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Interaction of a three-level atom with an SU(2) coherent state

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In this paper we investigate the sensitivity of coupled atomic transitions to correlations between field modes. In particular, we show that a three-level atom interacting with a cavity field supporting two quantized modes can be highly sensitive to the nature of the correlations between the modes. Correlated two-mode states include the two-mode squeezed state, whereas the SU(2) coherent state is an example of an anticorrelated state. We study in detail the dynamics of a three-level atom in the λ configuration, interacting with the two modes of an SU(2) coherent state. We show that for certain choices of the atom-field coupling constants, the generalized two-photon Rabi frequency for the system becomes independent of the photon number, leading to a purely periodic behavior in time of the atomic population probabilities; for other choices of the atom-field coupling constants, we observe the usual collapse and revival of the Rabi oscillations. We study also the dynamics of the field, in particular the correlations between the modes. Finally, for comparison purposes, we present a parallel study of the case in which the field modes are initially uncorrelated.

I. INTRODUCTION

The Jaynes-Cummings model [1–3] (JCM), composed of a two-level atom interacting with a single model of the electromagnetic field, has received considerable interest over recent years. This interest stems from the fact that this model is exactly solvable in the rotating-wave approximation and yields nontrivial results due to the quantized nature of the radiation field. These include the collapse and revival of the atomic inversion [4], the production of squeezed states of the electromagnetic field [5], and sub-Poissonian photon-counting statistics [6]. Recent advances in the construction of microwave cavities have led to the observation of some of these effects when Rydberg atoms interact with quantized fields in high-Q cavities [7].

A natural extension of the JCM would be to consider a three-level atom interacting with one or two modes of the electromagnetic field [8]. In the latter one can study the effect of intermode field correlations on the dynamics of the atom, and vice versa. This particular aspect of the problem will form the basis of this paper. We will take an SU(2) coherent state as our example of a two-mode correlated field state, and a three-level atom in the λ configuration as our atomic transition sequence used to sense such correlations. The SU(2) coherent state [9] is characterized by the property that if n photons are observed in one mode, then $m-n$ photons will be observed in the other mode, where m is the maximum possible number of photons in both modes. We show that this special correlation between the photon numbers of the individual modes (provided that the atom-field coupling constants are equal) leads to generalized two-photon Rabi frequencies that are *independent* of the photon number. This means that the population probabilities will exhibit purely periodic behavior. The usual collapse and revival of the Rabi oscillations as in the ordinary JCM are observed when the atom-field coupling constants are not equal. Moreover, because we have a three-level system in which one- and two-photon processes are possible, there will be two types of revivals of the population probabilities, with one occurring at half the period of the other.

For comparison purposes, we present a parallel study of the case in which the initial field is in a two-mode uncorrelated state. Yoo and Eberly [8] used the two-mode coherent state in their studies. In our case, we have taken a two-mode state in which each mode exhibits the same photon distribution as in the corresponding modes of the SU(2) coherent state. We have found that the population probabilities exhibit collapses and revivals for *all* choices of the atom-field coupling constants, and that the revivals occur at much later times than those associated with the SU(2) coherent state. In a further paper we address the problem of a three-level system coupled to a two-mode squeezed state where the field modes are correlated.

The plan of this paper is as follows. In Sec. II we present our model of the three-level atom interacting with a two-mode field, followed in Sec. II A by a résumé of the properties of the SU(2) coherent state. Then in Secs. III and IV, we present the results of a numerical study of the atomic population probabilities and photon correlations, ending in Sec. V with a summary.

II. THE MODEL

The scheme of the three-level atomic system, as shown in Fig. 1, consists of two allowed transitions, $0\leftrightarrow1$ and $1\leftrightarrow2$, each interacting with a different mode of the field. We assume for simplicity the full resonance condition, so that we can write the interaction Hamiltonian in the rotating-wave approximation as

$$H_I=\hbar[g_a(a^\dagger R_{01}+R_{10}a)+g_g(b^\dagger R_{21}+R_{12}b)]\ , \qquad (2.1)$$

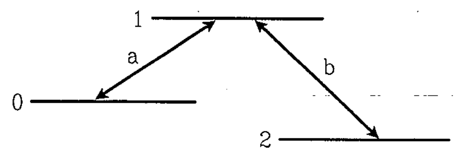

FIG. 1. Energy diagram of a three-level atom in the λ configuration interacting with two quantized cavity modes.

where $R_{ij} \equiv |i\rangle\langle j|$ are the atomic raising and lowering operators, a, b and $a^\dagger, b^\dagger$ are the annihilation and creation operators of the respective field modes, and g_a and g_b are the atom-field coupling constants.

We suppose that the initial-state vector of the field may be written in the form

$$|\Psi(0)\rangle_f = \sum_{n_a, n_b} \tilde{Q}_{n_a,n_b} |n_a, n_b\rangle , \tag{2.2}$$

where $|n_a, n_b\rangle \equiv |n_a\rangle \otimes |n_b\rangle$. The joint photon distribution $\tilde{P}_f(n_a, n_b)$ for the state (2.2) is related to the expansion coefficients $\tilde{Q}_{n_a,n_b}$ by

$$\tilde{P}_f(n_a, n_b) = |\tilde{Q}_{n_a,n_b}|^2 . \tag{2.3}$$

Later on we will take (2.3) either to represent the photon distribution for the SU(2) coherent state or the two-mode uncorrelated binomial state.

Assuming that the atom starts in the state $|0\rangle$, we may write the state vector of the total atom-field system at time t as

$$\begin{aligned} |\Psi(t)\rangle = \sum_{n_a,n_b} \tilde{Q}_{n_a,n_b} [& C_0(n_a,n_b,t)|0;n_a,n_b\rangle \\ & + C_1(n_a,n_b,t)|1;n_a-1,n_b\rangle \\ & + C_2(n_a,n_b,t)|2;n_a-1,n_b+1\rangle] , \end{aligned} \tag{2.4}$$

where the coefficients $C_i(n_a,n_b,t)$ are obtained from the time-dependent Schrödinger equation, and are given as follows:

$$\begin{aligned} C_0(n_a,n_b,t) &= \frac{\Omega^2_{n_b}}{\Omega^2_{n_a-1,n_b}} + \frac{\Omega^2_{n_a-1}}{\Omega^2_{n_a-1,n_b}} \cos\Omega_{n_a-1,n_b} t , \\ C_1(n_a,n_b,t) &= -i\frac{\Omega_{n_a-1}}{\Omega_{n_a-1,n_b}} \sin\Omega_{n_a-1,n_b} t , \\ C_2(n_a,n_b,t) &= -\frac{\Omega_{n_a-1}\Omega_{n_b}}{\Omega^2_{n_a-1,n_b}} (1-\cos\Omega_{n_a-1,n_b} t) . \end{aligned} \tag{2.5}$$

The quantities $\Omega_{n_{a(b)}}$ and Ω_{n_a,n_b} in Eqs. (2.5) are the one- and two-photon Rabi frequencies defined by

$$\begin{aligned} \Omega_{n_{a(b)}} &= g_{a(b)}(n_{a(b)}+1)^{1/2} , \\ \Omega_{n_a,n_b} &= (\Omega^2_{n_a} + \Omega^2_{n_b})^{1/2} . \end{aligned} \tag{2.6}$$

In the following sections we will use the solutions (2.5) to study the atomic and field dynamics under two initial fields, namely their SU(2) coherent field and the uncorrelated two-mode binomial field. But first, we briefly review the properties of the former.

A. Résumé of the SU(2) coherent state

The SU(2) coherent state has been studied by many authors in connection with superradiance in atomic systems [10]. In its bosonic representation, it takes the form [9]

$$|m,\tau\rangle = (1+|\tau|^2)^{-m/2} \sum_{n=0}^{m} \begin{bmatrix} m \\ n \end{bmatrix}^{1/2} \tau^n |n, m-n\rangle , \tag{2.7}$$

where m is the maximum possible number of photons and τ is a complex parameter related to the partition of photons in the SU(2) coherent-state field modes. The SU(2) coherent state can be generated in a linear directional coupler in which a pure number state $|m\rangle$ is launched into one port of the coupler, and the vacuum into the other [11].

By forming the density matrix and tracing out the appropriate mode, we obtain the following reduced density matrices:

$$\begin{aligned} \tilde{\rho}_a &= \sum_{n_a=0}^{m} \begin{bmatrix} m \\ n_a \end{bmatrix} s^{n_a}(1-s)^{m-n_a} |n_a\rangle\langle n_a| , \\ \tilde{\rho}_{ab} &= \sum_{n_b=0}^{m} \begin{bmatrix} m \\ n_b \end{bmatrix} (1-s)^{n_b} s^{m-n_b} |n_b\rangle\langle n_b| . \end{aligned} \tag{2.8}$$

Here the partition parameter s is related to τ by $s = |\tau|^2/(1+|\tau|^2)$ and governs the mean number of photons in each mode. For $s=1$, all photons are in mode a and the vacuum in mode b. The SU(2) coherent state is thus decoupled in this limit, with one mode projected into a pure number state and the other into the vacuum state. For $s=\frac{1}{2}$, each mode will have the same mean photon number but anticorrelated with respect to its partner.

It is clearly seen from Eqs. (2.7) and (2.8) that the joint and marginal photon distributions are described by the binomial distributions

$$\begin{aligned} \tilde{P}(n_a,n_b) &= \begin{bmatrix} m \\ n_a \end{bmatrix} s^{n_a}(1-s)^{m-n_a} \delta_{n_b, m-n_a} , \\ \tilde{P}_a(n) &= \begin{bmatrix} m \\ n \end{bmatrix} s^n (1-s)^{m-n} , \\ \tilde{P}_b(n) &= \tilde{P}_a(m-n) , \end{aligned} \tag{2.9}$$

with mean and variance given, respectively, by

$$\begin{aligned} &\langle n_a\rangle = sm, \quad \langle n_b\rangle = (1-s)m , \\ &\langle \Delta n_a^2\rangle = \langle \Delta n_b^2\rangle = s(1-s)m , \end{aligned} \tag{2.10}$$

where $\langle \Delta n^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2$. From Eqs. (2.10) it follows that the statistics in each mode are sub-Poissonian.

A very interesting property of the SU(2) coherent state is the anticorrelation between the modes, that is, there is no tendency for photons of both modes to appear simultaneously. We quantify this anticorrelation by defining the normalized cross-correlation function

$$g^{(2)} = \frac{\langle n_a, n_b \rangle}{\langle n_a \rangle \langle n_b \rangle} , \tag{2.11}$$

which for the SU(2) coherent state is given by

$$g^{(2)} = \frac{m-1}{m} . \tag{2.12}$$

As this quantity is less than unity, the modes are clearly anticorrelated, the magnitude of the anticorrelation being determined only by the total possible number of excitations in the field.

III. ATOMIC POPULATION PROBABILITIES

By measuring the occupation probabilities of the various atomic levels, one can gain an insight into the photon statistical properties of the exciting field. In the three-level–two-mode problem, these quantities are given by

$$P_i(t) = \sum_{n_a, n_b} \tilde{P}_f(n_a, n_b) P_i(n_a, n_b, t), \quad i = 0,1,2 , \tag{3.1}$$

where $\tilde{P}_f(n_a, n_b)$ is the initial joint photon distribution of the field and $P_i(n_a, n_b, t)$ are the occupation probabilities of the various atomic levels in the (n_a, n_b)th manifold, which are obtained from Eqs. (2.5) as

$$\begin{aligned} P_0(n_a,n_b,t) &= \frac{\Omega^4_{n_b}}{\Omega^2_{n_a-1,n_b}} + \frac{2\Omega^2_{n_a-1}\Omega^2_{n_b}}{\Omega^4_{n_a-1,n_b}} \cos\Omega_{n_a-1,n_b} t \\ &\quad + \frac{\Omega^4_{n_a-1}}{\Omega^4_{n_a-1,n_b}} \cos^2\Omega_{n_a-1,n_b} t , \\ P_1(n_a,n_b,t) &= \frac{\Omega^2_{n_a-1}}{\Omega^2_{n_a-1,n_b}} \sin^2\Omega_{n_a-1,n_b} t , \\ P_2(n_a,n_b,t) &= \frac{\Omega^2_{n_a-1}\Omega^2_{n_b}}{\Omega^4_{n_a-1,n_b}} (1 - 2\cos\Omega_{n_a-1,n_b} t \\ &\quad + \cos^2\Omega_{n_a-1,n_b} t) . \end{aligned} \tag{3.2}$$

From Eqs. (3.2) it is clear that there are two Rabi frequencies, $\frac{1}{2}\Omega_{n_a-1,n_b}$ and Ω_{n_a-1,n_b}, which govern the atomic dynamics. These are attributed to one- and two-photon processes, respectively, and as we show later, they lead to two types of revivals in the oscillations of the occupation probabilities.

We now treat two cases of the initial field, namely a field in the SU(2) coherent state and a field in the two-mode uncorrelated binomial state.

A. SU(2) coherent state

When the initial field is in the SU(2) coherent state, the double summation in Eq. (3.1) reduces to a single summation due to the tight photon correlation between the modes,

$$P_i(t) = \sum_{n=0}^{m} \tilde{P}_f(n) P_i(n, m-n, t) . \tag{3.3}$$

Here $\tilde{P}_f(n)$ is given by the binomial distribution

$$\tilde{P}_f(n) = \begin{bmatrix} m \\ n \end{bmatrix} s^n (1-s)^{m-n} . \tag{3.4}$$

In Figs. 2 and 3 we display the occupation probabilities of the various atomic levels as a function of time, for varying values of the atomic-field coupling constants and partition parameter s. The former is for the case in which the atom-field coupling constants are equal, and as we have mentioned earlier on, this leads to a purely *periodic* behavior of the atomic occupation probabilities due to the independence of the generalized two-photon Rabi frequency on the photon number. We further observe that increasing the mean number of photons in mode a or b leads to an enhancement of one-photon transitions, as seen in Fig 1(b), where 90% of the photons are initially in mode a. When the atom-field coupling constants are not equal, we observe collapse and revival of the Rabi oscillations (Fig. 3). Due to the possibility of one- and two-photon processes, two types of revivals are seen. The ones with smaller amplitude are associated with one-photon processes, while those of larger amplitude are associated with two-photon processes. As in the case of equal atom-field coupling constants, an enhancement of the one-photon processes is observed when there is an imbalance of mean photon number between the modes [see Fig. 3(b)].

Following a procedure analogous to that given in Refs. [4] and [8], we can estimate the revival times of the Rabi oscillations of the atomic occupation probabilities. We suppose that the dominant Rabi frequency is the one for which $n \simeq \bar{n}$, and that the interval between revivals t_r is obtained when two neighboring components with $n = \bar{n}$ and $n = \bar{n}+1$ differ in phase by 2π (for the two-photon revivals), that is,

$$(\Omega_{\bar{n}-1,m-\bar{n}} - \Omega_{\bar{n},m-\bar{n}-1}) t_r = 2\pi , \tag{3.5}$$

or more simply

$$t_r \approx \begin{cases} \dfrac{4\pi\Omega_{\bar{n},m-\bar{n}-1}}{g_b^2 - g_a^2}, & g_b > g_a \\[2ex] \dfrac{4\pi\Omega_{\bar{n}-1,m-\bar{n}}}{g_a^2 - g_b^2}, & g_a > b_g . \end{cases} \tag{3.6}$$

The interval between revivals of the one-photon Rabi oscillations occurs at half these times. These predictions are in agreement with the numerical plots of Fig. 3.

Finally we investigate the mean behavior of the atomic dynamics, as determined by the time-averaged occupation probabilities defined by

$$\bar{P}_i = \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, P_i(t) \,. \tag{3.7}$$

Using Eq. (3.3) we find

$$\bar{P}_i = \sum_{n=0}^{m} \tilde{P}_f(n) \bar{P}_i(n_a,n_b) \,, \tag{3.8}$$

where

$$\bar{P}_i(n_a,n_b) = \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, P_i(n_a,n_b,t) \,, \tag{3.9}$$

from which we obtain

$$\bar{P}_0(n_a,n_b) = \frac{2\Omega_{n_b}^4 + \Omega_{n_a-1}^4}{2\Omega_{n_a-1,n_b}^4} \,,$$
$$\bar{P}_1(n_a,n_b) = \frac{\Omega_{n_a-1}^2}{2\Omega_{n_a-1,n_b}^2} \,, \tag{3.10}$$
$$\bar{P}_2(n_a,n_b) = \frac{3\Omega_{n_a-1}^2 \Omega_{n_b}^2}{2\Omega_{n_a-1,n_b}^4} \,.$$

The time-average occupation probabilities (3.8) are plotted in Fig. 4 as a function of the parition parameter s and for varying values of the atom-field coupling constants. We observe that as we increase the number of mean pho-

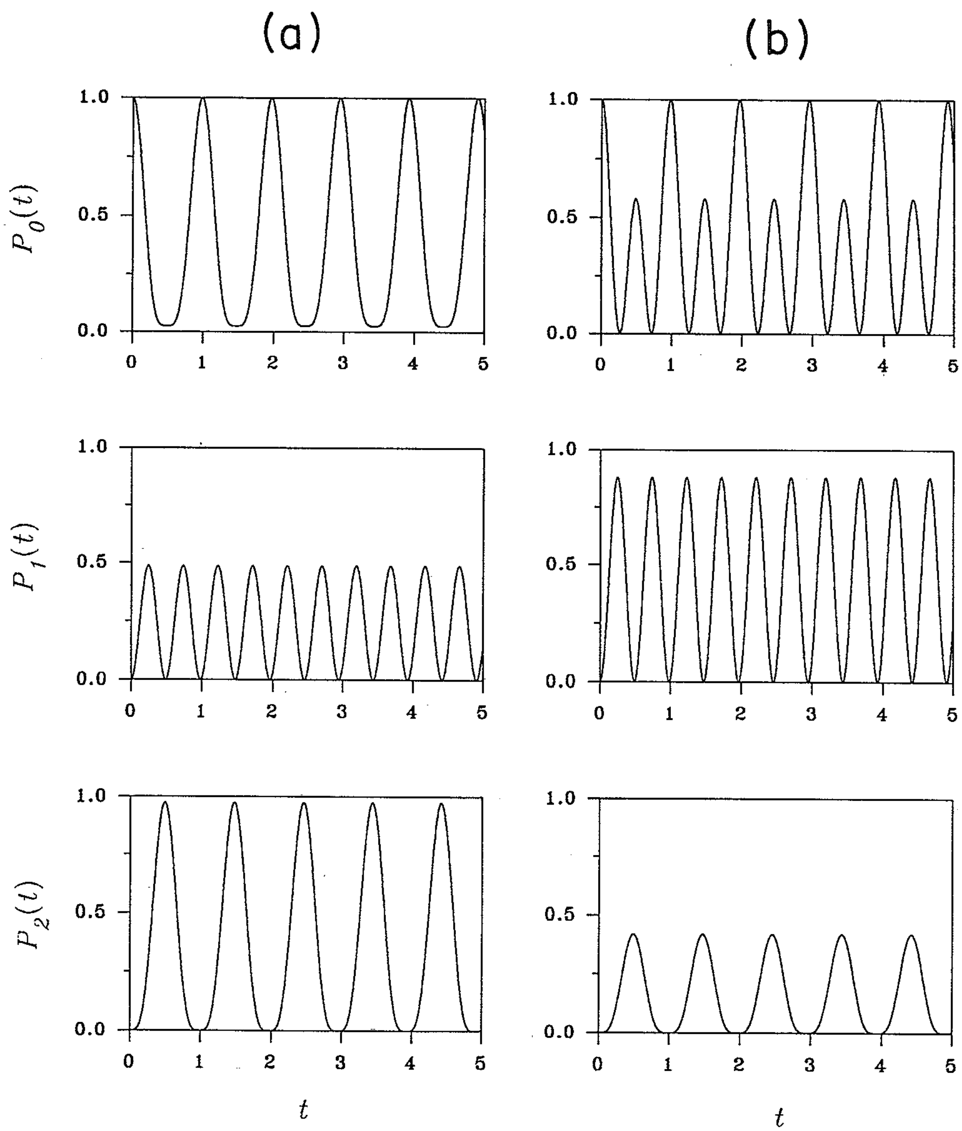

FIG. 2. Time evolution of the atomic occupation probabilities when the initial field is in the SU(2) coherent state with a maximum total m of 40 photons. The atom-field coupling constants are $g_a = g_b = 1$, and the partition parameter (a) $s = 0.5$ and (b) $s = 0.9$.

tons in mode a, at the expense of a decrease in mode b, the one-photon transitions are enhanced, as shown by an increase in the occupation probability of level 1. Of course we will observe the same effect when we increase the number of mean photons in mode b, at the expense of a decrease in mode a. We note that the time-averaged occupation probabilities of levels 0 and 2 have a minimum and a maximum value, respectively. These points occur when the Rabi frequencies of the respective modes are comparable in magnitude with each other.

B. Two-mode uncorrelated binomial state

In the two-mode uncorrelated binomial state, the occupation probabilities of the atomic levels take the form

$$P_i(t)=\sum_{n_a=0}^{m}\sum_{n_b=0}^{m}\tilde{P}_f(n_a,n_b)P_i(n_a,n_b,t)\ , \tag{3.11}$$

where $\tilde{P}_f(n_a,n_b)$ is given by the joint binomial photon distribution

$$\tilde{P}_f(n_a,n_b)=\begin{bmatrix} m \\ n_a \end{bmatrix} s^{n_a}(1-s)^{m-n_a}\begin{bmatrix} m \\ n_b \end{bmatrix}(1-s)^{n_b}s^{m-n_b}$$
$$=P_f^a(n_a)P_f^b(n_b)\ . \tag{3.12}$$

We have chosen the distribution (3.12) such that the mean and variance in photon number are the same as those in the corresponding modes of the SU(2) coherent state [see Eq. (2.10)].

In Fig. 5 we display the occupation probabilities of the various atomic levels for two choices of the atom-field coupling constants. We immediately notice distinct differences between this case and the case in which the field is initially prepared in an SU(2) coherent state. First, there is no purely periodic behavior in time of the

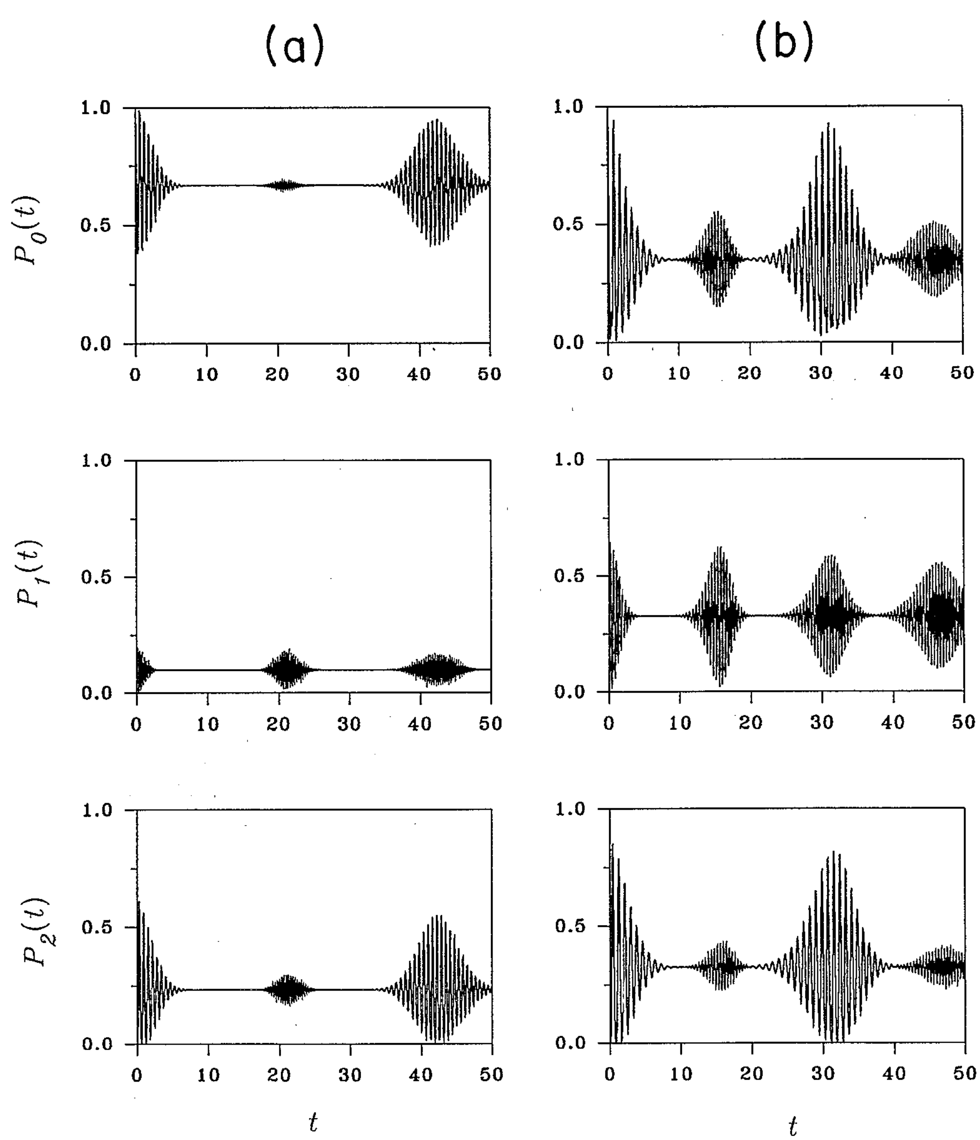

FIG. 3. As in Fig. 2 but with $g_a=1$ and $g_b=2$.

atomic occupation probabilities. Second, although there are still two types of revivals, these occur at much later times. We also note that the amplitude of the Rabi oscillations are smaller than those associated with the corresponding SU(2) coherent state.

To estimate the interval between revivals we adopt an analogous procedure given in Ref. 12. We write the two-photon component of the occupation probability of level 0 in the form

$$P_0'(t)=\mathrm{Re}\left[\sum_{n_a,n_b}\tilde{P}_f(n_a,n_b)\frac{2\Omega^2_{n_a-1}\Omega^2_{n_b}}{\Omega^4_{n_a-1,n_b}}\exp(i\Omega_{n_a-1,n_b}t)\right]. \quad (3.13)$$

We now suppose that the dominant Rabi frequency is that for which $n_a=\bar{n}_a$ and $n_b=\bar{n}_b$, so that we may expand Ω_{n_a-1,n_b} in a Taylor series,

$$\begin{aligned}\Omega_{n_a-1,n_b}&\approx\Omega_{n_a-1,n_b-1}\\&\approx\Omega_{\bar{n}_a-1,\bar{n}_b-1}+\tfrac{1}{2}g_a^2\Omega^{-1}_{\bar{n}_a-1,\bar{n}_b-1}(n_a-\bar{n}_a)\\&\quad+\tfrac{1}{2}g_b^2\Omega^{-1}_{\bar{n}_a-1,\bar{n}_b-1}(n_b-\bar{n}_b)+\cdots .\end{aligned} \quad (3.14)$$

Substituting this back into the expression (3.13) gives

$$\begin{aligned}P_0'(t)=\mathrm{Re}\Bigg[&\exp(i\Omega_{\bar{n}_a-1,\bar{n}_b-1}t)\sum_{n_a,n_b}\tilde{P}_f(n_a,n_b)\frac{2\Omega^2_{n_a-1}-\Omega^2_{n_b}}{\Omega^4_{n_a-1,n_b}}\\&\times\exp[i\tfrac{1}{2}g_a^2\Omega^{-1}_{\bar{n}_a-1,\bar{n}_b-1}(n_a-\bar{n}_a)t]\exp[i\tfrac{1}{2}g_b^2\Omega^{-1}_{\bar{n}_a-1,\bar{n}_b-1}(n_b-\bar{n}_b)t]\Bigg].\end{aligned} \quad (3.15)$$

Now the times of revivals t_r of the two-photon Rabi oscillations occur when the two phases in the exponentials inside the sum are in phase with each other, that is, when

$$\begin{aligned}&\tfrac{1}{2}g_a^2\Omega^{-1}_{\bar{n}_a-1,\bar{n}_b-1}=2\pi k,\quad k=0,1,2,\ldots,\\&\tfrac{1}{2}g_b^2\Omega^{-1}_{\bar{n}_a-1,\bar{n}_b-1}t_r=2\pi l,\quad l=0,1,2,\ldots .\end{aligned} \quad (3.16)$$

Multiplying and dividing these two expressions, we obtain the conditions for the revival times as follows:

$$\begin{aligned}&t_r=\frac{4\pi\sqrt{m}\,\Omega_{\bar{n}_a-1,\bar{n}_b-1}}{g_a g_b},\quad m=kl=0,1,2,\ldots,\\&\frac{g_a^2}{g_b^2}=\frac{k}{l}.\end{aligned} \quad (3.17)$$

For the values used in the numerical plots of Fig. 5 we estimate the revival times of the two-photon Rabi oscillations as

$$t_r\approx\begin{cases}80, & g_a=g_b=1\\126, & g_a=1,\ g_b=2 .\end{cases} \quad (3.18)$$

The revival times of the one-photon Rabi oscillations occur at half these values. These are in agreement with the numerical results.

IV. FIELD STATISTICS

In this section we turn our attention to a study of the field dynamics, in particular the time evolution of the photon distributions and the correlations (or anticorrelations) between the field modes. We start by calculating the joint photon distribution at time t, which from Eq. (2.4) is given by

$$P_f(n_a,n_b,t)=\tilde{P}_f(n_a,n_b)P_0(n_a,n_b,t)+\tilde{P}_f(n_a+1,n_b)P_1(n_a+1,n_b,t)+\tilde{P}_f(n_a+1,n_b-1)P_2(n_a+1,n_b-1,t). \quad (4.1)$$

The marginal photon distributions are obtained by tracing out the appropriate variable:

$$P_f^{a(b)}(n_{a(b)},t)=\sum_{n_{b(a)}}P_f(n_a,n_b,t). \quad (4.2)$$

For the SU(2) coherent state (2.7), Eqs. (4.1) and (4.2) become

$$P_f(n_a,n_b,t)=\tilde{P}_f(n_a,m-n_a)P_0(n_a,m-n_a)\delta_{n_b,m-n_a}$$
$$+\tilde{P}_f(n_a+1,m-n_a-1)P_1(n_a+1,m-n_a-1,t)\delta_{n_b,m-n_a-1}$$
$$+\tilde{P}_f(n_a+1,m-n_a-1)P_2(n_a+1,m-n_a-1,t)\delta_{n_b,m-n_a}, \quad (4.3)$$

$$P_f^a(n,t)=\tilde{P}_f^a(n)P_0(n,m-n,t)+\tilde{P}_f^a(n+1)[1-P_0(n+1,m-n-1,t)],$$

$$P_f^b(n,t)=\tilde{P}_f^b(n)[1-P_2(m-n,n,t)]+\tilde{P}_f^b(n-1)P_2(m-n+1,n-1,t),$$

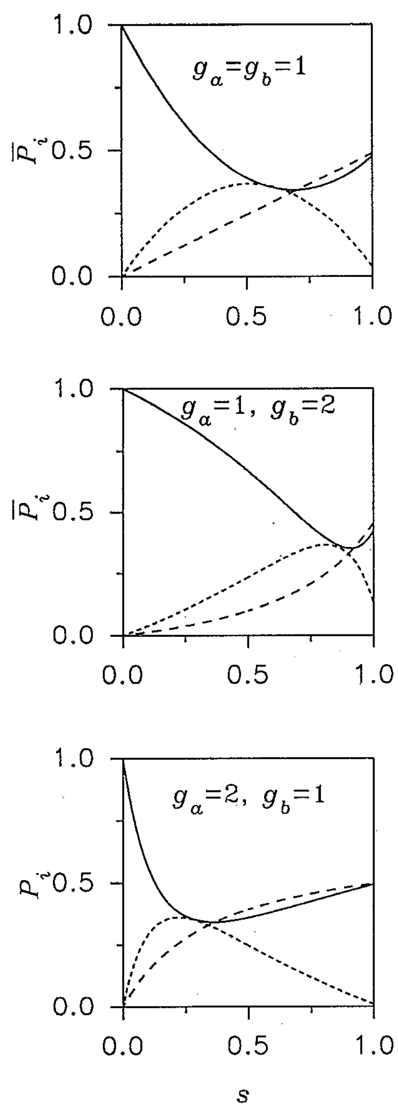

FIG. 4. Time-averaged values of the atomic occupation probabilities as a function of the partition parameter s when the initial field is in the SU(2) coherent state with a maximum total m of 40 excitations. The solid, long-dashed, and short-dashed lines represent levels 0, 1, and 2, respectively.

where the initial photon distributions are given by Eqs. (2.9). For the two-mode uncorrelated binomial state, we replace $\tilde{P}_f(n_a,n_b)$ in Eqs. (4.1) and (4.2) by the joint photon distribution (3.12). The photon distributions are thus greatly altered from their initial binomial forms by the atom-field interaction.

The correlation or anticorrelation between the modes as mentioned before is described by the normalized cross-correlation function defined by

$$g^{(2)}(t)=\frac{\langle n_a n_b\rangle_t}{\langle n_a\rangle_t\langle n_b\rangle_t}. \quad (4.4)$$

Written in this form, the cross-correlation function is a measure of the coincidence counting of a and b photons at time t. If $g^{(2)}$ is less than unity, we say that the photons of mode a and b are anticorrelated, otherwise they are correlated.

For the SU(2) coherent state the relevant quantities in the expression for $g^{(2)}(t)$ are given as follows:

$$\langle n_a\rangle_t=sm-\sum_{n=0}^{m}\tilde{P}_f^a(n)[1-P_0(n,m-n,t)],$$
$$\langle n_b\rangle_t=(1-s)m+\sum_{n=0}^{m}\tilde{P}_f^b(n)P_2(m-n,n,t), \quad (4.5)$$
$$\langle n_a n_b\rangle_t=s(1-s)m(m-1)$$
$$-\sum_{n=0}^{m}\tilde{P}_f(n,m-n)$$
$$\times[(m-n)P_1(n,m-n,t)$$
$$+(m-2n+1)P_2(n,m-n,t)].$$

On the other hand, for the two-mode uncorrelated binomial state, we have

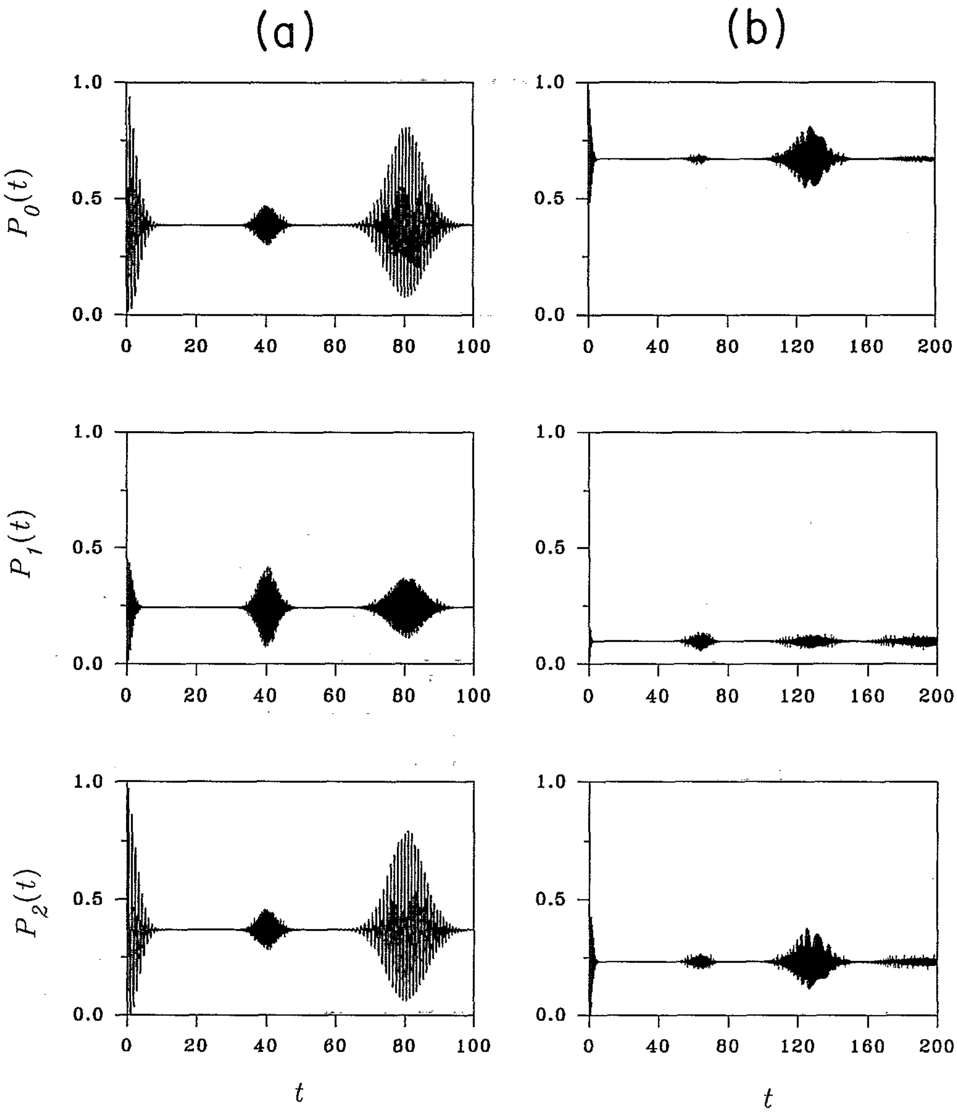

FIG. 5. Time evolution of the atomic occupation probabilities when the initial field is in the two-mode uncorrelated binomial state with a maximum total m of 40 excitations. The atom-field coupling constants are (a) $g_a = g_b = 1$ and (b) $g_a = 1$, $g_b = 2$, and the partition parameter $s = 0.5$.

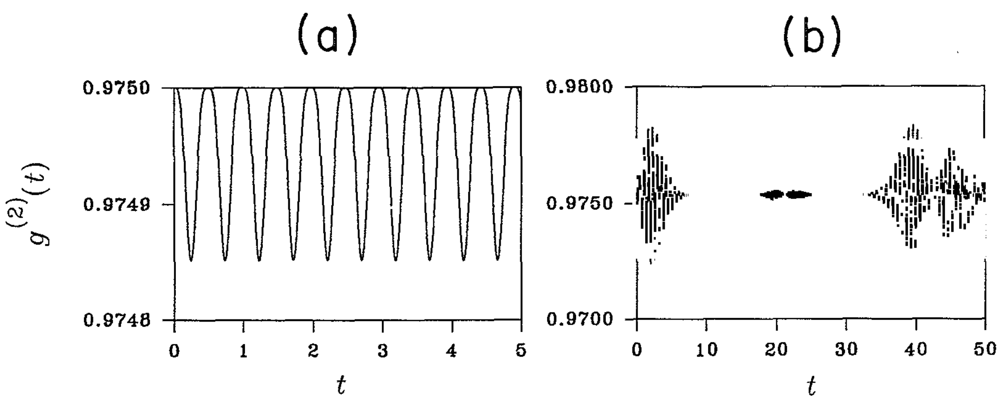

FIG. 6. Time evolution of the cross-correlation function when the initial field is in the SU(2) coherent state with a maximum total m of 40 excitations. The atom-field coupling constants are (a) $g_a = g_b = 1$ and (b) $g_a = 1$, $g_b = 2$, and the partition parameter $s = 0.5$.

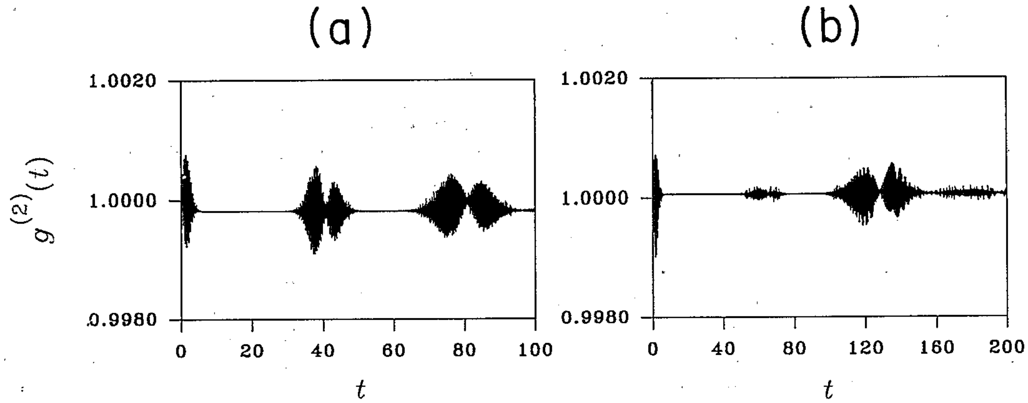

FIG. 7. As in Fig. 6, but with the field in the two-mode uncorrelated binomial state.

$$\begin{aligned}\langle n_a\rangle_t&=\langle n_a\rangle_0-\sum_{n_a=0}^{m}\sum_{n_b=0}^{m}\tilde{P}_f(n_a,n_b)[1-P_0(n_a,n_b,t)],\\ \langle n_b\rangle_t&=\langle n_b\rangle_0+\sum_{n_a=0}^{m}\sum_{n_b=0}^{m}\tilde{P}_f(n_a,n_b)P_2(n_a,n_b,t)\ ,\\ \langle n_a n_b\rangle_t&=\langle n_a n_b\rangle_0\\ &\quad-\sum_{n_a=0}^{m}\sum_{n_b=0}^{m}\tilde{P}_f(n_a,n_b)\\ &\qquad\times[n_bP_1(n_a,n_b,t)\\ &\qquad\quad-(n_a-n_b-1)P_2(n_a,n_b,t)]\ .\end{aligned}\tag{4.6}$$

In Figs. 6 and 7 we display the time evolution of the cross-correlation function for two sets of atom-field coupling constants. We note that for the SU(2) coherent state (Fig. 6), the cross-correlation function evolves in a purely periodic manner when the atom-field coupling constants are equal, and that it oscillates between its initial value and some lower value; thus the effect of the atom-field interaction is to *increase* the anticorrelation between the modes in this case. We can trace this increase in anticorrelation back to the periodic removal of a photon from mode a and the emission of the same photon into mode b. Had we started in the atomic state $|1\rangle$ we would have observed a decrease in anticorrelation due to the periodic emission of an additional photon into mode a or b. For nonequal atom-field coupling constants, oscillations of the cross-correlation exhibit collapses and revivals similar to those of the atomic occupation probabilities. However, there is one difference; the revivals occur in pairs. One can readily understand this from the expression for $\langle n_a n_b\rangle_t$ which depends on the occupation probabilities of two atomic levels, rather than one as in the case of $\langle n_a\rangle_t$ or $\langle n_b\rangle_t$. We note that, unlike the case of equal atom-field coupling constants, the time-averaged anticorrelation between the modes is slightly weakened by the atom-field interaction. Similar remarks apply for the two-mode uncorrelated binomial state, except that the cross correlation does not exhibit purely periodic behavior for any choice of the atom-field coupling constants.

V. SUMMARY

We have studied the dynamics of a three-level atom interacting with an SU(2) coherent state and a two-mode uncorrelated binomial state. We showed that there are marked differences between the two cases. In particular, we found that for the SU(2) coherent state, the atomic occupation probabilities exhibit purely periodic behavior for equal atom-field coupling constants. This is entirely absent when the field is in the two-mode uncorrelated binomial state or any other state except for the number state. Furthermore, we have found that the time of revivals of the Rabi oscillations when the initial field is in the SU(2) coherent state is much shorter than those when the field is in the two-mode uncorrelated binomial state, even though the photon statistics of the *individual* modes are the same as those in the latter. Finally, we studied the influence of the atom-field interaction on the field dynamics. We showed that the photon distributions are greatly altered from their initial forms by the interaction. We examined the anticorrelation between the modes and found that its time evolution follows a similar pattern to that of the atomic occupation probabilities.

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