We say an operator $\xi \in \mathcal{T}(\mathcal{H}^{\mathscr{X}} \otimes \mathcal{H}^{\mathscr{Y}})$ is block-positive (denoted as $\xi_{\rm BP}^{\mathscr{X}|\mathscr{Y}}$) if $\langle x \otimes y|\xi|x \otimes y\rangle \geqslant 0$ for all $|x\rangle \in \mathcal{H}^{\mathscr{X}}$, $|y\rangle \in \mathcal{H}^{\mathscr{Y}}$. Then, $\{\Lambda^S$ is positive$\} \Leftrightarrow \{\Omega_\Lambda^{SS'}$ is block-positive$\}$ [30].

A. EB channels

Suppose subsystem $\mathscr{A}$ is subjected to a quantum channel $\Phi^{\mathscr{A}}$ whose Kraus operators are rank-1 projectors, i.e., $A_k \propto |\varphi_k\rangle\langle\psi_k|$ with $|\psi_k\rangle \in \mathcal{H}_{\rm in}^{\mathscr{A}}$ and $|\varphi_k\rangle \in \mathcal{H}_{\rm out}^{\mathscr{A}}$. In this case, we deal with a measure-and-prepare procedure, i.e., the channel of Holevo form [10,14,15]. Such a channel $\Phi^{\mathscr{A}}$ is EB and disentangles $\mathscr{A}$ from all the rest of the subsystems $\mathscr{B},\mathscr{C},\ldots(=\mathscr{R})$ because it contains a stage of classical information transfer [depicted by double line in Fig. 1(c)]. Surprisingly, the converse statement is also true, i.e., $\{\Phi$ is EB$\} \Leftrightarrow \{$there exists a diagonal sum representation of Φ with rank-1 Kraus operators$\}$. Alternative characterization of EB channels exploits the property of the Choi matrix: $\{\Phi^{\mathscr{A}}$ is EB$\} \Leftrightarrow \{\Omega_\Phi^{\mathscr{A}\mathscr{A}'} \in \mathcal{S}(\mathcal{H}_{\rm out}^{\mathscr{A}} \otimes \mathcal{H}_{\rm in}^{\mathscr{A}'})$ is separable with respect to partition $\mathscr{A}|\mathscr{A}'\}$ [13–15].

As far as EB channels $\Phi_{\rm EB}^{\mathscr{A}\mathscr{B}}$ acting on a composite system $\mathscr{A}\mathscr{B}$ are concerned, the Choi state $\Omega_\Phi^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ is to be separable with respect to partition $\mathscr{A}\mathscr{B}|\mathscr{A}'\mathscr{B}'$ but can still be entangled with respect to partitions $\mathscr{A}|\mathscr{B}\mathscr{A}'\mathscr{B}'$ and $\mathscr{B}|\mathscr{A}\mathscr{A}'\mathscr{B}'$ (for instance, if $\Omega_\Phi^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ is the Smolin state of four qubits [31]). In this case, the channel disentangles $(\mathscr{A}\mathscr{B})$ from any other systems $\mathscr{C},\mathscr{D},\ldots$, but the entanglement between $\mathscr{A}$ and $\mathscr{B}$ can be preserved. However, if the channel $\Phi^{\mathscr{A}\mathscr{B}}$ has a local structure $\Phi_{\rm local}^{\mathscr{A}\mathscr{B}} = \Phi_1^{\mathscr{A}} \otimes \Phi_2^{\mathscr{B}}$, then $\{\Phi_{\rm local}^{\mathscr{A}\mathscr{B}}$ is EB$\} \Leftrightarrow \{\Phi_1^{\mathscr{A}}$ is EB and $\Phi_2^{\mathscr{B}}$ is EB$\}$, which follows immediately from the particular form of the maximally entangled state $|\Psi_+^{\mathscr{A}\mathscr{B}|\mathscr{A}'\mathscr{B}'}\rangle := |\Psi_+^{\mathscr{A}\mathscr{A}'}\rangle \otimes |\Psi_+^{\mathscr{B}\mathscr{B}'}\rangle$.

B. EA channels

In contrast to EB channels, EA channels by definition act on composite systems. For bipartite systems one can use the Horodecki criterion [32] to formulate a necessary and sufficient condition for the map to be EA.

Lemma 1. Suppose $\Phi^{\mathscr{A}\mathscr{B}} : \mathcal{T}(\mathcal{H}_{\rm in}^{\mathscr{A}\mathscr{B}}) \rightarrow \mathcal{T}(\mathcal{H}_{\rm out}^{\mathscr{A}\mathscr{B}})$ is a channel. Then $\{\Phi^{\mathscr{A}\mathscr{B}}$ is EA$\} \Leftrightarrow \{(\mathrm{Id}^{\mathscr{A}} \otimes \Lambda^{\mathscr{B}}) \circ \Phi^{\mathscr{A}\mathscr{B}}$ is a positive map for any positive map $\Lambda^{\mathscr{B}} : \mathcal{T}(\mathcal{H}_{\rm out}^{\mathscr{B}}) \rightarrow \mathcal{T}(\mathcal{H}_{\rm out}^{\mathscr{A}})\}$.

Unfortunately, Lemma 1 is not quite operational, which makes it difficult to apply. However, in the case of two qubits ($d_{\rm out}^{\mathscr{A},\mathscr{B}} = 2$), Lemma 1 turns out to be rather fruitful because without loss of generality the positive map $\Lambda^{\mathscr{B}}$ can be chosen to be either a transposition [33] or a reduction map [34]. This fact was exploited in characterization of local two-qubit EA channels in Ref. [28]. Particularly interesting in the case of bipartite local channels $\Phi^{\mathscr{A}\mathscr{B}}$ with $d^{\mathscr{A}} = d^{\mathscr{B}}$ are those that form $\Phi \otimes \Phi$ describing the physical situations when both parties experience the same noise. Following [16], if $\Phi \otimes \Phi$ is EA, we will refer to a "generating" channel Φ as a 2-locally EA channel (2LEA).

C. Structure of linear bipartite maps

To investigate the structure of EA channels it turns out to be instructive to introduce the concept of positive entanglement-annihilating (PEA) linear maps. In particular, a map $\Phi^{\mathscr{A}\mathscr{B}}$ is PEA if it is positive and $\Phi^{\mathscr{A}\mathscr{B}}[\varrho]$ belongs to a cone of states separable with respect to partition $\mathscr{A}|\mathscr{B}$ for all $\varrho \in \mathcal{S}(\mathcal{H}_{\rm in}^{\mathscr{A}\mathscr{B}})$. The set of PEA maps is convex and its intersection with CPT maps gives exactly all EA channels, i.e., EA = PEA ∩ CPT.

Consider an example of 2-locally unital qubit linear trace-preserving maps, i.e., maps of the form $\Upsilon \otimes \Upsilon$ with $\Upsilon[I] = I$. Up to a unitary preprocessing and postprocessing, the map Υ can be written [35] in the form $\Upsilon[X] = \frac{1}{2}\sum_{j=0}^3 \lambda_j \mathrm{tr}[\sigma_j X]\sigma_j$, where $\{\lambda_j\}$ are real numbers, $\sigma_0 = I$, and $\{\sigma_i\}_{j=1}^3$ is a conventional set of Pauli operators (in an appropriate basis). Due to the trace-preserving condition, $\lambda_0 = 1$. The remaining three parameters $\{\lambda_j\}_{j=1}^3$ are scaling coefficients of Bloch ball axes. The map Υ is given by a point in the Cartesian coordinate system $(\lambda_1,\lambda_2,\lambda_3)$ and the following relations hold: (i) $\{\Upsilon$ is positive$\} \Leftrightarrow \{|\lambda_j| \leqslant 1, j = 1,2,3\}$; (ii) $\{\Upsilon$ is CP$\} \Leftrightarrow \{\Upsilon \otimes \Upsilon$ is CP$\} \Leftrightarrow 1 \pm \lambda_3 \geqslant |\lambda_1 \pm \lambda_2|$; (iii) $\{\Upsilon$ is EB$\} \Leftrightarrow \{\Upsilon \otimes \Upsilon$ is EB$\} \Leftrightarrow \{|\lambda_1| + |\lambda_2| + |\lambda_3| \leqslant 1\}$; (iv) $\{\Upsilon \otimes \Upsilon$ is

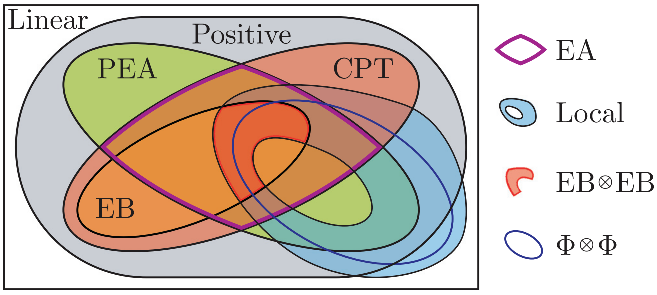

FIG. 2. (Color online) Venn diagram of linear bipartite maps $\Phi^{\mathscr{A}\mathscr{B}}$. Convex figures correspond to convex sets.

positive} $\Leftrightarrow$ {Υ^2 is CP} $\Leftrightarrow$ $1 \pm \lambda_3^2 \geqslant |\lambda_1^2 \pm \lambda_2^2|$; (v) {$\Upsilon \otimes \Upsilon$ is PEA} $\Leftrightarrow$ {Υ^2 is EB} $\Leftrightarrow$ {$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 \leqslant 1$}; (vi) {$\Upsilon \otimes \Upsilon$ is EA} $\Leftrightarrow$ {Υ is CP and Υ^2 is EB}. Clearly (vi) = (ii) $\cap$ (v). Analogies [(ii)$\sim$(iii)] and [(iv)$\sim$(v)] stimulated us to extend the concept of entanglement annihilation to positive maps. Indeed, definitions of both the CP and EB maps require extensions of the channel action, whereas the concepts of positive and PEA maps do not require for their definition any additional physical system. The structure of linear bipartite maps is illustrated in Fig. 2.

III. CRITERIA

The appealing simplicity of item (v) above is not sudden and holds due to a general property that the cone of PEA maps is closed under a left composition by partially positive maps $\mathrm{Id}^{\mathscr{A}} \otimes \Lambda^{\mathscr{B}}$ (left PP invariant), which follows from Lemma 1. This fundamental property enables us to characterize PEA maps.

Proposition 1. The map $\Phi^{\mathscr{A}\mathscr{B}}$ is PEA if and only if

$$\mathrm{tr}\big[\big(\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}} \otimes \varrho^{\mathscr{A}'\mathscr{B}'}\big)\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}\big] \geqslant 0 \tag{3}$$

for all block-positive $\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}}$ and $\varrho^{\mathscr{A}'\mathscr{B}'} \in \mathcal{S}(\mathcal{H}^{\mathscr{A}'} \otimes \mathcal{H}^{\mathscr{B}'})$.

Proof. Using the extension of Lemma 1 for positive maps, we get {$\Phi^{\mathscr{A}\mathscr{B}}$ is PEA} $\Leftrightarrow$ {$(\mathrm{Id}^{\mathscr{A}} \otimes \Lambda^{\mathscr{B}}) \circ \Phi^{\mathscr{A}\mathscr{B}}$ is a positive map for any positive map $\Lambda^{\mathscr{B}}$}, which is equivalent to the block positivity of matrix $\Omega_{(\mathrm{Id}\otimes\Lambda)\circ\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'} \equiv (\mathrm{Id}^{\mathscr{A}\mathscr{A}'\mathscr{B}'} \otimes \Lambda^{\mathscr{B}})[\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}]$. By the definition of block positivity,

$$\begin{aligned} &\mathrm{tr}\big\{|\varphi^{\mathscr{A}\mathscr{B}} \otimes \chi^{\mathscr{A}'\mathscr{B}'}\rangle\langle\varphi^{\mathscr{A}\mathscr{B}} \otimes \chi^{\mathscr{A}'\mathscr{B}'}| \\ &\quad\times (\mathrm{Id}^{\mathscr{A}\mathscr{A}'\mathscr{B}'} \otimes \Lambda^{\mathscr{B}})\big[\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}\big]\big\} \\ &\equiv \mathrm{tr}\big\{(\mathrm{Id}^{\mathscr{A}} \otimes \Lambda^{\dagger\mathscr{B}})[|\varphi^{\mathscr{A}\mathscr{B}}\rangle\langle\varphi^{\mathscr{A}\mathscr{B}}|] \\ &\quad\otimes|\chi^{\mathscr{A}'\mathscr{B}'}\rangle\langle\chi^{\mathscr{A}'\mathscr{B}'}|\ \Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}\big\} \geqslant 0, \end{aligned}$$

where $\Lambda^{\dagger}$ denotes the dual map $\mathrm{tr}[X\Lambda[Y]] \equiv \mathrm{tr}[\Lambda^{\dagger}[X]Y]$. Since the dual of a positive map is also positive (see, e.g., [36]), $\Lambda^{\dagger}$ is a positive map and the operator $(\mathrm{Id}^{\mathscr{A}} \otimes \Lambda^{\dagger\mathscr{B}})[|\varphi^{\mathscr{A}\mathscr{B}}\rangle\langle\varphi^{\mathscr{A}\mathscr{B}}|]$ is block positive (equals $\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}}$). Taking into account arbitrariness of $\Lambda, |\varphi\rangle, |\chi\rangle$ and remembering the convex structure of density operators, we obtain formula (3). ■

Proposition 1 says (in terms of Choi matrices) that the cone of PEA maps is dual to the cone of operators $\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}} \otimes \varrho^{\mathscr{A}'\mathscr{B}'}$ inducing [via formula (2)] maps of the form $\Phi_{\mathrm{d.c.}}^{\mathscr{A}\mathscr{B}}[X] = \sum_k \mathrm{tr}[F_k X]\xi_{\mathrm{BP}k}^{\mathscr{A}|\mathscr{B}}$, $F_k \geqslant 0$. Moreover, using $\mathrm{tr} = \mathrm{tr}_{\mathscr{A}\mathscr{B}} \circ \mathrm{tr}_{\mathscr{A}'\mathscr{B}'}$, we obtain alternative forms of the condition in Eq. (3). In particular, $\Phi^{\mathscr{A}\mathscr{B}}$ is PEA if and only if for all $\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}}$ the operator $\mathrm{tr}_{\mathscr{A}\mathscr{B}}[(\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}} \otimes I^{\mathscr{A}'\mathscr{B}'})\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}]$ is positive, i.e., belongs to $\mathrm{Cone}(\mathcal{S}(\mathcal{H}^{\mathscr{A}'\mathscr{B}'}))$, or equivalently, if the operator $\langle\chi^{\mathscr{A}'\mathscr{B}'}|\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}|\chi^{\mathscr{A}'\mathscr{B}'}\rangle$ belongs to a cone of separable states (with respect to partition $\mathscr{A}|\mathscr{B}$) for all $|\chi^{\mathscr{A}'\mathscr{B}'}\rangle$.

We already know that EA = PEA $\cap$ CPT, therefore the complete characterization of EA channels is as follows:

Corollary 1. The linear map $\Phi^{\mathscr{A}\mathscr{B}}$ is an EA channel if and only if its Choi matrix $\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ satisfies (3), $\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'} \geqslant 0$, and $\mathrm{tr}_{\mathscr{A}\mathscr{B}}\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'} = (d^{\mathscr{A}}d^{\mathscr{B}})^{-1}I^{\mathscr{A}'\mathscr{B}'}$.

Proof. The three requirements guarantee that $\Phi \in$ PEA, $\Phi \in$ CP, and Φ is trace preserving, respectively. ■

Although Proposition 1 provides the necessary and sufficient condition for the map to be PEA, it is challenging to apply it to a given map. The following proposition provides a nontrivial sufficient condition which is quite useful as we demonstrate later.

Proposition 2. If $\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ can be written as a convex sum of operators $\zeta_{\mathrm{BP}}^{\mathscr{A}|\mathscr{A}'\mathscr{B}'} \otimes \varrho^{\mathscr{B}}$ and $\varrho^{\mathscr{A}} \otimes \zeta_{\mathrm{BP}}^{\mathscr{B}|\mathscr{A}'\mathscr{B}'}$, where ζ_{BP} is block positive with respect to corresponding cut and ϱ is positive, then the map $\Phi^{\mathscr{A}\mathscr{B}}$ is PEA.

Proof. Substituting $\zeta_{\mathrm{BP}}^{\mathscr{A}|\mathscr{A}'\mathscr{B}'} \otimes \varrho^{\mathscr{B}}$ for $\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ in (3), we obtain that $\mathrm{tr}_{\mathscr{A}'\mathscr{B}'}[\zeta_{\mathrm{BP}}^{\mathscr{A}|\mathscr{A}'\mathscr{B}'}|\chi^{\mathscr{A}'\mathscr{B}'}\rangle\langle\chi^{\mathscr{A}'\mathscr{B}'}|] = \tilde{\varrho}^{\mathscr{A}} \geqslant 0$ and $\mathrm{tr}_{\mathscr{A}\mathscr{B}}[\xi_{\mathrm{BP}}^{\mathscr{A}|\mathscr{B}}\tilde{\varrho}^{\mathscr{A}} \otimes \varrho^{\mathscr{B}}] \geqslant 0$; thus, Eq. (3) holds. By exchanging $\mathscr{A} \leftrightarrow \mathscr{B}$ it is clear that the operator $\varrho^{\mathscr{A}} \otimes \zeta_{\mathrm{BP}}^{\mathscr{B}|\mathscr{A}'\mathscr{B}'}$ also satisfies the requirement (3). ■

Define $A^{\mathscr{B}} = |\varphi^{\mathscr{B}}\rangle\langle\psi^{\mathscr{B}}|$, then $\zeta_{\mathrm{BP}}^{\mathscr{A}|\mathscr{A}'\mathscr{B}'} \otimes |\varphi^{\mathscr{B}}\rangle\langle\varphi^{\mathscr{B}}| = (I^{\mathscr{A}\mathscr{A}'\mathscr{B}'} \otimes A^{\mathscr{B}})\Xi_{\mathrm{BP}}^{\mathscr{A}\mathscr{B}|\mathscr{A}'\mathscr{B}'}(I^{\mathscr{A}\mathscr{A}'\mathscr{B}'} \otimes A^{\mathscr{B}\dagger})$ for a suitable $\Xi_{\mathrm{BP}}^{\mathscr{A}\mathscr{B}|\mathscr{A}'\mathscr{B}'}$. Consequently, the map corresponding to $\zeta_{\mathrm{BP}}^{\mathscr{A}|\mathscr{A}'\mathscr{B}'} \otimes |\varphi^{\mathscr{B}}\rangle\langle\varphi^{\mathscr{B}}|$ is a concatenation of a positive map $\Lambda^{\mathscr{A}\mathscr{B}}$ (given by Choi matrix $\Xi_{\mathrm{BP}}^{\mathscr{A}\mathscr{B}|\mathscr{A}'\mathscr{B}'}$) followed by an EB operation $O_{\mathrm{EB}}[\bullet] = A \bullet A^{\dagger}$ acting on subsystem $\mathscr{B}$. Similarly, $\varrho^{\mathscr{A}} \otimes \zeta_{\mathrm{BP}}^{\mathscr{B}|\mathscr{A}'\mathscr{B}'}$ describes a positive map on $\mathscr{A}\mathscr{B}$ followed by some EB operation applied to subsystem $\mathscr{A}$. As a result, the subset of PEA maps characterized by Proposition 2 can be understood as mixture of concatenations of positive maps with EB operations applied on one of the subsystems (see also Fig. 3):

$$\Phi^{\mathscr{A}\mathscr{B}} = \sum_k \big(O_{\mathrm{EB}k}^{\mathscr{A}(\mathscr{B})} \otimes \mathrm{Id}^{\mathscr{B}(\mathscr{A})}\big) \circ \Lambda_k^{\mathscr{A}\mathscr{B}}. \tag{4}$$

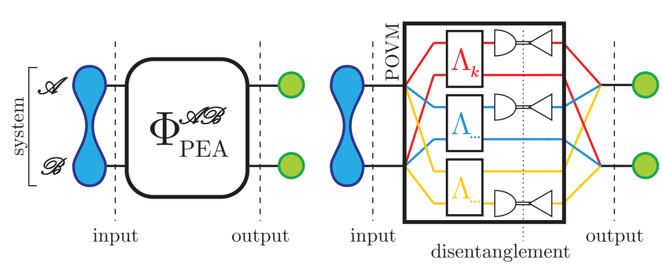

FIG. 3. (Color online) PEA maps and physical meaning of Proposition 2 (positive maps $\{\Lambda_k\}$ are followed by one-sided EB operations).

Moreover, if we replace in Proposition 2 the block-positive operators $\zeta_{\rm BP}^{\mathscr{A}|\mathscr{A}'\mathscr{B}'}$ and $\zeta_{\rm BP}^{\mathscr{B}|\mathscr{A}'\mathscr{B}'}$ by positive ones $\varrho^{\mathscr{A}\mathscr{A}'\mathscr{B}'}$ and $\varrho^{\mathscr{B}\mathscr{A}'\mathscr{B}'}$, respectively, then the corresponding Choi matrix will automatically be positive and the associated map will be a fair CP map.

Corollary 2. If $\mathrm{tr}_{\mathscr{A}\mathscr{B}}\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'} = (d^{\mathscr{A}}d^{\mathscr{B}})^{-1}I^{\mathscr{A}'\mathscr{B}'}$ and $\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ is a convex sum of density operators $\varrho^{\mathscr{A}|\mathscr{B}\mathscr{A}'\mathscr{B}'}$ and $\varrho^{\mathscr{B}|\mathscr{A}\mathscr{A}'\mathscr{B}'}$ (separable with respect to partitions $\mathscr{A}|\mathscr{B}\mathscr{A}'\mathscr{B}'$ and $\mathscr{B}|\mathscr{A}\mathscr{A}'\mathscr{B}'$, respectively), then $\Phi^{\mathscr{A}\mathscr{B}}$ is an EA channel.

Let us note that such states $\Omega_{\Phi}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ belong to a family of so-called biseparable states (convex hull of states separable with respect to some bipartite cut). Unfortunately, only a little is known about biseparability detection [37–40]; however, Corollary 2 encourages its deeper investigation (see, e.g., a recent approach in Ref. [41]).

IV. CASE STUDY: DEPOLARIZING CHANNELS

Given a quantum channel $\Phi^{\mathscr{A}\mathscr{B}}$, one can settle the question of its being EA in the affirmative by finding either the resolution (4) or the resolution of Corollary 2. Once the resolution is found, it guarantees that $\Phi^{\mathscr{A}\mathscr{B}}$ is a PEA map, and, consequently, the channel is EA. As an example we examine a family of depolarizing channels which can act either locally or globally on the system reflecting the physical situation of individual or common baths, respectively.

The depolarizing channel on a d-dimensional system is defined through $\Phi_q = q\mathrm{Id} + (1-q)\mathrm{Tr}$, where $\mathrm{Tr}[X] = \mathrm{tr}[X]\frac{1}{d}I_d$ is the trace map and $q \in [-\frac{1}{d^2-1}, 1]$. Note that Φ_q is EB if and only if $-\frac{1}{d^2-1} \leqslant q \leqslant \frac{1}{d+1}$ (see Appendix B). A bipartite system $\mathscr{A}\mathscr{B}$ can be affected by a local depolarizing noise of the form $\Phi_{q_1}^{\mathscr{A}} \otimes \Phi_{q_2}^{\mathscr{B}}$, or a global depolarizing noise of the form $\Phi_q^{\mathscr{A}\mathscr{B}}$.

First, we illustrate the efficiency of the derived criteria by examples of 2×2 and 3×2 systems for which the exact solutions can be readily found thanks to the Peres-Horodecki criterion [32,33]: in the case $d^{\mathscr{A}} = d^{\mathscr{B}} = 2$, $\Phi_{q_1}^{\mathscr{A}} \otimes \Phi_{q_2}^{\mathscr{B}}$ is EA if $q_1q_2 \leqslant \frac{1}{3}$ and $\Phi_q^{\mathscr{A}\mathscr{B}}$ is EA if $q \leqslant \frac{1}{3}$; in the case $d^{\mathscr{A}} = 3$ and $d^{\mathscr{B}} = 2$, $\Phi_{q_1}^{\mathscr{A}} \otimes \Phi_{q_2}^{\mathscr{B}}$ is EA if $q_1(9q_2 - 1) \leqslant 2$ and $\Phi_q^{\mathscr{A}\mathscr{B}}$ is EA if $q \leqslant \frac{1}{4}$. The resolution (4) holds true for all the above two-qubit EA channels, i.e., Proposition 2 reproduces the exact results (see Fig. 4(a) and Appendices D and F). As far as Corollary 2 is concerned, our analysis shows that it allows us to detect the EA property of a smaller set of maps $\Phi_{q_1}^{\mathscr{A}} \otimes \Phi_{q_2}^{\mathscr{B}}$ [see Appendix E and Fig. 4(a)]. Analyzing channels acting on qutrit-qubit systems, we succeeded in constructing resolution (4) for a subset of EA channels which is slightly smaller than the whole set of EA channels [see Appendix D and Fig. 4(b) for local channels and Appendix F for global ones]. In what follows, we consider bipartite systems $\mathscr{A}\mathscr{B}$ with $d^{\mathscr{A}} = d^{\mathscr{B}} = d$, where d is arbitrary.

For a local channel $\Phi_{q_1}^{\mathscr{A}} \otimes \Phi_{q_2}^{\mathscr{B}}$ it is possible to find resolution (4) explicitly (see Appendix D) if

$$(d^2-1)\,q_1q_2 \leqslant 1 + \frac{(d-2)(d+1)}{d+2}(q_1+q_2). \tag{5}$$

Hence, for these values of parameters q_1 and q_2 the channel is EA. Putting $q_{1,2} = q$ in (5), we obtain that Φ_q is 2LEA if $q \leqslant q_{\rm EA}^{\rm local} = \frac{d-2+d\sqrt{\frac{2d}{d+1}}}{(d-1)(d+2)}$ [see Fig. 5(a)], which determines a larger set than the EB condition $q \leqslant \frac{1}{d+1}$.

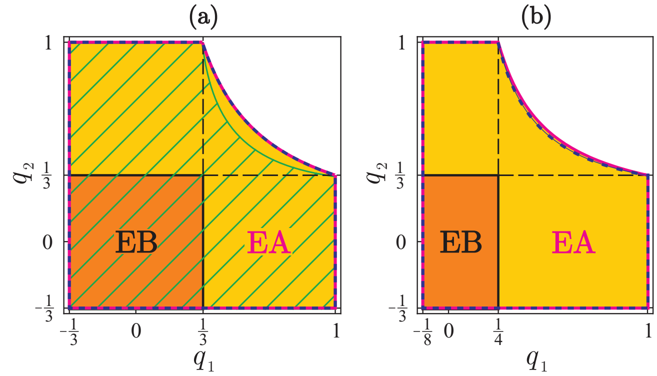

FIG. 4. (Color online) Area of parameters (q_1,q_2) where the local-depolarizing channel $\Phi_{q_1}^{\mathscr{A}} \otimes \Phi_{q_2}^{\mathscr{B}}$ is EA: (a) two-qubit system, $d^{\mathscr{A}} = d^{\mathscr{B}} = 2$; (b) qutrit-qubit system, $d^{\mathscr{A}} = 3$, $d^{\mathscr{B}} = 2$. The validity of Proposition 2 is justified inside the dashed region, which provides all EA channels in case (a) and a subset of EA channels in case (b). Being applied to case (a), Corollary 2 detects EA behavior inside the green hatching.

Consider a global depolarizing channel $\Phi_q^{\mathscr{A}\mathscr{B}}$ acting on a pair of d-dimensional subsystems $\mathscr{A}$ and $\mathscr{B}$ simultaneously. Such a noise is EB if and only if $q \leqslant \frac{1}{d^2+1}$. However, the noise disentangles $\mathscr{A}$ from $\mathscr{B}$, hence it is EA, if $q \leqslant q_{\rm EA}^{\rm global} = (d+2)/[(d+1)(d^2-d+2)]$ [see Fig. 5(b)], which we showed by an explicit construction of resolution (4) in Appendix F.

Finally, we would like to give a counterintuitive example of an entangled state which turns out to be more robust in the discussed dissipative dynamics than the maximally entangled state $|\Psi_+^{\mathscr{A}\mathscr{B}}\rangle = \frac{1}{\sqrt{d}}\sum_{i=1}^d |i\otimes i\rangle$. It can be readily seen that the state $(\Phi_q^{\mathscr{A}} \otimes \Phi_q^{\mathscr{B}})[|\Psi_+^{\mathscr{A}\mathscr{B}}\rangle\langle\Psi_+^{\mathscr{A}\mathscr{B}}|] \equiv (\Phi_{q^2}^{\mathscr{A}} \otimes \mathrm{Id}^{\mathscr{B}})[|\Psi_+^{\mathscr{A}\mathscr{B}}\rangle\langle\Psi_+^{\mathscr{A}\mathscr{B}}|]$ becomes separable if $\Phi_{q^2}^{\mathscr{A}}$ is EB, i.e., $q \leqslant q_{\rm MES}^{\rm local} = \frac{1}{\sqrt{d+1}}$. Similarly, $\Phi_q^{\mathscr{A}\mathscr{B}}[|\Psi_+^{\mathscr{A}\mathscr{B}}\rangle\langle\Psi_+^{\mathscr{A}\mathscr{B}}|] \equiv (\Phi_q^{\mathscr{A}} \otimes \mathrm{Id}^{\mathscr{B}})[|\Psi_+^{\mathscr{A}\mathscr{B}}\rangle\langle\Psi_+^{\mathscr{A}\mathscr{B}}|]$ becomes separable if $\Phi_q^{\mathscr{A}}$ is EB, i.e., $q \leqslant q_{\rm MES}^{\rm global} = \frac{1}{d+1}$. Consider now a state $|\gamma^{\mathscr{A}\mathscr{B}}\rangle = \frac{1}{\sqrt{2}}(|1\otimes 1\rangle + |d\otimes d\rangle)$ which is not maximally entangled (if

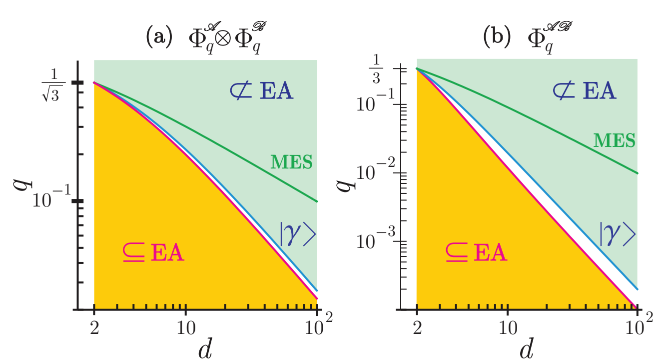

FIG. 5. (Color online) Local (a) and global (b) depolarizing channels that surely annihilate or preserve entanglement of $d\times d$ systems. Entanglement of the state $|\gamma\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |dd\rangle)$ is more robust than that of the maximally entangled state $|\Psi_+\rangle$ (MES).

$d>2$). Surprisingly, the states $(\Phi_q^{\mathscr{A}}\otimes\Phi_q^{\mathscr{B}})[|\gamma^{\mathscr{A}\mathscr{B}}\rangle\langle\gamma^{\mathscr{A}\mathscr{B}}|]$ and $\Phi_q^{\mathscr{A}\mathscr{B}}[|\gamma^{\mathscr{A}\mathscr{B}}\rangle\langle\gamma^{\mathscr{A}\mathscr{B}}|]$ remain nonpositive under partial transposition [32,33] and, consequently, are entangled if $q > q_{\rm nEA}^{\rm local} = \frac{1+\sqrt{3}}{d+1+\sqrt{3}}$ and $q > q_{\rm nEA}^{\rm global} = \frac{2}{d^2+2}$, respectively. These results are depicted in Fig. 5. A narrow gap between channels that are surely EA and channels that are definitely not EA underlines the importance of the state $|\gamma^{\mathscr{A}\mathscr{B}}\rangle$ in identifying potentially dangerous noises in applications.

V. SUMMARY

The analogy between the definitions of EB channels and CP maps based on the consideration of map extensions stimulates us to introduce the concept of PEA maps as counterparts of positive maps acting on a composite system $\mathscr{A}\mathscr{B}$. The cone of PEA maps is invariant under concatenation with partially positive maps. This fact enabled us to find the necessary and sufficient conditions for PEA maps as well as to find the explicit form of the dual cone of maps $\Phi_{\rm d.c.}^{\mathscr{A}\mathscr{B}}[X] = \sum_k {\rm tr}[F_k X]\xi_{{\rm BP}k}^{\mathscr{A}|\mathscr{B}}$, $F_k \geqslant 0$. This form resembles measure-and-prepare procedures (being EB) but differs in the use of block-positive operators. Based on these criteria and in analogy with the entanglement theory, one may introduce the concept of EA witnesses. Imposing the conditions of CP and trace preservation on PEA maps we formulated sufficient criteria for EA channels possessing a clear physical interpretation illustrated in Fig. 3. The derived criteria were used in the analysis of local and global depolarizing channels, for which we identified maximum noise levels; going beyond those levels leads to entanglement annihilation.

ACKNOWLEDGMENTS

This work was supported by the EU integrated project 2010-248095 (Q-ESSENCE), COST Action MP1006, APVV-0646-10 (COQI), and VEGA 2/0127/11 (TEQUDE). S.N.F. acknowledges support from the National Scholarship Programme of the Slovak Republic, the Dynasty Foundation, and the Russian Foundation for Basic Research under Project No. 12-02-31524-mol-a.

APPENDIX A: MATRIX REPRESENTATION OF MAPS

A linear map $\Phi : \mathcal{T}(\mathcal{H}_{\rm in}) \to \mathcal{T}(\mathcal{H}_{\rm out})$ can also be defined via the $d_{\rm out}^2 \times d_{\rm in}^2$ matrix $\mathcal{E}_\Phi$ with entries $(\mathcal{E}_\Phi)_{jk} = ({\rm tr}[o_j^\dagger o_j]{\rm tr}[\iota_k^\dagger \iota_k])^{-1}{\rm tr}[o_j^\dagger \Phi[\iota_k]]$, where $\{\iota_k\}_{k=0}^{d_{\rm in}^2-1}$ and $\{o_j\}_{j=0}^{d_{\rm out}^2-1}$ are orthogonal operator bases in $\mathcal{T}(\mathcal{H}_{\rm in})$ and $\mathcal{T}(\mathcal{H}_{\rm out})$, respectively. As a basis, we use normalized generalized Pauli (Gell-Mann) matrices $\{\gamma_j\}_{j=0}^{d^2-1}$ satisfying the relations $\gamma_j^\dagger = \gamma_j$, ${\rm tr}[\gamma_j\gamma_k] = \delta_{jk}$, and $\gamma_0 = \frac{1}{d}I_d$. Using such a basis, one can readily see that for $d_{\rm in,out} = d$, the matrix representation of depolarizing channel Φ_q reads $\mathcal{E}_{\Phi_q} = {\rm diag}(1,q,\ldots,q)$.

In a matrix representation, a concatenation of maps corresponds to a conventional matrix product: $\mathcal{E}_{\Upsilon\circ\Phi} = \mathcal{E}_\Phi\mathcal{E}_\Upsilon$. Also, $\mathcal{E}_{\Phi\otimes\Upsilon} = \mathcal{E}_\Phi \otimes \mathcal{E}_\Upsilon$. These properties are especially pleasing for diagonal matrices (depolarizing maps).

APPENDIX B: EB DEPOLARIZING CHANNELS

Let us make a change of variable $q = [d(2\mu-1)-1]/(d^2-1)$, then the Choi matrix Ω_q of the depolarizing map Φ_q is equal to the partially transposed Werner state ϱ_μ^Γ, where $\varrho_\mu = \mu\frac{2}{d(d+1)}P_+ + (1-\mu)\frac{2}{d(d-1)}P_-$ is a convex combination of projectors onto symmetric and antisymmetric subspaces of $\mathcal{H}_d\otimes\mathcal{H}_d$ [42]. The state ϱ_μ is known to be separable if and only if it is positive under partial transposition, i.e., $\frac{1}{2}\leqslant\mu\leqslant 1$ [42]. It means that Ω_q is separable and, consequently, Φ_q is EB if $q\in[-\frac{1}{d^2-1},\frac{1}{d+1}]$.

APPENDIX C: POSITIVE BIPARTITE MAPS

Since positive maps on operators $\mathcal{T}(\mathcal{H}_d\otimes\mathcal{H}_d)$ are quite needed, we define a two-parametric map Λ_{st} by the following matrix representation:

$$\Lambda_{st} = {\rm diag}(1,\underbrace{s,\ldots,s}_{d^2-1\text{ times}};\underbrace{s,\underbrace{t,\ldots,t}_{d^2-1\text{ times}};\cdots;s,\underbrace{t,\ldots,t}_{d^2-1\text{ times}}}_{d^2-1\text{ times}}). \quad \text{(C1)}$$

The map (C1) is surely positive if

$$0\leqslant s\leqslant t\leqslant\frac{1}{d-1}+\left(1-\frac{1}{d-1}\right)s, \quad \text{(C2)}$$

which is validated by checking the block positivity of its Choi matrix $\Omega_{\Lambda_{st}}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ via the method of Ref. [43] [positivity of operators $\langle y^{\mathscr{A}'\mathscr{B}'}|\Omega_{\Lambda_{st}}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}|y^{\mathscr{A}'\mathscr{B}'}\rangle\in\mathcal{T}(\mathcal{H}^{\mathscr{A}\mathscr{B}})$].

For systems $\mathscr{A}\mathscr{B}$, where $d^{\mathscr{A}}\neq d^{\mathscr{B}}$, one can use a straightforward modification of (C1) with an appropriate number of terms. Such a map will be positive if (C2) is fulfilled for $d=\max(d^{\mathscr{A}},d^{\mathscr{B}})$.

APPENDIX D: LOCAL DEPOLARIZING EA CHANNELS

For $d\times d$ systems, the local depolarizing channel $\Phi_{q_1}^{\mathscr{A}}\otimes\Phi_{q_2}^{\mathscr{B}}$ is compatible with resolution (4) and, consequently, is EA whenever q_1 and q_2 satisfy inequality (5). The resolution takes the form

$$\Phi_{q_1}^{\mathscr{A}}\otimes\Phi_{q_2}^{\mathscr{B}} = \mu\big(\Phi_p^{\mathscr{A}}\otimes{\rm Id}^{\mathscr{B}}\big)\circ\Lambda_{s_1t_1}^{\mathscr{A}\mathscr{B}} + (1-\mu)\big({\rm Id}^{\mathscr{A}}\otimes\Phi_p^{\mathscr{B}}\big)\circ\Lambda_{s_2t_2}^{\mathscr{A}\mathscr{B}},$$

where

$$\mu = \frac{1}{2}+\frac{d+1}{2d}(q_2-q_1), \quad -\frac{1}{d^2-1}\leqslant p\leqslant\frac{1}{d+1},$$

i.e., Φ_p is EB. Inequality (5) transforms into an equality if $p=\frac{1}{d+1}$. The maps $\Lambda_{s_it_i}$, $i=1,2$, are given by formula (C1), where

$$s_{1,2} = \frac{2(d+1)}{d+2}\,\frac{(d+1)q_{2,1}-q_{1,2}}{d+(d+1)(q_{2,1}-q_{1,2})},$$

$$t_{1,2} = \frac{1}{d-1}+\left(1-\frac{1}{d-1}\right)s_{1,2}.$$

For a qutrit-qubit system, one should substitute the corresponding EB maps $\Phi_{p_1\leqslant 1/4}^{\mathscr{A}}$ and $\Phi_{p_2\leqslant 1/3}^{\mathscr{B}}$ for Φ_p. Numerical optimization over parameters μ, $s_{1,2}$, and $t_{1,2}$ results in the area of parameters (q_1,q_2) shown in Fig. 4(b).

APPENDIX E: APPLICATION OF COROLLARY 2 TO LOCAL DEPOLARIZING TWO-QUBIT CHANNELS

In the case of two qubits, we now find parameters q_1 and q_2 such that the Choi matrix $\Omega_{\Phi_{q_1}\otimes\Phi_{q_2}}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'}$ can be represented as a convex sum of density operators separable with respect to partitions $\mathscr{A}|\mathscr{B}\mathscr{A}'\mathscr{B}'$ and $\mathscr{B}|\mathscr{A}\mathscr{A}'\mathscr{B}'$, i.e.,

$$\Omega_{\Phi_{q_1}\otimes\Phi_{q_2}}^{\mathscr{A}\mathscr{B}\mathscr{A}'\mathscr{B}'} = \frac{1}{k_{\max}}\sum_{k=1}^{k_{\max}}\big[\mu|\psi_k^{\mathscr{A}}\rangle\langle\psi_k^{\mathscr{A}}|\otimes\varrho_k^{\mathscr{B}\mathscr{A}'\mathscr{B}'} + (1-\mu)|\psi_k^{\mathscr{B}}\rangle\langle\psi_k^{\mathscr{B}}|\otimes\tilde{\varrho}_k^{\mathscr{A}\mathscr{A}'\mathscr{B}'}\big].$$

This resolution takes place if the operators $\frac{1}{2}|\psi_k\rangle\langle\psi_k|$ form a symmetric informationally complete positive operator-valued measure (SIC-POVM) ($k=1,\ldots,4$) or the vectors $\{|\psi_k\rangle\}$ are elements of a full set of mutually unbiased bases ($k=1,\ldots,6$) (see, e.g., [44]),

$$\varrho_k^{\mathscr{B}\mathscr{A}'\mathscr{B}'} = \big(a|\psi_k^{*\mathscr{A}'}\rangle\langle\psi_k^{*\mathscr{A}'}| + b|\psi_{k\perp}^{*\mathscr{A}'}\rangle\langle\psi_{k\perp}^{*\mathscr{A}'}|\big)\otimes|\Psi_+^{\mathscr{B}\mathscr{B}'}\rangle\langle\Psi_+^{\mathscr{B}\mathscr{B}'}| + cI^{\mathscr{B}\mathscr{A}'\mathscr{B}'},$$

$$\tilde{\varrho}_k^{\mathscr{A}\mathscr{A}'\mathscr{B}'} = \big(a|\psi_k^{*\mathscr{B}'}\rangle\langle\psi_k^{*\mathscr{B}'}| + b|\psi_{k\perp}^{*\mathscr{B}'}\rangle\langle\psi_{k\perp}^{*\mathscr{B}'}|\big)\otimes|\Psi_+^{\mathscr{A}\mathscr{A}'}\rangle\langle\Psi_+^{\mathscr{A}\mathscr{A}'}| + cI^{\mathscr{A}\mathscr{A}'\mathscr{B}'},$$

$\mu = (1-q_1)q_2/(q_1+q_2-2q_1q_2)$, $a=\frac{1}{2}(q_1+q_2+8q_1q_2)$, $b=\frac{1}{2}(q_1+q_2-4q_1q_2)$, and $c=\frac{1}{8}(1-q_1-q_2+q_1q_2)$. However, the operators ϱ_k and $\tilde{\varrho}_k$ are positive only if $c\geqslant 0$, $a+c\geqslant 0$, and $b+c\geqslant 0$. These restrictions specify the region of parameters q_1 and q_2 by the inequality $1+3(q_1+q_2)-15q_1q_2\geqslant 0$, which is depicted in Fig. 4(a).

APPENDIX F: GLOBAL DEPOLARIZING EA CHANNELS

For $d\times d$ systems, the global depolarizing channel $\Phi_q^{\mathscr{A}\mathscr{B}}$ is compatible with resolution (4) and, consequently, is EA whenever $q\leqslant (d+2)/[(d+1)(d^2-d+2)]$. The resolution takes the form

$$\Phi_q^{\mathscr{A}\mathscr{B}} = \tfrac{1}{2}\big(\Phi_p^{\mathscr{A}}\otimes\mathrm{Id}^{\mathscr{B}} + \mathrm{Id}^{\mathscr{A}}\otimes\Phi_p^{\mathscr{B}}\big)\circ\Lambda_{st}^{\mathscr{A}\mathscr{B}},$$

where $-\frac{1}{d^2-1}\leqslant p\leqslant\frac{1}{d+1}$ (i.e., Φ_p is EB) and Λ_{st} is given by formula (C1) with $s=2/(d^2-d+2)$, $t=(d+2)s$.

For a qutrit-qubit system, the weight factors $(\frac{1}{2},\frac{1}{2})$ should be replaced by $(\mu,1-\mu)$, a single positive map Λ_{st} should be split into two ($\Lambda_{s_1t_1}$ and $\Lambda_{s_2t_2}$), and the maps Φ_p should be replaced by the corresponding EB maps $\Phi_{p_1\leqslant 1/4}^{\mathscr{A}}$ and $\Phi_{p_2\leqslant 1/3}^{\mathscr{B}}$. Numerical optimization over parameters μ, $s_{1,2}$, and $t_{1,2}$ shows that $\Phi_q^{\mathscr{A}\mathscr{B}}$ is surely EA if $q\leqslant 0.21$, which is slightly less than the exact value $\frac{1}{4}$.

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