QUANTUM DYNAMICS OF OPEN SYSTEMS FROM THE POINT OF INFORMATION TRANSFER

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One of the most complex phenomena in quantum theory is the dynamics of open systems. In general, one can assume an open system S to be a part of a larger closed system composed of the open system and the environment R. The whole $S+R$ system evolves unitarily, and the question of irreversibility of dynamics of open systems is then a great issue. How do irreversible dynamics of the system S emerge from a unitary evolution of the $S+R$ system? We present an analysis of irreversibility from the point of view of information transfer between the system qudit and reservoir qudits.

1 Introduction

Let us consider the following model: an open system, S, represented by a single qudit (quantum system with d-dimensional Hilbert space) initially prepared in an unknown state $\varrho_S^{(0)}$, and the environment called the reservoir, R, composed of N qudits all prepared in the state ξ, which is arbitrary but the same for all reservoir qudits. We will enumerate the qudits of the reservoir and denote the state of the k-th qudit as ξ_k. From the definition of the reservoir it follows that initially $\xi_k = \xi$ for all k, so the initial state of the reservoir is described by the density matrix $\xi^{\otimes N}$. We will assume that at each time step the system qudit interacts with just a single qudit from the reservoir and let U be a unitary operator representing the interaction between a system qudit and one of the reservoir qudits (see Fig. 1). In what follows we will discuss two different types of evolution: the *deterministic* collision model and the *random* collision model.

2 Deterministic model

We will restrict the evolution in the following way. We will assume that the system qudit can interact with each of the reservoir qudits at most once. Moreover, the reservoir qudits are not allowed to interact with one another. These two additional assumptions lead to a deterministic model of evolution in which the system particle interacts with the reservoir via a sequence of interactions between the system qudit and N reservoir qudits all prepared in the state ξ. The final state of the system is then described by the density

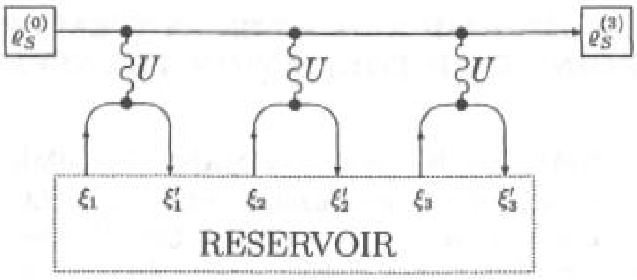

Figure 1. A simple collision-like model of homogenization with just three reservoir qudits involved.

operator

$$\varrho_S^{(N)} = \mathrm{Tr}_R \left[U_N \dots U_1 \left(\varrho_S^{(0)} \otimes \xi^{\otimes N} \right) U_1^\dagger \dots U_N^\dagger \right] , \tag{1}$$

where $U_k := U \otimes (\bigotimes_{j \neq k} \mathbf{1}_j)$ describes the interaction between the k-th qudit of the reservoir and the system qudit. So far we have not specified the unitary transformation U representing the interaction between qudits. Instead, we pose another condition on the evolution of the open system S and the reservoir R. We assume that states of the reservoir qudits change just a little, and that after N interactions the state of the system qudit becomes close to the initial state of the reservoir qudits. Formally,

$$\forall k, 1 \leq k \leq N \dots\dots D(\xi_k', \xi) \leq \delta; \tag{2}$$

$$\forall N \geq N_\delta \dots\dots D(\varrho_S^{(N)}, \xi) \leq \delta , \tag{3}$$

where $D(.,.)$ denotes some distance (e.g., a trace norm) between the states under consideration, $\delta > 0$ is a small parameter which is chosen a priori to determine the degree of the homogeneity and $\xi_k' := \mathrm{Tr}_S[U \varrho_S^{(k-1)} \otimes \xi U^\dagger]$ is the state of the k-th reservoir qudit after its interaction with the system qudit. Since all final states are almost identical, we can consider the process to be a quantum homogenization:[1,2] out of N qudits prepared in the same state ξ and a single object prepared in an arbitrary state $\rho_S^{(0)}$ we obtain $N+1$ homogenized objects.

Quantum homogenization that satisfies the conditions given by Eq.(2) and Eq.(3) can be realized by the partial swap operation[1,2]

$$P_{kj}(\eta) = \mathbf{1}_k \otimes \mathbf{1}_j \cos(\eta) + i \sin(\eta) S_{kj} , \tag{4}$$

where S_{kj} is the swap operation that acts on the qudits k and j as follows, $S_{kj}|\psi_k\rangle \otimes |\phi\rangle_j = |\phi\rangle_k \otimes |\psi\rangle_j$.

3 Random collision model

The conditions imposed in the case of the deterministic model are a bit restrictive. Therefore now we drop some of them in order to obtain a more realistic model. First of all, we allow the system particle to interact with any of the reservoir particles an arbitrary number of times. Secondly, we allow the reservoir particles to interact with one another. Thus the unitary transformation representing the interaction between two qudits is still described by the partial swap operation (4), but now the sequence of interactions may include all possible pairs of qudits. Such a model is usually called a *random* collision model. The immediate consequence of this is that there is more than one possible sequence for how the system particle can interact with the reservoir. (We are not counting as different those sequences which differ only by permutation of the reservoir particles, because we are free to relabel the reservoir particles.) In other words, the model becomes random and we have to take into account all possible paths or equivalently all possible scenarios of interactions between individual particles.

To illustrate our physical model, let us consider a situation in which a single system qubit is initially prepared in the state $|1\rangle$, while N reservoir qubits are prepared in the state $|0\rangle^{\otimes N}$, where we consider $\{|0\rangle, |1\rangle\}$ to be qubit basis vectors. Due to the sequence of k partial-swap operations (4), the system qubit is changed and is described by the density operator $\varrho^{(k)} = z|1\rangle\langle 1| + (1-z)|0\rangle\langle 0|$, where the parameter z depends on the specific sequence of partial swap interactions. In Fig. 2 we plot the parameter z as a function of the number k of interactions between the system and the reservoir qubits.

4 Reversibility

In both the deterministic as well as the random collision models at the end of the homogenization process all the particles are essentially in the same state (equal to the initial state of the reservoir particles). Since the whole process is unitary, the original information encoded in the state of the system particle cannot be lost — it has simply been transferred into the correlations between the interacting particles.[1,3] So the question is can it be recovered? That is, can we reverse the whole process of the homogenization? It turns out (for more details see Refs. 1,4) that it can be reversed providing the *classical* information about the sequence of interactions is available. If so, then the original *quantum* information, that is the initial state of the system qudit, can be completely recovered from the homogenized system by the specific sequence of inverse partial-swap operations. On the other hand, if the classical information about the order of interactions is not available, then the original quantum information is irreversibly lost in the homogenized system. We conclude that the necessary condition for the recovery of quantum information is the availability of the specific classical information.[4]

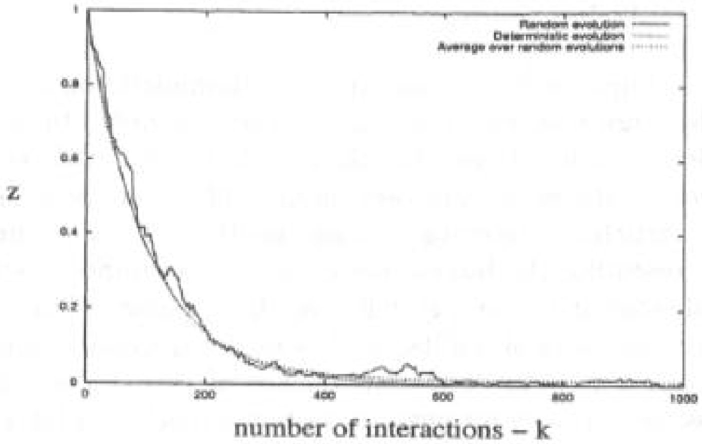

Figure 2. Results of numerical simulations of a quantum homogenization model. We assume a system qubit initially prepared in the state $|1\rangle$, while N reservoir qubits are prepared in the state $|0\rangle^{\otimes N}$. Due to the sequence of k partial-swap operations (4), the system qubit is changed and is described by the density operator $\varrho^{(k)} = z|1\rangle\langle 1| + (1-z)|0\rangle\langle 0|$ We plot the parameter z as a function of number of interactions k between the system and the reservoir qubits for three different cases: the deterministic model (dotted line), a specific trajectory with a random model (solid line), and the outcome of random model averaged over 1000 trajectories (dashed line).

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