Intrinsic decoherence in the atom-field interaction

H. Moya-Cessa*
Optics Section, The Blackett Laboratory, Imperial College, London SW7 2BZ, England

V. Bužek† and M. S. Kim
Physics Department, Sogang University, CPO Box 1142, Seoul 100-611, Korea

P. L. Knight
Optics Section, The Blackett Laboratory, Imperial College, London SW7 2BZ, England

(Received 3 May 1993)

Milburn has recently proposed [Phys. Rev. A **44**, 5401 (1991)] a variation of the Schrödinger equation which models decoherence as the system evolves through intrinsic mechanisms beyond conventional quantum mechanics rather than dissipative interaction with an environment. In this paper we give an exact solution of this equation and apply it to the Jaynes-Cummings model of atom-field interaction with nontrivial dynamics. We show that the intrinsic decoherence is responsible for deterioration of quantum coherence effects in this model, such as revivals of the atomic inversion. We discuss the applicability of the Milburn equation.

PACS number(s): 42.50.−p, 32.80.−t, 03.65.Bz

I. INTRODUCTION

The relation between macroscopic (i.e., mainly classical) and microscopic (quantum) phenomena is one of the most discussed problems in quantum mechanics. To be more specific, the problem is, "Why on the macroscopic level we do not observe quantum coherences which emerge as a direct consequence of quantum interference between quantum amplitudes?" One possible answer (the Copenhagen interpretation of quantum mechanics [1], see also Refs. [2,3]) is based on an assumption that there exists a *classical* measurement apparatus which during the measurement process simply destroys quantum coherences by projecting quantum states onto the pointer basis. Even though this explanation is quite convincing, the assumption of a classical apparatus makes quantum mechanics internally inconsistent. The other approach to solve the "micro-macro" antagonism has been advocated by Leggett [4], Zurek [3,5], and others (see Refs. [1–3]). This alternative approach is based on the observation that all quantum-mechanical systems are embedded in large systems (i.e., into systems with many degrees of freedom, which are called reservoirs). The interaction of the quantum system with a reservoir means that quantum coherences spread over many reservoir degrees of freedom, so that these coherences effectively decay [4,5]. It is quite important to note that, generally speaking, quantum coherences decay much faster than the dissipation rate of the energy of the quantum system.

Recently there have been several proposals to solve the decoherence problem by modifying the Schrödinger equation in such a way that the quantum coherences are automatically "destroyed" as the quantum system evolves, which means that at the macroscopic level the system behaves "classically." This *intrinsic decoherence* approach has recently been studied in frameworks of several models [6]. In particular, Milburn [7] has proposed a simple modification of standard quantum mechanics based on an assumption that on *sufficiently* short time steps the system does not evolve continuously under unitary evolution but rather in a *stochastic* sequence of identical unitary transformations. This assumption leads (see below) to a modification of the Schrödinger equation which contains a term responsible for the decay of quantum coherence in the energy eigenstate basis, without the intervention of a reservoir and therefore without the usual energy dissipation associated with normal decay. The decay is entirely of phase dependence only (akin to the dephasing decay of coherences produced by impact-theory collisions or by fluctuations in the phase of a laser in laser spectroscopy). Milburn's analysis considers only the free evolution of a given (prepared) quantum system. In what follows we consider a *dynamical* evolution: we study the interaction of two subsystems and the coherences which establish themselves as a consequence of the interaction and their "intrinsic decoherence."

It is generally accepted that all nonclassical effects in quantum optics emerge as a consequence of quantum interference between components of superposition states of light, i.e., nonclassical effects have their origin in quantum coherence. Therefore the decay of quantum coherences results in the deterioration of nonclassical effects. In those situations when it is difficult to observe *directly* nonclassical behavior of the light field it is convenient to study the dynamics of other quantum systems coupled to the light field under consideration. The best-known example of this coupling is an interaction between a two-level atom and the single-mode cavity field in a lossless one-atom micromaser [8]. In the micromaser, the atoms are used as an active medium as well as the "measurement" apparatus, and are very sensitive to quantum coherences of the cavity field. The simplified version of the atom-field interaction can be described in the framework of the Jaynes-Cummings model (JCM) when only one two-level atom interacts with the single-mode cavity field in a lossless cavity. The dynamics of the JCM is governed by the Hamiltonian which in the dipole and the rotating-wave approximations takes the form [8]:

$$\hat{H}=\hbar\omega_F(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})+\hbar\omega_A\hat{\sigma}_3+\lambda(\hat{a}^\dagger\hat{\sigma}_-+\hat{a}\hat{\sigma}_+)\ , \quad (1.1)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and the creation operators of photons of the field mode and $\hat{\sigma}_3$ and $\hat{\sigma}_\pm$ are the atomic spin-flip operators. In this paper we will assume exact resonance between the field (ω_F) and the atomic (ω_A) frequencies (i.e. $\omega_F=\omega_A\equiv\omega$); λ is the real coupling constant.

It is well known that the quantum coherences which are built up during the interaction with the atom significantly affect the dynamics of the atom [10]. In particular, because of these coherences one can observe collapses and revivals of the atomic inversion [11]. The relevant coherences in the Jaynes-Cummings model are such that at the half-revival time a macroscopic superposition of coherent states is established to good approximation. It is the coherent recombination of the components of this superposition at the revival time which is responsible for the restoration of oscillations in the inversion. So any intrinsic decoherence will not only reduce the superposition to a statistical mixture, it will also eliminate the revival consequent upon the survival of the superposition. In this paper we will study the Jaynes-Cummings system governed by the Milburn equation and we will show how the intrinsic decoherence modifies the time evolution of the atomic inversion.

The paper is organized as follows: In Sec. II we give a brief description of Milburn's model. In Sec. III we present the *exact* solution of the Milburn equation with the Jaynes-Cummings Hamiltonian. In Sec. IV we analyze our solution and discuss some consequences of the results obtained.

II. MILBURN'S MODEL

In standard quantum mechanics the dynamics of a conservative system described by the density operator $\hat{\rho}$ is governed by the evolution operator $\hat{U}(t)=\exp(-it\hat{H}/\hbar)$, where $\hat{H}$ is the corresponding Hamiltonian. The change in the state of the quantum system in a time interval $(t,t+\tau)$ is given by the unitary transformation

$$\hat{\rho}(t+\tau)=\hat{U}(\tau)\hat{\rho}(t)\hat{U}^\dagger(\tau)$$
$$=\exp\left[-\frac{i}{\hbar}\tau\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar}\tau\hat{H}\right]\ , \quad (2.1)$$

which is valid for arbitrarily large or small values of τ. Milburn has replaced the above paradigm with three new postulates:

(1) For sufficiently short time steps the system does not evolve continuously under the unitary transformation (2.1) but rather it changes stochastically. The probability that the state of the system is changed is $p(\tau)$, reflecting quantum jumps in the state of the system.

(2) Given that the state of the system is undergoing some changes, then the density operator is changed according to the relation

$$\hat{\rho}(t+\tau)=\exp\left[-\frac{i}{\hbar}\theta(\tau)\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar}\theta(\tau)\hat{H}\right]\ , \quad (2.2)$$

where $\theta(\tau)$ is some function of τ. In standard quantum mechanics we have $p(\tau)=1$ and $\theta(\tau)=\tau$. In the generalized model proposed by Milburn we only require that $p(\tau)\to 1$ and $\theta(\tau)\to\tau$ for values of τ which are sufficiently large.

(3) In Milburn's model it is postulated that

$$\lim_{\tau\to 0}\theta(\tau)=\theta_0\ . \quad (2.3)$$

This last postulate effectively introduces a minimum time step in the Universe [12]. The inverse of this time step is equal to the mean frequency of the unitary step, $\gamma=1/\theta_0$.

The rate of change of $\hat{\rho}(t)$ in Milburn's model is given by the equation (for details see the original paper by Milburn [7])

$$\frac{d}{dt}\hat{\rho}(t)=\gamma\left\{\exp\left[-\frac{i}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar\gamma}\hat{H}\right]-\hat{\rho}(t)\right\}\ , \quad (2.4)$$

which is equivalent to the assumption that on a very short time scale the probability that the system evolves is $p(\tau)=\gamma\tau$. Equation (2.4) is the proposed generalized equation which alters the Schrödinger dynamics. In the limit $\gamma\to\infty$ (i.e., when the fundamental time step goes to zero) Eq. (2.4) reduces to the ordinary von Neumann equation for the density operator. The stochastic element introduced by the effective jumps in Eq. (2.3) is responsible for the appearence of an "arrow of time" in the evolution.

Expanding Eq. (2.4) to first order in γ^{-1}, Milburn obtained the following dynamical equation:

$$\frac{d}{dt}\hat{\rho}(t)=-\frac{i}{\hbar}[\hat{H},\hat{\rho}]-\frac{1}{2\hbar^2\gamma}[\hat{H},[\hat{H},\hat{\rho}]]. \quad (2.5)$$

As shown by Milburn [7], the first-order correction in Eq. (2.5) leads to diagonalization of the density operator in the energy eigenstate basis. Moreover, this term induces diffusion in variables that do not commute with the Hamiltonian. However, *all* constants of motion commute with the Hamiltonian and thus remain unaffected. In the Jaynes-Cummings model this will cause the excitation number (and hence the energy) to be preserved while the revivals which depend on coherences are dephased. In the following section we present the exact solution of Eq. (2.5) with the Jaynes-Cummings Hamiltonian (1.1).

III. EXACT SOLUTION OF THE MILBURN EQUATION FOR THE JCM

We express the solution for the density operator $\rho(t)$ of the Milburn equation (2.5) applied to the Hamiltonian (1.1) in terms of three superoperators $\hat{J}$, $\hat{S}$, and $\hat{L}$:

$$\hat{\rho}(t)=e^{\hat{J}t}e^{\hat{S}t}e^{\hat{L}t}\hat{\rho}(0)\ , \tag{3.1}$$

where $\hat{\rho}(0)$ is the density operator of the initial atom-field system. We assume that initially the field is prepared in the coherent state $|\alpha\rangle$:

$$\begin{aligned}|\alpha\rangle&=\exp(\alpha\hat{a}^\dagger-\alpha^*\hat{a})|0\rangle\\&=\sum\exp(-|\alpha|^2/2)\frac{\alpha^n}{\sqrt{n!}}|n\rangle\\&\equiv\sum_{n=0}^{\infty}Q_n|n\rangle\ ,\end{aligned} \tag{3.2}$$

and the atom was prepared in its excited state $|e\rangle$, so that

$$\hat{\rho}(0)=|\alpha\rangle\langle\alpha|\otimes|e\rangle\langle e|. \tag{3.3}$$

The superoperators are defined through their action on the density operator (in what follows we assume units such that $\hbar=1$):

$$\hat{J}\hat{\rho}=\frac{1}{\gamma}\hat{H}\hat{\rho}\hat{H}\ , \tag{3.4}$$

$$\hat{S}\hat{\rho}=-i(\hat{H}\hat{\rho}-\hat{\rho}\hat{H})\ , \tag{3.5}$$

and

$$\hat{L}\hat{\rho}=\frac{1}{2\gamma}(\hat{H}^2\hat{\rho}+\hat{\rho}\hat{H}^2)\ . \tag{3.6}$$

In a two-dimensional atomic basis the Jaynes-Cummings Hamiltonian (1.1) can be expressed as a sum of diagonal ($\hat{H}_0$) and off-diagonal terms ($\hat{H}_I$):

$$\hat{H}=\hat{H}_0+\hat{H}_I,\quad[\hat{H}_0,\hat{H}_I]=0\ , \tag{3.7}$$

where

$$\hat{H}_0=\omega\begin{bmatrix}(\hat{n}+1)&0\\0&\hat{n}\end{bmatrix},\quad\hat{H}_I=\lambda\begin{bmatrix}0&\hat{a}\\\hat{a}^\dagger&0\end{bmatrix}, \tag{3.8}$$

where we have assumed exact resonance between the field and atomic transition frequencies. Similarly, the square of the Jaynes-Cummings Hamiltonian can be expressed as a sum of diagonal and off-diagonal terms:

$$\hat{H}^2=\hat{A}+\hat{B},\quad[\hat{A},\hat{B}]=0\ , \tag{3.9}$$

with

$$\hat{A}=\begin{bmatrix}\hat{\beta}_{n+1}&0\\0&\hat{\beta}_n\end{bmatrix},\quad\hat{B}=2\omega\lambda\begin{bmatrix}0&\hat{a}\hat{n}\\\hat{n}\hat{a}^\dagger&0\end{bmatrix}, \tag{3.10}$$

where

$$\hat{\beta}_n=\omega^2\hat{n}^2+\lambda^2\hat{n}\ . \tag{3.11}$$

Taking into account the initial condition (3.3) we can write down the following expression:

$$e^{\hat{S}t}e^{\hat{L}t}\hat{\rho}(0)=e^{-i\hat{H}_It}e^{-\epsilon\hat{B}}\hat{\rho}_1(t)e^{-\epsilon\hat{B}}e^{i\hat{H}_It}\equiv\hat{\rho}_2(t)\ , \tag{3.12}$$

where

$$\hat{\rho}_1(t)=|\Psi(t)\rangle\langle\Psi(t)|\otimes|e\rangle\langle e|\ , \tag{3.13}$$

with

$$|\Psi(t)\rangle=\exp(-\epsilon\hat{\beta}_{n+1})|\alpha e^{-i\omega t}\rangle\ , \tag{3.14}$$

and $\epsilon=t/2\gamma$.

If we notice that the powers of the off-diagonal operator $\hat{B}$ can be written as

$$\hat{B}^{2k}=\begin{bmatrix}[2\omega\lambda(\hat{n}+1)^{3/2}]^{2k}&0\\0&[2\omega\lambda\hat{n}^{3/2}]^{2k}\end{bmatrix}, \tag{3.15a}$$

$$\hat{B}^{2k+1}=\begin{bmatrix}0&\frac{[2\omega\lambda\hat{n}^{3/2}]^{2k+1}}{\sqrt{\hat{n}}}\\\frac{[2\omega\lambda\hat{n}^{3/2}]^{2k+1}}{\sqrt{\hat{n}}}\hat{a}^\dagger&0\end{bmatrix}, \tag{3.15b}$$

then we can write the operator $e^{-\epsilon\hat{B}}$ in the form

$$e^{-\epsilon\hat{B}}=\begin{bmatrix}\hat{X}_n(t)&-\hat{a}\frac{\hat{Y}_{n-1}(t)}{\sqrt{\hat{n}}}\\-\frac{\hat{Y}_{n-1}(t)}{\sqrt{\hat{n}}}\hat{a}^\dagger&\hat{X}_{n-1}(t)\end{bmatrix}, \tag{3.16}$$

where

$$\begin{aligned}\hat{X}_n(t)&=\cosh\left[\frac{\lambda t\omega}{\gamma}(\hat{n}+1)^{3/2}\right],\\\hat{Y}_n(t)&=\sinh\left[\frac{\lambda t\omega}{\gamma}(\hat{n}+1)^{3/2}\right].\end{aligned} \tag{3.17}$$

Analogously, we can write the operator $e^{-i\hat{H}_It}$ in the two-dimensional atomic basis as

$$e^{-i\hat{H}_It}=\begin{bmatrix}\hat{C}_n(t)&-i\frac{\hat{S}_n(t)}{\sqrt{\hat{n}+1}}\hat{a}\\-i\hat{a}^\dagger\frac{\hat{S}_n(t)}{\sqrt{\hat{n}+1}}&\hat{C}_{n-1}(t)\end{bmatrix}, \tag{3.18}$$

with the operators $\hat{C}_n(t)$ and $\hat{S}_n(t)$ defined as

$$\hat{C}_n(t)=\cos\sqrt{\hat{n}+1}\lambda t\ ,\quad\hat{S}_n(t)=\sin\sqrt{\hat{n}+1}\lambda t\ . \tag{3.19}$$

Combining expressions (3.16) and (3.18) we find that

$$e^{-\hat{H}_It}e^{-\epsilon\hat{B}}=\begin{bmatrix}\hat{R}_n(t)&\frac{\hat{V}_n(t)}{\sqrt{\hat{n}+1}}\hat{a}\\\hat{a}^\dagger\frac{\hat{V}_n(t)}{\sqrt{\hat{n}+1}}&\hat{R}_{n-1}(t)\end{bmatrix}, \tag{3.20}$$

with

$$\hat{R}_n(t)=\hat{C}_n(t)\hat{X}_n(t)+i\hat{S}_n(t)\hat{Y}_n(t) \tag{3.21a}$$

and

$$\hat{V}_n(t)=-\hat{C}_n(t)\hat{Y}_n(t)-i\hat{S}_n(t)\hat{X}_n(t)\ . \tag{3.21b}$$

Using Eq. (3.20) we can find an explicit expression for the operators $\hat{\rho}_2(t)$ given in Eq. (3.12):

$$\hat{\rho}_2(t)=\begin{bmatrix}|\Psi_1(t)\rangle\langle\Psi_1(t)| & |\Psi_1(t)\rangle\langle\Psi_2(t)| \\ |\Psi_2(t)\rangle\langle\Psi_1(t)| & |\Psi_2(t)\rangle\langle\Psi_2(t)|\end{bmatrix}, \quad (3.22)$$

where the state vectors $|\Psi_i(t)\rangle$ are defined as

$$|\Psi_1(t)\rangle=\hat{R}_n(t)|\Psi(t)\rangle, \quad |\Psi_2(t)\rangle=\hat{a}^\dagger\hat{V}_n(t)|\Psi(t)\rangle, \quad (3.23)$$

with the state vector $|\Psi(t)\rangle$ given by Eq. (3.14).

Finally, we have to evaluate the action of the operator $e^{\hat{J}t}$ on the "density" operator $\hat{\rho}_2(t)$, i.e.,

$$\hat{\rho}(t)=e^{\hat{J}t}\hat{\rho}_2(t) . \quad (3.24)$$

Taking into account the definition of the superoperator $\hat{J}$, we can rewrite Eq. (3.24) in the form

$$\hat{\rho}(t)=\sum_{k=0}^{\infty}\frac{(\lambda^2t/\gamma)^k}{k!}\hat{H}^k\hat{\rho}_2(t)\hat{H}^k. \quad (3.25)$$

Equation (3.25) describes the exact solution of the Milburn equation (2.5) with the Jaynes-Cummings Hamiltonian (1.1). Using this solution we can evaluate the explicit expression for the time evolution of the atomic inversion $W(t)=\mathrm{Tr}[\hat{\rho}(t)\hat{\sigma}_3]$ and the expression for the photon number distribution at time t: $P_n(t)=\langle n|\mathrm{Tr}_A\hat{\rho}(t)|n\rangle$. For the functions $W(t)$ and $P_n(t)$ we find

$$W(t)=\sum_{n=0}^{\infty}|Q_n|^2\exp[-2(n+1)\lambda^2t/\gamma]\cos(2\sqrt{n+1}\lambda t) \quad (3.26)$$

and

$$P_n(t)=|Q_n|^2\left\{C_n^2(t)\exp[-2(n+1)\lambda^2t/\gamma]+\frac{1-\exp[-2(n+1)\lambda^2t/\gamma]}{2}\right\}+|Q_{n-1}|^2\left\{S_{n-1}^2(t)\exp[-2n\lambda^2t/\gamma]+\frac{1-\exp[-2n\lambda^2t/\gamma]}{2}\right\}, \quad (3.27)$$

where the probability amplitudes Q_n are given by Eq. (3.2) and the functions $C_n(t)$ and $S_n(t)$ are defined by Eq. (3.19). We see that both Eqs. (3.26) and (3.27) in the limit $\gamma\to\infty$ reduce to the well-known expressions for the atomic inversion and the photon number distribution in the standard Jaynes-Cummings model governed by the von Neumann equation [9,11].

It is well known that the revivals of the atomic inversion [9–11] as well as the oscillations in the photon number distribution [13] arise as a consequence of quantum interference in phase space. In other words, these nonclassical effects have their origin in quantum coherences established during the interaction between the atom and the cavity field. From our solution (3.26) it follows that the additional term in the evolution, Eq. (2.5), which destroys quantum coherences, leads to the appearance of "decay" factors exp $[-2(n+1)\lambda^2t/\gamma]$ in Eq. (3.26), which are responsible for the destruction of revivals of the atomic inversion. In other words, with the decrease of the parameter γ, i.e., with a more rapid suppression of quantum coherences, we can observe rapid deterioration of revivals of the atomic inversion. In Fig. 1 we plot the time evolution of the atomic inversion for three values of the parameter λ^2/γ. From these figures it follows that the larger is the "fundamental" time step (i.e., the smaller is the parameter γ), the more pronounced is the suppression of the first revival. These figures illustrate the decay of quantum coherences due to the very specific time evolution described by Eq. (2.5), i.e., due to the intrinsic decoherence. Of course the system remains conservative, so there is no dissipation of energy and the inversion "relaxes" to a zero value appropriate to an equal mixture of upper and lower levels. It is interesting to compare this dephasing with the effects of genuine dissipation to an external reservoir, either due to field damping or spontaneous emission decay [14]. Obviously in this case the energy of the atom-field system is not a constant of motion. Such external dissipation affects both coherences

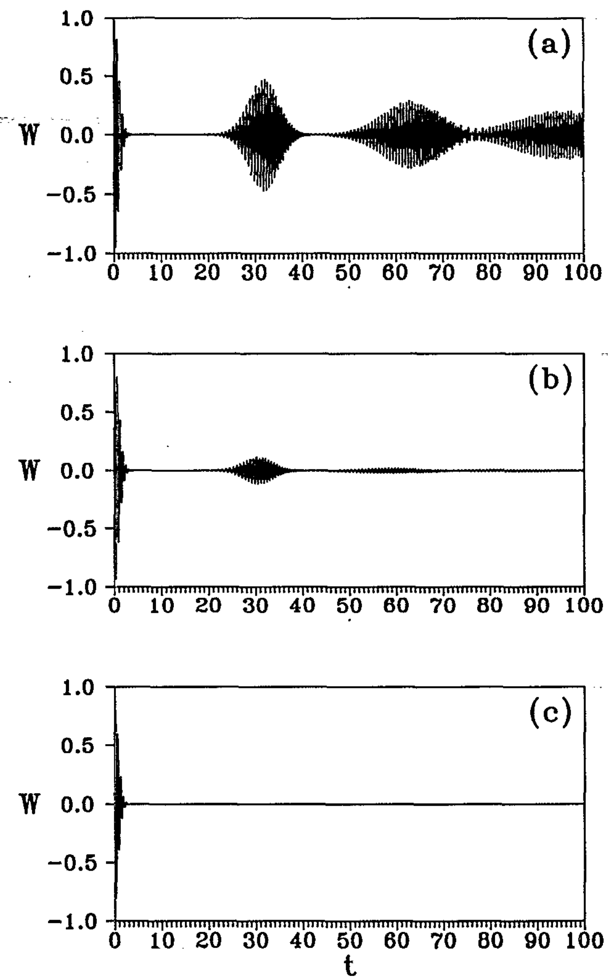

FIG. 1. Time evolution of the atomic inversion $W(t)$ of the atom initially prepared in the excited state interacting with the coherent field $|\alpha\rangle$ ($|\alpha|^2=25$) for various values of the parameter λ^2/γ: (a) $\lambda^2/\gamma=10^{-4}$, (b) $\lambda^2/\gamma=10^{-3}$, and (c) $\lambda^2/\gamma=10^{-2}$. (We have set $\lambda=1$; the quantities t and W are dimensionless.)

and energy, both of which decay. Nevertheless, the decay of quantum coherences is much faster than the energy decay [14]: even normal dissipation wipes out coherences in the Jaynes-Cummings model (and hence revivals) well before any visible effect is made on the energy (i.e., on the value of the inversion in the collapse region).

From Eq. (3.27) it follows that the intrinsic decoherence leads to a suppression of oscillations in the photon number distribution. In the standard Jaynes-Cummings model these oscillations appear as a consequence of quantum coherences which are dynamically established during the atom-field interaction. If the dynamics of the atom-field system is governed by the Milburn equation (2.5) then the intrinsic decoherence suppresses quantum coherences and the oscillations in the photon number distribution. Moreover, the field at one-half of the revival time, $t=t_R/2\simeq\pi\bar{n}^{1/2}/\lambda$ (where $\bar{n}=|\alpha|^2$) is not in a pure quantum-mechanical superposition state [10], but in a statistical mixture state.

IV. ANALYSIS AND CONCLUSIONS

We have found the exact solution of the Milburn equation (2.5) for the Jaynes-Cummings Hamiltonian (1.1). We have shown that intrinsic decoherence in the atom-field interaction is responsible for deterioration of quantum effects, such as the revival of the atomic inversion.

To obtain a more clear understanding of the nature of the Milburn equation we have to stress here that the revivals of the atomic inversion depend on the establishment of quantum coherences in the energy (Fock) basis of the cavity field (i.e., the revivals depend on the off-diagonal terms of the density operator in the Fock basis). The Milburn equation describes perfectly well the decay of the off-diagonal terms in the Fock basis and therefore describes well the suppression of quantum effects which are related to the existence of the off-diagonal terms of the density operator in the Fock basis, such as squeezing (see below). On the other hand, there are nonclassical effects such as sub-Poissonian photon statistics which have their origin in quantum interference (i.e., they depend on the existence of quantum coherences) [15] but which do not depend on the off-diagonal terms in the Fock basis. These effects (which can depend on quantum coherences in the coherent-state basis) are not suppressed by the Fock-basis intrinsic decoherence as described by Milburn. On the other hand, these nonclassical effects are very sensitive to the influence of dissipative processes [15] which generally destroy quantum coherences.

From the above it seems that the application of the Milburn equation is of interest in those cases when we are interested in the observation of nonclassical effects which emerge as a consequence of the existence of off-diagonal terms of the field-density operator in the Fock basis. To illustrate this statement we briefly analyze the time evolution of a harmonic oscillator (a single mode of the quantized electromagnetic field) with the Hamiltonian $\hat{H}=\omega(\hat{a}^\dagger\hat{a}+\frac{1}{2})$ governed by the Milburn equation (2.5). In this case the exact solution for the matrix elements of the density operator in the Fock basis is [7]

$$\rho_{nm}(t)=\rho_{nm}(0)\exp\left[-it\omega(n-m)-\frac{t}{2\gamma}\omega^2(n-m)^2\right]. \tag{4.1}$$

Let us assume the initial state of the harmonic oscillator is a quantum-mechanical superposition of two coherent states with the same amplitude but with a phase difference equal to π. Quantum interference between these two component states can lead to various nonclassical effects. For instance, in the case of the even coherent state $|\alpha\rangle_e$ (we assume α to be real):

$$|\alpha\rangle_e=A_e^{1/2}(|\alpha\rangle+|-\alpha\rangle),\quad A_e^{-1}=2[1+\exp(-2\alpha^2)], \tag{4.2}$$

one can observe quadrature squeezing, i.e., the reduction of fluctuations of quadrature operators $\hat{a}_1$ and $\hat{a}_2$, which are defined as

$$\hat{a}_1=\frac{\hat{a}e^{i\omega t}+\hat{a}^\dagger e^{-i\omega t}}{2},\quad \hat{a}_2=\frac{\hat{a}e^{i\omega t}-\hat{a}^\dagger e^{-i\omega t}}{2i}. \tag{4.3}$$

The degree of the reduction (squeezing) of the quadrature fluctuations can be measured by two parameters S_i:

$$S_i=4[\langle(\Delta\hat{a}_i)^2\rangle-1]. \tag{4.4}$$

The state is said to be quadrature squeezed if S_1 or S_2 is less than zero. For the even coherent state we find [15], for $t=0$,

$$S_1=\frac{4\alpha^2}{1+\exp(-2\alpha^2)}>0,\quad S_2=\frac{-4\alpha^2\exp(-2\alpha^2)}{1+\exp(-2\alpha^2)}<0, \tag{4.5}$$

which means that this state is quadrature squeezed. The reduction of quadrature fluctuations results as a consequence of the quantum interference between component states $|\alpha\rangle$ and $|-\alpha\rangle$. Formally this effect is related to the existence of off-diagonal terms in the density matrix. Due to the dynamics described by the Milburn equation the initial even coherent state of the harmonic oscillator is transformed at time $t\omega^2/(2\gamma)\gg1$ into a state for which

$$S_1=S_2=2\alpha^2\frac{1-\exp(-2\alpha^2)}{1+\exp(-2\alpha^2)}>0, \tag{4.6}$$

which means that the suppression of quantum coherences leads to deterioration of squeezing.

On the other hand, the quantum interference between two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ which constitute the odd coherent state $|\alpha\rangle_0$:

$$|\alpha\rangle_0=A_0^{1/2}(|\alpha\rangle-|-\alpha\rangle),\quad A_0^{-1}=2[1-\exp(-2\alpha^2)], \tag{4.7}$$

leads to appearance of the sub-Poissonian photon statistics (i.e., to the reduction of quantum fluctuations of the mean photon number) [15]. The degree of the sub-Poissonian photon statistics is measured by the Mandel Q parameter:

$$Q=\frac{\langle(\Delta\hat{n})^2\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle}, \tag{4.8}$$

which in the case of the sub-Poissonian state is less than zero. For the odd coherent state we find that

$$Q=\frac{-4\alpha^2\exp(-2\alpha^2)}{1-\exp(-4\alpha^2)}, \tag{4.9}$$

from which it follows that the odd coherent state is a nonclassical state with reduced fluctuations in the mean photon number. This nonclassical effect has its origin in the quantum interference. Although this quantum effect has its origin in quantum coherences, nevertheless it is not deteriorated during the time evolution of the harmonic oscillator governed by the Milburn equation, that is, the Mandel Q parameter is constant during the evolution. The Milburn dynamics does not affect diagonal terms in the Fock basis and consequently does not change the photon number distribution of the harmonic oscillator.

We can conclude that the intrinsic decoherence as described by Milburn is rather selective. It leads to deterioration of only those nonclassical effects which have their origin in the existence of off-diagonal terms in the Fock basis. Finally, we draw attention to the fact that *both* quadrature squeezing as well as sub-Poissonian photon statistics can be destroyed very rapidly under the influence of normal dissipative processes [15], which in general lead to destruction of quantum coherences in the coherent-state basis as well as in the Fock basis. Nevertheless, as shown by Zurek *et al.* [16], coherent states form a more robust pointer basis when environmental dissipation is included than the Fock basis. Consequently, one can expect that the intrinsic decoherence process in the Fock basis as described by Milburn should be altered by the intrinsic decoherence process in the coherent-state basis.

ACKNOWLEDGMENTS

This work was supported in part by the Grant Agency of the Slovak Academy of Sciences Grant No. GA SAV 2/999143/93, the Mexican Consejo Nacional de Ciencia y Tecnologia (CONACyT), the British Overseas Research Student Awards Scheme, the UK Science and Engineering Research Council, and by the Nondirected Research Fund KRF. H.M.-C. would like to acknowledge the Slovak Academy of Sciences for hospitality and V.B. would like to acknowledge financial support from the Korean Science and Engineering Foundation.

---

*On leave from: Instituto Nacional de Astrofisica Optica y Electronica Apartado Postal 51 y 216, Puebla, Puebla, Mexico.

†Permanent address: Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Slovakia. Electronic address: buzek@savba.savba.cs

[1] N. Bohr, Nature **121**, 580 (1928).

[2] J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement* (Princeton University Press, Princeton, NJ, 1986).

[3] W. H. Zurek, Phys. Today **36** (10), 36 (1991).

[4] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zweger, Rev. Mod. Phys. **59**, 1 (1987); A. O. Caldeira and A. J. Leggett, Phys. Rev. A **31**, 1057 (1985).

[5] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981); **26**, 1862 (1982); W. G. Unruh and W. H. Zurek, *ibid.* **40**, 4059 (1989).

[6] C. M. Caves and G. J. Milburn, Phys. Rev. D **36**, 3543 (1987); L. Diosi, Phys. Rev. A **40**, 1165 (1989); G. C. Ghirardi, P. Pearle, and A. Rimini, *ibid.* **42**, 78 (1990).

[7] G. J. Milburn, Phys. Rev. A **44**, 5401 (1991).

[8] D. Meschede, H. Walther, and G. Müller, Phys. Rev. Lett. **54**, 551 (1985); P. Filipowicz, J. Javanainen, and P. Meystre, Phys. Rev. A **34**, 3077 (1986).

[9] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963); for a recent review see B. W. Shore and P. L. Knight, J. Mod. Opt. **40**, 1195 (1993).

[10] J. Eiselt and H. Risken, Opt. Commun. **72**, 351 (1989); S. J. D. Phoenix and P. L. Knight, Ann. Phys. (N.Y.) **186**, 381 (1988); J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990); Phys. Rev. A **44**, 5913 (1991).

[11] J. H. Eberly, N. B. Narozhny, and J. J. Sánchez-Mondragón, Phys. Rev. Lett. **44**, 1323 (1980); N. B. Narozhny, J. J. Sánchez-Mondragón, and J. H. Eberly, Phys. Rev. A **23**, 236 (1981).

[12] V. G. Kadyshevsky, Nucl. Phys. B **141**, 477 (1978); S. Golden, Phys. Rev. A **46**, 6905 (1992), and references therein.

[13] V. Bužek, H. Moya-Cessa, P. L. Knight, and S. J. D. Phoenix, Phys. Rev. A **45**, 8190 (1992).

[14] Tran Quang, P. L. Knight, and V. Bužek, Phys. Rev. A **44**, 6092 (1991), and references therein.

[15] V. Bužek, A. Vidiella-Barranco, and P. L. Knight, Phys. Rev. A **45**, 6570 (1992).

[16] W. H. Zurek, S. Habib, and J. P. Paz, Phys. Rev. Lett. **70**, 1187 (1993).