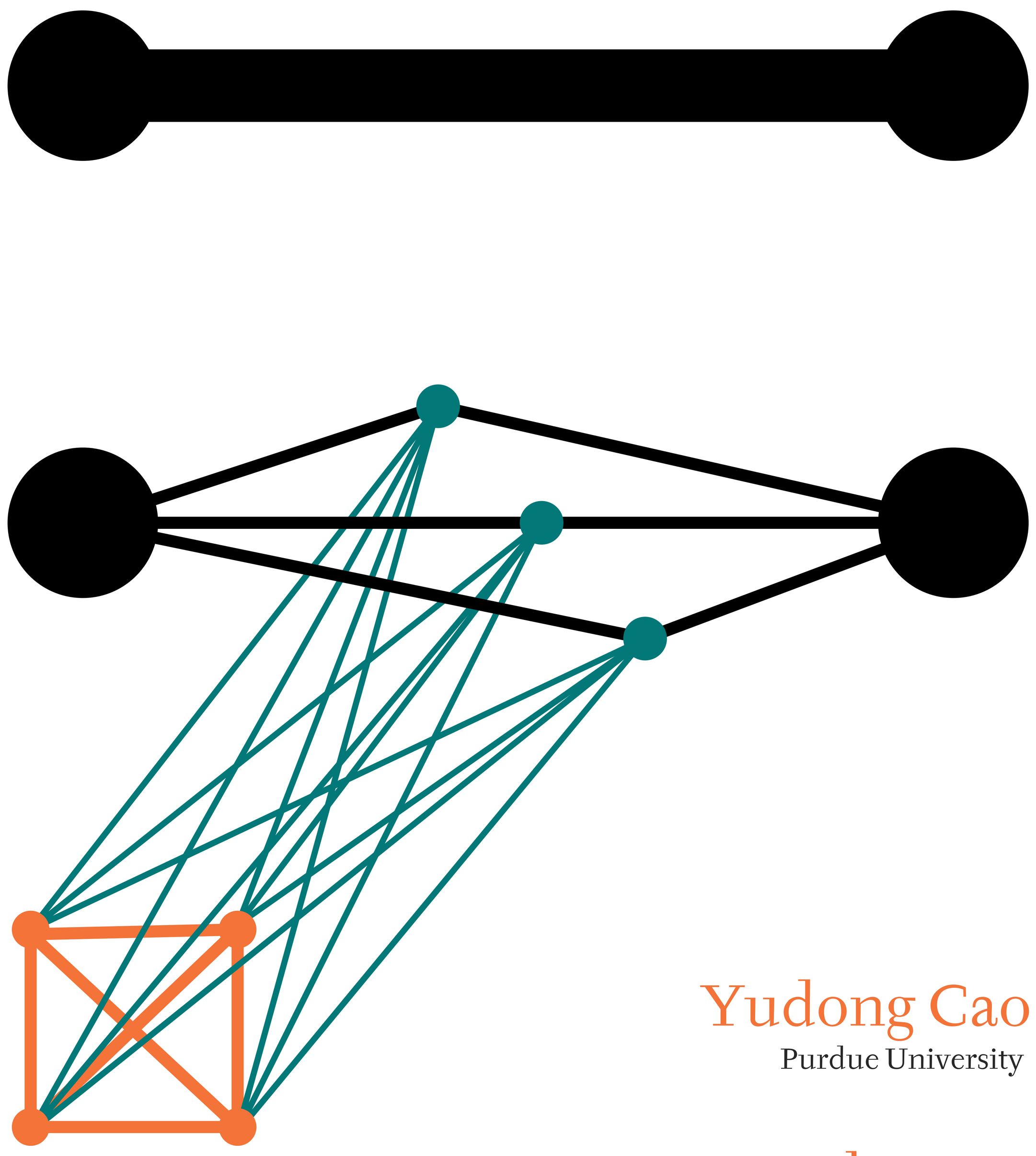

perturbative gadgets without STRONG interactions

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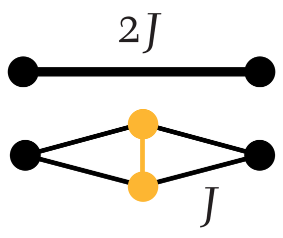
Perturbative gadgets let us find a Hamiltonian whose low-energy subspace approximates a given quantum Hamiltonian up to some absolute error ε. Typically, gadget constructions have terms with **large interaction strengths** $O(\mathrm{poly}(\epsilon^{-1}))$.

We present a 2-body gadget construction for approximating a target many-body Hamiltonian with O(1) interactions up to absolute error ϵ with one that has **much weaker interactions** $O(\epsilon)$. It relies on a new condition for the convergence of the perturbation series, allowing us to apply gadgets in parallel on multiple many-body terms without costly overhead.

The price we pay for avoiding strong interactions is a large overhead in the number of ancilla qubits, and the number of interaction terms per particle, both of which scale as $O(\mathrm{poly}(\epsilon^{-1}))$.

1 Classical gadgets

- Mimicking an interaction by others (decompose, etc.)



2 Quantum perturbation gadgets

- Build effective interactions: put ancillas in a very large z-field. Treat interactions with the target spins as a perturbation. $\tilde{H} = H + V$
- The approximation: $|\lambda_j(H_{\mathit{eff}}) - \lambda_j(\tilde{H}_-)| \le \epsilon$ (desired; actual low spectrum)
  Eigenvectors: ϵ-close to H_{eff}'s, with 0's on ancillas.
- Condition 1: the perturbation can't bring the energy of former high-energy states too low (the subspace condition). $\tilde{\mathcal{L}}_- \cap \mathcal{L}_+ = \{0\}$

  Condition 2: the self-energy is related to some effective Hamiltonian. $\|\Sigma_-(z) - H_{\mathit{eff}}\| \le \epsilon$
- Resolvent: $\tilde{G}(z) = (z\mathbb{I} - \tilde{H})^{-1}$
  Self-energy: $\Sigma_-(z) = z\mathbb{I} - (\tilde{G}_-(z))^{-1}$
  Perturbation expansion:
  $$\begin{aligned}\Sigma_-(z) &= H_- + V_- \\ &+ V_{-+}G_+(z)V_{+-} \\ &+ V_{-+}G_+(z)V_+G_+(z)V_{+-} \\ &+ \cdots\end{aligned}$$

3 The standard 2-body gadget

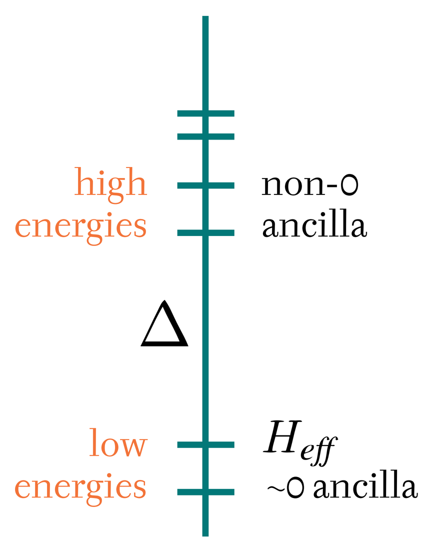
- Target H_{eff}: a Pauli interaction $A \otimes B$.
  Tools: indirect interaction with an ancilla.

The cost: $\Delta = \Theta(\epsilon^{-1})$ for ϵ-approximation.

- The self-energy expansion for $z \ll \Delta$ (2nd order):

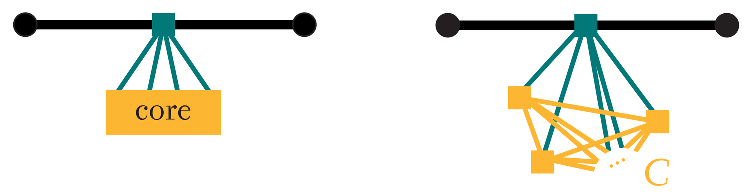
$$\sqrt{\frac{\Delta}{2}}\,(A \otimes |0\rangle\langle 1|) \quad \frac{1}{\Delta - z} \quad \sqrt{\frac{\Delta}{2}}\,(|1\rangle\langle 0| \otimes B) \approx A \otimes B$$

+ a term with A/B switched

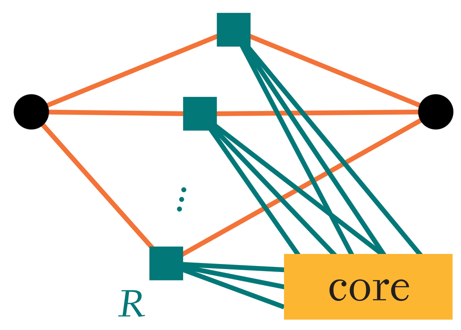


4 Building our weak-interaction gadget

- Replace the strong field by coupling to a core (size C, ferromagnetic).

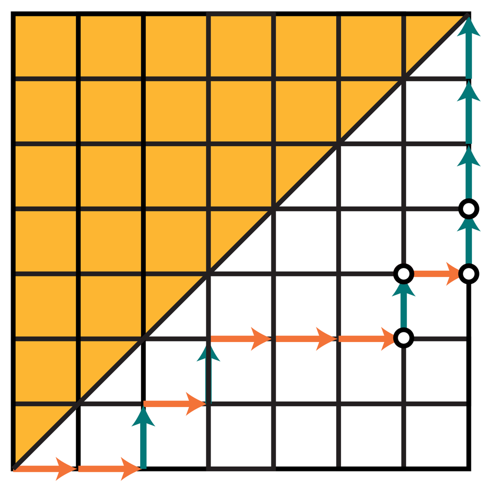


- Replace the strong interaction by coupling target spins to many (R) ancillas, each of which is connected to the core.



5 The gadget works: convergence

- We bound the higher-order terms in the perturbation series by counting ancilla flips & their contributions.

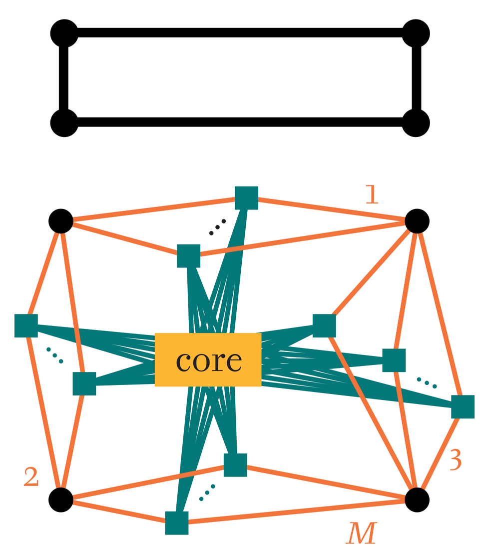


k-th order = grid size k

→ flip an ancilla bit
↑ unflip an ancilla
○ being in a state y with $h(y)$ flipped bits costs $1/h(y)\Delta$
○→ the number of flip choices is $N-h(y)$
○↑ the number of flip choices $h(y)$

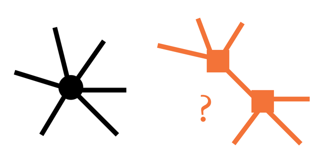
6 Using the gadgets in parallel

- We meet the subspace condition: after adding the perturbation V, the high-energy states of H alone don't combine into low energy states.
- Essential for combining the gadgets in parallel, not relying on $|V| \ll \Delta$.
- For M gadgets/interactions, we can make it work up to error ϵ, with interaction strength $O(\epsilon)$ and using $\mathrm{poly}(\epsilon^{-1}, M)$ qubits.



7 Conclusions & applications

- QMA hardness of restricted norm/form Hamiltonians.
  A generalized area law counterexample (gap amplification).
  Effective interactions using an intermediary atom cloud?
- *Quantum degree reduction (few interactions/spin)?*



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Some references on quantum perturbation gadgets

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