Bužek, Orszag, and Roško Reply: The main result of our original Letter [1] has been that an interaction of two-level atoms with a single-mode electromagnetic (em) field induces bipartite entanglement between arbitrary two atoms from the set. Specifically, we have analyzed the Dicke model (DM) in the rotating-wave approximation (RWA) that is described by the Hamiltonian $H_0 = (\omega_A/2)\sum_j \sigma_j^z + \omega_F a^\dagger a + \kappa\sum_j(\sigma_j^+ a + \sigma_j^- a^\dagger)$, under the resonant condition $\omega_F = \omega_A$. We have shown that, due to the fact that this Hamiltonian has an integral of motion equal to the number of excitations in the system, at specific values of κ the number of excitation in the ground state of the DM changes abruptly and the first derivative of the energy of the ground state with respect to κ is discontinuous (see the black solid line in Fig. 1). At these values of κ, the concurrence (a measure of bipartite entanglement between two atoms) changes abruptly as well (see the black solid line in Fig. 2). As correctly pointed out by Rzazewski and Wodkiewicz in Ref. [2], the fact that the ground-state energy of the system described by the Hamiltonian H_0 in the large-κ limit is not bounded from below is related with the choice of the Hamiltonian H_0. Interestingly enough, even if one adds counterrotating terms into the Hamiltonian H_0, i.e., if one considers the DM without the RWA described by the Hamiltonian $H_1 = (\omega_A/2)\sum_j \sigma_j^z + \omega_F a^\dagger a + \kappa\sum_j(\sigma_j^+ a + \sigma_j^- a^\dagger + \sigma_j^+ a^\dagger + \sigma_j^- a)$, the ground state of the DM is still not bounded from below (see the red dashed line in Fig. 1). Nevertheless, as one can see from Fig. 2, the effect of interest persists: The interaction of a set of atoms with an em field induces bipartite entanglement between atoms, though the degree of entanglement is weaker compared to the case of H_0. However, it does not exhibit abrupt changes (we note that this is related to the fact that the number of excitations is not an integral of motion of the Hamiltonian H_1), and its maximum is achieved for smaller values of κ.

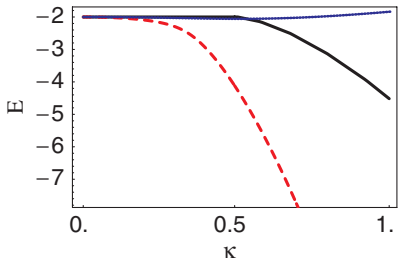

FIG. 1 (color online). The ground-state energy E (measured in units of $\hbar\omega$) of the ground state of the DM with $N = 4$ atoms as a function of the scaled coupling parameter κ/ω. We plot three lines corresponding to different Hamiltonians associated with the DM: the black solid line is related to the energy of H_0, the red dashed line corresponds to H_1, and the blue dotted line corresponds to H_2 (in this case, we consider $\lambda = 1, 7$). We clearly see that, while in the case of the Hamiltonians H_0 and H_1 the energy of the ground state is unbounded from below in the limit $\kappa \to \infty$, the energy of the ground state of the total Hamiltonian H_2 is bounded as a consequence of the field "self-interacting" term $\lambda\kappa^2(a + a^\dagger)^2$.

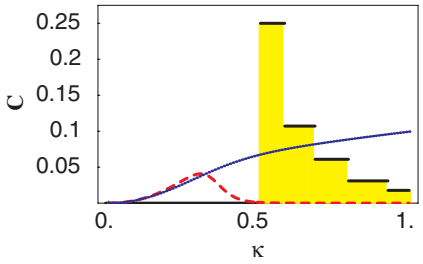

FIG. 2 (color online). The bipartite concurrence between two atoms in the ground state of the DM as a function of the scaled coupling parameter κ/ω. We use the same notation as in Fig. 1. We can conclude that the interaction with the field mode induces a bipartite entanglement between atoms in the ground state of the Dicke model irrespective of the choice of the Hamiltonian.

Finally, when the full Hamiltonian $H_2 = (\omega_A/2)\sum_j \sigma_j^z + \omega_F a^\dagger a + \kappa\sum_j(\sigma_j^+ a + \sigma_j^- a^\dagger + \sigma_j^+ a^\dagger + \sigma_j^- a) + \lambda\kappa^2(a + a^\dagger)^2$ that includes a specific self-interaction term of the field mode $\lambda\kappa^2(a + a^\dagger)^2$ with a coupling parameter $\lambda\kappa^2$ that is derived via a rigorous quantization procedure [3] is considered, it can be found that the energy of the ground state is bounded from below for an arbitrary value of κ (see the blue dotted line in Fig. 1). The effect of the field self-interacting term is not only that it regularizes the energy of the ground state of the DM. Comparing with the results for the Hamiltonian H_1, we see that this term also leads to an enhancement of bipartite entanglement between atoms that interact with the field. This is a rather interesting effect which might have a more general application—a self-interaction of the "mediator" of entanglement can increase the degree of quantum correlations. We conclude that the main result of our Letter, the existence of bipartite entanglement in the ground state of the Dicke model, is valid. Some particular features of this effect depend on a specific choice of the Hamiltonian.

V. Bužek,[1] M. Orszag,[2] and M. Roško[1]
[1]Research Center for Quantum Information
Slovak Academy of Sciences
Dúbravská cesta 9, Bratislava, Slovakia
[2]Facultad de Física
Pontificia Universidad Católica de Chile
Casilla 306, Santiago, Chile

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