

# Notes on optimality of direct characterisation of quantum dynamics

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We argue that the claimed optimality of a new process tomography method suggested in [1] is based on not completely fair comparison that does not take into account the available information in an equal way. We also argue that the method is not a new process tomography scheme, but rather represents an interesting modification of ancilla assisted process tomography method. In our opinion these modifications require deeper understanding and further investigation.

## I. PROBLEM

Authors in [1] proposed a scheme for process reconstruction, which in their opinion is more optimal than the other schemes. Here we would like to question this optimality and even several other points made by authors. The following classification of different reconstruction schemes is used (according to [1]):

1. SQPT (Standard quantum process tomography) In this scheme one analyze how a collection of test states (i.e. a collection of linearly independent states  $\varrho_k \in \mathcal{S}(\mathcal{H}_S)$  forming a basis of the set of operators) is affected by the action of the unknown device. In particular, as a result one gets a set of assignments  $\varrho_k \rightarrow \varrho'_k$  that due to linearity of quantum dynamics specify the channel completely.
2. AAPT (Ancilla-assisted process tomography) In this scheme we employ a suitable ancillary system. In particular, we prepare a state  $\Omega \in \mathcal{S}(\mathcal{H}_S \otimes \mathcal{H}_A)$ , we let the device acts on the system part of the state (ancilla evolves either trivially, or in some known way), and finally, we perform the measurements on the output joint state  $\Omega' = \mathcal{E} \otimes \mathcal{I}[\Omega]$ . With a suitable choice of the input state (it must be from the family of so called faithful states [2]) one can deduce the action of the device  $\mathcal{E}$  from the single state assignment  $\Omega \rightarrow \Omega'$ .
3. DCQD (Direct characterization of quantum dynamics) This scheme is proposed by authors. It is based on AAPT scheme and a simple idea that instead of doing several measurements of the output state one can use a fixed single measurement, but preparing several different input states. This enables us to access to all the parameters of the quantum device in a similar way as in the previous schemes, i.e. we are able to deduce the action of  $\mathcal{E}$  on operators forming the operator basis.

## A. First comment: Quantification of resources.

Authors conclude that their scheme (DCQD) is more optimal than the remaining two. This result is illustrated in the Table II of their paper. They claim the following (for  $n$  qubits):

scheme	$N_{\text{in}}$	$N_{\text{out}}$	$N_{\text{total}}$
SQPT	$4^n$	$4^n$	$16^n$
AAPT	1	$16^n$	$16^n$
DCQD	$4^n$	1	$4^n$

In this table  $N_{\text{in}}$  stands for number of ensembles of different input states,  $N_{\text{out}}$  stands for number of output measurements per one ensemble, and  $N_{\text{total}} = N_{\text{in}}N_{\text{out}}$  is the total number of measurements in the scheme. The optimality of DCQD is based on the comparison of  $N_{\text{total}}$  for all schemes. However, this table is not only not completely precise, and also not completely fair. Each quantum device acting on  $d$  dimensional quantum system is described by  $d^2(d^2 - 1)$  independent parameters that has to be specified in arbitrary (complete) process tomography scheme. For  $n$  qubits this means that we need to specify  $N = 2^{2n}(2^{2n} - 1) = 16^n - 4^n$ . This is exactly the number of independent parameters we are specifying in all the mentioned schemes.

Let us get back to the main point of our comment that the authors did not correctly quantify the minimal number of measurements needed for particular tomography schemes. For SQPT they calculated the number of single qubit measurements. Each qubit measurement effectively provides us with only one number (mean value) that corresponds to one the parameters of the device map  $\mathcal{E}$ . The same logic is used also for calculating the number of measurements in the case of AAPT. Again they assume that each measurement (called by authors joint single qubit measurement) specify only one number. But this is not true, because each measurement now has  $2^n$  different results and the measured probability distribution contains  $2^n - 1$  independent parameters. As we will see exactly this effect stands behind the claimed optimality of DCQD method, because in that case they

consider Bell-state measurements and take into account the information contained in the whole probability distribution (not only in mean value). Therefore, this measurement allows authors to gain more information about the transformation, which finally results in the smaller total number of measurements (in comparison with AAPT scheme not treated in the same way). From this point of view the conclusion is not that surprising, since they did not use the available resources in the same manner. The problem is that for SQPT scheme for  $d$  dimensional system each measurement gives  $d - 1$  independent numbers, whereas for AAPT and DCQD each measurement gives  $d^2 - 1$  independent numbers. For suitable choices of measurements these numbers specifies the corresponding number of parameters of the device map  $\mathcal{E}$ . The correct version of the table for a single qubit (the generalization for arbitrary dimensional system is obvious) is the following:

scheme	$N_{\text{in}}$	$N_{\text{out}}$	$N_{\text{total}}$
SQPT	4	3	12
AAPT	1	4	4
DCQD	4	1	4

Thus, we see that AAPT and DCQD are (after proper evaluation) of the same quality. However, also such conclusion is, in our opinion, not correct and fair. To really compare the resources for different tomography schemes one would need to be able to quantify the complexity of the preparations and measurements. In our opinion such quantification will result in mutual equivalence of all these schemes. To see some differences one should indeed include and discuss particular physical systems, or discuss process reconstructions from incomplete data, or small statistical samples.

In the above table it is easy to see that each scheme provides us with the same number of numbers. We remind that for AAPT and DCQD schemes the measurements have 4 outcomes, i.e. the measured probabilities contain 3 independent numbers. Therefore, for total number of specified parameters we obtain: for AAPT  $1 \times 4 \times 3 = 12$  and for DCQD  $4 \times 1 \times 3 = 12$ . This is not surprising and only means that there is no redundant information in all of the schemes.

Finally, let us consider POVM instead of von Neumann measurements and perform similar comparison of resources. In this case the state tomography can be accomplished by a single so called informationally complete POVM, i.e.  $N_{\text{out}} = 1$  for all possible reconstruction schemes. The quantification of the quality of schemes in the number of POVMs would lead to the following table for the qubit:

scheme	$N_{\text{in}}$	$N_{\text{out}}$	$N_{\text{total}}$
SQPT	4	1	4
AAPT	1	1	1
DCQD	4	1	4

Thus the DCQD is as efficient as SQPT scheme and AAPT scheme is much better when one calculate the resources in terms of realized POVMs. However, let us remind that we do not think that this is the way how to compare different reconstruction schemes. In fact, to perform the reconstruction experimentally one really needs to perform the tomography of input states as well. This also should be counted among the needed resources.

### B. Second comment: usage of quantumness.

In our view the DCQD scheme is just a modification of the AAPT scheme, in which instead of performing several measurements we use different preparations. We do not share the opinion of authors that only in DCQD scheme the quantumness is used. The meaning of quantumness is equivalent to the necessity of entangled input states and entangled measurements in the scheme. Similarly, like AAPT scheme can be realized with, or without entanglement, the same holds also for DCQD method.

### C. Third comment: direct state tomography.

Authors claimed that the suggested DCQD scheme access the process directly, i.e. without any usage of state tomography. This is indeed true that in this scheme we do not have sufficient knowledge about the output state, and therefore we can perform only partial state tomography. In fact, neither for AAPT scheme we really perform complete state tomography, since we need to specify only  $d^4 - d^2$  parameters instead of  $d^4 - 1$  required for  $d^2$  dimensional system. The others parameters describe the local state of the ancilla system which is not changed and known. We agree that in AAPT and SQPT one can in principle perform complete state tomography, but we do not think this is somehow important. To learn anything about the quantum process one needs to perform measurements on states. It is not surprising that one can do even more, i.e. we can exclude also the complete knowledge about the input state preparators. It is sufficient only to analyze (for a fixed measurement) the transformation of input/output probability distributions. However, we would not call it new process tomography method. In our opinion there are logically only two different methods: 1) SQPT and 2) AAPT. It is our choice how we will use the particular resources, i.e. the preparators and the measurements. A similar process tomography method (as DCQD scheme is related to AAPT scheme) can be designed also for SQTP scheme. That is, the fact that we have access to particular state assignments (state tomography) is not that important in the process tomography method and does not represent some redundant information.

## II. CONCLUSION

Let us conclude that in our opinion the authors did not succeed to show that their scheme is more optimal. It seems that the scheme can be more efficient in cases when we do not have the opportunity to perform all measurements. However, since the tomography of input states (calibration of preparators) is a necessary part of process tomography, these measurements (allowing complete state tomography) must be available. It can be very difficult to experimentally perform some measurements and only in this sense the DCQD scheme can be more optimal. The modifications of SQPT and AAPT schemes are very interesting options that can be very useful in particular process estimation problems (for instance incomplete

reconstructions) and therefore they deserve further investigation. The proposed DCQD scheme represents an interesting example within such program and provides an important estimation algorithm that can be very useful and optimal in particular physical realizations.

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