PHYSICAL REVIEW A VOLUME 39, NUMBER 6 MARCH 15, 1989

Brief Reports

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Jaynes-Cummings model with intensity-dependent coupling interacting with Holstein-Primakoff SU(1,1) coherent state

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(Received 29 September 1988; revised manuscript received 8 November 1988)

We show that in the Jaynes-Cummings model with the intensity-dependent coupling interacting with the Holstein-Primakoff SU(1,1) coherent state the revivals of the radiation squeezing are strictly periodical for any value of initial squeezing. The expression for the atomic population inversion exhibiting the exact periodicity of the population revivals is obtained.

I. INTRODUCTION

Recently Gerry[1] has studied the time evolution of the variances of the field quadratures for a squeezed vacuum state, described as an SU(1,1) coherent state (CS),[2–4] interacting with a two-photon generalization[5] of the Jaynes-Cummings model (JCM).[6,7] The field operators in the Hamiltonian for the two-photon JCM have been written in terms of the same SU(1,1) generators [see Eq. (2.7)] as those on which the SU(1,1) CS has been built. Gerry has found that if the variance of the field quadrature is squeezed at the initial moment, then the squeezing of the variance can recur at later times. These revivals of squeezing are not periodical in the two-photon JCM; nevertheless, it can be observed, that the higher the initial squeezing, the more regular the oscillations become.

In the present paper we want to turn attention to another generalization of the JCM, namely, on the JCM with the intensity-dependent coupling,[8,9] whose interaction terms can be expressed through the SU(1,1) generators. The realization of the SU(1,1) Lie algebra in the present paper [see Eq. (2.4)] is different from that considered by Gerry. In fact, we consider the special case of the Holstein-Primakoff realization of the SU(1,1) Lie algebra.[10,11] We will study the time evolution of the field quadratures of an SU(1,1) CS built on this Holstein-Primakoff realization of the SU(1,1) Lie algebra.[12–14] We will show, that in such a model the revivals of squeezing are strictly periodical for any value of the initial squeezing.

II. THE MODEL

We will suppose the Hamiltonian for the JCM with the intensity-dependent coupling in the rotating-wave approximation ($\hbar=1$),

$$H=\omega_0\sigma_3+\omega a^\dagger a+\lambda(R^\dagger\sigma_-+R\sigma_+) , \tag{2.1}$$

where $\sigma_3,\sigma_\pm$ are the pseudospin atomic operators. The constant λ is a real number; ω and ω_0 are the frequencies of the atom and the field, respectively. The operators R and $R^\dagger$ are constructed from the single-mode field operators a and $a^\dagger$ ($[a,a^\dagger]=1$),

$$R=a\sqrt{N} \ , \quad R^\dagger=\sqrt{N}a^\dagger , \tag{2.2}$$

with the commutation relations

$$[R,R^\dagger]=2N+1, \quad [R,N]=R, \quad [R^\dagger,N]=-R^\dagger , \tag{2.3}$$

where $N=a^\dagger a$ is the photon number operator. One can say that the Hamiltonian (2.1) effectively describes the intensity-dependent coupling between the atom and the radiation field.

Since the SU(1,1) Lie algebra for a single-mode field may be realized as

$$K_+=R^\dagger, \ K_-=R, \quad K_0=N+\tfrac{1}{2} , \tag{2.4}$$

with the commutation relations for the generators K_0 and $K_\pm$,

$$[K_-,K_0]=2K_0, \quad [K_0,K_\pm]=\pm K_\pm , \tag{2.5}$$

the Hamiltonian (2.1) may be rewritten in the following way:

$$H=\omega_0\sigma_3+\omega(K_0-\tfrac{1}{2})+\lambda(K_+\sigma_-+K_-\sigma_+)\ . \tag{2.6}$$

The form of the Hamiltonian (2.6) is the same (up to constant factors) as that in Gerry's paper.[1] But the realization of the operators K_0 and $K_\pm$ is fundamentally different—Gerry has analyzed the case with

$$K_0=(a^\dagger a+\tfrac{1}{2})/2,\ \ K_+=(a^\dagger)^2/2,\ \ K_-=a^2/2\ . \tag{2.7}$$

III. SU(1,1) COHERENT STATES

We will define the SU(1,1) CS in the usual way,[2,15]

$$|\xi\rangle=(1-|\xi|^2)^k\sum_{m=0}^{\infty}\left[\frac{\Gamma(m+2k)}{m!\Gamma(2k)}\right]^{1/2}\xi^m|m,k\rangle\ , \tag{3.1}$$

where $|m,k\rangle$ are the eigenvectors of the Casimir operator $C=K_0^2-(K_+K_-+K_-K_+)/2=k(k-1)I$, where k is the so-called Bargmann index, and $\xi=|\xi|\exp(i\phi)$ $(0\le|\xi|\le1)$.

The standard oscillator realization (2.7) of the SU(1,1) Lie algebra is labeled by $k=\frac{1}{4}$ and $k=\frac{3}{4}$. The $k=\frac{1}{4}$ representation is what was used by Gerry[1] and the corresponding SU(1,1) CS is the usual squeezed vacuum state[3] with the Gaussian wave function.

The realization (2.4) of the SU(1,1) Lie algebra used in the present paper is labeled by $k=\frac{1}{2}$. The basis $|m,\frac{1}{2}\rangle$ in this representation is formed by the usual oscillator number states $|n\rangle$, and for the relevant SU(1,1) CS we obtain

$$|\xi\rangle=(1-|\xi|^2)^{1/2}\sum_{n=0}^{\infty}\xi^n|n\rangle\equiv\sum_{n=0}^{\infty}Q_n|n\rangle\ . \tag{3.2}$$

The wave function of this SU(1,1) CS is not a Gaussian one. It should be mentioned that the SU(1,1) CS (3.2) is the special (one-photon) case of the multiboson Holstein-Primakoff SU(1,1) CS as discussed by Katriel and co-workers.[12–14] In Sec. V we will show that the state (3.2) can be a squeezed state.

IV. EVOLUTION OF THE SYSTEM

Following Gerry's idea we assume the initial state of the radiation field to be an SU(1,1) CS built on the representation of the SU(1,1) Lie algebra in terms of which the Hamiltonian of the system is written. In particular, for the realization (2.4) the initial state of the field is defined by (3.2). If the atom is supposed to be in the ground state $|-\rangle$ at the initial moment $(t=0)$, then the initial-state vector $|\psi(t=0)\rangle$ of the system can be written as

$$|\psi(t=0)\rangle=|\xi\rangle\otimes|-\rangle=\sum_{n=0}^{\infty}Q_n|-,n\rangle\ . \tag{4.1}$$

Due to the fact that in the rotating-wave approximation the excitation number operator $N+\sigma_+\sigma_-$ is the integral of motion, we can find the exact solution of the time-dependent Schrödinger equation,

$$i\frac{d}{dt}|\psi(t)\rangle=H|\psi(t)\rangle\ , \tag{4.2}$$

for the state vector $|\psi(t)\rangle$. In the resonant case $(\omega=\omega_0)$ for $|\psi(t)\rangle$ we obtain

$$|\psi(t)\rangle=\sum_{n=0}^{\infty}e^{-i(E_-+n\omega)t}Q_n[\cos(n)\tau|-,n\rangle-i\sin(n)\tau|+,n-1\rangle]\ , \tag{4.3}$$

where $\tau=\lambda t$ and E_- is the energy of the ground state of the atom.

To see how the system under consideration evolves we calculate first of all the mean photon number $\bar{n}(t)=\langle a^\dagger a\rangle\equiv A_0(t)$ and the atomic population inversion (API) $W(t)=+\langle\sigma_+\sigma_--\sigma_-\sigma_+\rangle$, for which the following compact expressions can be obtained:

$$W(t)=-(1-|\xi|^2)\frac{1-|\xi|^2\cos2\tau}{1-2|\xi|^2\cos2\tau+|\xi|^4} \tag{4.4}$$

and

$$A_0(t)=\frac{|\xi|^2}{1-|\xi|^2}-\tfrac{1}{2}[1+W(t)]\ . \tag{4.5}$$

We can conclude that the API, as well as the mean photon number, are periodical functions of time with the period $T=\pi/\lambda$. The time evolution of the API for $|\xi|=0.6$ and 0.9 is given in Fig. 1.

V. SQUEEZING OF THE RADIATION FIELD

To analyze the squeezing properties of the radiation field we introduce two Hermitian time-dependent quadrature operators

$$a_1=(ae^{i\omega t}+a^\dagger e^{-i\omega t})/2\ ,\qquad a_2=(ae^{i\omega t}-a^\dagger e^{-i\omega t})/2i. \tag{5.1}$$

Since the squeezed states of the radiation field are defined as the states with smaller uncertainty in one quadrature than that associated with the coherent field, it is convenient to define the functions $S_i(t)$ $(i=1,2)$,

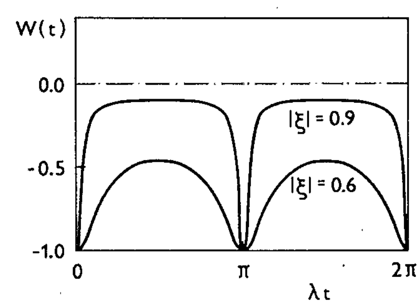

FIG. 1. Time evolution of the atomic population inversion $W(t)$ with $|\xi|=0.6$ and 0.9.

$$S_i(t)=\frac{\langle(\Delta a_i)^2\rangle-\langle(\Delta a_i)^2\rangle_{\text{coh}}}{\langle(\Delta a_i)^2\rangle_{\text{coh}}}=4\langle(\Delta a_i)^2\rangle-1\ , \qquad (5.2)$$

where $\langle(\Delta a_i)^2\rangle=\langle a_i^2\rangle-\langle a_i\rangle^2$ and $\langle(\Delta a_i)^2\rangle_{\text{coh}}=\frac{1}{4}$. The squeezing condition now looks very simply to be

$$S_i(t)<0\ . \qquad (5.3)$$

Due to the fact that the variances of the quadrature operators can be expressed through the mean values of the photon operators $\langle a\rangle=e^{-i(\omega t-\phi)}A_1(t)$, $\langle a^2\rangle=e^{-2i(\omega t-\phi)}A_2(t)$, and $\langle a^\dagger a\rangle=A_0(t)$ (4.4), where

$$A_1(t)=|\xi|\sum_{n=0}^{\infty}P_n[\sqrt{n}\,\sin(n)\tau\sin(n+1)\tau+\sqrt{n+1}\cos(n)\tau\cos(n+1)\tau]\ , \qquad (5.4)$$

$$A_2(t)=|\xi|^2\sum_{n=0}^{\infty}P_n[\sqrt{n(n+1)}\sin(n)\tau\sin(n+2)\tau+\sqrt{(n+1)(n+2)}\cos(n)\tau\cos(n+2)\tau]\ , \qquad (5.5)$$

and $P_n=|Q_n|^2=(1-|\xi|^2)|\xi|^{2n}$, the functions $S_i(t)$ can be written as

$$S_1(t)=2[A_0(t)-A_2(t)]+4\cos^2\phi[A_2(t)-A_1(t)]\ , \qquad (5.6)$$

$$S_2(t)=2[A_0(t)-A_2(t)]+4\sin^2\phi[A_2(t)-A_1(t)]\ . \qquad (5.7)$$

Now we can study the squeezing properties of the SU(1,1) CS (3.2), which is supposed to be the initial state of the radiation field. The squeezing of the variance in the first quadrature is described by the function $S_1(t=0)$. The dependence of this function on the parameter ϕ is shown in Fig. 2. From here it is seen that maximum squeezing can be obtained for $\phi=\pi/2$. In Fig. 3 the dependence of the function $S_1(t=0)$ (with $\phi=\pi/2$) on the parameter $|\xi|$ is plotted. We see that the bigger the photon number $[\bar{n}=|\xi|^2/(1-|\xi|^2)]$, the stronger the squeezing that can be obtained. Maximum (100%) squeezing ($S_1\rightarrow-1$) is obtained for $\bar{n}\rightarrow\infty$ ($|\xi|\rightarrow1$). So we can conclude that the SU(1,1) CS (3.2) for $\phi=\pi/2$ is the squeezed state.

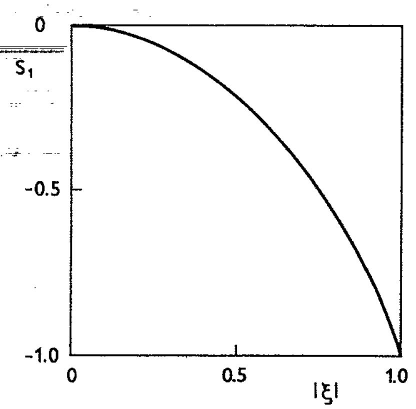

FIG. 3. The function $S_1(t=0)$ vs $|\xi|$ with $\phi=\pi/2$.

Further, we will analyze the time evolution of the functions $S_i(t)$ with $\phi=\pi/2$. As seen from the explicit expressions for $A_i(t)$ (4.4) and (5.4) and (5.5), $S_i(t)$ are *periodic* functions (with the period $T=\pi/\lambda$) for *any* value of the initial squeezing.

The time evolution of the function $S_1(t)$ for $|\xi|=0.6$ and 0.9 is plotted in Fig. 4. From this figure it is seen that in the first moments of the evolution the initial squeezing becomes destroyed. Moreover, the stronger the initial squeezing, the more rapidly the squeezing is revoked. To see this, we have to calculate the first and second time derivatives of the function $S_1(t)$ at $t=0$. The first derivative is equal to zero and for the second

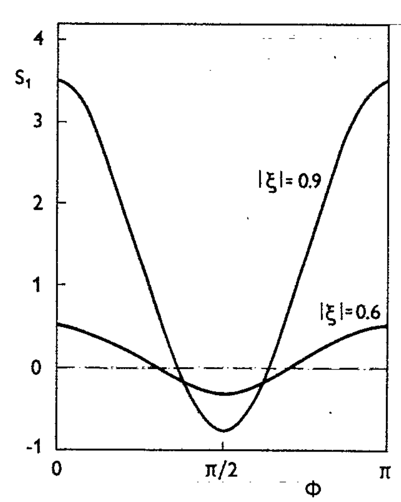

FIG. 2. The function $S_1(t=0)$ for $|\xi|=0.6$ and 0.9 vs the parameter ϕ.

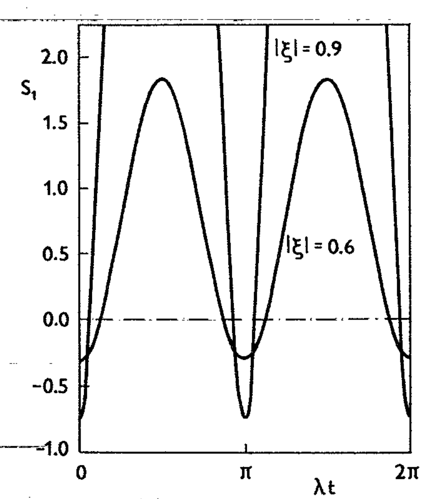

FIG. 4. Time evolution of the function $S_1(t)$ for $|\xi|=0.6$ and 0.9.

one the relation

$$\frac{\partial}{\partial|\xi|}\left[\left.\frac{\partial^2 S_1}{\partial\tau^2}\right|_{\tau=0}\right]>0 \tag{5.8}$$

is valid, which proves the above statement.

After reaching its maximum value at $t=T/2$, the function $S_1(t)$ tends again to its initial value. The initial squeezing is completely restored at $t=T$.

We conclude that in the intensity-dependent coupling JCM described above the exact periodicity of the physical quantities, particularly of the API and the squeezing, can be destroyed only if the radiation field interacts with a system of more than one two-level atom, or when more than two levels of the single-atom model are taken into account.

ACKNOWLEDGMENTS

I would like to thank Professor C. C. Gerry for turning my attention to the problem of the Holstein-Primakoff SU(1,1) CS interacting with the two-level atom.

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