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Three-level atoms in phase-sensitive broadband correlated reservoirs

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A three-level atom with two competing transitions can sense correlations between modes of an electromagnetic field with which it interacts. We examine the sensitivity of three-level atoms in ladder, Λ, and V configurations to correlations between modes in a broadband correlated reservoir of field modes. Our model of a correlated reservoir includes pairwise mode correlations around a center frequency. When such correlations are set to zero, the reservoir becomes a simple thermal heat bath, but in the opposite extreme when the modes are perfectly correlated, we recover a minimum-uncertainty broadband squeezed vacuum. We show that atomic populations are extremely sensitive to these correlations, and can, in the ladder system, become inverted as a result of the phase-sensitive noise. We also show how atomic population trapping is affected by mode correlations.

I. INTRODUCTION

Heat baths are traditionally associated with relaxation processes. A quantum system prepared in some superposition state, or any other kind of pure state, and then allowed to interact with such a bath generally dissipates its coherence and relaxes to a steady state which for a nonzero occupation number (or temperature) is a *mixed* state independent of the initial state of the quantum system [1]. The finite temperature state of a broadband multimode light field is a good example of such a phase-insensitive heat bath or reservoir. However, it is now possible to construct *correlated* multimode reservoirs [2], sometimes called "rigged reservoirs" [3] based on the establishment of a squeezed light field [4]. For example, a degenerate parametric amplifier creates a single-mode squeezed state when operated in an appropriate cavity [5]; the coupling of the single cavity field mode to an infinite number of external "output" modes transfers this squeezing into correlations between sidebands of the multimode light field [6]. Even though the output is a dissipative process, the broadband squeezed vacuum output remains in a *pure* state [7]. This pure state is characterized by the mean photon number N at frequency ω, and by the correlation M between modes symmetrically displaced about some center frequency. An ideal squeezed state [4] is characterized by the equality $M^2=N(N+1)$; for a nonideal but correlated state $M^2\leq N(N+1)$, and for an uncorrelated state, $M=0$. An atom interacting with a correlated reservoir will be sensitive to the correlations between the modes and in general will relax in a phase-sensitive fashion to a final state that will reflect such correlations. In this paper, we investigate how a three-level atom, variously in ladder (Ξ), lambda (Λ), and vee (V) configurations interactions with a broadband correlated reservoir. A three-level atom, with two competing transitions, is ideally suited for such correlation studies [8]. We will show how the response of these atoms is sensitive to the effects of mode occupation N and of intermode correlations M: atomic steady states can be dramatically altered even to the point of establishing population inversions; coherent superpositions can be established in steady state instead of statistical mixtures: conversely coherent effects in the atoms such as population trapping [8] can be eliminated or preserved, depending on the nature of the mode correlations and the atomic transition dipoles.

Two-level atoms interacting with a broadband squeezed vacuum are known to exhibit phase-sensitive dipole decay [9], but relax individually to a final state that is a statistical mixture indistinguishable from a *thermal* state independent of any mode correlations in the field. Nevertheless *two* two-level atoms sufficiently close together and excited by a correlated reservoir do relax to a state that reflects the mode correlations and which can be a pure state with maximal internal correlations [10]. The establishment of correlations between subsystems interacting with a broadband squeezed reservoir is a general property of such systems under appropriate conditions of resonance [11]. The interaction of many two-level atoms with such a reservoir can lead to a highly pairwise-correlated final state critically dependent on the existence of mode correlations [12]. We will show that many of these features can be seen in the relaxation of a *single* three-level atoms. The plan of this paper is as follows: in Sec. II we introduce the three-level atom configurations and the description of the correlated reservoir. In Sec. III we examine in detail the most interesting atomic configuration, the ladder system, and show how the steady-state populations reflect the mode correlations, and study the transition from thermal equilibrium to a partially inverted final state as the correlations increase until an SU(2) atomic coherent state [13] is established for an ideally correlated reservoir. In Sec. IV we examine the behavior of the V system in a similar reservoir; here we see an insensitivity to mode correlations, but the persistence of interesting atomic coherences. In Sec. V, we examine the Λ system, with similar results to the V system but with an interesting preservation of population trapping. Three appendices describe (A) the properties of correlated reservoirs, (B) the shifts and widths of atomic transitions in correlated reservoirs, and (C) the properties of the SU(2) coherent states for three-level systems.

II. THE MODEL

There are three distinct atomic-level configurations of three-level atoms. These are the Ξ configuration, the Λ configuration, and the V configuration. We will study in detail the interaction of a broadband correlated reservoir with atoms of all three types. In this section we will define the relevant atom-field Hamiltonians which will be used in later sections in the paper.

A. Ξ configuration

The Hamiltonian for a Ξ-type atom [see Fig. 1(a)] interacting with the electromagnetic field in the dipole and rotating-wave approximations may be written as

$$\hat{H}=\sum_{j=1}^{3} E_j\hat{R}_{jj}(t)+\sum_{\mathbf{k},\lambda}\hbar\omega_{\mathbf{k}}\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{a}_{\mathbf{k}\lambda}(t)+\hat{H}_{\text{int}} \tag{1}$$

with an atom-field interaction

$$\begin{aligned}\hat{H}_{\text{int}}=i\hbar\sum_{\mathbf{k},\lambda}[&g_{12}(\mathbf{k},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{12}(t)\\ &-g_{21}(\mathbf{k},\lambda)\hat{R}_{21}(t)\hat{a}_{\mathbf{k}\lambda}(t)]\\ +i\hbar\sum_{\mathbf{k},\lambda}[&g_{23}(\mathbf{k},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{23}(t)\\ &-g_{32}(\mathbf{k},\lambda)\hat{R}_{32}(t)\hat{a}_{\mathbf{k}\lambda}(t)]\ ,\end{aligned} \tag{2}$$

where $\hat{R}_{mn}(t)$ are the atomic operators in the Heisenberg representation obeying the canonical commutation rule

$$[\hat{R}_{ij}(t),\hat{R}_{mn}(t)]=\delta_{jm}\hat{R}_{in}(t)-\delta_{in}\hat{R}_{mj}(t)\ . \tag{3}$$

The energies E_j of the unperturbed atomic states $|j\rangle$ $(j=1,2,3)$ of the atom in the Ξ configuration are ordered as indicated in Fig. 1(a), i.e., $E_3>E_2>E_1$. The Heisenberg-picture commutation relation for the photon creation $[\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)]$ and annihilation $[\hat{a}_{\mathbf{k}\lambda}(t)]$ operators for mode $(\mathbf{k},\lambda)$ is

$$[\hat{a}_{\mathbf{k}\lambda}(t),\hat{a}^{\dagger}_{\mathbf{q}\sigma}(t)]=\delta^3_{\mathbf{k},\mathbf{q}}\delta_{\lambda,\sigma}\ . \tag{4}$$

In this paper we employ the Coulomb gauge [14], when $\mathbf{k}\cdot\mathbf{e}_{\mathbf{k}\lambda}=0$ where $\mathbf{e}_{\mathbf{k}\lambda}$ is the polarization vector (we will suppose this vector to be real) with $\lambda=1,2$. The frequency of the mode $(\mathbf{k},\lambda)$ is defined as usual: $\omega_{\mathbf{k}}=c|\mathbf{k}|$, where the wave vector $\mathbf{k}$ in the finite quantization volume V (for example a cube with side L) has components $(k_x,k_y,k_z)=(2\pi/L)(n_x,n_y,n_z)$ with n_x, n_y, n_z integer. The broadband correlated radiation field [2] is characterized by the following expectation values for the annihilation and creation operators:

$$\langle\hat{a}^{\dagger}_{\mathbf{k}\lambda}\hat{a}_{\mathbf{q}\sigma}\rangle=N(\mathbf{k},\lambda)\delta^3_{\mathbf{k},\mathbf{q}}\delta_{\lambda,\sigma}\ , \tag{5}$$

where $N(\mathbf{k},\lambda)$ is the number of photons per unit bandwidth. In what follows we will suppose $N(\mathbf{k},\lambda)=N(\omega_{\mathbf{k}})/|\mathbf{k}|^2$. In correspondence with the broadband correlated reservoir defined in a one-dimensional space (for details see Appendix A) we define a two-photon correlation function $\langle\hat{a}_{\mathbf{k}\lambda}\hat{a}_{\mathbf{q}\sigma}\rangle$ as

$$\begin{aligned}\langle\hat{a}_{\mathbf{k}\lambda}\hat{a}_{\mathbf{q}\sigma}\rangle=&[M(2\Omega-\omega_{\mathbf{k}})/|\mathbf{k}|\cdot|\mathbf{q}|]\delta_{\lambda,\sigma}\delta(2\Omega-\omega_{\mathbf{k}}-\omega_{\mathbf{q}})\\ &\times\delta(\phi_{\mathbf{k}}-\phi_{\mathbf{q}})\delta(\theta_{\mathbf{k}}-\theta_{\mathbf{q}})\ ,\end{aligned} \tag{6}$$

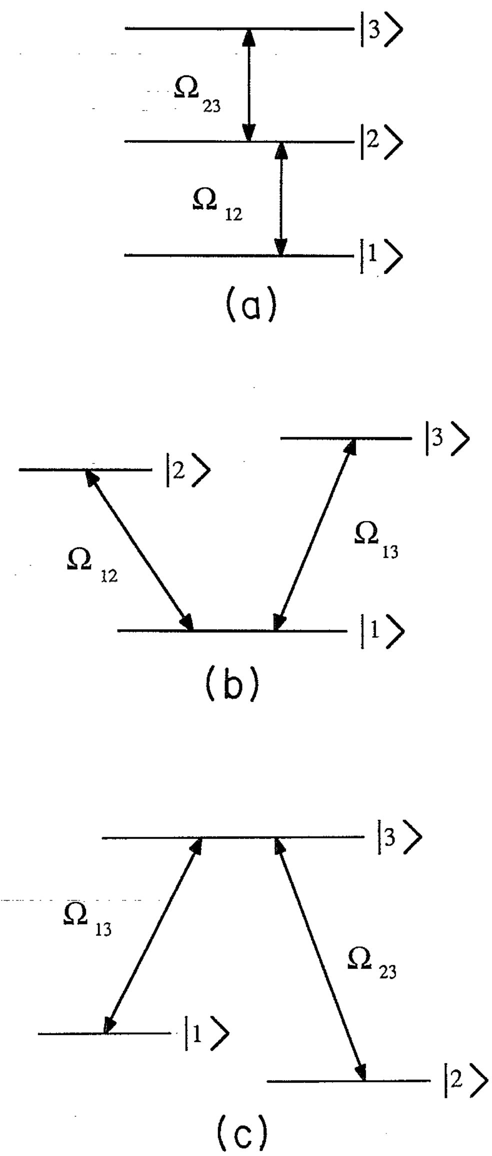

FIG. 1. The energy level diagrams for a three-level atom in (a) Ξ configuration, (b) V configuration, and (c) Λ configuration.

where we have used polar variables to describe the wave vectors **k** and **q** and the wave vector of the central carrier mode with frequency Ω. It should be noticed here that our model is isotropic and therefore the polar angles ϕ and θ do not play any role and we can consider just correlations with respect to frequencies. In what follows we will suppose for simplicity M to be real and positive.

The atom-field coupling constants $g_{ij}(\mathbf{k},\lambda)$ in the dipole approximation [14,15] are defined as

$$g_{ij}(\mathbf{k},\lambda)=\left[\frac{2\pi\omega_{\mathbf{k}}}{\hbar V}\right]^{1/2}(\mu_{ij}\cdot\mathbf{e}_{\mathbf{k},\lambda}) , \qquad (7)$$

where $\mu_{ij}=\langle i|e\mathbf{r}|j\rangle$ is the electric-dipole-moment transition matrix element between the atomic states $|i\rangle$ and $|j\rangle$ (e is the electron charge); if there is an allowed electric-dipole transition between states $|i\rangle$ and $|j\rangle$, the $\mu_{ij}\neq 0$. In what follows we will suppose the coupling constants $g_{ij}(\mathbf{k},\lambda)$ to be isotropic, i.e., $g_{ij}(\mathbf{k},\lambda)=g_{ij}(\omega_{\mathbf{k}},\lambda)$. In all that follows, we make the rotating-wave approximation; our frequency shifts are therefore in need of correction, but we will assume such shifts are sufficiently unimportant to warrant their neglect.

B. V configuration and Λ configuration

The energies E_j of the unperturbed atomic states $|j\rangle$ of the V-type atom are such that $E_1<E_2,E_3$ [see Fig. 1(b)] and the interaction part of the Hamiltonian is

$$\begin{aligned}\hat{H}_{\text{int}}=i\hbar\sum_{\mathbf{k},\lambda}&[g_{12}(\mathbf{k},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{12}(t)\\&-g_{21}(\mathbf{k},\lambda)\hat{R}_{21}(t)\hat{a}_{\mathbf{k}\lambda}(t)]\\&+i\hbar\sum_{\mathbf{k},\lambda}[g_{13}(\mathbf{k},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{13}(t)\\&-g_{31}(\mathbf{k},\lambda)\hat{R}_{31}(t)\hat{a}_{\mathbf{k}\lambda}(t)] .\end{aligned} \qquad (8)$$

For the Λ-type atom [Fig. 1(c)] the energies E_j are related as follows $E_1,E_2<E_3$ and the corresponding interaction Hamiltonian is

$$\begin{aligned}\hat{H}_{\text{int}}=i\hbar\sum_{\mathbf{k},\lambda}&[g_{13}(\mathbf{k},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{13}(t)\\&-g_{31}(\mathbf{k},\lambda)\hat{R}_{31}(t)\hat{a}_{\mathbf{k}\lambda}(t)]\\&+i\hbar\sum_{\mathbf{k},\lambda}[g_{23}(\mathbf{k},\lambda)a^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{23}(t)\\&-g_{32}(\mathbf{k},\lambda)\hat{R}_{32}(t)\hat{a}_{\mathbf{k}\lambda}(t)] .\end{aligned} \qquad (9)$$

All notations are the same as for the Ξ-type atom.

III. Ξ-TYPE ATOM DECAY IN A CORRELATED RESERVOIR

The dynamics of the Ξ-type atom interaction with a broadband correlated reservoir is governed by the Heisenberg equations of motion for the atomic and field operators [15]. In order to define the dynamics properly we suppose that before the interaction [i.e., at the initial moment $t=0$; $\hat{a}_{\mathbf{k}\lambda}(t=0)\equiv\hat{a}_{\mathbf{k}\lambda}$ and $\hat{R}_{mn}(t=0)\equiv\hat{R}_{mn}$] field and atomic operators commute:

$$[\hat{a}_{\mathbf{k}\lambda},\hat{R}_{mn}]=0 ,$$

so that at the initial moment the atom and the field can be supposed as independent systems. Because the dynamics are governed by a unitary operator, we see that the equal-time commutators of field and atomic operators are always equal to zero.

We will write the Heisenberg equations of motion with normally ordered field operators as

$$\frac{d}{dt}\hat{a}_{\mathbf{k}\lambda}(t)=-i\omega_{\mathbf{k}}\hat{a}_{\mathbf{k}\lambda}(t)+g_{12}(\omega_{\mathbf{k}},\lambda)\hat{R}_{12}(t)+g_{23}(\omega_{\mathbf{k}},\lambda)\hat{R}_{23}(t) , \qquad (10)$$

$$\frac{d}{dt}\hat{R}_{11}(t)=\sum_{\mathbf{k},\lambda}[g_{12}(\omega_{\mathbf{k}},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{12}(t)+g_{21}(\omega_{\mathbf{k}},\lambda)\hat{R}_{21}(t)\hat{a}_{\mathbf{k}\lambda}(t)] , \qquad (11)$$

$$\frac{d}{dt}\hat{R}_{33}(t)=-\sum_{\mathbf{k},\lambda}[g_{23}(\omega_{\mathbf{k}},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{23}(t)+g_{32}(\omega_{\mathbf{k}},\lambda)\hat{R}_{32}(t)\hat{a}_{\mathbf{k}\lambda}(t)] , \qquad (12)$$

$$\begin{aligned}\frac{d}{dt}\hat{R}_{12}(t)=&-i\Omega_{12}\hat{R}_{12}(t)\\&-\sum_{\mathbf{k},\lambda}g_{21}(\omega_{\mathbf{k}},\lambda)[2\hat{R}_{11}(t)+\hat{R}_{33}(t)-1]\hat{a}_{\mathbf{k}\lambda}(t)\\&+\sum_{\mathbf{k},\lambda}g_{23}(\omega_{\mathbf{k}},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{13}(t) ,\end{aligned} \qquad (13)$$

$$\begin{aligned}\frac{d}{dt}\hat{R}_{23}(t)=&-i\Omega_{23}\hat{R}_{23}(t)\\&+\sum_{\mathbf{k},\lambda}g_{32}(\omega_{\mathbf{k}},\lambda)[2\hat{R}_{33}(t)+\hat{R}_{11}(t)-1]\hat{a}_{\mathbf{k}\lambda}(t)\\&-\sum_{\mathbf{k},\lambda}g_{12}(\omega_{\mathbf{k}},\lambda)\hat{a}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{R}_{13}(t) ,\end{aligned} \qquad (14)$$

$$\begin{aligned}\frac{d}{dt}\hat{R}_{13}(t)=&-i\Omega_{13}\hat{R}_{13}(t)+\sum_{\mathbf{k},\lambda}g_{21}(\omega_{\mathbf{k}},\lambda)\hat{R}_{23}(t)\hat{a}_{\mathbf{k}\lambda}(t)\\&-\sum_{\mathbf{k},\lambda}g_{32}(\omega_{\mathbf{k}},\lambda)\hat{R}_{12}(t)\hat{a}_{\mathbf{k}\lambda}(t) ,\end{aligned} \qquad (15)$$

where the atomic transition frequencies are defined as $\hbar\Omega_{ij}=E_j-E_i$. The rest of the equations can be found easily using the relations

$$\hat{R}_{11}(t)=1-\hat{R}_{22}(t)-\hat{R}_{33}(t),\quad \hat{R}^{\dagger}_{ij}(t)=\hat{R}_{ji}(t) .$$

Formally integrating the atomic and field equations of motion gives

$$\hat{a}_{\mathbf{k}\lambda}(t)=\hat{a}_{\mathbf{k}\lambda}\exp(-i\omega_{\mathbf{k}}t)+\int_0^t ds\,\exp[i\omega_{\mathbf{k}}(s-t)][g_{12}(\omega_{\mathbf{k}},\lambda)\hat{R}_{12}(s)+g_{23}(\omega_{\mathbf{k}},\lambda)\hat{R}_{23}(s)]\ , \tag{16}$$

$$\hat{R}_{11}(t)=\hat{R}_{11}+\sum_{\mathbf{k},\lambda}\int_0^t ds[g_{12}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(s)\hat{R}_{12}(s)+g_{21}(\omega_{\mathbf{k}},\lambda)\hat{R}_{21}(s)\hat{a}_{\mathbf{k}\lambda}(s)]\ , \tag{17}$$

$$\hat{R}_{33}(t)=\hat{R}_{33}-\sum_{\mathbf{k},\lambda}\int_0^t ds[g_{23}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(s)\hat{R}_{23}(s)-g_{32}(\omega_{\mathbf{k}},\lambda)\hat{R}_{32}(s)\hat{a}_{\mathbf{k}\lambda}(s)]\ , \tag{18}$$

$$\hat{R}_{12}(t)=\hat{R}_{12}\exp(-i\Omega_{12}t)-\sum_{\mathbf{k},\lambda}\int_0^t ds\{g_{21}(\omega_{\mathbf{k}},\lambda)[2\hat{R}_{11}(s)+\hat{R}_{33}(s)-1]\hat{a}_{\mathbf{k}\lambda}(s)-g_{23}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(s)\hat{R}_{13}(s)\}\exp[i\Omega_{12}(s-t)]\ , \tag{19}$$

$$\hat{R}_{23}(t)=\hat{R}_{23}\exp(-i\Omega_{23}t)+\sum_{\mathbf{k},\lambda}\int_0^t ds\{g_{23}(\omega_{\mathbf{k}},\lambda)[2\hat{R}_{33}(s)+\hat{R}_{11}(s)-1]\hat{a}_{\mathbf{k}\lambda}(s)-g_{12}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(s)\hat{R}_{13}(s)\}\exp[i\Omega_{23}(s-t)]\ , \tag{20}$$

$$\hat{R}_{13}(t)=\hat{R}_{13}\exp(-i\Omega_{13}t)+\sum_{\mathbf{k},\lambda}\int_0^t ds[g_{21}(\omega_{\mathbf{k}},\lambda)\hat{R}_{23}(s)\hat{a}_{\mathbf{k}\lambda}(s)-g_{32}(\omega_{\mathbf{k}},\lambda)\hat{R}_{12}(s)\hat{a}_{\mathbf{k}\lambda}(s)]\exp[i\Omega_{13}(s-t)]\ . \tag{21}$$

In the adiabatic approximation (equivalent to the Weisskopf-Wigner, or Markov approximation), i.e., when the atom-field interaction is supposed to be weak (for details see, for instance, Refs. [15] and [16]), the atomic operators $\hat{R}_{ij}(t)$ should evolve very nearly according to their free evolution:

$$\hat{R}_{ij}(s)\simeq\hat{R}_{ij}(t)\exp[-i\Omega_{ij}(s-t)]\ . \tag{22}$$

Substituting these "zero-order" solutions [with respect to the coupling constants $g_{ij}(\omega_{\mathbf{k}},\lambda)$] for the atomic operators as well as the "zero-order" solution for the field operator $\hat{a}_{\mathbf{k}\lambda}(t)$:

$$\hat{a}_{\mathbf{k}\lambda}(s)\simeq\hat{a}_{\mathbf{k}\lambda}(t)\exp[-i\omega_{\mathbf{k}}(s-t)] \tag{23}$$

into the relations (16)–(21), we obtain the "first-order" approximation for the operators under consideration:

$$\hat{a}_{\mathbf{k}\lambda}(t)=\hat{a}_{\mathbf{k}\lambda}\exp(-i\omega_{\mathbf{k}}t)+g_{12}(\omega_{\mathbf{k}},\lambda)\hat{R}_{12}(t)\xi(\omega_{\mathbf{k}}-\Omega_{12};t)+g_{23}(\omega_{\mathbf{k}},\lambda)\hat{R}_{23}(t)\xi(\omega_{\mathbf{k}}-\Omega_{23};t)\ , \tag{24}$$

$$\hat{R}_{11}(t)=\hat{R}_{11}+\sum_{\mathbf{k},\lambda}g_{12}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(t)\hat{R}_{12}(t)\xi(\omega_{\mathbf{k}}-\Omega_{12};t)+\sum_{\mathbf{k},\lambda}g_{21}(\omega_{\mathbf{k}},\lambda)\hat{R}_{21}(t)\hat{a}_{\mathbf{k}\lambda}(t)\xi^*(\omega_{\mathbf{k}}-\Omega_{12};t)\ , \tag{25}$$

$$\hat{R}_{33}(t)=\hat{R}_{33}-\sum_{\mathbf{k}\lambda}g_{23}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(t)\hat{R}_{23}(t)\xi(\omega_{\mathbf{k}}-\Omega_{23};t)+\sum_{\mathbf{k},\lambda}g_{32}(\omega_{\mathbf{k}},\lambda)\hat{R}_{32}(t)\hat{a}_{\mathbf{k}\lambda}(t)\xi^*(\omega_{\mathbf{k}}-\Omega_{23};t)\ , \tag{26}$$

$$\hat{R}_{12}(t)=\hat{R}_{12}\exp(-i\Omega_{12}t)-\sum_{\mathbf{k},\lambda}g_{21}(\omega_{\mathbf{k}},\lambda)[2\hat{R}_{11}(t)+\hat{R}_{33}(t)-1]\hat{a}_{\mathbf{k}\lambda}(t)\xi^*(\omega_{\mathbf{k}}-\Omega_{12};t)+\sum_{\mathbf{k},\lambda}g_{23}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(t)\hat{R}_{13}(t)\xi(\omega_{\mathbf{k}}-\Omega_{23};t)\ , \tag{27}$$

$$\hat{R}_{23}(t)=\hat{R}_{23}\exp(-i\Omega_{23}t)+\sum_{\mathbf{k},\lambda}g_{23}(\omega_{\mathbf{k}},\lambda)[2\hat{R}_{33}(t)+\hat{R}_{11}(t)-1]\hat{a}_{\mathbf{k}\lambda}(t)\xi^*(\omega_{\mathbf{k}}-\Omega_{23};t)-\sum_{\mathbf{k},\lambda}g_{12}(\omega_{\mathbf{k}},\lambda)\hat{a}^\dagger_{\mathbf{k}\lambda}(t)\hat{R}_{13}(t)\xi(\omega_{\mathbf{k}}-\Omega_{12};t)\ , \tag{28}$$

$$\hat{R}_{13}(t)=\hat{R}_{13}\exp(-i\Omega_{13}t)+\sum_{\mathbf{k},\lambda}g_{21}(\omega_{\mathbf{k}},\lambda)\hat{R}_{23}(t)\hat{a}_{\mathbf{k}\lambda}(t)\xi^*(\omega_{\mathbf{k}}-\Omega_{12};t)-\sum_{\mathbf{k}\lambda}g_{32}(\omega_{\mathbf{k}},\lambda)\hat{R}_{12}(t)\hat{a}_{\mathbf{k}\lambda}(t)\xi^*(\omega_{\mathbf{k}}-\Omega_{23};t)\ , \tag{29}$$

where

$$\xi(x;t)=\int_0^t ds\,\exp[ix(s-t)]\ . \tag{30}$$

Now we substitute the first-order solutions (24)–(29) into Eqs. (10)–(15). Keeping only the terms of order g^2 and averaging over the field operators we obtain from the Heisenberg equations of motion (10)–(15) the Bloch equations for the expectation values of the atomic operators. It can be seen that the system of equations for the mean values of atomic operators $\langle\hat{R}_{ij}(t)\rangle$ decouples into two independent systems of equations for $\langle\hat{R}_{11}(t)\rangle$, $\langle\hat{R}_{33}(t)\rangle$, $\langle\hat{R}_{13}(t)\rangle$, and $\langle\hat{R}_{31}(t)\rangle$ and for the rest of the variables, i.e., for $\langle\hat{R}_{12}(t)$, $\langle\hat{R}_{21}(t)\rangle$, $\langle\hat{R}_{23}(t)\rangle$, and $\langle\hat{R}_{32}(t)\rangle$. Because we are interested mainly in the atomic-level populations in what follows we will analyze only the variables $\langle\hat{R}_{11}(t)\rangle$, $\langle\hat{R}_{33}(t)\rangle$, $\langle\hat{R}_{13}(t)\rangle$, and $\langle\hat{R}_{31}(t)\rangle$. In writing down the set of equations for these operators we neglect all nonsecular terms, i.e., all terms oscillating too fast in a rotating frame [17]. These equations in matrix form can be written as follows:

$$\frac{d}{dt}\mathbf{A}=\underline{M}\,\mathbf{A}+\mathbf{B}\ , \tag{31}$$

where the column vectors **A** and **B** are given by the relations

$$\mathbf{A}=\begin{bmatrix}\langle\hat{R}_{11}(t)\rangle\\ \langle\hat{R}_{33}(t)\rangle\\ \langle\hat{R}'_{13}(t)\rangle\\ \langle\hat{R}'_{31}(t)\rangle\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}2[1+N(\Omega_{12})]\gamma_1(\Omega_{12})\\ 2N(\Omega_{23})\gamma_2(\Omega_{23})\\ -[M(\Omega_{23})\Phi_{12}(\Omega_{23})+M(\Omega_{12})\Phi_{12}(\Omega_{12})]\exp[i(\Omega_{13}-2\Omega)t]\\ -[M(\Omega_{23})\Phi^*_{12}(\Omega_{23})+M(\Omega_{12})\Phi^*_{12}(\Omega_{12})]\exp[-i(\Omega_{13}-2\Omega)t]\end{bmatrix}, \tag{32}$$

and the matrix $\underline{M}$ is defined as

$$\underline{M}=\begin{bmatrix}M_{11} & M_{12} & M_{13} & M_{14}\\ M_{21} & M_{22} & M_{23} & M_{24}\\ M_{31} & M_{32} & M_{33} & M_{34}\\ M_{41} & M_{42} & M_{43} & M_{44}\end{bmatrix}, \tag{33}$$

where

$$\begin{aligned}
&M_{11}=-2[1+2N(\Omega_{12})]\gamma_1(\Omega_{12})\ ,\\
&M_{12}=-2[1+N(\Omega_{12})]\gamma_1(\Omega_{12})\ ,\\
&M_{13}=M(\Omega_{23})\Phi^*_{12}(\Omega_{23})\exp[i(2\Omega-\Omega_{13})t]=M^*_{14}\ ,\\
&M_{21}=-2N(\Omega_{23})\gamma_2(\Omega_{23})\ ,\\
&M_{22}=-2[1+2N(\Omega_{23})]\gamma_2(\Omega_{23})\ ,\\
&M_{23}=M(\Omega_{12})\Phi^*_{12}(\Omega_{12})\exp[i(2\Omega-\Omega_{13})t]=M^*_{24}\ ,\\
&M_{31}=[2M(\Omega_{12})\Phi_{12}(\Omega_{12})+M(\Omega_{23})\Phi_{12}(\Omega_{23})]\exp[-i(2\Omega-\Omega_{13})t]=M^*_{41}\ ,\\
&M_{32}=[M(\Omega_{12})\Phi_{12}(\Omega_{12})+2M(\Omega_{23})\Phi_{12}(\Omega_{23})]\exp[-i(2\Omega-\Omega_{13})t]=M^*_{42}\ ,\\
&M_{33}=-\Phi_2(\Omega_{23})-\Phi_1^{(N)}(\Omega_{12})N(\Omega_{12})-\Phi_2^{(N)}(\Omega_{23})N(\Omega_{23})=M^*_{44}\ ,\\
&M_{34}=M_{43}=0\ .
\end{aligned} \tag{34}$$

In the above equations the slowly varying operator $\langle\hat{R}'_{13}(t)\rangle$ is defined by the relation

$$\langle\hat{R}'_{13}(t)\rangle=\langle\hat{R}_{13}(t)\rangle\exp(i\Omega_{13}t)\ , \tag{35}$$

and the other parameters are defined as

$$\Phi_i(\Omega_{mn})=\gamma_i(\Omega_{mn})-i\delta_i(\Omega_{mn})\ , \tag{36}$$

$$\Phi_i^{(N)}(\Omega_{mn})=\gamma_i(\Omega_{mn})-i\delta_i^{(N)}(\Omega_{mn})\ , \tag{37}$$

$$\Phi_{12}(\Omega_{mn})=\gamma_{12}(\Omega_{mn})-i\delta_{12}(\Omega_{mn})\ . \tag{38}$$

The physical meaning of these parameters is as follows: $\gamma_1(\Omega_{12})$ is the spontaneous decay rate of the transition $|2\rangle\to|1\rangle$ and $\delta_1(\Omega_{12})$ is the corresponding frequency shift; $\gamma_2(\Omega_{23})$ is the spontaneous decay rate of the transition $|3\rangle\to|2\rangle$ with the frequency shift $\delta_2(\Omega_{23})$; $N(\Omega_{12})\gamma_1(\Omega_{12})$ is the decay rate due to stimulated transitions between levels $|2\rangle$ and $|1\rangle$ and $N(\Omega_{12})\delta_1^{(N)}(\Omega_{12})$ is the corresponding frequency shift; similarly $N(\Omega_{23})\gamma_2(\Omega_{23})$ is the decay rate due to stimulated transitions between levels $|3\rangle$ and $|2\rangle$ and $N(\Omega_{23})\delta_2^{(N)}(\Omega_{23})$ is the corresponding frequency shift. Finally, $M(\Omega_{mn})\gamma_{12}(\Omega_{mn})$ is the decay rate between upper ($|n\rangle$) and lower ($|m\rangle$) levels induced by the correlations between various modes in the broadband correlated reservoir; $M(\Omega_{mn})\delta_{12}(\Omega_{mn})$ is the corresponding frequency shift. Explicit expressions for decay rates and shifts are presented in Appendix B.

The correlations between pairs of modes of the reservoir lead to very complex time evolution of the mean values of the atomic operators involved in Eq. (31). In particular, the right-hand side of this equation is explicitly time dependent, which means that the matrix equation (31) cannot be solved straightforwardly. To overcome this problem we assume the carrier frequency of the reservoir to be such that the two-photon-resonance condition $2\Omega=\Omega_{13}$ is satisfied. In this case the matrix $\underline{M}$ (33) does not depend on time and Eq. (31) can be solved. In addition we make one further simplifying assumption which allows us to derive results which draw out in a clear way the essential physics (of course not at the expense of losing the most characteristic features of the interaction between the three-level atom and the correlated reservoir); we impose an additional one-photon-resonance condition: $\Omega_{12}=\Omega_{23}=\Omega$, which means that the carrier mode of the reservoir is in resonance with both transitions $|1\rangle\leftrightarrow|2\rangle$ and $|2\rangle\leftrightarrow|3\rangle$. Nevertheless the transition matrix elements μ_{12} and μ_{23} are not equal, so the decay rates γ_1 and γ_2 are not equal. In this case the decay rate γ_{12} can be written as $\gamma_{12}^2=\gamma_1\gamma_2$. Hereafter we omit the frequency argument Ω in all parameters and we can rewrite Eq. (31) in the following form:

$$\frac{d}{dt}\mathbf{A}'=\underline{M}'\mathbf{A}'+\mathbf{B}' , \tag{39}$$

where

$$\mathbf{A}'=\begin{bmatrix}\langle\hat{R}_{11}(t)\rangle\\ \langle\hat{R}_{33}(t)\rangle\\ \langle\hat{X}_{13}(t)\rangle\\ \langle\hat{Y}_{13}(t)\rangle\end{bmatrix},\quad \mathbf{B}'=\begin{bmatrix}2(1+N)\gamma_1\\ 2N\gamma_2\\ -2M\gamma_{12}\\ 2M\delta_{12}\end{bmatrix}, \tag{40}$$

where $\langle\hat{X}_{13}(t)\rangle$ and $\langle\hat{Y}_{13}(t)\rangle$ are the real and imaginary part of $\langle\hat{R}'_{13}(t)\rangle$, respectively. The matrix $\underline{M}'$ is defined as follows:

$$\underline{M}'=\begin{bmatrix}-2(1+2N)\gamma_1 & -2(1+N)\gamma_1 & 2M\gamma_{12} & 2M\delta_{12}\\ -2N\gamma_2 & -2(1+2N)\gamma_2 & 2M\gamma_{12} & 2M\delta_{12}\\ 3M\gamma_{12} & 3M\gamma_{12} & -[(1+N)\gamma_2+N\gamma_1] & -[\delta_2+N(\delta_2^{(N)}+\delta_1^{(N)})]\\ -3M\delta_{12} & -3M\delta_{12} & \delta_2+N(\delta_2^{(N)}+\delta_1^{(N)}) & -[(1+N)\gamma_2+N\gamma_1]\end{bmatrix}. \tag{41}$$

From the above equations it is seen that the dynamics of the atom-field system is sensitive to the presence of frequency shifts induced by spontaneous transitions, stimulated transitions, and shifts induced by correlations in the broadband reservoir [18]. In what follows we neglect all frequency shifts (i.e., we put $\delta_i=\delta_i^{(N)}=\delta_{12}=0$) because they are insignificant compared with decay rates. In this case the imaginary part of the function $\langle\hat{R}'_{13}(t)\rangle$ is decoupled from the rest of the variables [i.e., $\langle\hat{R}_{11}(t)\rangle$, $\langle\hat{R}_{33}(t)\rangle$, and $\langle\hat{X}_{13}(t)\rangle$] and is governed by the equation

$$\frac{d}{dt}\langle\hat{Y}_{13}(t)\rangle=-[(1+N)\gamma_2+N\gamma_1]\langle\hat{Y}_{13}(t)\rangle . \tag{42}$$

The equations of motion for the other variables now take the form (39) but with the column vectors $\mathbf{A}'$ and $\mathbf{B}'$ defined as

$$\mathbf{A}'=\begin{bmatrix}\langle\hat{R}_{11}(t)\rangle\\ \langle\hat{R}_{33}(t)\rangle\\ \langle\hat{X}_{13}(t)\rangle\end{bmatrix},\quad \mathbf{B}'=\begin{bmatrix}2(1+N)\gamma_1\\ 2N\gamma_2\\ -2M\gamma_{12}\end{bmatrix}, \tag{43}$$

and the matrix $\underline{M}'$,

$$\underline{M}'=\begin{bmatrix}-2(1+2N)\gamma_1 & -2(1+N)\gamma_1 & 2M\gamma_{12}\\ -2N\gamma_2 & -2(1+2N)\gamma_2 & 2M\gamma_{12}\\ 3M\gamma_{12} & 3M\gamma_{12} & -[(1+N)\gamma_2+N\gamma_1]\end{bmatrix}. \tag{44}$$

The time evolution of the atomic operators can now be studied straightforwardly. Nevertheless in what follows instead of studying the detailed time evolution of the atomic-level populations we will analyze their stationary solutions

$$\lim_{t\to\infty}\langle\hat{R}_{ij}(t)\rangle=\langle\hat{R}_{ij}\rangle_{\rm st} .$$

A. Stationary solutions for Ξ-type atom

Stationary solutions for the mean values of the atomic operators of interest can be found from Eq. (39) with the left-hand side set equal to zero. After some algebra we find

$$\langle\hat{R}_{11}\rangle_{\rm st}=\frac{M^2\gamma_{12}^2[(2+N)\gamma_2+(1+N)\gamma_1]-[(1+N)\gamma_2+N\gamma_1](1+N)^2\gamma_1\gamma_2}{[(1+N)\gamma_2+N\gamma_1][3M^2\gamma_{12}^2-\gamma_1\gamma_2(1+3N+3N^2)]}, \tag{45}$$

$$\langle \hat{R}_{33} \rangle_{st} = \frac{M^2\gamma_{12}^2[N\gamma_2+(N-1)\gamma_1]-[(1+N)\gamma_2+N\gamma_1]N^2\gamma_1\gamma_2}{[(1+N)\gamma_2+N\gamma_1][3M^2\gamma_{12}^2-\gamma_1\gamma_2(1+3N+3N^2)]} , \tag{46}$$

$$\langle \hat{R}_{22} \rangle_{st} = \frac{M^2\gamma_{12}^2-(1+N)N\gamma_1\gamma_2}{[3M^2\gamma_{12}^2-\gamma_1\gamma_2(1+3N+3N^2)]} , \tag{47}$$

$$\langle \hat{X}_{13} \rangle_{st} = \frac{-M\gamma_{12}\gamma_1\gamma_2}{[(1+N)\gamma_2+N\gamma_1][3M^2\gamma_{12}^2-\gamma_1\gamma_2(1+3N+3N^2)]} . \tag{48}$$

In order to understand more clearly the role of correlations in the broadband reservoir on the decay of the three-level atom, we will first of all present the results for the stationary level populations of the three-level atom in a thermal reservoir.

1. Thermal reservoir

One can obtain from Eqs. (45)–(48) results for the stationary level populations of a three-level atom interacting with a thermal reservoir by setting M equal to zero. In a thermal reservoir the numbers of photons per unit of bandwidth is governed by the Planck thermal distribution, so we have

$$N=[\exp(\hbar\Omega/kT)-1]^{-1} , \tag{49}$$

where k is the Boltzmann constant and T is the temperature of the reservoir.

Stationary solutions for $\langle \hat{R}_{ii} \rangle_{st}$ are ($\langle \hat{X}_{13} \rangle_{st}$ is equal to zero):

$$\langle \hat{R}_{11} \rangle_{st} = \frac{(1+N)^2}{(1+3N+3N^2)} , \tag{50}$$

$$\langle \hat{R}_{22} \rangle_{st} = \frac{(1+N)N}{(1+3N+3N^2)} , \tag{51}$$

and

$$\langle \hat{R}_{33} \rangle_{st} = \frac{N^2}{(1+3N+3N^2)} . \tag{52}$$

Using the expression for the photon number distribution in the thermal reservoir (49) we find that

$$\frac{\langle \hat{R}_{33} \rangle_{st}}{\langle \hat{R}_{11} \rangle_{st}} = e^{-(E_3-E_1)/kT} , \quad \frac{\langle \hat{R}_{22} \rangle_{st}}{\langle \hat{R}_{11} \rangle_{st}} = e^{-(E_2-E_1)/kT} , \tag{53}$$

which reflects the well-known fact that in the thermal reservoir the level populations have a Boltzmann distribution [19] and $\langle \hat{R}_{11} \rangle_{st} > \langle \hat{R}_{22} \rangle_{st} > \langle \hat{R}_{33} \rangle_{st}$. From (53) it also follows that for high enough temperatures ($N \gg 1$) the levels are occupied with equal probability: $\langle \hat{R}_{ii} \rangle_{st} = \frac{1}{3}$. As we will see later this is true also for weakly correlated reservoirs when $M \ll N$ and $N \gg 1$.

Finally we should underline the fact that the level populations (50)–(52) of the three-level Ξ-type atom in a thermal reservoir do not depend on the particular values of the decay rates and are valid for $\gamma_1 = \gamma_2$ as well as for $\gamma_1 \neq \gamma_2$. In the correlated reservoir the situation is quite different, and depends intimately on the decay rates, that is, on the precise mechanism by which equilibrium is attained.

2. Effects of correlations. 1. $\gamma_1 = \gamma_2$

Here we will study the influence of correlations between pairs of field modes in the reservoir on the level populations of the three-level Ξ-type atom. For simplicity firstly we will suppose the decay rates γ_1 and γ_2 to be equal. In this case from relations (45)–(48) we find that

$$\langle \hat{R}_{11} \rangle_{st} = \frac{M^2(2N+3)-(1+2N)(N+1)^2}{(1+2N)(3M^2-1-3N-3N^2)} , \tag{54}$$

$$\langle \hat{R}_{22} \rangle_{st} = \frac{M^2-(1+N)N}{(3M^2-1-3N-3N^2)} , \tag{55}$$

$$\langle \hat{R}_{33} \rangle_{st} = \frac{M^2(2N-1)-(1+2N)N^2}{(1+2N)(3M^2-1-3N-3N^2)} , \tag{56}$$

$$\langle \hat{X}_{13} \rangle_{st} = \frac{-M}{(1+2N)(3M^2-1-3N-3N^2)} . \tag{57}$$

The nonzero stationary mean value $\langle \hat{X}_{13} \rangle_{st}$ reflects the fact that the correlations in the reservoir induce a stationary correlation between the levels $|1\rangle$ and $|3\rangle$. We will discuss this correlation between atomic levels later on and now we turn our attention to the level populations.

In Fig. 2 we plot the stationary level populations $\langle \hat{R}_{ii} \rangle_{st}$ as functions of the correlation M^2. From this figure several conclusions can be deduced.

(1) Level populations are almost unchanged for $M^2 < N^2$, which means that even if the reservoir is correlated to some extent, but is not squeezed then the level populations remain almost the same as in the case of a thermal (noncorrelated) reservoir.

(2) Very significant differences between the level populations in the case of a correlated reservoir and a thermal reservoir for larger values of the correlation factor M^2 are manifested by the population of level $|3\rangle$ exceeding the population of level $|2\rangle$, which means that the level populations do not obey the Boltzmann distribution. From the relation

$$\langle \hat{R}_{22} \rangle_{st} = \langle \hat{R}_{33} \rangle_{st} \tag{58}$$

we can evaluate simply the critical value of $M = M_{cr}$ at which a population inversion between levels $|3\rangle$ and $|2\rangle$ appears. Using Eq. (58) and relations (55) and (56) we find for M_{cr} the following expression:

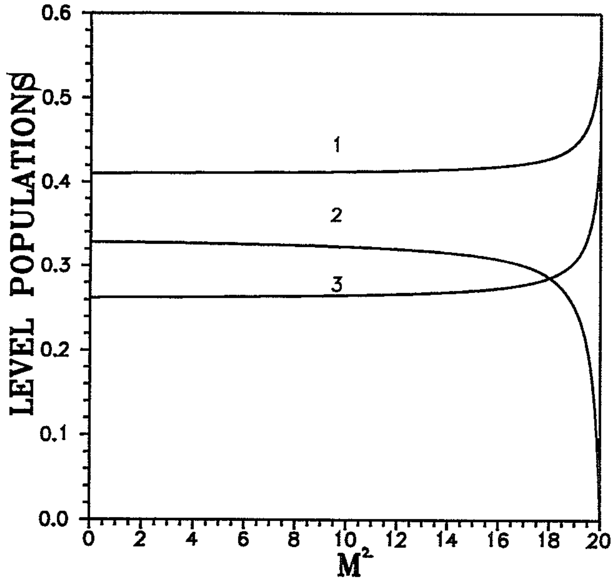

FIG. 2. The steady-state level populations of a three-level atom in the Ξ configuration interacting with a correlated broadband reservoir with mean photon number per unit bandwidth (N) equal to 4. The level populations are plotted vs the square of the correlation parameter M. We see that for $M^2<N^2=16$ (i.e., no squeezing) the level populations are almost identical to their thermal values (with $M^2=0$). Population inversion between levels $|2\rangle$ and $|1\rangle$ can be observed for $M^2>M_{\rm cr}^2=N(1+2N)/2=18$, which corresponds to the squeezing of reservoir higher than 49%. For $M^2=M_{\rm ideal}^2=N(N+1)=20$ (corresponding to the maximum squeezing obtainable for given value of N^2, which in the present case is equal to 95%) the atom is in the pure SU(2) coherent state.

$$M_{\rm cr}=\left[\frac{(1+2N)N}{2}\right]^{1/2}, \tag{59}$$

which corresponds to the following (critical) degree of squeezing of the reservoir in the Y quadrature (for details see Appendix A):

$$S_Y^{\rm cr}=2\left[N-\left[\frac{(1+2N)N}{2}\right]^{1/2}\right], \tag{60}$$

while the level populations are

$$\langle\hat{R}_{22}\rangle_{\rm st}|_{M=M_{\rm cr}}=\langle\hat{R}_{33}\rangle_{\rm st}|_{M=M_{\rm cr}}=\frac{N}{3N+2} \tag{61a}$$

and

$$\langle\hat{R}_{11}\rangle_{\rm st}|_{M=M_{\rm cr}}=\frac{N+2}{3N+2}. \tag{61b}$$

(3) It is very interesting to note that the degree of critical squeezing of the reservoir is, for all values of N larger than 1, equal to approximately 50% (for $N=1$ the degree of critical squeezing is 45%, while for $N\to\infty$ the degree of squeezing is equal to 50%). We can conclude that when the degree of squeezing in the multimode reservoir exceeds 50% one can expect significant changes in the level populations, including the inversion of level populations between levels $|3\rangle$ and $|2\rangle$.

(4) If the reservoir is ideally squeezed, i.e., $M=[N(N+1)]^{1/2}$, then the stationary level populations $\langle\hat{R}_{ii}\rangle_{\rm st}$ are

$$\langle\hat{R}_{11}\rangle_{\rm st}=\frac{N+1}{1+2N},\quad \langle\hat{R}_{22}\rangle_{\rm st}=0,\quad \langle\hat{R}_{33}\rangle_{\rm st}=\frac{N}{1+2N}. \tag{62}$$

From above we see that the three-level atom in the ideally squeezed reservoir (the reservoir is in a minimum-uncertainty state with squeezing given by the relation $S_Y=2\{N-[N(N+1)]^{1/2}\}$) decays into a highly correlated pure state, i.e., correlations and phase information are transformed from the reservoir to the atomic system exactly as for the two two-level atoms described in terms of Dicke states [10]. The stationary state of the Ξ-type three-level atom in an ideally squeezed reservoir can be written as the pure state

$$|\Psi\rangle_{\rm st}=\cos\left[\frac{\theta}{2}\right]|1\rangle+\sin\left[\frac{\theta}{2}\right]|3\rangle, \tag{63}$$

where

$$\cos\left[\frac{\theta}{2}\right]=\left[\frac{N+1}{2N+1}\right]^{1/2},\qquad \sin\left[\frac{\theta}{2}\right]=\left[\frac{N}{2N+1}\right]^{1/2}.$$

As shown in Appendix C, state (63) can be identified with the SU(2) atomic coherent state [13] for a three-level atom with a degree of dipole [SU(2)] squeezing (for details see Appendix C) given by the relation

$$S_1=-\frac{2N}{2N+1}, \tag{64}$$

which means that as the degree of squeezing of the reservoir is increased the dipole squeezing similarly increases. In the limit $N\to\infty$ both the reservoir *and* the atom exhibit 100% squeezing. It should be mentioned at this point that we have neglected the reservoir-induced frequency shifts in this derivation of relaxation to a pure-state superposition; this result does not hold when shifts are included, although the situation is hardly changed from the above if the shifts are small [10].

(5) Finally, from Eq. (61) it follows that for high enough N the level populations at $M=M_{\rm cr}$ are equal: $\langle\hat{R}_{ii}\rangle_{\rm st}=\frac{1}{3}$. This means that for high photon number N, level populations are exactly the same as in the case of a thermal field with $N\gg1$, i.e., $\langle\hat{R}_{ii}\rangle_{\rm st}$ are almost *constant* in the interval $0\le M\le M_{\rm cr}$ and then in the interval $M_{\rm cr}\le M\le M_{\rm ideal}=[N(N+1)]^{1/2}$ the correlations become dominant and the correlated atomic state (63) is produced. We can easily find that

$$\lim_{N\to\infty}\frac{M_{\rm cr}}{M_{\rm ideal}}=1,$$

which means that for high enough N a sudden *jump* from an uncorrelated (thermal) state of the atom to the highly correlated atomic state can be observed. This is true also for the degree of squeezing of the reservoir measured by the function S_Y which changes almost "instantaneously" from 50% to 100% as $M_{\rm cr}\to M_{\rm ideal}$. This shows that the field squeezing *and* the atomic response changed very dramatically for quite small changes of M around $M_{\rm cr}$. We should thus be cautious about relying solely on the qualitative size of the correlations to characterize the light field near $M_{\rm cr}$: a fairly precise evaluation is required to obtain a detailed understanding.

3. *Effects of correlations. 2.* $\gamma_1\neq\gamma_2$

From the above we have learned that an atomic population inversion appears between levels $|3\rangle$ and $|2\rangle$ when the squeezing of the reservoir exceeds 50% for $\gamma_1=\gamma_2$ and that for an ideally squeezed reservoir ($M=M_{\rm ideal}$) the relation

$$\frac{\langle\hat R_{33}\rangle_{\rm st}}{\langle\hat R_{11}\rangle_{\rm st}}=\frac{N}{N+1}$$

holds, which means that in the case of 100% squeezing ($N\gg1$) levels $|3\rangle$ and $|1\rangle$ are equally occupied with a probability $\frac{1}{2}$.

Here two natural questions arise: (1) Is it possible to obtain inversion between levels $|3\rangle$ and $|2\rangle$ for a degree of squeezing less than 50%? and (2) can an inversion between levels $|3\rangle$ and $|1\rangle$ be created (i.e., $\langle\hat R_{33}\rangle_{\rm st}/\langle\hat R_{11}\rangle_{\rm st}>1$)? To answer these two questions we turn our attention back to the model with $\mu_{12}\neq\mu_{23}$, when the coupling constants γ_1 and γ_2 are *not* equal (in what follows we suppose $\gamma_{12}^2=\gamma_1\gamma_2$). From

$$\langle\hat R_{22}\rangle_{\rm st}=\langle\hat R_{33}\rangle_{\rm st} \tag{65}$$

with $\langle\hat R_{22}\rangle_{\rm st}$ and $\langle\hat R_{33}\rangle_{\rm st}$ given by Eqs. (46) and (47) we can easily derive the critical value of M, when an inversion between levels $|3\rangle$ and $|2\rangle$ appears:

$$M_{\rm cr}^2=N\left[N+\frac{1}{N(1+\alpha)}\right], \tag{66}$$

where $\alpha=\gamma_1/\gamma_2$. Now we can evaluate the degree of field squeezing corresponding to this critical value of M by rewriting $M_{\rm cr}$ in terms of N and α using Eq. (66),

$$S_Y^{\rm cr}=2\left[N-N\left[1+\frac{1}{N(1+\alpha)}\right]^{1/2}\right], \tag{67}$$

which for large N is

$$S_Y^{\rm cr}\simeq-\frac{1}{1+\alpha}. \tag{68}$$

From the last equation it follows that for $\gamma_1>\gamma_2$ the inversion between levels $|3\rangle$ and $|2\rangle$ can be observed for a field squeezing which is less than 50%. In fact for $\gamma_1\gg\gamma_2$ inversion appears as soon as the reservoir is squeezed. It is also important to note that

$$\langle\hat R_{22}\rangle_{\rm st}|_{M=M_{\rm cr}}=\langle\hat R_{33}\rangle_{M=M_{\rm cr}}=\frac{N\alpha}{1+\alpha+3N\alpha}, \tag{69}$$

which means that for large enough N we can observe a thermal level-population distribution in the interval $0\le M\le M_{\rm cr}$.

To answer the second question we evaluate the relation between $\langle\hat R_{33}\rangle_{\rm st}$ and $\langle\hat R_{11}\rangle_{\rm st}$ in the case of ideal field squeezing but $\gamma_1\neq\gamma_2$. From Eqs. (46) and (47) we find that

$$\frac{\langle\hat R_{33}\rangle_{\rm st}}{\langle\hat R_{11}\rangle_{\rm st}}=\alpha\frac{N}{1+N}, \tag{70}$$

while $\langle\hat R_{22}\rangle_{\rm st}=0$. Now it is clear that for α large enough we can obtain an inversion between levels $|3\rangle$ and $|1\rangle$ even for a finite degree of squeezing of ideally correlated (squeezed) reservoir. If the reservoir is 100% squeezed (i.e., $N\to\infty$) the level $|3\rangle$ is α times more populated than level $|1\rangle$. From our results it follows that the steady-state population inversion can be created via the decay process in a correlated broadband reservoir. Finally we should stress the fact that the highest possible atomic squeezing for a particular value of N can be obtained for $\alpha=1$.

IV. V-TYPE ATOM DECAY IN A CORRELATED RESERVOIR

In the preceding section we have studied the phase-sensitive population decay of a Ξ-type three-level atom in a correlated reservoir. As we see in this section the phase sensitivity in the population decay is absent in the case of a V-type atom.

The Hamiltonian for the V-type three-level atom [see Fig. 1(b)] is given by Eqs. (1) and (8) with the energies of the atomic levels related as follows: $E_1<E_2,E_3$. Following the same procedure as described in the preceding section we can write down the set of equations for the mean values of the atomic operators $\langle\hat R_{22}(t)\rangle$, $\langle\hat R_{33}(t)\rangle$, $\langle\hat R'_{23}(t)\rangle$, and $\langle\hat R'_{32}(t)\rangle$, where

$$\langle\hat R'_{23}(t)\rangle=\langle\hat R_{23}(t)\rangle\exp(i\Omega_{23}).$$

We will write these equations in the matrix form (31), where

$$\mathbf{A}=\begin{bmatrix}\langle\hat R_{22}(t)\rangle\\ \langle\hat R_{33}(t)\rangle\\ \langle\hat R'_{23}(t)\rangle\\ \langle\hat R'_{32}(t)\rangle\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}2N(\Omega_{12})\gamma_1(\Omega_{12})\\ 2N(\Omega_{13})\gamma_2(\Omega_{13})\\ +[N(\Omega_{13})\Theta_{12}^{(N)*}(\Omega_{13})+N(\Omega_{12})\Theta_{12}^{(N)}(\Omega_{12})]\exp(i\Omega_{23}t)\\ +[N(\Omega_{13})\Theta_{12}^{(N)}(\Omega_{13})+N(\Omega_{12})\Theta_{12}^{(N)*}(\Omega_{12})]\exp(-i\Omega_{23}t)\end{bmatrix} \tag{71}$$

and the matrix elements of the matrix $\underline{M}$ are

$$
\begin{aligned}
M_{11}&=-2[1+2N(\Omega_{12})]\gamma_1(\Omega_{12}) ,\\
M_{12}&=-2N(\Omega_{12})\gamma_1(\Omega_{12}) ,\\
M_{13}&=-[\Theta_{12}(\Omega_{13})+N(\Omega_{13})\Theta_{12}^{(N)}(\Omega_{13})]\exp(-i\Omega_{23}t)=M_{14}^* ,\\
M_{21}&=-2N(\Omega_{13})\gamma_2(\Omega_{13}) ,\\
M_{22}&=-2[1+2N(\Omega_{13})]\gamma_2(\Omega_{13}) ,\\
M_{23}&=-[\Theta_{12}^*(\Omega_{12})+N(\Omega_{12})\Theta_{12}^{(N)*}(\Omega_{12})]\exp(-i\Omega_{23}t)=M_{24}^* ,\\
M_{31}&=-[\Theta_{12}(\Omega_{12})+2N(\Omega_{12})\Theta_{12}^{(N)}(\Omega_{12})+N(\Omega_{13})\Omega_{12}^{(N)*}(\Omega_{13})]\exp(i\Omega_{23}t)=M_{41}^* ,\\
M_{32}&=-[\Theta_{12}^*(\Omega_{13})+2N(\Omega_{13})\Theta_{12}^{(N)*}(\Omega_{13})+N(\Omega_{12})\Theta_{12}^{(N)}(\Omega_{12})]\exp(i\Omega_{23}t)=M_{42}^* ,\\
M_{33}&=-[\Phi_1^*(\Omega_{12})+N(\Omega_{12})\Phi_1^{(N)*}(\Omega_{12})+\Phi_2(\Omega_{13})+N(\Omega_{13})\Phi_2^{(N)*}(\Omega_{13})]=M_{44}^* ,\\
M_{34}&=M_{43}=0 ,
\end{aligned}
\tag{72}
$$

where in addition to the parameters defined in the preceding section and Appendix B, we also introduce the new parameters $\Theta_{12}(\Omega_{mn})$ and $\Theta_{12}^{(N)}(\Omega_{mn})$:

$$\Theta_{12}(\Omega_{mn})=\kappa_{12}(\Omega_{mn})-i\phi_{12}(\Omega_{mn}) , \tag{73}$$

$$N(\Omega_{mn})\Theta_{12}^{(N)}(\Omega_{mn})=N(\Omega_{mn})[\kappa_{12}(\Omega_{mn})-i\phi_{12}^{(N)}(\Omega_{mn})] . \tag{74}$$

The expressions for the decay rates κ_{12} and shifts ϕ_{12} and $\phi_{12}^{(N)}$ can be found in Appendix B. The parameter Φ_2 is defined as in the case of the Ξ-type atom but with μ_{13} instead of μ_{23}; κ_{12} is equal to γ_{12} with μ_{13} instead of μ_{23} (for details see Appendix B).

The first important fact we can derive from the above equations is that the mean values of the atomic operators involved do not depend on correlations between pairs of modes in the reservoir and this leads to an absence of phase sensitivity in the population decay. In what follows we will proceed in the same way as in the preceding section, i.e., we will assume the frequency shifts to be negligible and the atomic transition frequencies to be in resonance with the carrier frequency of the reservoir: $\Omega_{12}=\Omega_{31}=\Omega$, i.e., $\Omega_{23}=0$. In this case we can rewrite the equation of motion in terms of variables $\langle \hat{R}_{22}(t)\rangle$, $\langle \hat{R}_{33}(t)\rangle$, $\langle \hat{X}_{23}(t)\rangle$, and $\langle \hat{Y}_{23}(t)\rangle$, where $\langle \hat{R}'_{23}(t)\rangle =\langle \hat{X}_{23}(t)\rangle +i\langle \hat{Y}_{23}(t)\rangle$. We find that the imaginary part of $\langle \hat{R}'_{23}(t)\rangle$ is decoupled from the rest of the variables and is governed by the equation

$$\frac{d}{dt}\langle \hat{Y}_{23}(t)\rangle =-(1+N)(\gamma_1+\gamma_2)\langle \hat{Y}_{23}(t)\rangle , \tag{75}$$

while the rest of the equations can be written in the matrix form (39) with the column vectors $\mathbf{A}'$ and $\mathbf{B}'$ defined as

$$\mathbf{A}'=\begin{bmatrix}\langle \hat{R}_{22}(t)\rangle\\ \langle \hat{R}_{33}(t)\rangle\\ \langle \hat{X}_{23}(t)\rangle\end{bmatrix},\quad \mathbf{B}'=\begin{bmatrix}2N\gamma_1\\ 2N\gamma_2\\ 2N\kappa_{12}\end{bmatrix}, \tag{76}$$

and matrix $\underline{M}'$,

$$\underline{M}'=\begin{bmatrix}-2(1+2N)\gamma_1 & -2N\gamma_1 & -2(1+N)\kappa_{12}\\ -2N\gamma_2 & -2(1+2N)\gamma_2 & -2(1+N)\kappa_{12}\\ -(1+3N)\kappa_{12} & -(1+3N)\kappa_{12} & -(1+N)(\gamma_2+\gamma_1)\end{bmatrix} . \tag{77}$$

Instead of studying the detailed time evolution of the atomic-level populations we will concentrate on analyzing their stationary solutions.

A. $\mu_{12}\neq\mu_{13}$

First we will study the case when the transition matrix elements μ_{12} and μ_{13} are not equal, i.e., the decay rates between levels $|3\rangle$ and $|1\rangle$ and between levels $|2\rangle$ and $|1\rangle$ are different. From the equation

$$\underline{M}'\mathbf{A}'+\mathbf{B}'=0$$

with matrix $\underline{M}'$ and vectors $\mathbf{A}'$ and $\mathbf{B}'$ defined by Eqs. (76) and (77), we easily find the stationary values for level populations:

$$\langle \hat{R}_{22}\rangle_{\rm st}=\langle \hat{R}_{33}\rangle_{\rm st}=\frac{N}{1+3N} \quad\text{and}\quad \langle \hat{R}_{11}\rangle_{\rm st}=\frac{1+N}{1+3N} . \tag{78}$$

The stationary value of $\langle \hat{X}_{23}\rangle_{\rm st}$ is equal to zero. We see that these stationary solutions depend *only* on the number of photons per unit bandwidth. If the atom is embedded in a thermal reservoir, then $N(\Omega)$ is given by Eq. (49) and we see that the population probabilities (78) obey a Boltzmann distribution law independently of particular values of the decay rates (we suppose $\gamma_1 \neq \gamma_2$). In the limit $N \to \infty$ all levels are occupied with equal probability.

B. $\mu_{12}=\mu_{13}$

When the Ξ-type atom or V-type atom with $\mu_{12} \neq \mu_{13}$ interacts with the broadband reservoir the stationary solutions for level populations do not depend on the initial state of the atom. This is in accordance with the assumption about the Markovian character of the atom-field interaction [15,16]. Therefore it is quite surprising to find that the stationary solutions for the level populations of the V-type atom with equal decay rates (i.e., $\gamma_1=\gamma_2$) depend on the particular initial state of the atom.

(1) Let us suppose the atom is prepared initially in the ground state $|1\rangle$. In this case we find the following stationary solutions:

$$\langle \hat{R}_{22}\rangle_{\rm st}=\langle \hat{R}_{33}\rangle_{\rm st}=\frac{N}{2(1+2N)}=\langle \hat{X}_{23}\rangle_{\rm st}\,, \qquad \langle \hat{R}_{11}\rangle_{\rm st}=\frac{1+N}{1+2N}\,. \tag{79}$$

We should underline here the fact that the stationary value of $\langle \hat{X}_{23}\rangle_{\rm st}$ is not zero. Before we explain this strange behavior we display stationary solutions for other initial states of the atom.

(2) If the atom is initially in the state $|2\rangle$ or $|3\rangle$ then

$$\langle \hat{R}_{22}\rangle_{\rm st}=\langle \hat{R}_{33}\rangle_{\rm st}=\frac{1+3N}{4(1+2N)}\,, \qquad \langle \hat{R}_{11}\rangle_{\rm st}=\frac{1+N}{2(1+2N)}\,, \tag{80}$$

and

$$\langle \hat{X}_{23}\rangle_{\rm st}=-\frac{1+N}{4(1+2N)}\,.$$

(3) If the atom is initially in the symmetric state $|S\rangle$ defined as

$$|S\rangle=\frac{|2\rangle+|3\rangle}{\sqrt{2}}\,, \tag{81}$$

then the solutions are *exactly* the same as for case (1) with the atom initially in the state $|1\rangle$.

(4) Finally, if the atom is initially in the antisymmetric state $|A\rangle$:

$$|A\rangle=\frac{|2\rangle-|3\rangle}{\sqrt{2}}\,, \tag{82}$$

then

$$\langle \hat{R}_{22}\rangle_{\rm st}=\langle \hat{R}_{33}\rangle_{\rm st}=\tfrac{1}{2}, \quad \langle \hat{R}_{11}\rangle_{\rm st}=0\,, \tag{83}$$

and

$$\langle \hat{X}_{23}\rangle_{\rm st}=-\tfrac{1}{2}\,.$$

We see that if the atom is initially prepared in the antisymmetric state (82) then at $t>0$ it does not evolve at all, i.e., the atom remains in its initial state. Agarwal [1] had noted effects related to this for ordinary spontaneous decay from a V system some years ago although he did not make the link with the mechanism now known as population trapping [8]. Our results reduce to those of Ref. [1] when $N \to 0$.

As we mentioned earlier the stationary solutions with the atom initially in the state $|1\rangle$ or $|S\rangle$ are identical. Moreover, the antisymmetric state $|A\rangle$ is the eigenstate of the interaction Hamiltonian describing the V-type atom (8), and therefore if the atom at $t=0$ is in the state $|A\rangle$ then it remains in this state for all times $t>0$. These two facts give us a clue to understand the dependence of stationary solutions on the initial state of the atom. Namely, the chosen atomic basis $|1\rangle$, $|2\rangle$, and $|3\rangle$ is inappropriate to describe the dynamics of the V-type atom under consideration [20]. If $\gamma_1=\gamma_2$ and $E_2=E_3$ then the states $|2\rangle$ and $|3\rangle$ are indistinguishable and therefore the appropriate atomic basis for the description of the atom consists of states $|1\rangle$, $|S\rangle$, and $|A\rangle$ [20]. To see this we introduce new projection operators

$$\hat{R}_{SS}=|S\rangle\langle S|=\tfrac{1}{2}(\hat{R}_{22}+\hat{R}_{33}+2\hat{X}_{23})\,, \tag{84}$$

$$\hat{R}_{AA}=|A\rangle\langle A|=\tfrac{1}{2}(\hat{R}_{22}+\hat{R}_{33}-2\hat{X}_{23})\,, \tag{85}$$

$$\hat{R}_{SA}=|S\rangle\langle A|=\tfrac{1}{2}(\hat{R}_{22}-\hat{R}_{33}-2i\hat{Y}_{23})=\hat{R}^{\dagger}_{AS}\,. \tag{86}$$

These operators are simply related to $\hat{R}_{11}$ and $\hat{X}_{23}$:

$$\hat{R}_{11}=1-(\hat{R}_{SS}+\hat{R}_{AA}), \quad \hat{X}_{23}=\frac{\hat{R}_{SS}-\hat{R}_{AA}}{2}\,. \tag{87}$$

One can easily find that

$$\frac{d}{dt}\langle \hat{R}_{AA}(t)\rangle=0\,, \tag{88}$$

which clearly illustrates that the population of the antisymmetric state is constant in time. From Eq. (87) it follows that $\langle \hat{R}_{SS}(t)\rangle$ is simply related to $\langle \hat{X}_{23}(t)\rangle$ and $\langle \hat{R}_{11}(t)\rangle$. We therefore need only to obtain the solution for $\langle \hat{R}_{SS}(t)\rangle$, and all other variables can be written in terms of this solution. We need therefore to solve

$$\frac{d}{dt}\langle \hat{R}_{SS}(t)\rangle=-4(1+2N)\gamma\langle \hat{R}_{SS}(t)\rangle +4\gamma N(1-\langle \hat{R}_{AA}\rangle)\,. \tag{89}$$

Now the stationary solutions of atomic operators can be found easily:

$$\langle \hat{R}_{SS}\rangle_{\rm st}=\frac{N}{1+2N}(1-\langle \hat{R}_{AA}\rangle)\,, \qquad \langle \hat{X}_{23}\rangle_{\rm st}=\frac{N-(1+3N)\langle \hat{R}_{AA}\rangle}{2(1+2N)}\,, \qquad \langle \hat{R}_{SA}\rangle_{\rm st}=\langle \hat{R}_{AS}\rangle_{\rm st}=0\,, \tag{90}$$

and

$$\langle \hat{R}_{11} \rangle_{\rm st} = \frac{1+N}{1+2N}(1-\langle \hat{R}_{AA} \rangle) ,$$
$$\langle \hat{R}_{22} \rangle_{\rm st} = \langle \hat{R}_{33} \rangle_{\rm st} = \frac{N+(1+N)\langle \hat{R}_{AA} \rangle}{2(1+2N)} . \tag{91}$$

From Eq. (91) we can reconstruct all of the solutions (79), (80), and (83). It is now clear why the stationary solutions in the atomic basis $|1,2,3\rangle$ are "sensitive" to the initial conditions: it is due to the fact that part of the atomic energy is trapped in the antisymmetric state. In the atomic basis $|1,S,A\rangle$ the V-type three-level atom behaves effectively as a two-level system with upper ($|S\rangle$) and lower ($|1\rangle$) levels [20], but with the total occupation probability equal to $1-\langle \hat{R}_{AA} \rangle$. In a thermal heat bath we find that

$$\frac{\langle \hat{R}_{SS} \rangle_{\rm st}}{\langle \hat{R}_{11} \rangle_{\rm st}} = \exp(-\hbar\Omega/kT) .$$

It is interesting to note that the atomic coherence responsible for population trapping is not disturbed by the applied broadband radiation field. Finally we should turn our attention to the fact that the population trapping described above can be observed *only* if the frequency shifts are negligible. Otherwise these shifts can break the symmetry with respect to states $|2\rangle$ and $|3\rangle$ which results in diminishing the trapping effect [10].

V. Λ-TYPE ATOM DECAY IN CORRELATED RESERVOIR

The level diagram of the Λ-type atom is shown in Fig. 1(c) where the atomic-level energies are ordered as follows $E_1, E_2 < E_3$. The interaction Hamiltonian describing the interaction of a Λ-type atom with the electromagnetic field is given by Eq. (9).

Following the same procedure as in the case of the Ξ- and V-type atoms we can derive the evolution equations for the population probabilities $\langle \hat{R}_{11}(t) \rangle$, $\langle \hat{R}_{22}(t) \rangle$, and the transition probability $\langle \hat{R}_{12}(t) \rangle$. Assuming one-photon resonance (i.e., $\Omega_{13}=\Omega_{23}=\Omega$) and taking the frequency shifts due to the spontaneous as well as due to the stimulated transitions to be zero, we find that the imaginary part of $\langle \hat{R}_{12}(t) \rangle$ is decoupled from the rest of the variables which are governed by the matrix equation (39) with

$$\mathbf{A}' = \begin{bmatrix} \langle \hat{R}_{11}(t) \rangle \\ \langle \hat{R}_{22}(t) \rangle \\ \langle \hat{X}_{12}(t) \rangle \end{bmatrix}, \quad \mathbf{B}' = \begin{bmatrix} 2(1+N)\gamma_1 \\ 2(1+N)\gamma_2 \\ 2(1+N)\kappa_{12} \end{bmatrix}, \tag{92}$$

and

$$\underline{M}' = \begin{bmatrix} -2(1+2N)\gamma_1 & -2(1+N)\gamma_1 & -2N\kappa_{12} \\ -2(1+N)\gamma_2 & -2(1+2N)\gamma_2 & -2N\kappa_{12} \\ -(2+3N)\kappa_{12} & -(2+3N)\kappa_{12} & -N(\gamma_2+\gamma_1) \end{bmatrix} . \tag{93}$$

The decay rates γ_1, γ_2, and κ_{12} are defined in Appendix B. As before the stationary value of the imaginary part of the transition operator $\langle \hat{R}_{12} \rangle_{\rm st}$ is equal to zero.

A. $\mu_{13} \neq \mu_{23}$

If the decay rates between levels $|3\rangle$ and $|1\rangle$ and between levels $|3\rangle$ and $|2\rangle$ are not equal, then we can obtain for the stationary solutions of the occupation probabilities the following expressions:

$$\langle \hat{R}_{11} \rangle_{\rm st} = \langle \hat{R}_{22} \rangle_{\rm st} = \frac{1+N}{2+3N}, \quad \langle \hat{R}_{33} \rangle_{\rm st} = \frac{N}{2+3N} . \tag{94}$$

The stationary expectation value of the real part of the transition operator $\langle \hat{X}_{12} \rangle_{\rm st}$ is zero. One can easily prove that if the reservoir has a thermal photon distribution, i.e., $N(\Omega)$ is given by (49), then

$$\frac{\langle \hat{R}_{33} \rangle_{\rm st}}{\langle \hat{R}_{11} \rangle_{\rm st}} = \frac{\langle \hat{R}_{33} \rangle_{\rm st}}{\langle \hat{R}_{22} \rangle_{\rm st}} = \exp(-\hbar\Omega/kT) , \tag{95}$$

which means that the atomic levels are occupied in accordance with the Boltzmann distribution law irrespectively of the correlations between the pairs of modes of the reservoir (i.e., irrespectively of the value of M). This is in contrast with the Ξ-type atom where the final equilibrium atomic state is far from being a state of thermal equilibrium.

B. $\mu_{13} = \mu_{23}$

Following the same arguments as in the case of the V-type atom we can find that if the decay rates between levels $|3\rangle$ and $|1\rangle$ and between levels $|3\rangle$ and $|2\rangle$ of the Λ-type atom are equal ($\gamma_1=\gamma_2$), then it is more convenient to describe the dynamics of the system under consideration in the atomic basis [20,21]

$$|3\rangle; |S\rangle = \frac{|1\rangle+|2\rangle}{\sqrt{2}} \text{ and } |A\rangle = \frac{|1\rangle-|2\rangle}{\sqrt{2}} , \tag{96}$$

where $|S\rangle$ and $|A\rangle$ are the symmetric and the antisymmetric states of the lower levels of the atom, respectively. Following the procedure described in the preceding section we find that $\langle \hat{R}_{AA} \rangle_{\rm st}$ is constant and equal to its initial value, so that

$$\langle \hat{R}_{AA} \rangle_{\rm st} = \langle \hat{R}_{AA} \rangle, \quad \langle \hat{R}_{SS} \rangle_{\rm st} = \frac{1+N}{1+2N}(1-\langle \hat{R}_{AA} \rangle) , \tag{97}$$

and

$$\langle \hat{R}_{33} \rangle_{\rm st} = \frac{N}{1+2N}(1-\langle \hat{R}_{AA} \rangle) ,$$
$$\langle \hat{R}_{11} \rangle_{\rm st} = \langle \hat{R}_{22} \rangle_{\rm st} = \frac{1+N+N\langle \hat{R}_{AA} \rangle}{2(2N+1)} , \tag{98}$$
$$\langle \hat{X}_{12} \rangle_{\rm st} = \frac{1+N-(2+3N)\langle \hat{R}_{AA} \rangle}{2(2N+1)} .$$

We can conclude that the Λ-type atom with equal transition matrix elements $\mu_{13}=\mu_{23}$ behaves effectively as a two-level atom with upper level $|3\rangle$ and lower level $|S\rangle$ and the total population probability equal to $1-\langle \hat{R}_{AA} \rangle$.

The stationary solutions for the V-type atom [(90) and (91)] and the Λ-type atom [(97) and (98)] are identical provided due recognition is given to the fact that in the first case the symmetric state $|S\rangle$ plays the role of an upper state, while in the second case it plays the role of a lower state.

Finally we note that if the reservoir has a thermal photon number distribution (49) in each mode, then irrespective of the correlations between the modes (i.e., irrespective of the value of M) we have

$$\frac{\langle \hat{R}_{33}\rangle_{\rm st}}{\langle \hat{R}_{SS}\rangle_{\rm st}}=\exp(-\hbar\Omega/kT)\ . \tag{99}$$

VI. CONCLUSIONS

In this paper we have studied the influence of correlations between pairs of reservoir modes on the decay of a three-level atom. We have derived the equations of motion for a three-level atom in a correlated reservoir and discussed the stationary solutions for level populations. We have studied in detail all possible atomic configurations, i.e., the Ξ, the V, and the Λ configurations. In particular, we have shown that the Ξ-type atom is excited to a steady state which depends critically on the intermode correlations and the coupling constants between the atomic levels. We have studied in detail the dependence of the level populations on the degree of correlation between the modes. We have shown that for the maximal correlations between modes (i.e., when the reservoir is in a broadband squeezed state) the atom is excited to a pure SU(2) coherent state exhibiting population inversion between upper and lower states. The degree of the inversion depends on the particular values of coupling constants between levels $|3\rangle\leftrightarrow|2\rangle$ and $|2\rangle\leftrightarrow|1\rangle$, respectively. In the case of the V- and the Λ-type atoms the stationary level populations do not exhibit phase sensitivity and their decay is analogous to decay in a heat bath at finite temperature.

Note added in proof. We recently became aware of related work on three-level systems in broadband squeezed vacua by Z. Ficek and P. D. Drummond [Phys. Rev. A **43**, 6247 (1991); *ibid.* **43**, 6258 (1991)].

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APPENDIX A: BROADBAND CORRELATED RESERVOIRS

We begin with the ordinary multimode vacuum $|0\rangle$ of the electromagnetic field, defined as $\hat{a}(\omega)|0\rangle=0$ (for simplicity we will discuss only the one-dimensional case with field modes characterized only by the frequency ω). This vacuum can be transformed by the unitary squeeze operator [2,4] $\hat{S}(\xi)$:

$$|\xi\rangle=\hat{S}(\xi)|0\rangle\ ,$$

where

$$\hat{S}(\xi)=\exp[\xi(\omega)\hat{a}^\dagger(\omega)\hat{a}^\dagger(2\Omega-\omega)-\xi^*(\omega)\hat{a}(\omega)\hat{a}(2\Omega-\omega)] \tag{A1}$$

and $\xi=r\exp(i\phi)$. The unitary transformation (A1) realizes the canonical Bogoliubov transformation

$$\hat{b}(\omega)=\hat{S}(\xi)\hat{a}(\omega)\hat{S}^\dagger(\xi)=\mu(\omega)\hat{a}(\omega)+\nu(\omega)\hat{a}^\dagger(2\Omega-\omega)\ ,$$

$$\hat{b}^\dagger(\omega)=\hat{S}(\xi)\hat{a}^\dagger(\omega)\hat{S}^\dagger(\xi)=\mu^*(\omega)\hat{a}^\dagger(\omega)+\nu^*(\omega)\hat{a}(2\Omega-\omega)\ ,$$

where

$$\mu(\omega)=\cosh r,\quad \nu(\omega)=e^{i\phi}\sinh r\ ,$$

and

$$|\mu(\omega)|^2-|\nu(\omega)|^2=1\ .$$

We see that the state $|\xi\rangle$ (the so-called *two-mode squeezed vacuum* [2,22]) is the vacuum state of the annihilation operators $\hat{b}(\omega)$ and $\hat{b}(2\Omega-\omega)$. The unitary transformation (A1) models the coupling of a field mode of frequency ω to its corresponding sideband at frequency $2\Omega-\omega$ with respect to the carrier frequency Ω. It is well known [2,4] that the variances of the generalized quadrature operators $\hat{X}$ and $\hat{Y}$,

$$\hat{X}=\frac{\hat{a}(\omega)+\hat{a}(2\Omega-\omega)+\hat{a}^\dagger(\omega)+\hat{a}^\dagger(2\Omega-\omega)}{2^{3/2}}\ ,\qquad \hat{Y}=\frac{\hat{a}(\omega)+\hat{a}(2\Omega-\omega)-\hat{a}^\dagger(\omega)-\hat{a}^\dagger(2\Omega-\omega)}{2^{3/2}i}\ , \tag{A2}$$

exhibit squeezing in the vacuum state $|\xi\rangle$. In particular, for the variance of the operator $\hat{X}$ in state $|\xi\rangle$ we obtain

$$\langle[\Delta\hat{X}(t)]^2\rangle=\tfrac{1}{4}[\exp(-2r)\cos^2(\phi/2)+\exp(2r)\sin^2(\phi/2)]$$

from which it is seen that if $\phi=0$ the variance $\langle[\Delta\hat{X}(t)]^2\rangle$ is reduced below the value of vacuum fluctuations. Moreover, one can show that the squeezed vacuum $|\xi\rangle$ is a minimum-uncertainty state, i.e.,

$$\langle[\Delta\hat{X}(t)]^2\rangle\langle[\Delta\hat{Y}(t)]^2\rangle=\tfrac{1}{16}\ .$$

On the other hand, if we evaluate expectation values for single-mode operators, such as $\hat{X}_1=[\hat{a}(\omega)+\hat{a}^\dagger(\omega)]/2$ and $\hat{Y}_1=[\hat{a}(\omega)-\hat{a}^\dagger(\omega)]/2i$ in the two-mode squeezed vacuum $|\xi\rangle$, we find that these expectation values are the same as for the thermal field with Bose-Einstein statistics [22]. In other words, tracing the two-mode squeezed vacuum over one mode we find that the reduced field statistics are identical to those of the thermal field. In this case the squeeze parameter r is related to the temperature T of the field as follows:

$$|\sinh r|^2=\frac{1}{\exp(\hbar\omega/kT)-1}\ .$$

Now we turn our attention to the fact that in the two-mode squeezed vacuum the following correlation functions hold:

$$\begin{aligned}
&\langle \hat{a}(\omega)\hat{a}(\omega')\rangle = \mu(\omega)\nu(\omega')\delta(2\Omega-\omega-\omega') ,\\
&\langle \hat{a}^\dagger(\omega)\hat{a}^\dagger(\omega')\rangle = \mu^*(\omega)\nu^*(\omega')\delta(2\Omega-\omega-\omega') ,\\
&\langle \hat{a}^\dagger(\omega)\hat{a}(\omega')\rangle = |\nu(\omega)|^2\delta(\omega-\omega') ,\\
&\langle \hat{a}(\omega)\hat{a}^\dagger(\omega')\rangle = |\mu(\omega')|^2\delta(\omega-\omega') .
\end{aligned} \tag{A3}$$

These correlation functions together with the definition of parameters μ and ν can serve as an alternative definition of the squeezed vacuum. Moreover if we suppose an arbitrary ω then the last relations serve as a definition of the broadband squeezed vacuum with pairs of correlated modes. In fact one generalize the previous correlation functions in the following way:

$$\begin{aligned}
&\langle \hat{a}^\dagger(\omega)\hat{a}(\omega')\rangle = N(\omega)\delta(\omega-\omega') ,\\
&\langle \hat{a}(\omega)\hat{a}^\dagger(\omega')\rangle = [N(\omega)+1]\delta(\omega-\omega') ,\\
&\langle \hat{a}(\omega)\hat{a}(\omega')\rangle = M(\omega,\omega')\delta(2\Omega-\omega-\omega') ,\\
&\langle \hat{a}^\dagger(\omega)\hat{a}^\dagger(\omega')\rangle = M^*(\omega,\omega')\delta(2\Omega-\omega-\omega') ,
\end{aligned} \tag{A4}$$

and use these correlation functions as a definition of multimode (broadband) correlated states. It is obvious that parameter $N(\omega)$ in (A4) plays the role of the number of photons per unit bandwidth while $M(\omega,\omega')$ describes the correlation between two modes. It can be shown (see, for instance, the paper by Drummond [23]) that these parameters are related as follows:

$$|M(\omega,\omega')|^2 \le N(\omega)N(\omega') + \min[N(\omega),N(\omega')] . \tag{A5}$$

If one supposes the function $N(\omega)$ to be symmetric with respect to the carrier frequency Ω then the last relation can be written as

$$|M(\omega)|^2 \le N(\omega)[N(\omega)+1] . \tag{A6}$$

The equality in (A6) holds when the state under consideration is a minimum-uncertainty state.

Using the correlation functions (A4) one can readily evaluate the variances of the quadrature operators $\hat{X}$ and $\hat{Y}$ [see Eq. (A2)]. To find whether these variances are squeezed (two-mode squeezed) it is convenient to introduce the parameters S_X and S_Y:

$$S_X = 4\langle [\Delta\hat{X}(t)]^2\rangle - 1, \quad S_Y = 4\langle [\Delta\hat{Y}(t)]^2\rangle - 1 . \tag{A7}$$

The squeezing condition becomes $S_X < 0$ or $S_Y < 0$. From (A4) we can find the explicit expression for these parameters,

$$S_X = 2(N+|M|), \quad S_Y = 2(N-|M|) . \tag{A8}$$

(In the final expressions we omit the argument ω.) From the above it follows that the correlated state is squeezed if $M > N$ (we suppose M to be positive). The maximum squeezing for a particular value of N can be achieved if $M = [N(N+1)]^{1/2}$ (i.e., for the squeezed vacuum) and 100% squeezing can be obtained when $N \to \infty$.

APPENDIX B: DECAY RATES AND SHIFTS IN BROADBAND CORRELATED RESERVOIR

Following Milonni [15] we find frequency shifts and decay rates due to the spontaneous emission from the imaginary and real parts of the complex self-energy expression

$$\Gamma_{lmnp}(\Omega_{pn};t) = \sum_{\mathbf{k},\lambda} g_{lm}(\omega_{\mathbf{k}},\lambda) g_{np}(\omega_{\mathbf{k}},\lambda)\xi(\omega_{\mathbf{k}}-\Omega_{pn};t) . \tag{B1}$$

Letting the quantization volume V go to infinity and averaging over possible polarization from (B1) we obtain

$$\Gamma_{lmnp}(\Omega_{pn};t) = \frac{2(\boldsymbol{\mu}_{lm}\cdot\boldsymbol{\mu}_{np})}{3\pi\hbar c^3}\int_0^\infty d\omega\, \omega^3 \xi(\omega-\Omega_{pn};t) . \tag{B2}$$

For times t long compared with Ω_{pn}^{-1} we can use the approximation [24]

$$\xi(x;t) = \int_0^t \exp[ix(s-t)]ds \simeq \pi\delta(x) - i\mathrm{P}\frac{1}{x} , \tag{B3}$$

where P stands for the principal value. Using this approximation we find from (B2) the following expression:

$$\begin{aligned}
\Gamma_{lmnp}(\Omega_{pn};t) &\simeq \Gamma_{lmnp}(\Omega_{pn})\\
&= \gamma_{lmnp}(\Omega_{pn}) - i\delta_{lmnp}(\Omega_{pn})
\end{aligned} \tag{B4}$$

where the decay rate is defined as

$$\gamma_{lmnp}(\Omega_{pn}) = \frac{2\Omega_{pn}^3(\boldsymbol{\mu}_{lm}\cdot\boldsymbol{\mu}_{np})}{3\hbar c^3} , \tag{B5}$$

and the frequency shifts are given by the relation

$$\delta_{lmnp}(\Omega_{pn}) = \frac{2(\boldsymbol{\mu}_{lm}\cdot\boldsymbol{\mu}_{np})}{3\pi\hbar c^3}\mathrm{P}\int \frac{\omega^3}{\omega-\Omega_{pn}}d\omega . \tag{B6}$$

The frequency shift and the decay rate due to the stimulated emission can be found from the relation

$$\begin{aligned}
\tilde{\Gamma}^{(N)}_{lmnp}(\Omega_{pn};t) = \sum_{\mathbf{k},\lambda} & g_{lm}(\omega_{\mathbf{k}},\lambda) g_{np}(\omega_{\mathbf{k}},\lambda)\\
&\times \xi^*(\omega_{\mathbf{k}}-\Omega_{pn};t) N(\mathbf{k},\lambda) .
\end{aligned} \tag{B7}$$

Using the same arguments as previously we can rewrite $\tilde{\Gamma}^{(N)}_{lmnp}(\Omega_{pn};t)$ in the form

$$\begin{aligned}
\tilde{\Gamma}^{(N)}_{lmnp}(\Omega_{pn};t) &\simeq N(\Omega_{pn})\Gamma^{(N)}_{lmnp}(\Omega_{pn};t)\\
&= N(\Omega_{pn})[\gamma_{lmnp}(\Omega_{pn}) - i\delta^{(N)}_{lmnp}(\Omega_{pn})] ,
\end{aligned} \tag{B8}$$

where $\gamma_{lmnp}(\Omega_{pn})$ is given by Eq. (B5) and [25]

$$\delta^{(N)}_{lmnp}(\Omega_{pn}) \cong -\frac{2(\boldsymbol{\mu}_{lm}\cdot\boldsymbol{\mu}_{np})}{3\pi\hbar c^3 N(\Omega_{pn})}\mathrm{P}\int \frac{\omega^3}{\omega-\Omega_{pn}}N(\omega)d\omega . \tag{B9}$$

We do not address here the divergence or renormalization properties of these shifts but refer the reader to Ref. [15] for a full account of such problems.

Finally the frequency shift and the decay rate due to the correlations between the pairs of modes in the reservoir [18] are

$$\tilde{\Gamma}^{(M)}_{lmnp}(\Omega_{pn};t) \simeq M(\Omega_{pn})\Gamma^{(M)}_{lmnp}(\Omega_{pn})\exp(-i2\Omega t)$$
$$= \sum_{\mathbf{k},\lambda}\sum_{\mathbf{q},\sigma} g_{lm}(\omega_{\mathbf{k}},\lambda)g_{np}(\omega_{\mathbf{q}},\sigma)\exp[-i(\omega_{\mathbf{k}}+\omega_{\mathbf{q}})t]\langle \hat{a}_{\mathbf{k}\lambda}\hat{a}_{\mathbf{q}\sigma}\rangle \xi^*(\omega_{\mathbf{q}}-\Omega_{pn};t)$$
$$\equiv M(\Omega_{pn})[\gamma^{(M)}_{lmnp}(\Omega_{pn})-i\delta^{(M)}_{lmnp}(\Omega_{pn})]\exp(-i2\Omega t)\ , \tag{B10}$$

where

$$\gamma^{(M)}_{lmnp}(\Omega_{pn}) = \frac{2(\boldsymbol{\mu}_{lm}\cdot\boldsymbol{\mu}_{np})}{3\hbar c^3}(2\Omega-\Omega_{pn})^{3/2}\Omega_{pn}^{3/2}\ , \tag{B11}$$

$$\delta^{(M)}_{lmnp}(\Omega_{pn}) \cong \frac{2(\boldsymbol{\mu}_{lm}\cdot\boldsymbol{\mu}_{np})}{3\pi\hbar c^3 M(\Omega_{pn})}\mathrm{P}\int \frac{\omega^{3/2}(2\Omega-\omega)^{3/2}}{\omega+\Omega_{pn}-2\Omega}M(2\Omega-\omega)d\omega\ . \tag{B12}$$

Notation used for description of the Ξ-type atom:

$$\begin{aligned}
&\Phi_1(\Omega_{12}) \equiv \Gamma_{1221}(\Omega_{12}) = \gamma_1(\Omega_{12}) - i\delta_1(\Omega_{12})\ ,\\
&\Phi_2(\Omega_{23}) \equiv \Gamma_{2332}(\Omega_{23}) = \gamma_2(\Omega_{23}) - i\delta_2(\Omega_{23})\ ,\\
&\gamma_1(\Omega_{12}) \equiv \gamma_{1221}(\Omega_{12}),\quad \gamma_2(\Omega_{23}) \equiv \gamma_{2332}(\Omega_{23})\ ,\\
&\delta_1(\Omega_{12}) \equiv \delta_{1221}(\Omega_{12}),\quad \delta_2(\Omega_{23}) \equiv \delta_{2332}(\Omega_{23})\ ,\\
&\Phi_1^{(N)}(\Omega_{12}) \equiv \Gamma^{(N)}_{1221}(\Omega_{12}) = \gamma_1(\Omega_{12}) - i\delta_1^{(N)}(\Omega_{12})\ ,\\
&\Phi_2^{(N)}(\Omega_{23}) \equiv \Gamma^{(N)}_{2332}(\Omega_{23}) = \gamma_2(\Omega_{23}) - i\delta_2^{(N)}(\Omega_{23})\ ,\\
&\delta_1^{(N)}(\Omega_{12}) \equiv \delta^{(N)}_{1221}(\Omega_{12}),\quad \delta_2^{(N)}(\Omega_{23}) \equiv \delta^{(N)}_{2332}(\Omega_{23})\ ,\\
&\Phi_{12}(\Omega_{23}) \equiv \Gamma^{(M)}_{2132}(\Omega_{23}) = \gamma_{12}(\Omega_{23}) - i\delta_{12}(\Omega_{23})\ ,\\
&\gamma_{12}(\Omega_{23}) \equiv \gamma^{(M)}_{2132}(\Omega_{23}),\quad \delta_{12}(\Omega_{23}) \equiv \delta^{(M)}_{2132}(\Omega_{23})\ .
\end{aligned} \tag{B13}$$

Notation used for description of the V-type atom:

$$\begin{aligned}
&\Phi_1(\Omega_{12}) \equiv \Gamma_{1221}(\Omega_{12}) = \gamma_1(\Omega_{12}) - i\delta_1(\Omega_{12})\ ,\\
&\Phi_2(\Omega_{13}) \equiv \Gamma_{1331}(\Omega_{13}) = \gamma_2(\Omega_{13}) - i\delta_2(\Omega_{13})\ ,\\
&\gamma_1(\Omega_{12}) \equiv \gamma_{1221}(\Omega_{12}),\quad \gamma_2(\Omega_{23}) \equiv \gamma_{1331}(\Omega_{13})\ ,\\
&\delta_1(\Omega_{12}) \equiv \delta_{1221}(\Omega_{12}),\quad \delta_2(\Omega_{13}) \equiv \delta_{1331}(\Omega_{13})\ .
\end{aligned} \tag{B14}$$

Other parameters are defined as

$$\begin{aligned}
&\Theta_{12}(\Omega_{mn}) \equiv \Gamma_{1213}(\Omega_{mn}) = \kappa_{12}(\Omega_{mn}) - i\phi_{12}(\Omega_{mn})\ ,\\
&\kappa_{12}(\Omega_{mn}) \equiv \gamma_{1213}(\Omega_{mn}),\quad \phi_{12}(\Omega_{mn}) \equiv \delta_{1213}(\Omega_{mn})\ ,\\
&\Theta^{(N)}_{12}(\Omega_{mn}) \equiv \Gamma^{(N)*}_{1213}(\Omega_{mn}) = \kappa_{12}(\Omega_{mn}) - i\phi^{(N)}_{12}(\Omega_{mn})\ ,\\
&\phi^{(N)}_{12}(\Omega_{mn}) \equiv -\delta^{(N)}_{1213}(\Omega_{mn})\ .
\end{aligned} \tag{B15}$$

Notation used for description of the Λ-type atom:

$$\begin{aligned}
&\Phi_1(\Omega_{12}) \equiv \Gamma_{1331}(\Omega_{13}) = \gamma_1(\Omega_{13}) - i\delta_1(\Omega_{13})\ ,\\
&\Phi_2(\Omega_{23}) \equiv \Gamma_{2332}(\Omega_{23}) = \gamma_2(\Omega_{23}) - i\delta_2(\Omega_{23})\ ,\\
&\gamma_1(\Omega_{13}) \equiv \gamma_{1331}(\Omega_{13}),\quad \gamma_2(\Omega_{23}) \equiv \gamma_{2332}(\Omega_{23})\ ,\\
&\delta_1(\Omega_{13}) \equiv \delta_{1331}(\Omega_{13}),\quad \delta_2(\Omega_{23}) \equiv \delta_{2332}(\Omega_{23})\ ,\\
&\Theta_{12}(\Omega_{mn}) \equiv \Gamma_{1323}(\Omega_{mn}) = \kappa_{12}(\Omega_{mn}) - i\phi_{12}(\Omega_{mn})\ ,\\
&\kappa_{12}(\Omega_{mn}) \equiv \gamma_{1323}(\Omega_{mn}),\quad \phi_{12}(\Omega_{mn}) \equiv \delta_{1323}(\Omega_{mn})\ ,\\
&\Theta^{(N)}_{12}(\Omega_{mn}) \equiv \Gamma^{(N)*}_{1323}(\Omega_{mn}) = \kappa_{12}(\Omega_{mn}) - i\phi^{(N)}_{12}(\Omega_{mn})\ ,\\
&\phi^{(N)}_{12}(\Omega_{mn}) \equiv -\delta^{(M)}_{1323}(\Omega_{mn})\ .
\end{aligned} \tag{B16}$$

APPENDIX C: SU(2) COHERENT STATES FOR A THREE-LEVEL ATOM

The atomic operators $\hat{R}_{13}$, $\hat{R}_{11}$, and $\hat{R}_{33}$ defined by Eq. (3) are related to the angular-momentum generators $\hat{J}_i$ of the SU(2) Lie algebra [13] with commutation relations

$$[\hat{J}_i,\hat{J}_j] = i\epsilon_{ijk}\hat{J}_k\ , \tag{C1}$$

where ϵ_{ijk} is an antisymmetric tensor. The relations between $\hat{R}_{ij}$ and $\hat{J}_k$ are as follows:

$$\begin{aligned}
\hat{J}_1 &= \frac{\hat{R}_{31}+\hat{R}_{13}}{2}\ ,\\
\hat{J}_2 &= \frac{\hat{R}_{31}-\hat{R}_{13}}{2i}\ ,\\
\hat{J}_3 &= \frac{\hat{R}_{33}-\hat{R}_{11}}{2}\ .
\end{aligned} \tag{C2}$$

Following the general procedure proposed by Perelomov [26] (see also Ref. [13]) one can define the SU(2) coherent state $|\eta\rangle$ as

$$|\eta\rangle = \exp(\eta\hat{J}_+ - \eta^*\hat{J}_-)|J,-J\rangle\ ,\quad \eta = \left[\frac{\theta}{2}\right]e^{i\phi}\ , \tag{C3}$$

where the ladder operators $\hat{J}_\pm = \hat{J}_1 \pm i\hat{J}_2$ are simply related to the atomic operators:

$$\hat{J}_- = \hat{R}_{13} \text{ and } \hat{J}_+ = \hat{R}_{31}\ .$$

The ladder operators select the vacuum state $|J,-J\rangle$ from the angular-momentum states $|J,M\rangle$:

$$\hat{J}_-|J,-J\rangle = 0\ .$$

For the particular realization (C2) of the angular-momentum operators one can identify the lowest atomic state $|1\rangle$ with the vacuum state $|J,-J\rangle$. It follows that the SU(2) coherent state for the three-level atom can be defined as

$$|\eta\rangle=\exp(\eta\hat{R}_{31}-\eta^*\hat{R}_{13})|1\rangle\equiv\hat{S}(\eta)|1\rangle\ . \quad \text{(C4)}$$

Using the disentangling theorem for the angular-momentum generators [13] we can rewrite the unitary operator $\hat{S}(\eta)=\exp(\eta\hat{R}_{31}-\eta^*\hat{R}_{13})$ in the following way:

$$\hat{S}(\eta)=\exp\left[e^{i\phi}\tan\left[\frac{\theta}{2}\right]\hat{R}_{31}\right]\times\exp\left\{\ln\left[\cos\left[\frac{\theta}{2}\right]\right]^{-1/2}\frac{\hat{R}_{33}-\hat{R}_{11}}{2}\right\}\times\exp\left[-e^{-i\phi}\tan\left[\frac{\theta}{2}\right]\hat{R}_{13}\right]\ . \quad \text{(C5)}$$

The action of $\hat{S}(\eta)$ on the state $|1\rangle$ can now be evaluated to give an explicit expression for the SU(2) coherent state for the three-level atom:

$$|\eta\rangle=\cos\left[\frac{\theta}{2}\right]|1\rangle+e^{-i\phi}\sin\left[\frac{\theta}{2}\right]|3\rangle\ , \quad \text{(C6)}$$

which means that $|\eta\rangle$ is a superposition of only two states.

SU(2) squeezing

One of the consequences of the commutation relation (C1) is the following uncertainty relation for the variances of the angular-momentum operators $\hat{J}_i$:

$$\langle[\Delta\hat{J}_1(t)]^2\rangle\langle[\Delta\hat{J}_2(t)]^2\rangle\geq\tfrac{1}{4}|\langle\hat{J}_3\rangle|^2\ , \quad \text{(C7)}$$

where the variances $\langle[\Delta\hat{J}_i(t)]^2\rangle$ are defined as usual:

$$\langle[\Delta\hat{J}_i(t)]^2\rangle=\langle\hat{J}_i^2\rangle-\langle\hat{J}_i\rangle^2\ .$$

It should be noted here that due to the fact that the right-hand side of the uncertainty relation (C7) is a state-dependent quantity, we can find states for which the left and right sides of (C7) are equal but where $|\langle\hat{J}_3\rangle|^2$ does not reach its local minimum. These states are *intelligent states* (for details see Refs. [27] and [13]). If a local minimum of both sides of the equality (C7) is reached then these states are *minimum-uncertainty states.*

Following Wódkiewicz and Eberly [13] we shall say that variances (fluctuations) of the operators $\hat{J}_1$ or $\hat{J}_2$ are squeezed [SU(2) *squeezing*] if

$$\langle[\Delta\hat{J}_1(t)]^2\rangle<\tfrac{1}{2}|\langle\hat{J}_3\rangle|\ \text{or}\ \langle[\Delta\hat{J}_2(t)]^2\rangle<\tfrac{1}{2}|\langle\hat{J}_3\rangle|\ . \quad \text{(C8)}$$

To measure the degree of SU(2) squeezing we introduce two parameters S_i $(i=1,2)$:

$$S_i=\frac{\langle[\Delta\hat{J}_i(t)]^2\rangle-\frac{1}{2}|\langle\hat{J}_3\rangle|}{\frac{1}{2}|\langle\hat{J}_3\rangle|}\ . \quad \text{(C9)}$$

In this case the squeezing condition can be written in the simple form

$$S_1<0\ \text{or}\ S_2<0$$

and maximum (100%) SU(2) squeezing corresponds to $S_i=-1$. Using the representation (C2) for the angular-momentum operators one can easily evaluate the expectation values and variances of the operators $\hat{J}_i$ in the three-level atom coherent state (C5):

$$\langle[\Delta\hat{J}_1(t)]^2\rangle=\tfrac{1}{4}(1-\sin^2\theta\cos^2\phi)\ ,$$
$$\langle[\Delta\hat{J}_2(t)]^2\rangle=\tfrac{1}{4}(1-\sin^2\theta\sin^2\phi)\ , \quad \text{(C10)}$$
$$\langle\hat{J}_3\rangle=-\tfrac{1}{2}\cos\theta\ .$$

From the above we see that if the phase ϕ is equal to 0, $\pm\pi$, or $\pm\pi/2$ then the three-level atom coherent state (C5) is an intelligent state (nevertheless it is not generally a minimum-uncertainty state). If $\phi=0,\pm\pi$ then SU(2) squeezing in the $\hat{J}_1$ quadrature can be seen:

$$S_1=\frac{\cos^2\theta-|\cos\theta|}{|\cos\theta|}<0\ ,$$

while if $\phi=\pm\pi/2$ then squeezing in the $\hat{J}_2$ quadrature appears:

$$S_2=\frac{\cos^2\theta-|\cos\theta|}{|\cos\theta|}<0$$

for any value of θ. In both cases the maximum (100%) squeezing can be obtained for $\theta=\pm\pi/2$. Moreover, we see that in this case the SU(2) coherent state under consideration is the minimum-uncertainty state that can be written as

$$|\eta\rangle=\frac{1}{\sqrt{2}}(|1\rangle\pm|3\rangle)\ \text{for}\ \phi=0,\pm\pi\ \text{and}\ \theta=\pi/2\ , \quad \text{(C11)}$$

$$|\eta\rangle=\frac{1}{\sqrt{2}}(|1\rangle\pm i|3\rangle)\ \text{for}\ \phi=\pm\pi/2\ \text{and}\ \theta=\pi/2\ . \quad \text{(C12)}$$

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