Amplification of superposition states in phase-sensitive amplifiers

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We study statistical properties of quantum superposition states (Schrödinger-cat states) amplified by phase-sensitive (squeezed) amplifiers. We show that the phase-sensitive amplifier with a properly chosen phase can preserve quantum coherences and nonclassical behavior of the Schrödinger-cat-state input even for a gain factor G larger than 2. In particular, we show that for an even coherent state (CS) phase-sensitive amplifiers can preserve squeezing for $G>2$ but simultaneously in the process of amplification the noise added by the amplifier leads to a rapid increase of fluctuations in the photon number. Because of the finite maximum degree of squeezing obtainable for the even CS the maximum gain factor G_m for which squeezing can still be observed in the output state is finite. The phase-sensitive amplifier with a properly chosen phase can also reduce fluctuations in the photon number of the initial even CS. Nevertheless, one cannot amplify the initial even CS with super-Poissonian photon statistics into the state with sub-Poissonian photon statistics.

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Recently Haroche and co-workers [1] have proposed a conceptually simple but elegant method to prepare quantum superposition states confined in a microwave cavity. In particular, the superposition of two coherent states $|\xi\rangle$ and $|-\xi\rangle$ which are 180° out of phase with respect to each other can be produced in the cavity. This pure quantum-mechanical superposition state may be of the form

$$|\xi\rangle_{\text{even}}=\mathcal{N}^{1/2}(|\xi\rangle+|-\xi\rangle)\,,\qquad \mathcal{N}^{-1}=2[1+\exp(-2|\xi|^2)]\,. \tag{1a}$$

The state (1a) is called the even coherent state (CS) and exhibits various nonclassical effects such as quadrature squeezing, higher-order squeezing, and oscillations in the photon number distribution.

Sherman and Kurizki [2] have pointed out a scheme of macroscopic quantum superpositions generation based on conditional measurements in the two-photon resonant Jaynes-Cummings model. This scheme allows the generation of superposition states with any relative phase of the form

$$|\Psi\rangle=\mathcal{N}^{1/2}(\theta)[e^{i\phi}|\xi e^{i\theta/2}\rangle+e^{-i\phi}|\xi e^{-i\theta/2}\rangle]\,,$$
$$\mathcal{N}^{-1}=2\{1+\cos(|\xi|^2\sin\theta+2\phi)\times\exp[-2|\xi|^2\sin^2(\theta/2)]\}\,. \tag{1b}$$

The even CS (1a) and the Sherman-Kurizki state (1b) represent particular realizations of a more general class of quantum superposition states,

$$|\Psi\rangle=\mathcal{N}^{1/2}\left[\sum_{j=1}^{N}e^{i\phi_j}|\xi_j\rangle\right]\,,\qquad \mathcal{N}^{-1}=\sum_{j,k=1}^{N}e^{i(\phi_j-\phi_k)}\langle\xi_k|\xi_j\rangle\,, \tag{2}$$

where $|\xi_j\rangle$ is the coherent state with the complex amplitude ξ_j [3]. The nonclassical effects mentioned above emerge as a direct consequence of the quantum interference between component coherent states $|\xi_j\rangle$ and $|\xi_k\rangle$ [4]. In particular, the even CS exhibits a substantial degree of quadrature squeezing. The maximum value of the squeezing can be observed for relatively small values of the amplitude $|\xi|$ of the component coherent states (for approximately $|\xi|\simeq 0.8$), while oscillations in the photon number distribution can be observed for any value of $|\xi|$ (see below). In the experiment proposed by Haroche and co-workers one can expect the amplitude $|\xi|$ of the order of unity [5] which is rather small. Therefore the natural question arises of whether it is possible to amplify the superposition states (1) and simultaneously preserve its nonclassical behavior.

It is well known that amplification degrades an optical signal and rapidly destroys quantum features that may have been associated with the signal. In particular, for an arbitrarily squeezed input the phase-insensitive amplifier provides a squeezed output only for a gain smaller than 2. To overcome this cloning limit phase-sensitive amplifiers have been proposed, for which squeezed output for a gain larger than 2 can be obtained [6].

In this paper we will analyze statistical properties of the even CS amplified by a phase-sensitive amplifier. We will study the influence of the phase-sensitive amplifier on the degree of the quadrature squeezing and on the photon statistics of the field mode under consideration.

The state of the quantum-mechanical system can be characterized by a set of expectation values of the system operators. In particular, those of a harmonic oscillator are described by the mean values of the bosonic operators $\hat{a}^\dagger$ and $\hat{a}$. The moments of the antinormally ordered bosonic operators can be evaluated with the help of the quasiprobability function $Q(\alpha,t)$ [7]:

$$\langle \hat{a}^m(\hat{a}^\dagger)^n\rangle = \int d^2\alpha\, \alpha^m(\alpha^*)^n Q(\alpha,t) , \tag{3}$$

where $Q(\alpha,t)$ is defined as

$$Q(\alpha,t)=\frac{1}{\pi}\langle \alpha|\hat{\rho}(t)|\alpha\rangle . \tag{4}$$

To measure the degree of the quadrature squeezing of the even CS, we introduce two parameters S_j (for review articles on squeezing see Refs. [8]):

$$S_j=\frac{\langle(\Delta\hat{a}_j)^2\rangle-\frac{1}{4}}{\frac{1}{4}} , \quad j=1,2 , \tag{5}$$

which are related to the variances $\langle(\Delta\hat{a}_j)^2\rangle = \langle(\hat{a}_j)^2\rangle - \langle\hat{a}_j\rangle^2$ of the quadrature operators

$$\hat{a}_1=\frac{\hat{a}+\hat{a}^\dagger}{2} , \quad \hat{a}_2=\frac{\hat{a}-\hat{a}^\dagger}{2i} , \tag{6}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the usual annihilation and creation operators of the single-mode field ($[\hat{a},\hat{a}^\dagger]=1$). We say that the state under consideration is squeezed if the variance of the quadrature operator is less than the vacuum limit $\frac{1}{4}$, i.e.,

$$\langle(\Delta\hat{a}_j)^2\rangle<\tfrac{1}{4} , \tag{7}$$

which means that the squeezing condition now reads $S_j^{(2)}<0$, and the maximum squeezing corresponds to $S_j^{(2)}=-1$.

For the even CS (1a) with the real amplitude ζ we find the following expressions for the squeezing parameters S_j:

$$S_1=\frac{4\zeta^2}{1+\exp(-2\zeta^2)} , \quad S_2=-\frac{4\zeta^2\exp(-2\zeta^2)}{1+\exp(-2\zeta^2)} , \tag{8}$$

from which it follows that the even CS with the given choice of ζ is squeezed in the $\hat{a}_2$ quadrature ($S_2<0$) and the maximum degree of squeezing is equal to approximately 50% for $\zeta\simeq0.8$. For the values of $\zeta\geq0.8$ the degree of squeezing is smaller than 50% and in the limit $\zeta\rightarrow\infty$ we find $S_2\rightarrow0$.

If we assume the even CS with $\zeta\simeq0.8$ as the input state of the amplifier then we have to expect that the degree of squeezing of the output state is deteriorated by two effects. First, the amplitude of the component states is amplified, i.e., $\zeta_{\rm out}=G^{1/2}\zeta$, which means that the degree of squeezing should be smaller than in the case $\zeta\simeq0.8$. Second, noise added by the amplifier inevitably destroys nonclassical features of the input state. The amount of noise transferred from the amplifier to the field mode depends on the nature of the amplifier. In what follows we consider the phase-sensitive amplifier which can be implemented as a stream of three-level atoms in a ladder configuration with equispaced levels injected into the cavity where the initial state of the electromagnetic field has been prepared. Each atom is initially prepared in a coherent superposition of the lowest and the uppermost states and the atomic transitions are in resonance with the field in the cavity [9]. We denote the population in the uppermost state by σ_{33}, the population in the lowest state by σ_{11} and the coherences between them by σ_{13} and σ_{31} ($=\sigma_{13}^*$). The dynamics of the field mode coupled to the phase-sensitive amplifier is in the Born and Markov approximation governed by the Fokker-Planck equation of the Q function, which in the interaction picture can be written as [10]

$$\frac{\partial Q(\alpha,t)}{\partial t}=\gamma\left[N\frac{\partial^2}{\partial\alpha^*\partial\alpha}-\frac{1}{2}\left[\frac{\partial}{\partial\alpha^*}\alpha^*+\frac{\partial}{\partial\alpha}\alpha\right]+\frac{M^*}{2}\frac{\partial^2}{\partial\alpha^2}+\frac{M}{2}\frac{\partial^2}{\partial\alpha^{*2}}\right]Q(\alpha,t) , \tag{9}$$

where

$$N=\frac{\sigma_{11}}{\sigma_{33}-\sigma_{11}} . \tag{10}$$

The parameter γ is proportional to the population inversion, $\sigma_{33}-\sigma_{11}$, and M is proportional to the atomic coherence σ_{13}. The parameter γ is always positive because $\sigma_{33}>\sigma_{11}$ for the amplifier. If the phase-sensitive parameter M is put equal to zero then the Fokker-Planck equation (9) reduces into the equation describing the phase-insensitive amplification of the single-mode field [11]. The squeezing parameter M has the limit determined by the value of N [12]:

$$|M|^2\leq N(N+1) . \tag{11}$$

The gain G of the amplifier is defined as [13]

$$G=\exp(\gamma t) . \tag{12}$$

In this paper we study in detail the evolution of the single-mode field which is initially prepared in the even CS (1) with the real amplitude ζ of the composition states $|\pm\zeta\rangle$. The squeezing parameter M is, in general, complex but to make our analytical results more transparent we will consider only the case of M real. The Q function for the even CS can be written as

$$Q(\alpha,0)=\frac{\mathcal{N}}{\pi}[Q_{\rm mix}(\alpha,0)+Q_{\rm int}(\alpha,0)] , \tag{13}$$

with

$$Q_{\rm mix}(\alpha,0)=\exp(-\alpha_i^2)\{\exp[-(\alpha_r-\zeta)^2]+\exp[-(\alpha_r+\zeta)^2]\}\ , \quad (14a)$$

$$Q_{\rm int}(\alpha,0)=2e^{-\zeta^2}\exp(-\alpha_r^2-\alpha_i^2)\cos(2\zeta\alpha_i)\ , \quad (14b)$$

where α_r and α_i are the real and imaginary parts of α. The mixture part $Q_{\rm mix}$ of the Q function of the even CS consists of two Gaussian peaks localized around $\alpha_r=\pm\zeta$. The interference part $Q_{\rm int}$ has an oscillatory behavior and has its maximum at the origin of phase space $\alpha=\{0,0\}$. This term arises as a direct consequence of the quantum interference between coherent states $|\zeta\rangle$ and $|-\zeta\rangle$ and is responsible for nonclassical behavior of the even CS.

It has been shown by Peřinová, Lukš, and Szlachetka [14] that if the initial Q function of the quantum system is Gaussian then the solution of the Fokker-Planck equation (9) is also Gaussian with time-dependent parameters. According to the superposition principle, if $Q_{\rm mix}(\alpha,t)$ and $Q_{\rm int}(\alpha,t)$ are solutions of the linear Fokker-Planck equation (9) then $C_1Q_{\rm mix}(\alpha,t)+C_2Q_{\rm int}(\alpha,t)$, where C_1 and C_2 are constant, is also a solution of this equation. The function $Q_{\rm mix}(\alpha,t)$ describing the mixture part of the state is easily obtained with the use of the above argument because the initial $Q_{mix}(\alpha,0)$ is Gaussian as shown by Eq. (14a). Although the interference part given by Eq. (14b), is not Gaussian it is a real part of the complex Gaussian function

$$Q_c(\alpha,0)=\exp[-2\zeta^2-\alpha_r^2-(\alpha_i-i\zeta)^2] \quad (15)$$

so that one can use Peřinova's argument to obtain the dynamics of the interference part of the Q function. To find $Q_{\rm int}(\alpha,t)$ we first evaluate the complex Gaussian quasiprobability $Q_c(\alpha,t)$ under the initial condition (15), and then we extract from it the real part representing $Q_{\rm int}(\alpha,t)$.

The solution of the Fokker-Planck equation (9) for the Q function with the initial condition (13) reads

$$Q(\alpha,t)=\frac{\mathcal{N}/\pi}{(a_qb_q)^{1/2}}\exp\left[\frac{-\alpha_i^2}{a_q}\right]\left\{\exp\left[\frac{-[\alpha_r-\zeta(t)]^2}{b_q}\right]+\exp\left[\frac{-[\alpha_r+\zeta(t)]^2}{b_q}\right]+2\exp\left[-2\zeta^2+\frac{\zeta^2(t)}{a_q}-\frac{\alpha_r^2}{b_q}\right]\cos\left[\frac{2\zeta(t)\alpha_i}{a_q}\right]\right\}, \quad (16)$$

where the total additional noise factors a_q and b_q are defined as

$$a_q=N(t)+G-M(t),\quad b_q=N(t)+G+M(t)\ , \quad (17)$$

and

$$N(t)=N(G-1),\quad M(t)=M(G-1)\ . \quad (18)$$

The time-dependent amplitude of the component states at $t>0$ is

$$\zeta(t)=\zeta\sqrt{G}\ . \quad (19)$$

From the Eqs. (16)–(18) it is clearly seen that the value of $N(t)+G$ blurs the Q function, which means that the noise is added inevitably as the gain increases. However, the nonzero value of $M(t)$ can slow down the blurring of the function in one axis (α_i when $M>0$, α_r when $M<0$) at the expense of the increased noise in the other axis. For this reason we call a_q and b_q the total additional noise factors keeping $N(t)+G$ as the phase-insensitive pure noise factor. If the modes of the amplifier are not correlated ($M=0$), the Q function (16) reduces to the equation describing the field mode which is amplified with the noise given in a phase-insensitive fashion.

Once the explicit form of the Q function is known one can readily derive the Wigner function $W(\alpha,t)$, which is more convenient for investigation and visualization of the coherence [4,7]. The form of the Wigner function is analogous to that of the Q function (16) with the modified total additional noise factors

$$a_w=N(t)+\exp(\gamma t)-M(t)-\tfrac{1}{2}\ ,$$
$$b_w=N(t)+\exp(\gamma t)+M(t)-\tfrac{1}{2}\ . \quad (20)$$

In other words, replacing a_q and b_q in Eq. (16) by a_w and b_w we obtain the Wigner function $W(\alpha,t)$. Using the Wigner function and the relation

$$\hat{\rho}=\frac{1}{\pi}\int d^2\alpha\, W(\alpha,t)\hat{T}(\alpha)\ , \quad (21)$$

one can find the density operator $\hat{\rho}$. The operator $\hat{T}(\alpha)$ is defined as

$$\hat{T}(\alpha)=\sum_{n,m=0}^{\infty}T_{nm}(\alpha)|n\rangle\langle m|\ , \quad (22a)$$

and

$$T_{nm}(\alpha)=\left[\frac{n!}{m!}\right]^{1/2}2^{m-n+1}(-1)^n(\alpha^*)^{m-n}\times\exp(-2|\alpha|^2)L_n^{(m-n)}(4|\alpha|^2)\ , \quad (22b)$$

where $L_n^{(m-n)}(x)$ is the Laguerre polynomial.

One of the nonclassical effects which have their origin in the quantum interference between coherent states is the presence of oscillations in the photon number distribution. The photon number distribution $P(n,t)$ of the field mode at time t can be expressed in terms of the Wigner function $W(\alpha,t)$ of the state under consideration and the Wigner function $W_n(\alpha)$ of the number state $|n\rangle$ [11]:

$$P(n,t)=\pi\int d^2\alpha\, W(\alpha,t)W_n(\alpha)\ . \tag{23}$$

Using Eq. (23) we can evaluate the explicit expression for $P(n,t)$ describing the time evolution of the photon number distribution of the field mode initially prepared in the even CS amplified by the phase-sensitive amplifier. The photon number distribution of the even CS exhibits significant oscillations:

$$P(n,0)=\frac{2\exp(-2\zeta^2)}{1+\exp(-2\zeta^2)}\frac{\zeta^{2n}}{n!}\ ,\quad \text{if } n=0,2,4,\ldots, \tag{24a}$$

$$P(n,0)=0\ ,\quad \text{if } n=1,3,5,\ldots\ . \tag{24b}$$

The oscillations in the photon number distribution described by Eqs. (24) are similar to those of the squeezed vacuum [8] and they can serve as a good indication that the state under consideration exhibits nonclassical behavior.

The amplification process adds noise to the quantum system which leads to deterioration of nonclassical effects. In particular, the oscillations in the photon number distribution vanish when the even CS is amplified. Nevertheless, the way in which the oscillations disappear depends on the nature of the amplifier. In Fig. 1 we plot the photon number distribution of the field mode initially prepared in the even CS (1) amplified by the factor $G=2.5$. We see that in the case of the phase-sensitive amplifier ($M\neq 0$) with a properly chosen phase of squeezing one can preserve a reminiscence of oscillations in the photon number distribution for a relatively high G [compare lines d and a in Fig. 1 plotted for $N=3$ and $M=\sqrt{12}$ and $M=-\sqrt{12}$, respectively]. The phase-insensitive amplifier deteriorates quantum coherences rapidly giving no chance to preserve nonclassical effects for significantly large gain factors (see lines b and c in Fig. 1). We note here that one can prove the above statement more rigorously by evaluating the entropy S:

$$S=-k\,\mathrm{Tr}\hat{\rho}\ln\hat{\rho}\ , \tag{25}$$

or the purity parameter $S_{\rm pur}$

$$S_{\rm pur}=1-\mathrm{Tr}\hat{\rho}^2\ , \tag{26}$$

of the field mode under consideration. The purity parameter $S_{\rm pur}$ can be written with the use of Eqs. (21) and (22) in the form

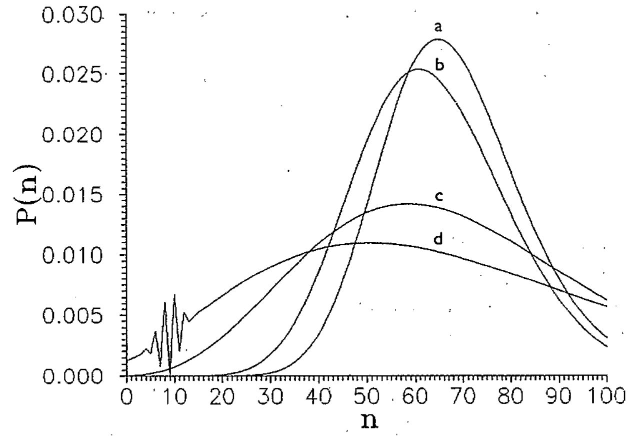

FIG. 1. The photon number distribution $P(n,t)$ of the amplified field mode for the gain of $G=2.5$ when the field is initially prepared in the even coherent state with $\zeta=5$. The phase-sensitive amplifier is characterized by $N=3$ and $M=-\sqrt{12}$ (curve a) and by $N=3$ and $M=\sqrt{12}\,(d)$; the phase-insensitive amplifier is characterized by $N=M=0\,(b)$ and by $N=3$ and $M=0\,(c)$.

$$S_{\rm pur}(t)=1-\frac{1}{\pi^2}\sum_{n,m=0}^{\infty}\left|\int d^2\alpha\, W(\alpha,t)T_{n,m}(\alpha)\right|^2 . \tag{27}$$

Using this expression one can ultimately determine whether the state under consideration is in the pure superposition ($S_{\rm pur}=0$) or in a statistical mixture ($S_{\rm pur}>0$), that is the increase of the function $S_{\rm pur}$ indicates the loss of the quantum coherence. The explicit evaluation of the expression (27) is not always straightforward and therefore some indirect indication of the loss of the quantum coherence can be very helpful. In particular, there exists a very good relation between the quantum coherence and squeezing (for instance, see Ref. [4]).

In our case it is instructive to study the time evolution of the squeezing parameter $S_2(t)$ of the initial even CS. Before we evaluate this function we present a general expression for the antinormally ordered moments (3) using the explicit solution (16) for the Q function:

$$\langle\hat{a}^m(\hat{a}^\dagger)^n\rangle=\sum_{\substack{0\le l\le m,\ 0\le k\le n\\ l+k=\text{even}}}\frac{2\mathcal{N}(-1)^k m!n!}{l!k!(m-l)!(n-k)!}\left\{i^{(l+k)}f!\zeta^f(l+k-1)!!\left[\frac{a_q}{2}\right]^{(l+k)/2}\sum_{j=0}^{f/2}\frac{1}{(f-2j)!j!}\left[\frac{b_q}{4\zeta^2(t)}\right]^j\right.$$
$$+e^{-2\zeta^2}(l+k)!\zeta^{(l+k)}(f-1)!!\left[\frac{b_q}{2}\right]^{f/2}$$
$$\left.\times\sum_{j=0}^{(l+k)/2}\frac{1}{(l+k-2j)!j!}\left[-\frac{a_q}{4\zeta^2(t)}\right]^j\right\}, \tag{28}$$

with $f=m+n-l-k$ when $m+n$ is even, otherwise $\langle \hat{a}^m(\hat{a}^\dagger)^n\rangle=0$. The squeeze parameter $S_2(t)$ is evaluated by substituting the appropriate moments of Eq. (28) into Eq. (5):

$$S_2(t)=2\left[-1+a_q+\frac{2e^{-2\zeta^2}}{1+e^{-2\zeta^2}}\zeta^2(t)\right], \quad (29)$$

where the time-dependent amplitude $\zeta(t)$ is given by Eq. (19). The initial even CS is noise reduced in the a_2 quadrature as shown in Eq. (8), i.e., $S_2<0$ for $t=0$. Because the noise reduction in one quadrature is achieved at the expense of the increased noise in the other quadrature we have $S_1>0$ at $t=0$. Due to the fact that the amplification is accompanied by the inevitable increase of noise in the light field the a_1 quadrature will remain superfluctuant for any $t>0$. The amount of noise in the initially squeezed quadrature will increase as well. Nevertheless this quadrature will remain squeezed up to a certain moment (i.e., up to a certain value of the gain parameter G).

In order to find the maximum gain G_m for which the quadrature squeezing can still be observed at the output for the given input even CS we set the left-hand side of Eq. (29) equal to zero and solve the equation for G:

$$G_m=\left[1-\frac{1}{1+N-M}\frac{2\zeta^2}{1+e^{2\zeta^2}}\right]^{-1}. \quad (30)$$

In Fig. 2 we plot the maximum gain for the various N and M values. We see that G_m is optimized when the initial amplitude ζ of the composite coherent state is around 0.8. When M is positive the rate of adding noise is reduced in the α_i axis (which is equivalent to the a_2 quadrature) so that for a given N the maximum gain G_m is larger as M is larger. When the amplifier is ideally squeezed with $M=\sqrt{N(N+1)}$ the maximum amplification without losing quadrature squeezing is larger than 2. On the other hand, if $M<0$ the quadrature squeezing is lost even for a small amplification gain.

Finally we turn our attention to the variance of the photon number. In particular, we will study the time evolution of the Mandel Q parameter defined as

$$Q=\frac{\langle(\Delta\hat{n})^2\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle} =\frac{\langle\hat{a}^2(\hat{a}^\dagger)^2\rangle-\langle\hat{a}\hat{a}^\dagger\rangle^2-\langle\hat{a}\hat{a}^\dagger\rangle}{\langle\hat{a}\hat{a}^\dagger\rangle-1}-1, \quad (31)$$

which measures the deviation from pure Poissonian number fluctuation. If $Q=0$ the light field has Poissonian photon statistics. When $Q>0$ ($Q<0$) the field has super- (sub-) Poissonian photon statistics.

For the field initially prepared in the even coherent state amplified by the phase-sensitive amplifier the Mandel Q parameter is readily evaluated as the appropriate moments of Eq. (28) are substituted into the definition (31). In Fig. 3 we plot the Mandel Q parameter for the field initially prepared in the even CS state amplified by the squeezed amplifier. The initial even CS exhibits super-Poissonian photon statistics. From Fig. 3 we see that the initial super-Poissonian state never becomes sub-Poissonian, which is in agreement with Barnett and Gilson [15], who have shown that in order to observe sub-Poissonian photon statistics at the output of the amplifier the input has to be sub-Poissonian, i.e., the photon number variance increases as the initial field is amplified. However as shown in Fig. 3 (curve b) for $N=3$ and $M=-\sqrt{12}$ (ideally squeezed amplifier) we see that the value of the Mandel Q parameter is reduced at the first moments of the amplification. On the other hand, the fastest increase of the fluctuations in the photon number can be seen in the case of the ideally squeezed amplifier with $M=\sqrt{12}$ [see line a in Fig. 3].

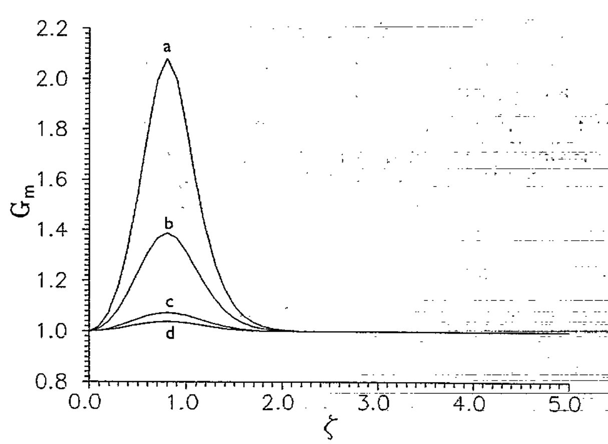

FIG. 2. The maximum gain G_m that permits a quadrature squeezing output. The field mode is initially prepared in the even coherent state with $\zeta=0.8$ interacting with the phase-sensitive amplifier characterized by $N=3$ and $M=\sqrt{12}$ (curve a), and by $N=3$ and $M=-\sqrt{12}$ (d). When the amplifier is phase insensitive the maximum gain is plotted for $N=M=0$ (b) and for $N=3$ and $M=0$ (c).

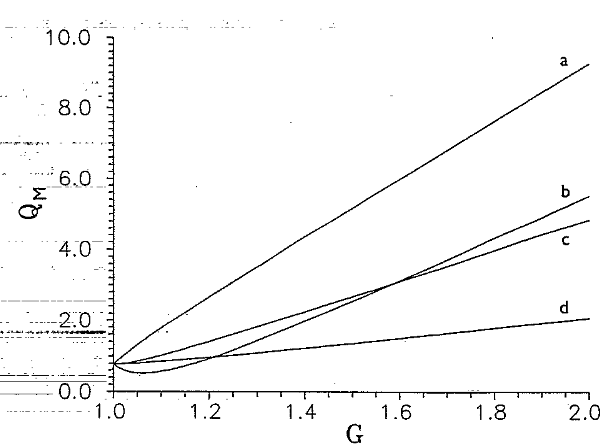

FIG. 3. The time evolution of the Q parameter of the field mode initially prepared in the even coherent state amplified by the interaction with the phase-sensitive amplifier characterized by $N=3$ and $M=\sqrt{12}$ (curve a) and by $N=3$ and $M=-\sqrt{12}$ (b). When the amplifier is phase insensitive the Q parameter is plotted for $N=3$ and $M=0$ (c) and for $N=M=0$ (d).

In conclusion, we have studied statistical properties of quantum superposition states amplified by phase-sensitive amplifiers. We have shown that quantum coherences are deteriorated by the action of the amplifier, which leads to the destruction of nonclassical properties of the superposition states. We have shown that for the phase-sensitive amplifier with the properly chosen phase the oscillations in the photon number distribution as well as quadrature squeezing can be observed even for gains larger than 2. Nevertheless, because the degree of squeezing of the even CS decreases as the amplitude of the composition state gets larger and the amplification adds noise, the maximum gain G_m for which the output state exhibits quadrature squeezing is finite and approximately equal to 2.1. In the case of phase-insensitive amplifiers quantum coherence is destroyed faster than in the case of squeezed amplifiers and, consequently, nonclassical effects of the amplified state are deteriorated faster. From the above argument it follows that it would be quite difficult to amplify experimentally quantum superposition states with their nonclassical properties being preserved.

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