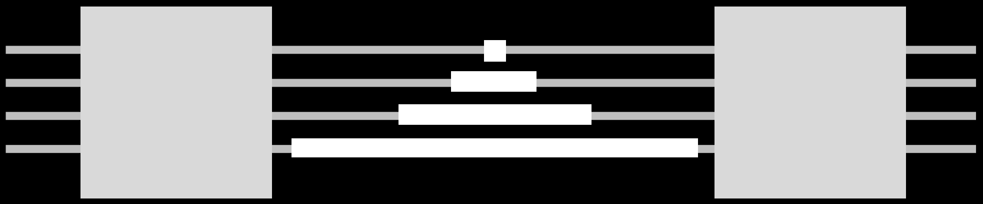
improving phase detection with boson sampling



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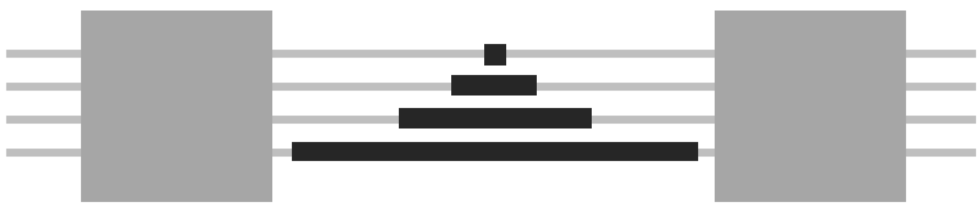
improving phase detection with boson sampling



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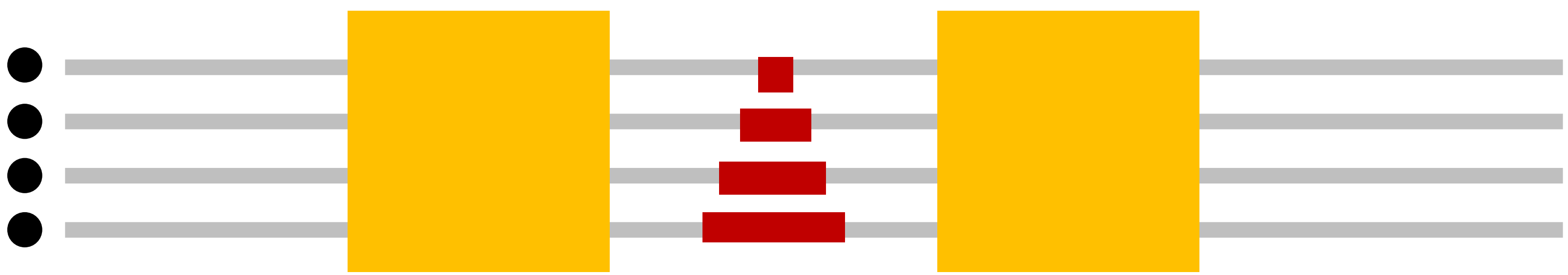
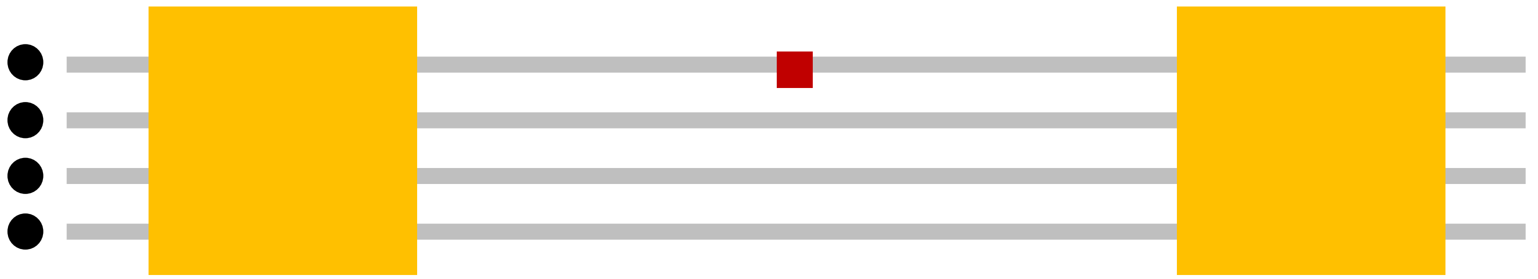


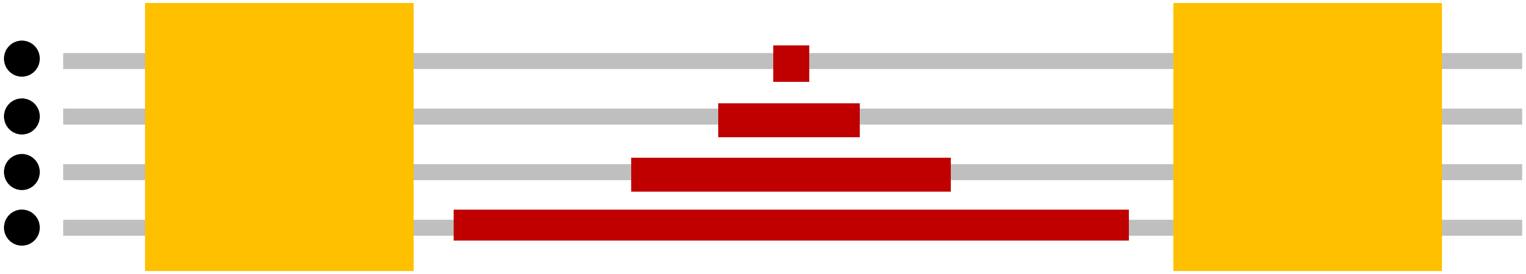
Motes, Olson, Rabeaux, Dowling, Olson, Rohde

Linear optical quantum metrology with single photons: Exploiting spontaneously generated entanglement to beat the shot-noise limit.

Phys. Rev. Lett. 114, 170802 (2015)

No good.



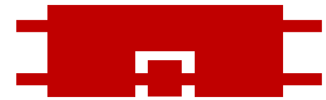


Pretty good.

1 quantum metrology

enhanced by entanglement

1 **quantum metrology**



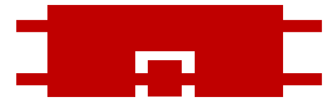
enhanced by entanglement

2 **a phase gradient**

between two Fouriers [Motes et al.]

1 quantum metrology

enhanced by entanglement

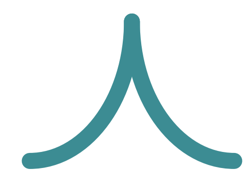


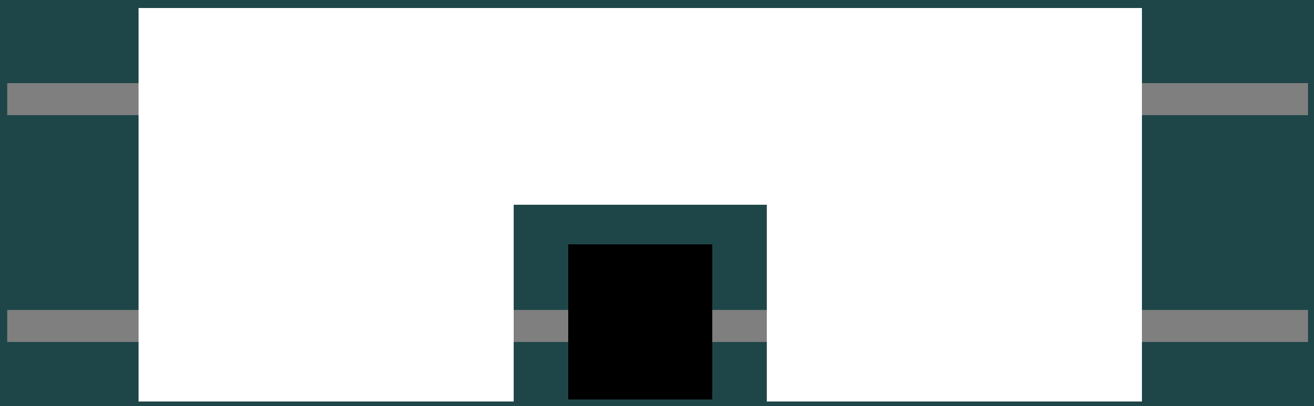
2 a phase gradient

between two Fouriers [Motes et al.]

3 the Heisenberg limit

from higher polynomial gradients

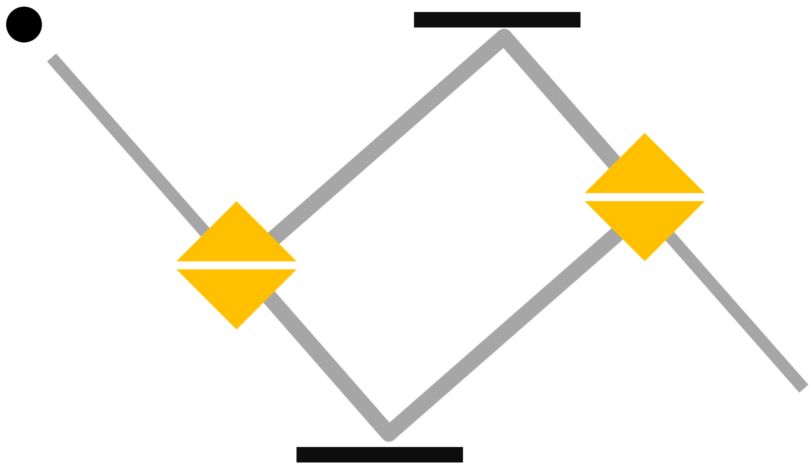




phase detection

1 Detecting a phase: single photons

- 1 photon, 2 beamsplitters



1 Detecting a phase: single photons

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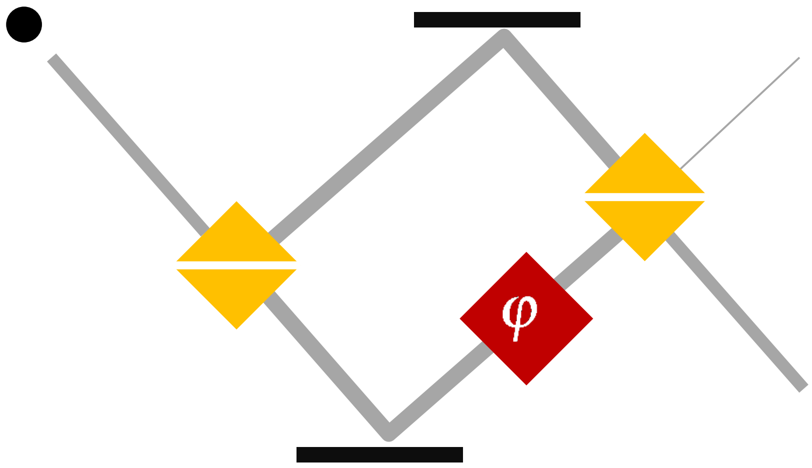
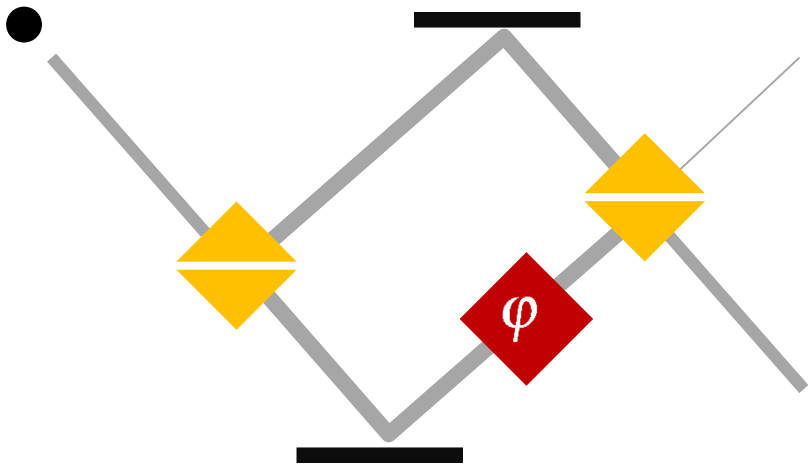


1 Detecting a phase: single photons

- 1 photon, 2 beamsplitters

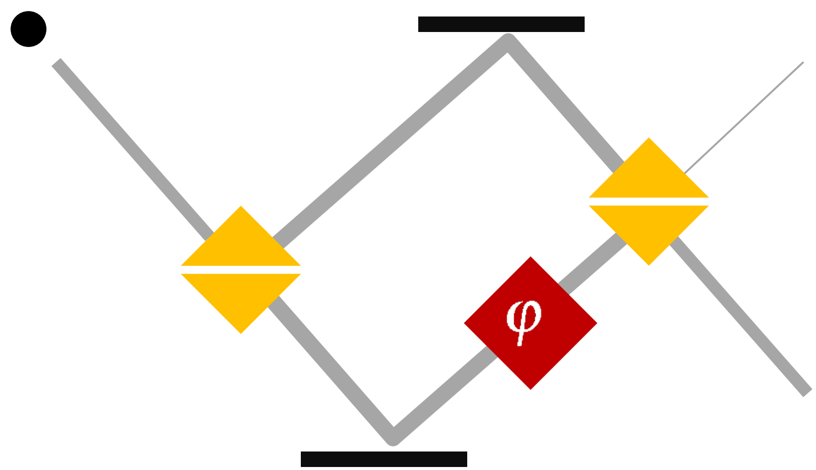


$$\Delta\varphi \propto \frac{1}{\sqrt{N}}$$

- shot noise phase sensitivity for repeated experiments

1 Detecting a phase: path entangled photons

- 1 photon, 2 beamsplitters

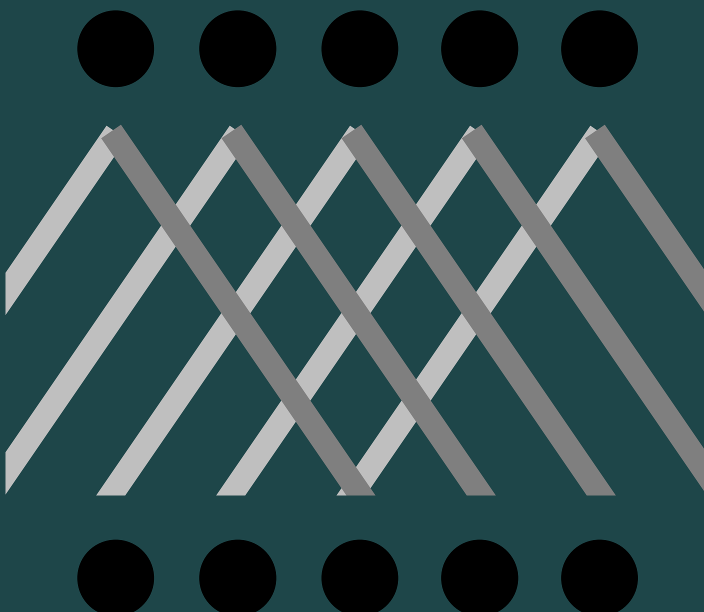


$$\Delta\varphi \propto \frac{1}{\sqrt{N}}$$

- NOON states $\frac{1}{\sqrt{2}}\left(|N,0\rangle + |0,N\rangle\right)$

  Heisenberg sensitivity
  hard to make (postselect)
  sensitive to dephasing

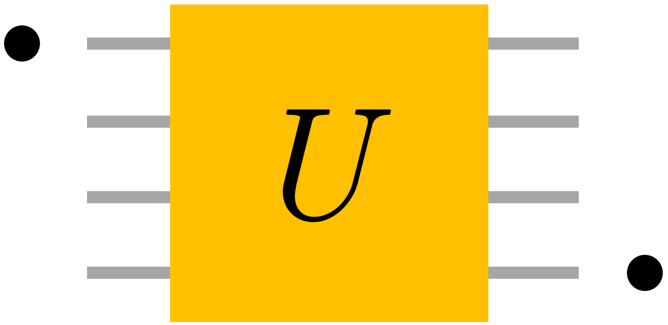
$$\Delta\varphi \propto \frac{1}{N}$$



boson sampling metrology

Motes et al., PRL 114, 170802 (2015)

2 Path entanglement from a boson sampler

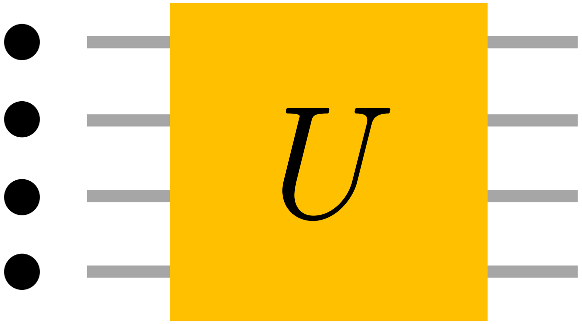


a multimode unitary

$$U_{41}$$

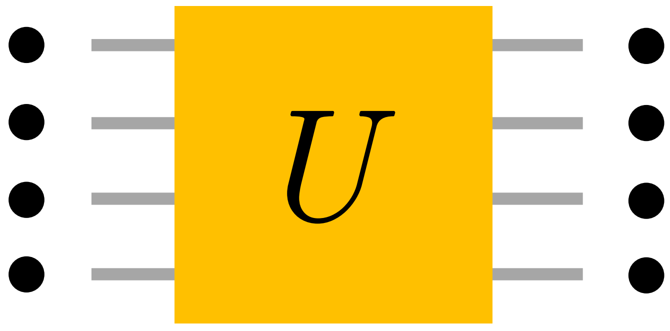
amplitude

2 Path entanglement from a boson sampler



a boson sampler

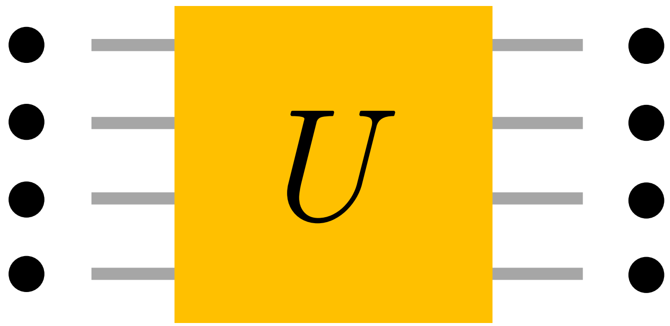
2 Path entanglement from a boson sampler



a boson sampler

$$U_{11}U_{22}U_{33}U_{44} + \ldots$$

2 Path entanglement from a boson sampler



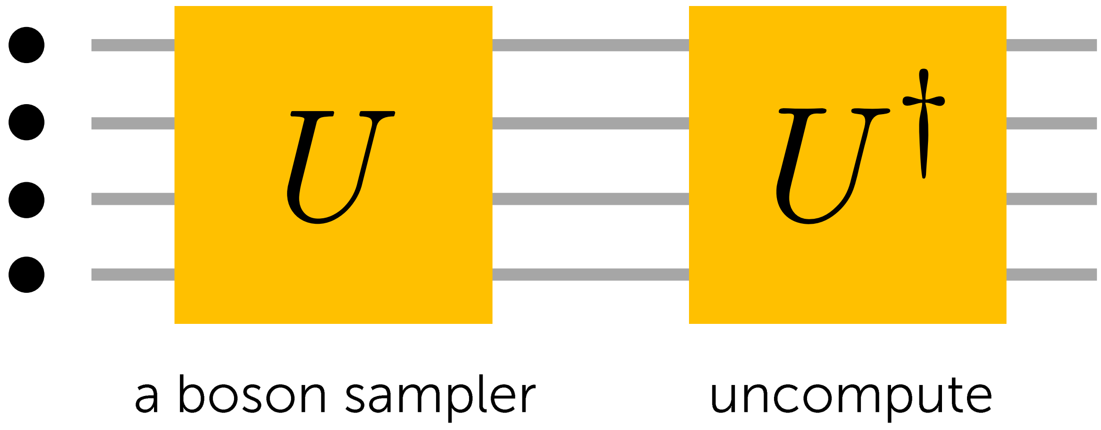
a boson sampler

$$\mathrm{Per}(U)$$

amplitude

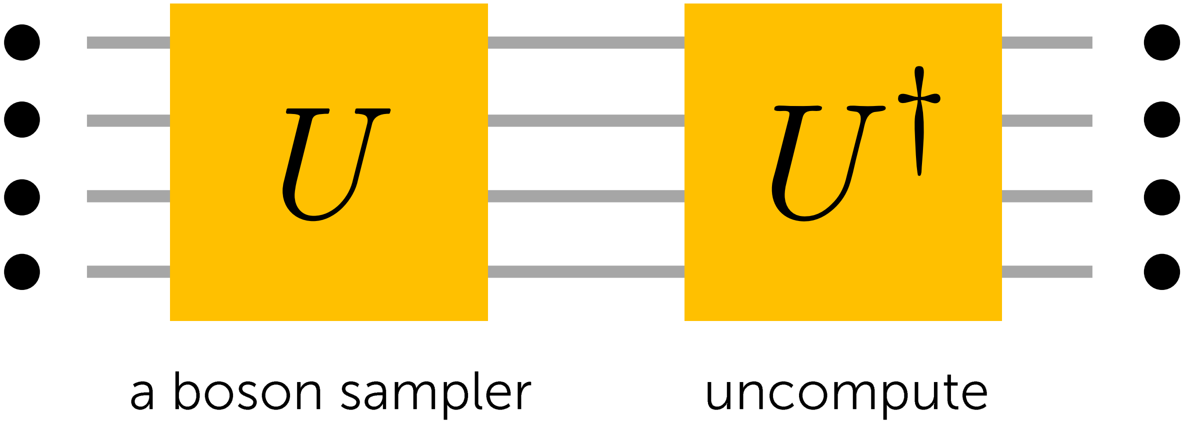
$$U_{11}U_{22}U_{33}U_{44} + \ldots$$

2 Path entanglement from a boson sampler



2 Path entanglement from a boson sampler

- a complicated way to do nothing



2 Path entanglement from a boson sampler

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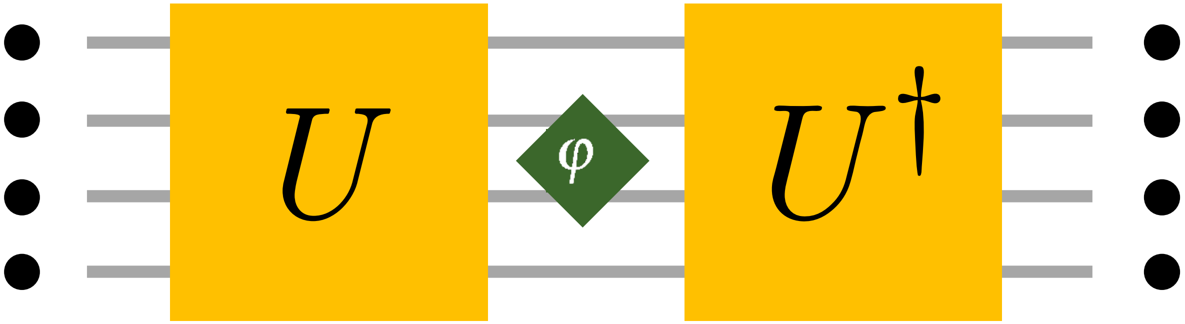
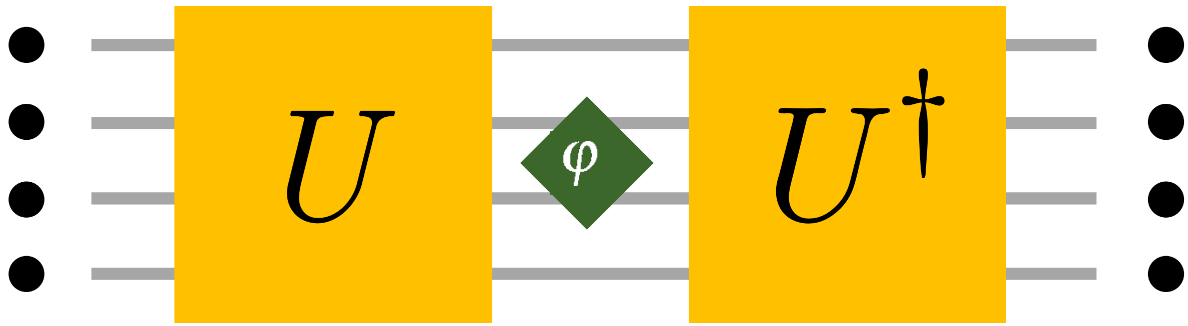


- sensitive to phase disturbances?

2 Path entanglement from a boson sampler

- a complicated way to do nothing

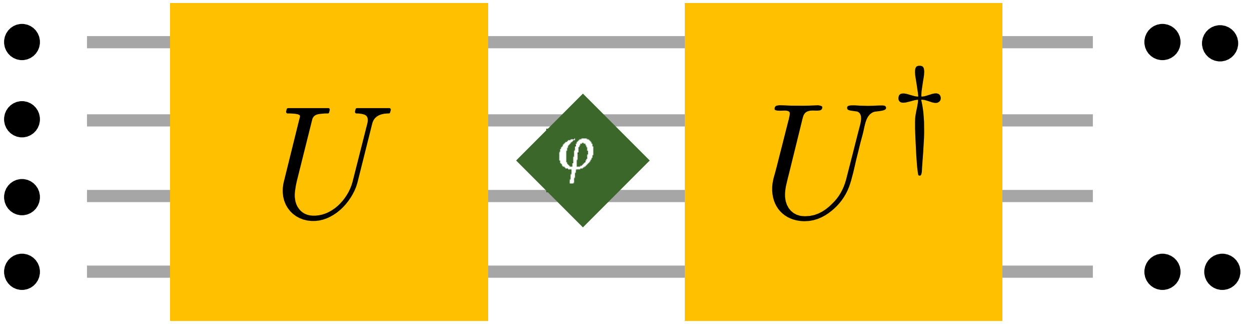


- sensitive to phase disturbances?

- still just one
significant amplitude $\left|\mathrm{Per}\left(U^\dagger V_\varphi U\right)\right|^2 \approx \left|\mathbf{Per}(\mathbb{I})\right|^2 = 1$

2 Path entanglement from a boson sampler

- Fourier transform, undo, check sampling



2 Path entanglement from a boson sampler

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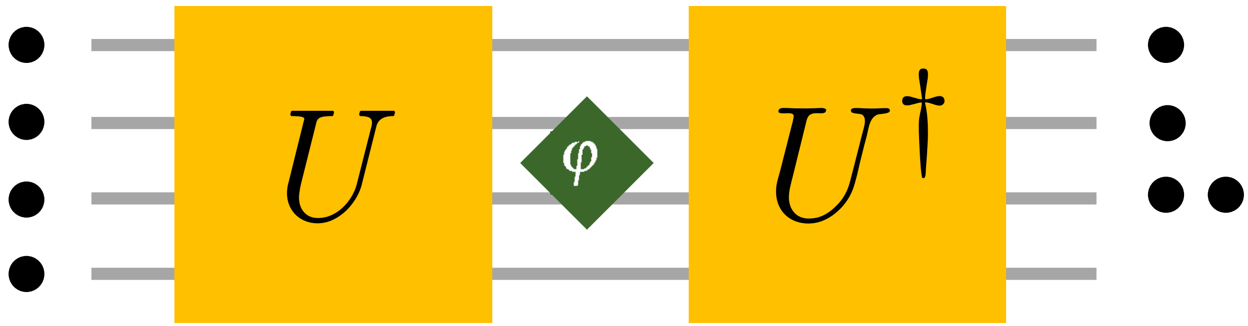

2 Path entanglement from a boson sampler

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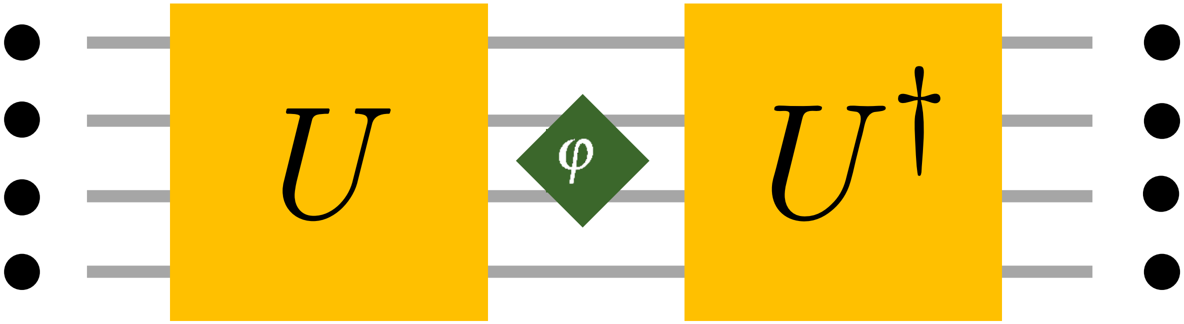
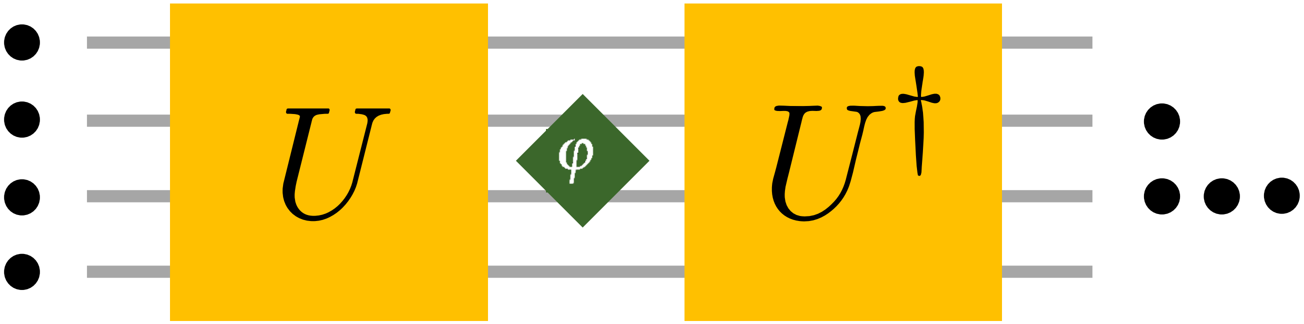


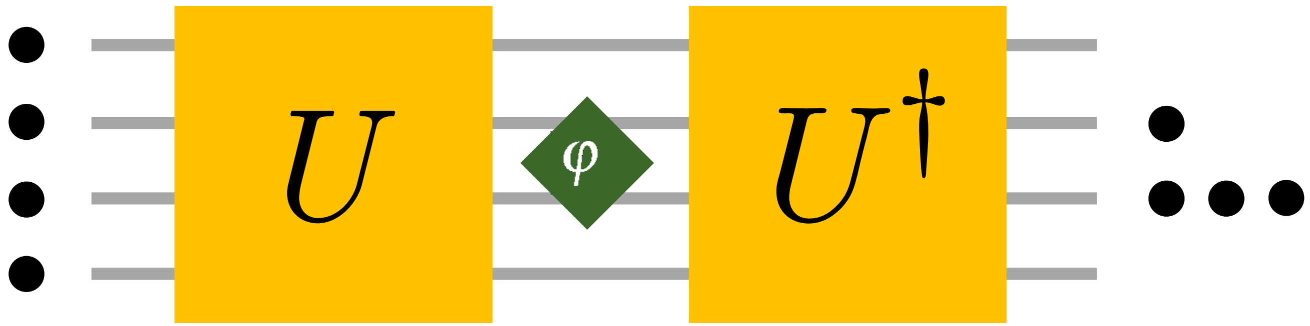
2 Path entanglement from a boson sampler

- Fourier transform, undo, check sampling



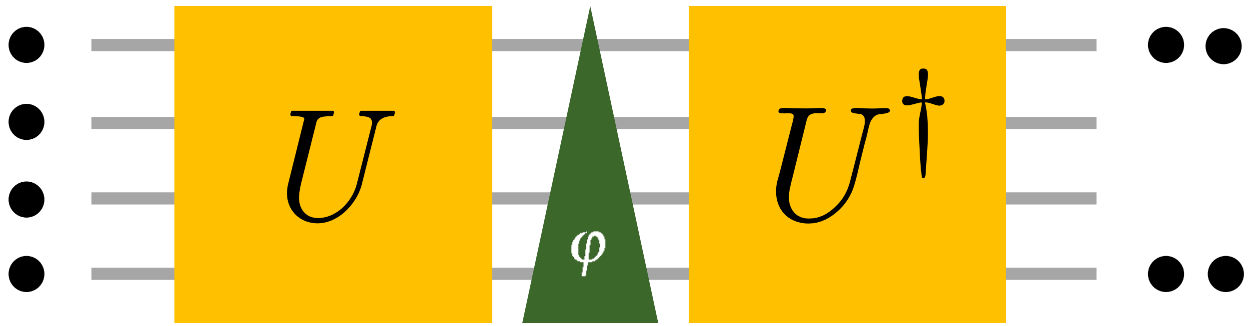
2 Path entanglement from a boson sampler

- Fourier transform, undo, check sampling



2 Path entanglement from a boson sampler

- Fourier transform, undo, check sampling



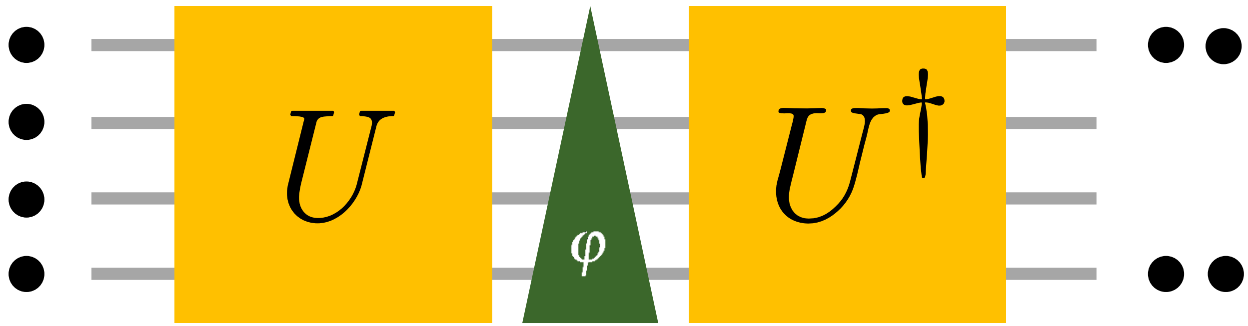
- sensitive to a phase **gradient**

$$\Delta\varphi \propto \frac{1}{M^{\frac{3}{4}}}$$

$$M = 1 + \frac{N(N-1)}{2}$$

2 Path entanglement from a boson sampler

- Fourier transform, undo, check sampling

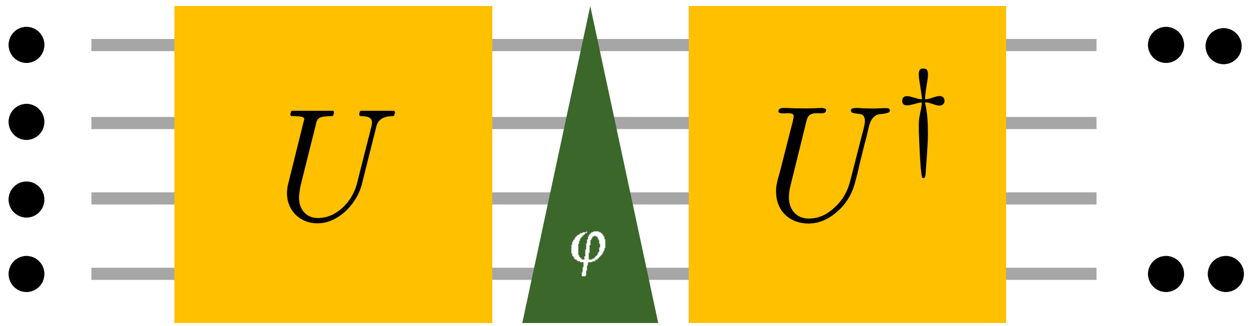


- sensitive to a phase **gradient** $\Delta\varphi \propto \frac{1}{M^{\frac{3}{4}}}$

- the probability of
  1 photon per output $\left|\mathrm{Per}\left(U^\dagger V_\varphi U\right)\right|^2$

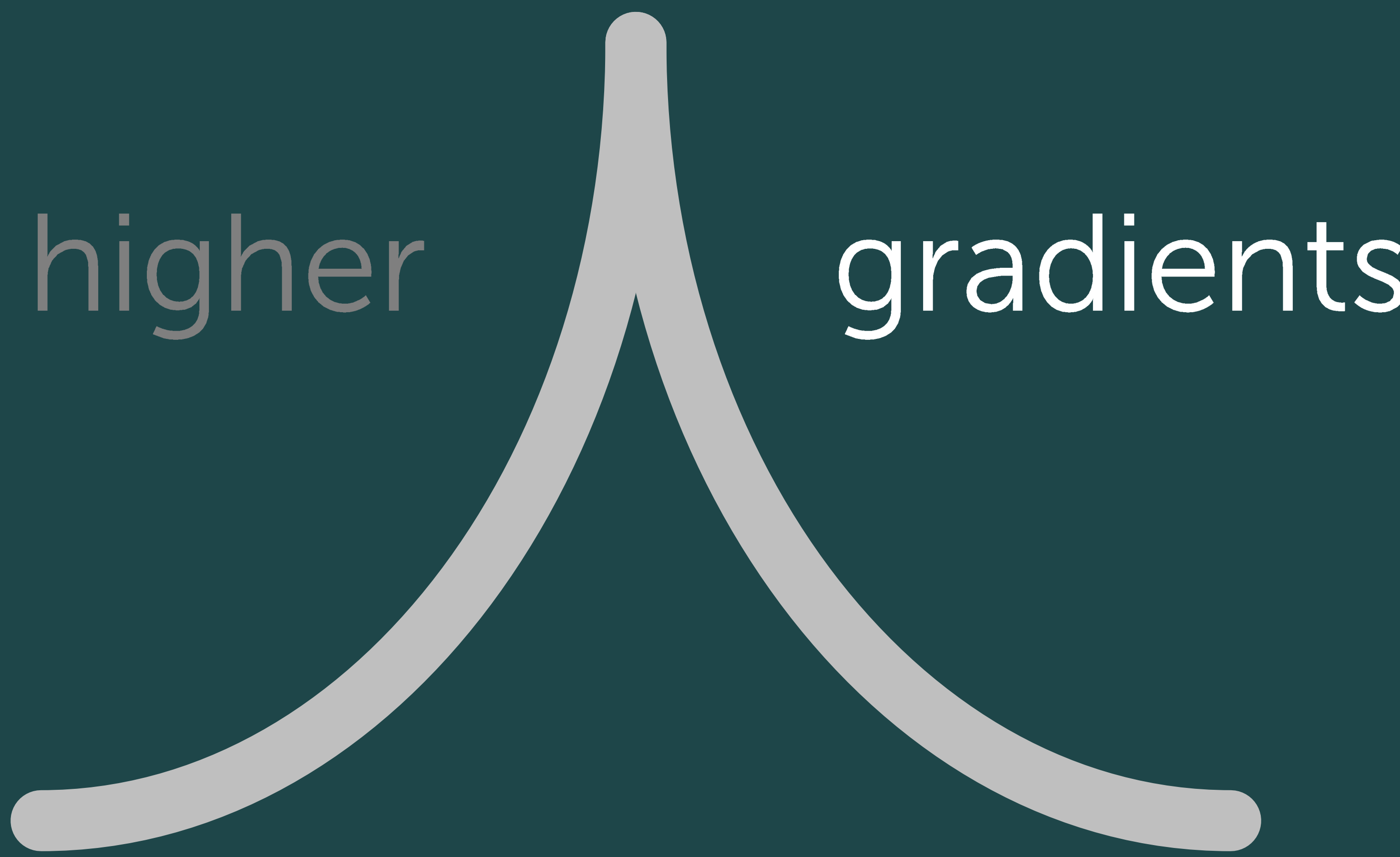
2 Path entanglement from a boson sampler

- Fourier transform, undo, check sampling



- sensitive to a phase **gradient** $\Delta\varphi \propto \frac{1}{M^{\frac{3}{4}}}$

- my questions:
  analytics? why gradient? $\left|\mathrm{Per}\left(U^\dagger V_\varphi U\right)\right|^2$


higher
gradients

3 Analytics for the permanent

- phase sensitivity $\Delta\varphi = \frac{\sqrt{P - P^2}}{\left|\frac{\partial P}{\partial \varphi}\right|} \approx \frac{1}{2\sqrt{k_N}}$

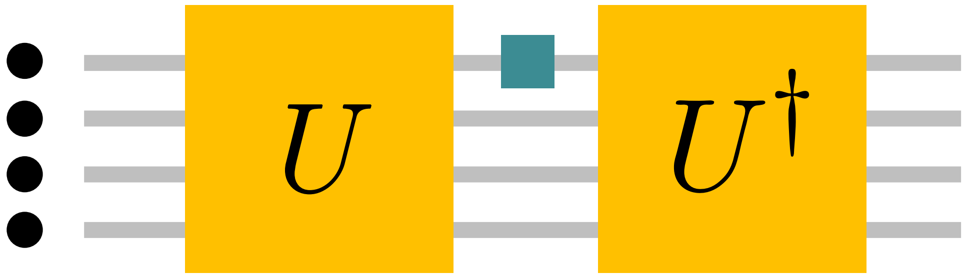
- approximate the permanent by a series

$$P = \left|\mathrm{Per}\left(U^\dagger V_\varphi U\right)\right|^2 \approx 1 - \varphi^2 k_N$$

2^{nd} order coefficient

3 Analytics for the permanent

- a single phase application: no gain



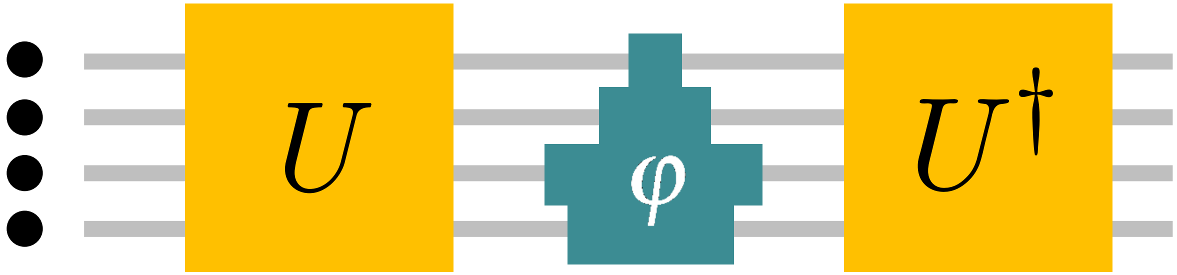
- phase sensitivity $\Delta\varphi = \frac{\sqrt{P - P^2}}{\left|\frac{\partial P}{\partial \varphi}\right|} \approx \frac{1}{2\sqrt{k_N}}$

- approximate the permanent by a series

$$P = \left|\mathrm{Per}\left(U^\dagger V_\varphi U\right)\right|^2 \approx 1 - \varphi^2 \mathrm{const.}$$

3 Analytics for the permanent

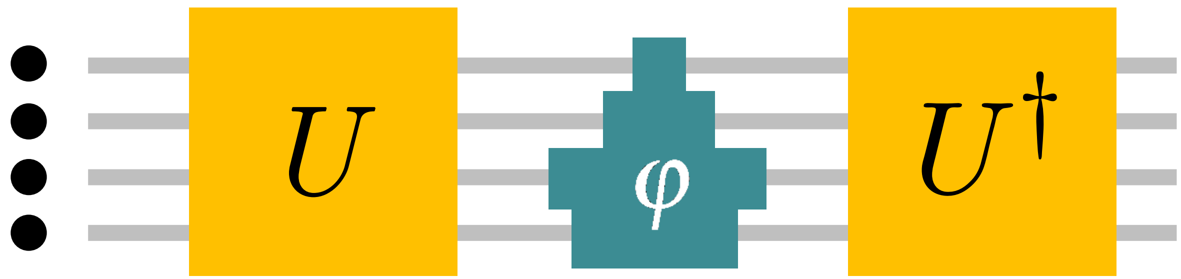
- varying the phase application: the fun stuff



$$\sum_x e^{i f_x \varphi} |x\rangle\langle x|$$

3 Analytics for the permanent

- varying the phase application: the fun stuff



$$\sum_x e^{i f_x \varphi} |x\rangle\langle x|$$

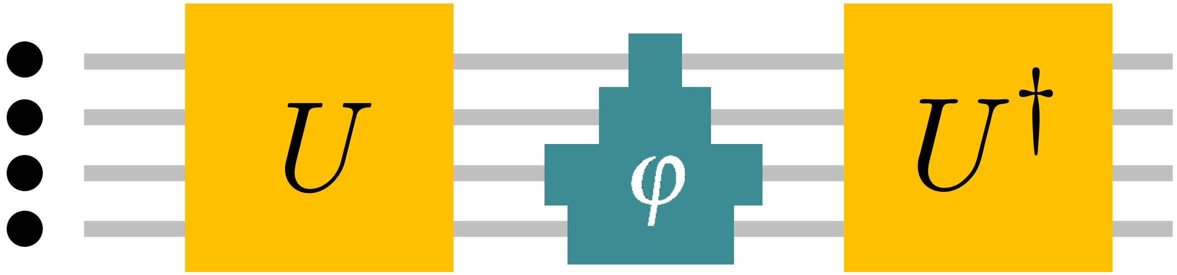
- approximate the permanent by a series

$$\approx \underbrace{1 + i\varphi \sum_x \sum_j f_j U^\dagger_{xj} U_{jx} - \frac{\varphi^2}{2} \sum_x \sum_j f_j^2 U^\dagger_{xj} U_{jx} - \frac{1}{2} \sum_{x \neq y} \varphi^2 \left(\sum_j f_j U^\dagger_{xj} U_{jx} \right) \left(\sum_k f_k U^\dagger_{yk} U_{ky} \right)}_{\text{from the diagonal}} - \varphi^2 \underbrace{\frac{1}{2} \sum_{x \neq y} \left(\sum_j f_j U^\dagger_{xj} U_{jy} \right) \left(\sum_k f_k U^\dagger_{yk} U_{kx} \right)}_{\text{using 2 off-diagonal terms}}$$

- the expansion works better in $z = e^{i\varphi} - 1$

3 Analytics for the permanent

- varying the phase application: the fun stuff



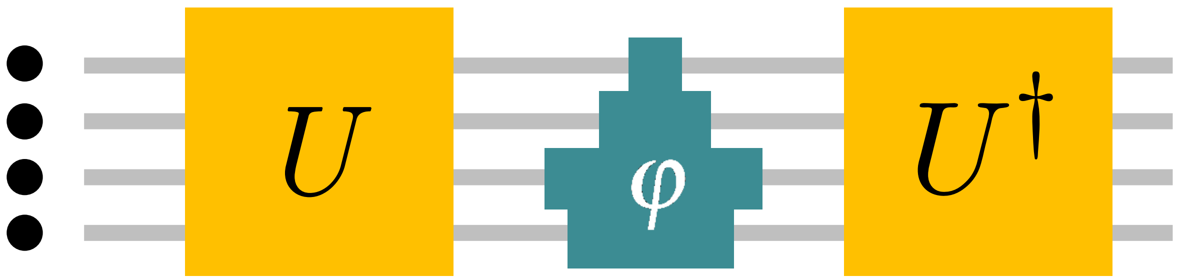
$$\sum_x e^{i f_x \varphi} |x\rangle\langle x|$$

- approximate the permanent by a series

$$P = \left|\mathbf{Per}\left(U^\dagger V_\varphi U\right)\right|^2 \approx 1 - \varphi^2 \Theta\left(N^{2a+1}\right)$$

3 Analytics for the permanent

- varying the phase application: the fun stuff



$$\sum_x e^{i f_x \varphi} |x\rangle\langle x|$$

- approximate the permanent by a series

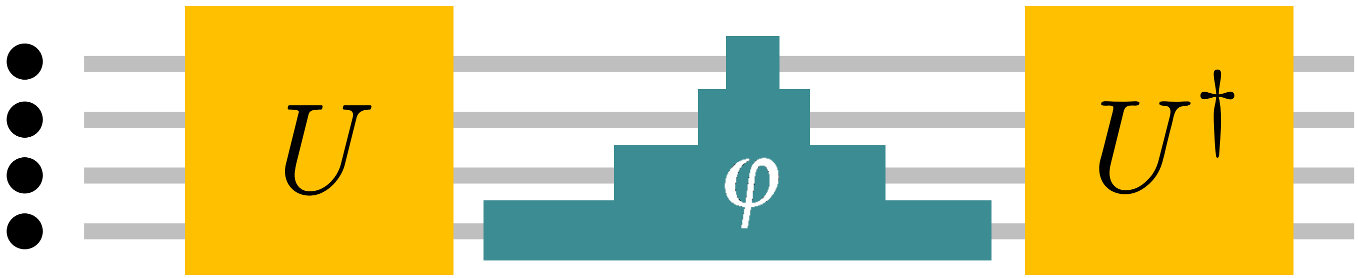
$$P = \left| \mathrm{Per}\left(U^\dagger V_\varphi U\right) \right|^2 \approx 1 - \varphi^2 \Theta\left(N^{2a+1}\right)$$

- for a well-“mixing” U (e.g. Fourier) $|U_{yx}|^2 = \frac{1}{N}$
  and small $\varphi \max_x \{f_x\} \ll 1$

3 Boson sampling to detect a phase

- apply the unknown phase for different "lengths"

$$f_j = j^a$$



$$\sum_x e^{i f_x \varphi} |x\rangle\langle x|$$

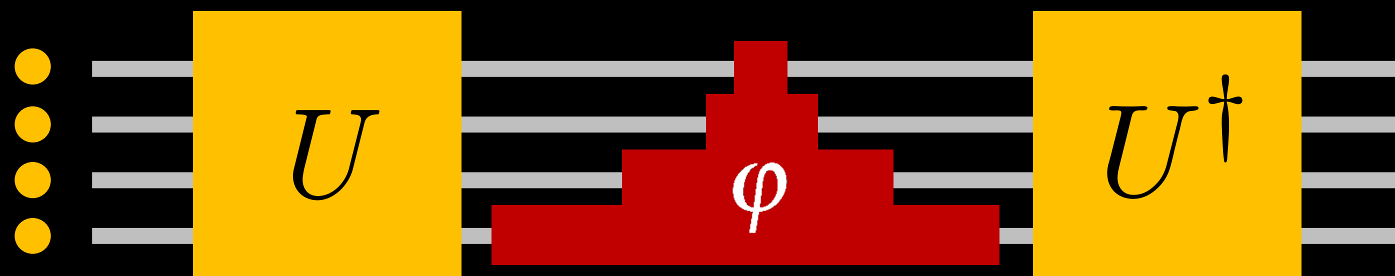
- the phase sensitivity goes up to the H.L. with higher a

$$\Delta\varphi = \Theta\left(M^{-1+\frac{1}{2(a+1)}}\right)$$

$$M = \sum_j f_j$$

number of "uses"

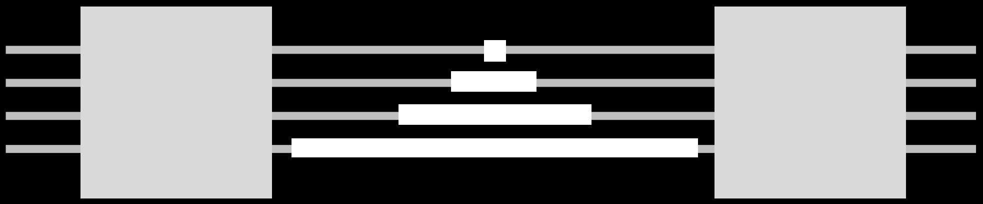
source inefficiency
noise sensitivity
Cramér-Rao bound
Fisher information



?

other perturbations
beyond the $U^{+}U$ arrangement
the other amplitudes
path entanglement applications
path entanglement classification

improving phase detection with boson sampling



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