PHYSICAL REVIEW B **85**, 241301(R) (2012)

Nonlinear spin to charge conversion in mesoscopic structures

Peter Stano,[1] Jaroslav Fabian,[2] and Philippe Jacquod[3,4,5]
[1]*Institute of Physics, Slovak Academy of Sciences, SK-845 11 Bratislava, Slovakia*
[2]*Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany*
[3]*Physics Department, University of Arizona, Tucson, Arizona 85721, USA*
[4]*College of Optical Sciences, University of Arizona, Tucson, Arizona 85721, USA*
[5]*Département de Physique Théorique, Université de Genève, CH-1205 Genève, Switzerland*
(Received 31 December 2011; revised manuscript received 29 March 2012; published 1 June 2012)

Motivated by recent experiments [I. J. Vera-Marun, V. Ranjan, and B. J. van Wees, Nat. Phys. **8**, 313 (2012)], we formulate a nonlinear theory of spin transport in quantum coherent conductors. We show how a mesoscopic constriction with energy-dependent transmission can convert a spin current injected by a spin accumulation into an electric signal, relying neither on magnetic nor exchange fields. When the transmission through the constriction is spin independent, the spin-charge coupling is nonlinear, with an electric signal that is quadratic in the accumulation. We estimate that gated mesoscopic constrictions have a sensitivity that allows to detect accumulations much smaller than a percent of the Fermi energy.

DOI: 10.1103/PhysRevB.85.241301 PACS number(s): 72.25.Dc, 73.23.−b, 73.50.Fq, 75.76.+j

Introduction. Spin current detection and measurement protocols are for the most part based on ferromagnetic contacts[1–7] or Zeeman fields.[8–11] While efficient and well controlled, these schemes are not optimized for miniaturization, because exchange and magnetic fields have low spatial resolution and because they cannot detect the weak spin accumulations achievable in two-dimensional electron gases such as GaAs heterostructures, the platform of choice for submicron spintronics. To unleash the full potential of spintronics at the nanoscale, it is therefore imperative to find novel, all-electric protocols. In submicron structures, however, reciprocity and other symmetry relations constrain the detection of spin currents in the linear response regime.[12–14] In the absence of time-reversal symmetry breaking field and focusing on two-terminal geometries, these constraining rules can only be waived by going beyond the linear response regime. At the nanoscale, this presents important theoretical challenges as local electric potentials must be determined self-consistently to ensure gauge invariance.[15]

In this Rapid Communication we construct a mean-field nonlinear theory of spin transport through submicron scale structures. We use it to propose a protocol which converts the spin current injected by a spin accumulation into a charge signal via the energy-dependent transmission of a mesoscopic structure. While our scheme is general, we focus our discussion on quantum point contacts and Coulomb-blockaded quantum dots, whose transmission is easily tunable by electric gate potentials. We show that the electric response is quadratic in the spin accumulation $\delta\mu$ when the transmission is energy dependent. A linear response arises only if the transmission is spin dependent, which usually requires an external magnetic field. We foresee that our scheme has sufficient sensitivity to detect weak spin accumulations such as those that can be generated magnetoelectrically in GaAs heterostructures,[16] to which the magnetic spin detection schemes are notoriously difficult to apply.

We consider the standard measurement setup depicted in Fig. 1. One aims to detect the nonequilibrium spin accumulation drop below the two terminals, $\delta\mu_{s1} \neq \delta\mu_{s2}$, as an electric signal. The spin accumulation origin is not specified, be it ferromagnetic, magnetoelectric, or optical. A recent pioneering work, Ref. 17, demonstrated that nonlinear effects make the detection possible in graphene even without using ferromagnetic terminals, on which the linear Johnson-Silsbee method relies.[3] A voltage quadratic in the spin accumulation arises due to the energy dependence of the graphene conductivity near the Dirac point. The bottom line of our theory is that, while the drift-diffusion approach of Ref. 17 is appropriate for bulk systems, it cannot be directly exported to submicron structures, where gauge invariance requires special care.[15] Furthermore, unlike in graphene, the density of states in GaAs heterostructures is mostly energy independent, thus nonlinear effects emerge only if further constrictions induce energy-dependent transmission $T(E)$. The constriction, such as a Coulomb-blockaded quantum dot, a resonant tunneling barrier, or a quantum point contact (QPC), is the active element in our scheme, converting the spin to charge in proportion to $\partial_E T(E)$. This quantity (and this is the crucial point) is fully tunable electrically and independently of the spin accumulation itself, providing our method with the versatility necessary for practical spintronics.

Theory calculation. We model the detection circuit in Fig. 1 as a quantum scatterer connected to two electron reservoirs, each with its own electrochemical potential and spin

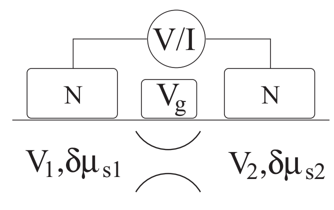

FIG. 1. Measurement scheme: A mesoscopic constriction separates two nonmagnetic terminals with spin accumulations $\delta\mu_{s1,2}$. The transmission coefficient through the constriction is gate tunable (voltage V_g), and when it is energy dependent, an electric signal that is nonlinear in $\delta\mu_{s1,2}$ arises.

accumulation, via two leads. We start by writing the current in lead $i = 1,2$ in the spin subband $\sigma = \uparrow, \downarrow$ (alternatively $\sigma = \pm 1$)[15]

$$I_i^\sigma = \frac{e}{h}\int dE \sum_{j\sigma'} \{N_i^\sigma \delta_{\sigma\sigma'}\delta_{ij} - T_{ij}^{\sigma\sigma'}[E,U(\mathbf{r})]\} f_j^{\sigma'}(E). \quad (1)$$

The transmission $T_{ij}^{\sigma\sigma'} = \sum_{\alpha\beta} T_{i\alpha j\beta}^{\sigma\,\sigma'}$ is a sum over probabilities that a particle with spin σ' injected from transversal channel β in lead j exits the system with spin σ into channel α in lead i. It depends on the particle energy E and the local electrostatic potential $U(\mathbf{r})$. We consider a spin conserving transmission $T^{\sigma\sigma'} \propto \delta_{\sigma\sigma'}$ with

$$T_{12}^{\sigma\sigma}(E) = T(E) + \sigma\delta T(E). \quad (2)$$

Neglecting spin-flip terms is consistent with the constriction dimensions being much smaller than the spin-orbit length (a few microns in GaAs). For a spin-insensitive structure, the transmission difference is zero, $\delta T = 0$, and $T = T^{\uparrow\uparrow} = T^{\downarrow\downarrow}$. Each lead is characterized by the number N_i^σ of transmission channels it carries (whose weak energy dependence we neglect), and the particle distribution $f_i^\sigma(E) = f(E - \mu_i^\sigma)$, at the corresponding terminal, with the Fermi function $f(x) = \{\exp[(x-\mu_F)/k_BT] + 1\}^{-1}$. The electrochemical potential of the spin subband σ, measured from the Fermi energy μ_F, is (e is the electron charge)

$$\mu_i^\sigma = eV_i + \sigma\,\delta\mu_{si}, \quad (3)$$

where V_i is the applied voltage.

Equation (1) is current conserving due to the unitarity condition $\sum_{j,\sigma} T_{ji}^{\sigma\sigma'} = N_i^{\sigma'}$. To guarantee gauge invariance (i.e., that currents are invariant under an overall voltage shift) one has to take into account that $U(\mathbf{r})$ is a function of the applied voltages. Up to second order in μ's, the current is[15]

$$\begin{aligned} I_1^\sigma = \frac{e}{h}\int dE\,[-\partial_E f(E)]\Big\{ & T_{12}^{\sigma\sigma}(E)\big(\mu_1^\sigma - \mu_2^\sigma\big) \\ & + (1/2)\partial_E T_{12}^{\sigma\sigma}(E)\big[\big(\mu_1^\sigma\big)^2 - \big(\mu_2^\sigma\big)^2\big] \\ & + \int d\mathbf{r}\,\big[\delta_{U(\mathbf{r})}T_{12}^{\sigma\sigma}(E)\big]\delta U(\mathbf{r})\big(\mu_1^\sigma - \mu_2^\sigma\big)\Big\}. \quad (4)\end{aligned}$$

Though the first linear term is explicitly gauge invariant, self-consistent conditions have to be imposed on $U(\mathbf{r})$ in order to ensure that the nonlinear terms also are gauge invariant. This is taken care of by the last term where the functional derivative of the transmission with respect to $U(\mathbf{r})$ couples to the deviation of the electrostatic profile from its equilibrium value $U(\mathbf{r}) = U_{\rm eq}(\mathbf{r}) + \delta U(\mathbf{r})$. The deviation $\delta U(\mathbf{r})$, which results from applied voltages and spin accumulations and the way they are injected into the scattering region, can be parametrized by characteristic potentials,[15] which generally speaking are determined by self-consistent solutions to the Schrödinger and Poisson equations. To restrain from these (here unnecessary) complications we neglect the spatial dependence of the potential changes $\delta U(\mathbf{r}) = \delta U$ and calculate the functional derivative of $T_{12}^{\sigma\sigma'}$ using the identity

$$\int d\mathbf{r}\big[\delta_{U(\mathbf{r})}T_{12}^{\sigma\sigma'}(E)\big] = -e\partial_E T_{12}^{\sigma\sigma'}(E). \quad (5)$$

To solve for δU, we assume a symmetric probe, with equal, spin-independent coupling to both leads,

$$e\,\delta U = (\mu_1^\uparrow + \mu_1^\downarrow + \mu_2^\uparrow + \mu_2^\downarrow)/4. \quad (6)$$

Using Eqs. (2)–(6) we get our main result, that the electrical current $I \equiv I_1^\uparrow + I_1^\downarrow$ is

$$\begin{aligned} I = {} & G_1\,e\,\delta V + G_2\big(\delta\mu_{s1}^2 - \delta\mu_{s2}^2\big) + G_3(\delta\mu_{s1} - \delta\mu_{s2}) \\ & + G_4(\delta\mu_{s1} + \delta\mu_{s2})e\,\delta V. \quad (7)\end{aligned}$$

The formula is explicitly gauge invariant, as it depends only on $\delta V = V_1 - V_2$. The calculation is furthermore current conserving, with $I_2 = -I_1$ obtained by substituting $\delta\mu_{s1} \leftrightarrow \delta\mu_{s2}$ and $\delta V \to -\delta V$. We see the emergence of a nonlinear spin to charge coupling term $G_2(\delta\mu_{s1}^2 - \delta\mu_{s2}^2)$, even in the absence of any rectification term $\propto \delta V^2$. That such a term is absent follows from our choice of a symmetric potential δU, in agreement with Ref. 15. There are four contributions to the current, with conductances

$$G_1 = \frac{2e}{h}\int dE(-\partial_E f)T(E), \quad (8a)$$

$$G_2 = \frac{e}{h}\int dE(-\partial_E f)[\partial_E T(E)], \quad (8b)$$

$$G_3 = \frac{2e}{h}\int dE(-\partial_E f)\delta T(E), \quad (8c)$$

$$G_4 = \frac{e}{h}\int dE(-\partial_E f)[\partial_E \delta T(E)], \quad (8d)$$

which we discuss in detail below, first for a spin-insensitive constriction, and second for a spin-sensitive constriction.

Spin-insensitive constriction. We first consider $\delta T = 0$ in Eq. (2), in which case $G_3 = 0 = G_4$, and focus our discussion on a gate-defined QPC in a two-dimensional electron gas (2DEG) GaAs heterostructure, with energy-dependent transmission[18]

$$T(E) = \{1 + \exp[-2\pi(E - e\alpha V_g)/\hbar\omega]\}^{-1}. \quad (9)$$

The transmission is easily tuned by an external gate voltage V_g, with a sensitivity set by the QPC characteristic energy scale $\hbar\omega$ and α the "lever arm" converting gate voltage into energy. We take typical values $\hbar\omega = 180$ μeV, corresponding to the Zeeman energy of 8 T field at the g factor $g = -0.39$, and $\alpha = 0.05$. We further fix $\mu_F = 8$ meV, $T = 0.1$ K, and spin accumulations $\delta\mu_{s2} = 0$ and $\delta\mu_{s1} \equiv \delta\mu_s = 0.1\%\mu_F$, which should be magnetoelectrically achievable.[16]

We are now ready to investigate the electric response of the circuit. First, we assume that both leads are held at the same potential. Even in this case, the spin accumulation generates a finite current, due to the second term in Eq. (7). Its magnitude is determined by the nonlinear conductance G_2 in Eq. (8b), which we plot in Fig. 2(a). It is proportional to $\partial_E T$, and thus maximal when the QPC is half open, $T = 0.5$. The current is then

$$I = G_2\delta\mu_s^2 \sim \frac{e/h}{\max(k_BT,\hbar\omega/2\pi)}\delta\mu_s^2, \quad (10)$$

and we plot it in Fig. 2(b). For our choice of parameters, the current is of the order of tens of pA, which is well above the experimental detection limit. The dependence of the current signal on $\max(k_BT,\hbar\omega/2\pi)$ is demonstrated in Figs. 2(c) and 2(d). Decreasing k_BT at fixed $\hbar\omega$ ($\hbar\omega$ at fixed k_BT), the signal first increases before it saturates when $k_BT \simeq \hbar\omega$.

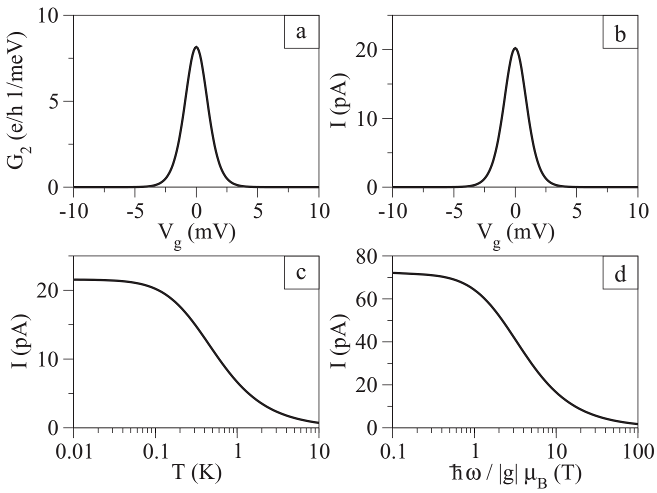

FIG. 2. (a) Nonlinear conductance G_2 given in Eq. (8b). (b)–(d) Current, Eq. (7) for zero bias, $\delta V = 0$, as a function of (b) gate voltage determining the QPC transmission [see Eq. (9)], (c) temperature, and (d) QPC energy resolution in units of the magnetic field.

Alternatively, terminal 2 can be operated as a floating probe. In this case, a finite voltage drop develops, which we find by setting $I_1 = 0$. One obtains

$$e\,\delta V = -G_2 \delta\mu_s^2 / G_1. \tag{11}$$

We see that the current is converted into a voltage by the linear conductance G_1, given in Eq. (8a) and plotted in Fig. 3(a). This agrees with the already mentioned absence of a rectification term in our symmetric QPC.[15] We plot the signal voltage in Fig. 3(b). In the region where the QPC is closed ($V_g \ll -\hbar\omega$), Eq. (11) gives an unphysical saturation of δV (dashed line). To remove this artifact, we add a small constant to G_1, which enforces that $\delta\mu_{s1}$ does not influence V_2 if the QPC is closed. Then the electric signals (current or voltage) in the two protocols just discussed behave similarly, $I, V \propto G_2 \delta\mu_s^2$. For spin-insensitive constrictions, we see that the electric response is quadratic in the spin accumulation. The qualitative picture for this effect is the following. At zero bias, $V = 0$, a finite spin accumulation $\delta\mu_s$ on one side of the constriction drives two counterpropagating currents in the two spin subbands. Since these two currents flow at different energies, they do not cancel exactly when the conductance is energy dependent.

This result is not specific to a QPC, which we next replace by a Coulomb-blockaded quantum dot. Neglecting inelastic processes and near resonance, its low-temperature transmission is given by[19] $T(E) = \Gamma^{(1)}\Gamma^{(2)}/[(E - E_0)^2/\hbar^2 + (\Gamma/2)^2]$, with the tunneling rates $\Gamma^{(1)}$ and $\Gamma^{(2)}$ of the resonant level to the left and right leads, $\Gamma = \Gamma^{(1)} + \Gamma^{(2)}$, and the resonance peak position E_0. At $E - E_0 = \pm\hbar\Gamma/2\sqrt{3}$, $\partial_E T(E)$ takes its maximal value $\pm 9\Gamma^{(1)}\Gamma^{(2)}/\sqrt{3}\Gamma^3$. Because $T(E)$ differs from its QPC expression, Eq. (9), in its energy dependence, the shape of the quantities plotted in Fig. 2 will be different; most notably, the signal changes sign upon crossing the resonance. However, the maximal current magnitude is still given by Eq. (10) with Γ replacing ω.

Spin-sensitive constriction. We next consider the case when $\delta T \neq 0$ in Eq. (2), when the QPC is made spin sensitive, e.g., by an external Zeeman field. We assume that the field is parallel

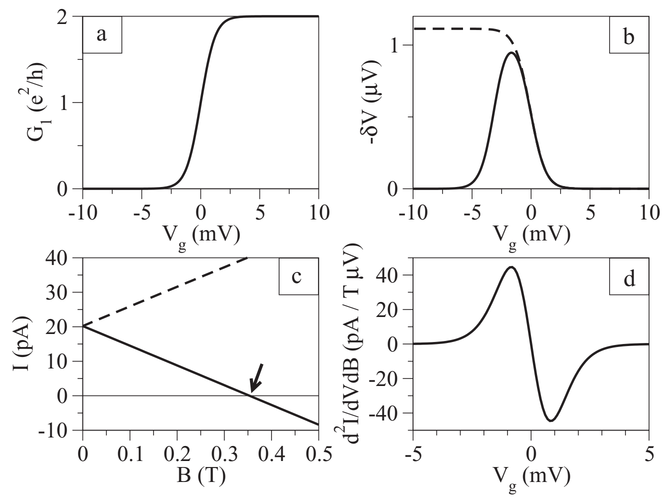

FIG. 3. (a) Linear conductance G_1, Eq. (8a). (b) Voltage, calculated according to Eq. (11) (dashed line) and adding a constant 0.1 e^2/h to G_1 (solid line). (c) Current in external magnetic field antiparallel (solid line) and parallel (dashed line) to the spin polarization direction. The arrow denotes the zero current position, and the thin line is a guide to the eye. (d) Second derivative of the current with respect to V and B.

to the spin accumulation $\delta\mu_{s1}$ and that it is sufficiently weak that the latter is not influenced. Equation (9) becomes

$$T^{\sigma\sigma}(E) = \{1 + \exp[-2\pi(E - \sigma\mu B - e\alpha V_g)/\hbar\omega]\}^{-1}, \tag{12}$$

where the Zeeman energy μB is added or subtracted from the electron's energy depending on its spin. Here, B is the magnetic field, $\mu = (g/2)\mu_B$ and μ_B is the Bohr magneton. To linear order in B, we then have $\delta T = -\mu B\, \partial_E T$. We see that a term linear in the spin accumulation has appeared, whose magnitude is given by the conductance G_3, Eq. (8c). This is a linear response term, similar to the one reported in Ref. 11, giving an odd magnetoresponse of the electric current through a QPC in the presence of a spin current. This term allows to find the sign of the spin accumulation, which is impossible for a spin-insensitive probe. This is demonstrated in Fig. 3(c), where we plot the current as a function of the magnetic field. Applying the field antiparallel (for $g < 0$ as in GaAs heterostructures) to the spin polarization direction, the Zeeman energy penalty compensates for larger transmission at higher energies. The compensation is exact (the current becomes zero) if

$$\delta\mu_s = g\mu_B B_c. \tag{13}$$

Remarkably, determining the compensation field B_c alone allows to measure both the magnitude and direction of the spin accumulation.

For spin-sensitive constrictions, the conductance G_4 gives an interesting contribution to the current, which is coupled to the average spin accumulation $\delta\mu_{s1} + \delta\mu_{s2}$ and the voltage bias δV. We rewrite this contribution as

$$G_4(\delta\mu_2 + \delta\mu_1)e\,\delta V = G_4\left(\frac{\mu_1^{\uparrow} + \mu_2^{\uparrow}}{2} - \frac{\mu_1^{\downarrow} + \mu_2^{\downarrow}}{2}\right)e\,\delta V, \tag{14}$$

which makes it clear that this term describes two different rectification currents in the two spin subbands which are uncompensated if (i) the transport in the subbands happens at different energies, (ii) there is a finite bias, and (iii) the probe transmission is both spin and energy sensitive. Experimentally, this contribution can be identified from the current derivative with respect to both the applied bias and the external field, as in Ref. 20. We plot this contribution in Fig. 3(d) where the antisymmetric shape of $G_4 \propto \partial_E^2 T$ strongly contrasts with the symmetric conductances G_2 and G_3.

Conclusions. We have shown how spin accumulations can be converted into electric signals in mesoscopic systems with energy-dependent, but spin-conserving transmission. When transport is spin-independent and in the absence of voltage bias, the conversion occurs in the nonlinear regime and the electric signal is quadratic in the spin accumulation. In submicron structures, such nonlinearities have to be treated self-consistently in local electrostatic potentials generated by the finite applied biases. We did that within a simplified mean-field approach, which resides in neglecting the spatial structure of the potential changes $\delta U(\mathbf{r})$. It is reassuring that the nonlinear signal arises within this approximation, which restrains from details of the constriction. Further, device specific, spin rectifying effects may arise from the spatial effects in $\delta U(\mathbf{r})$, along the lines of Ref. 15. We also note that for no applied bias our approximation becomes exact since $\delta U \to 0$.

In the case of transmission through a QPC, Eq. (10) suggests that it is its energy resolution, with $\hbar\omega \simeq 2$–3 K typically, rather than the temperature, which limits the signal magnitude and hence the sensitivity of our approach. Alternatively, a Coulomb-blockaded quantum dot can be used, where the resolution is given by the tunneling width which can easily reach $h\Gamma \simeq 0.1$ K or less (see, e.g., Ref. 21). Close to resonance, we would expect such a quantum dot to enhance the signal by at least one order of magnitude compared to the results shown in Fig. 2(d).

Finally, we estimate the minimal detectable spin accumulation. Being a DC measurement, the signal-to-noise ratio is ultimately limited by a device-specific $1/f$ noise. It sets a lower limit on the measurement bandwidth Δ, below which $1/f$ dominates the thermal and shot noises. In a $B = 0 = V$ measurement the signal is given by Eq. (10). For a small spin accumulation, $\delta\mu_s \leqslant 4k_BT$, the thermal noise dominates, $S = 4k_BTG_1$, with $G_1 \approx e^2/h$, where we used Eq. (8a) for a half-open QPC. The signal-to-noise ratio, $I/\sqrt{2\pi S\Delta}$, becomes one for

$$\delta\mu_s = \sqrt{2\max(\hbar\omega, 2\pi k_BT)\sqrt{\hbar\Delta k_BT}},$$

which gives $\delta\mu_s = 5\,\mu$eV using $T = 1$ K, $\hbar\omega = 180\,\mu$eV, and a realistic bandwidth $\Delta = 10$ kHz.

Acknowledgments. This work was supported by the EU project Q-essence, meta-QUTE ITMS NFP 26240120022, APVV-0646-10, CE SAS QUTE, the NSF under Grant No. DMR-0706319, and the Swiss Center of Excellence MANEP. J.F. acknowledges support from the DFG SFB 689 and SPP 1285. We would also like to thank K. Ensslin, Ch. Renner, Ch. Schönenberger and D. Zumbühl for useful discussions.

[1] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
[2] J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Žutić, Acta Phys. Slovaca **57**, 565 (2007); arXiv:0711.1461.
[3] R. H. Silsbee, Bull. Magn. Reson. **2**, 284 (1980); M. Johnson and R. H. Silsbee, Phys. Rev. Lett. **55**, 1790 (1985).
[4] F. J. Jedema, A. T. Filip, and B. J. van Wees, Nature (London) **410**, 345 (2002).
[5] S. P. Dash, S. Sharma, R. S. Patel, M. P. de Jong, and R. Jansen, Nature (London) **462**, 491 (2009).
[6] M. Ciorga, A. Einwanger, U. Wurstbauer, D. Schuh, W. Wegscheider, and D. Weiss, Phys. Rev. B **79**, 165321 (2009).
[7] X. Lou *et al.*, Nat. Phys. **3**, 197 (2007).
[8] J. A. Folk, R. M. Potok, C. M. Marcus, and V. Umansky, Science **299**, 679 (2003).
[9] S. M. Frolov, A. Venkatesan, W. Yu, J. A. Folk, and W. Wegscheider, Phys. Rev. Lett. **102**, 116802 (2009).
[10] E. J. Koop, B. J. van Wees, D. Reuter, A. D. Wieck, and C. H. van der Wal, Phys. Rev. Lett. **101**, 056602 (2008).
[11] P. Stano and Ph. Jacquod, Phys. Rev. Lett. **106**, 206602 (2011).
[12] F. Zhai and H. Q. Xu, Phys. Rev. Lett. **94**, 246601 (2005).
[13] A. A. Kiselev and K. W. Kim, Phys. Rev. B **71**, 153315 (2005).
[14] İ. Adagideli, G. E. W. Bauer, and B. I. Halperin, Phys. Rev. Lett. **97**, 256601 (2006).
[15] T. Christen and M. Büttiker, Europhys. Lett. **35**, 523 (1996); *Mesoscopic Electron Transport*, edited by L. Sohn, L. Kouwenhoven, and G. Schön, NATO Advanced Studies Institute, Series E: Applied Sciences (Kluwer Academic, Dordrecht, 1996), Vol. 345.
[16] V. M. Edelstein, Solid State Commun. **73**, 233 (1990).
[17] I. J. Vera-Marun, V. Ranjan, and B. J. van Wees, Nat. Phys. **8**, 313 (2012); Phys. Rev. B **84**, 241408 (2011).
[18] M. Büttiker, Phys. Rev. B **41**, 7906 (1990).
[19] A. D. Stone and P. A. Lee, Phys. Rev. Lett. **54**, 1196 (1985).
[20] D. M. Zumbühl, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **96**, 206802 (2006).
[21] S. R. Patel, S. M. Cronenwett, D. R. Stewart, A. G. Huibers, C. M. Marcus, C. I. Duruoz, J. S. Harris, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **80**, 4522 (1998).