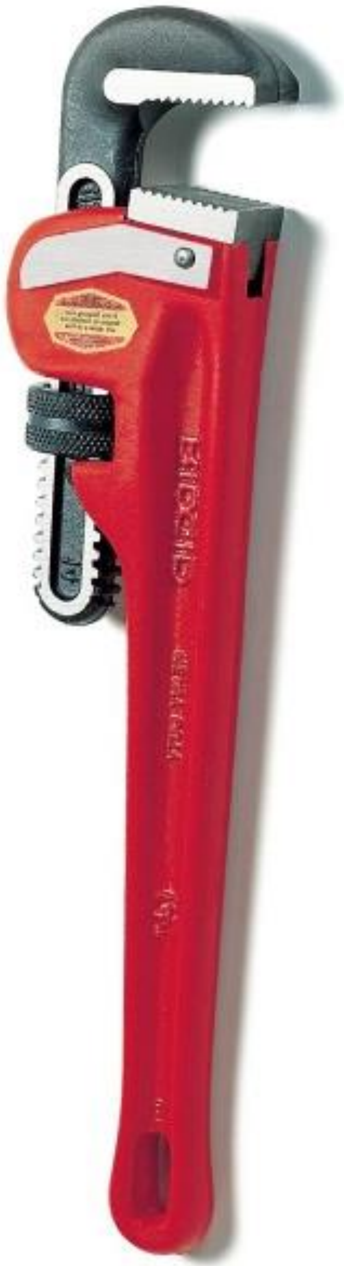


Introduction to **Quantum Computation**

Daniel Nagaj

letná škola FMFI UK, Svit, 9/2014

0 Review: quantum algorithms

- good for …

search

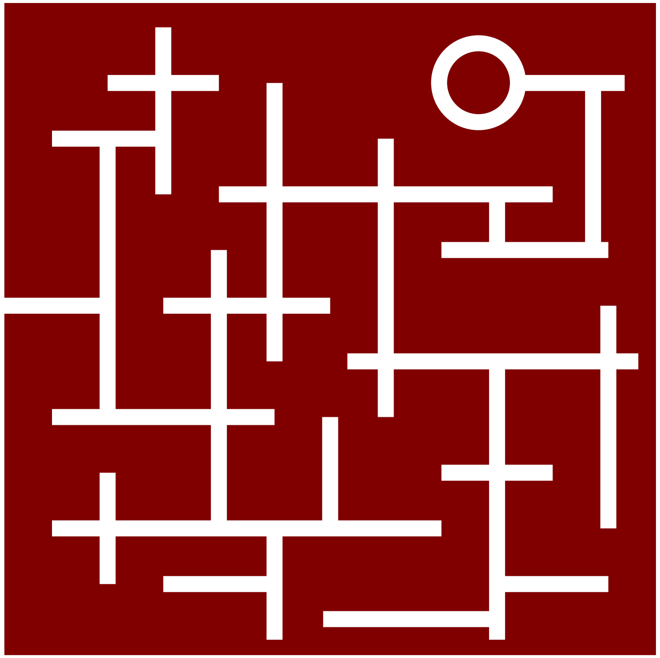
simulation

- the essentials …

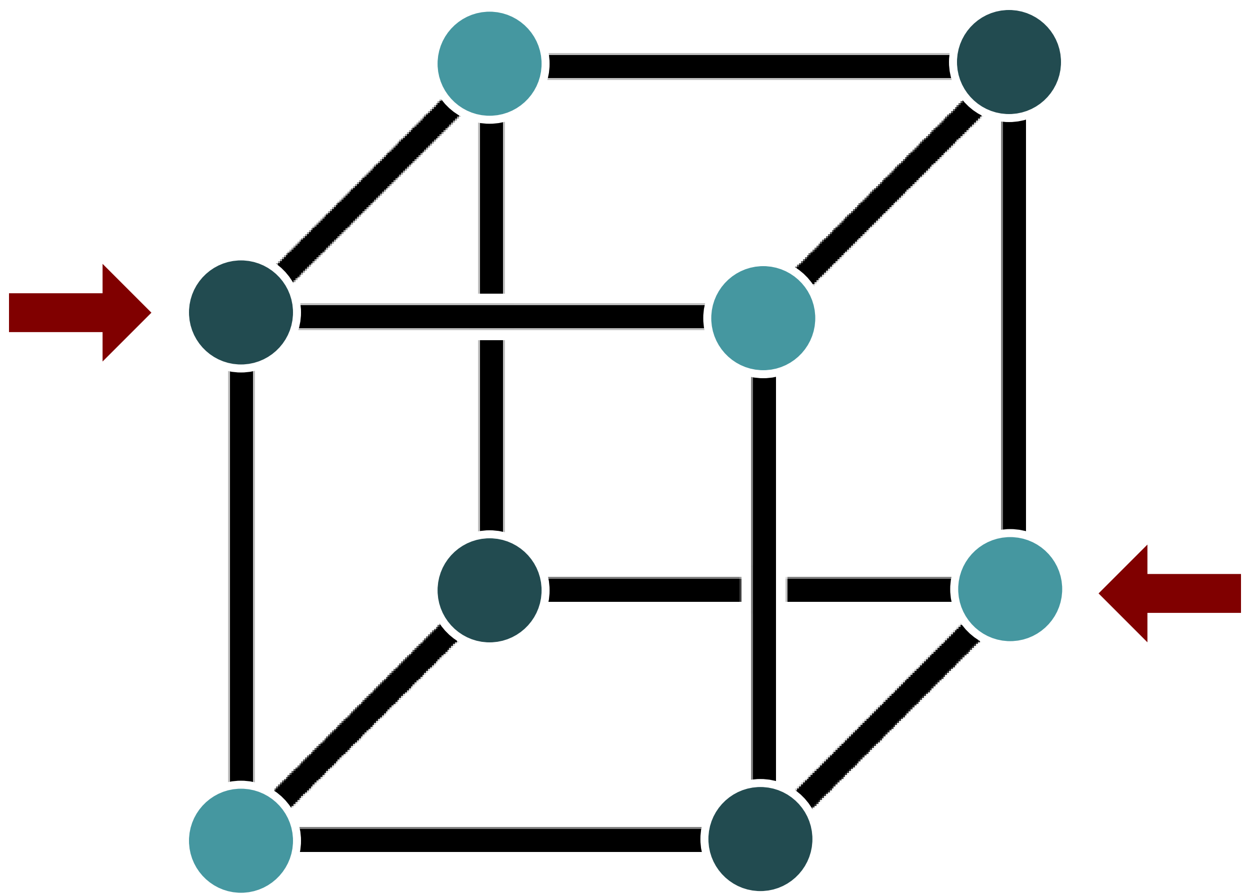
interference

symmetries

entanglement



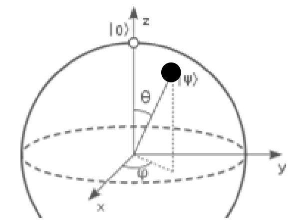
Traversing a (d-dimensional) hypercube



- a classical random walk would be stuck in the “middle”

1 **we need a qubit**

well, what can we do with it?

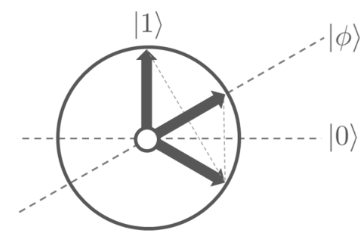


2 **EPR pairs**

and tricky 2-qubit protocols

3 **the algorithms**

that make quantum computing tick

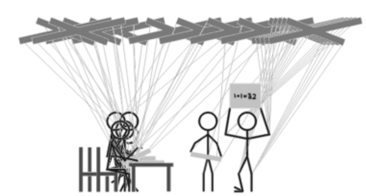


4 **error correction**

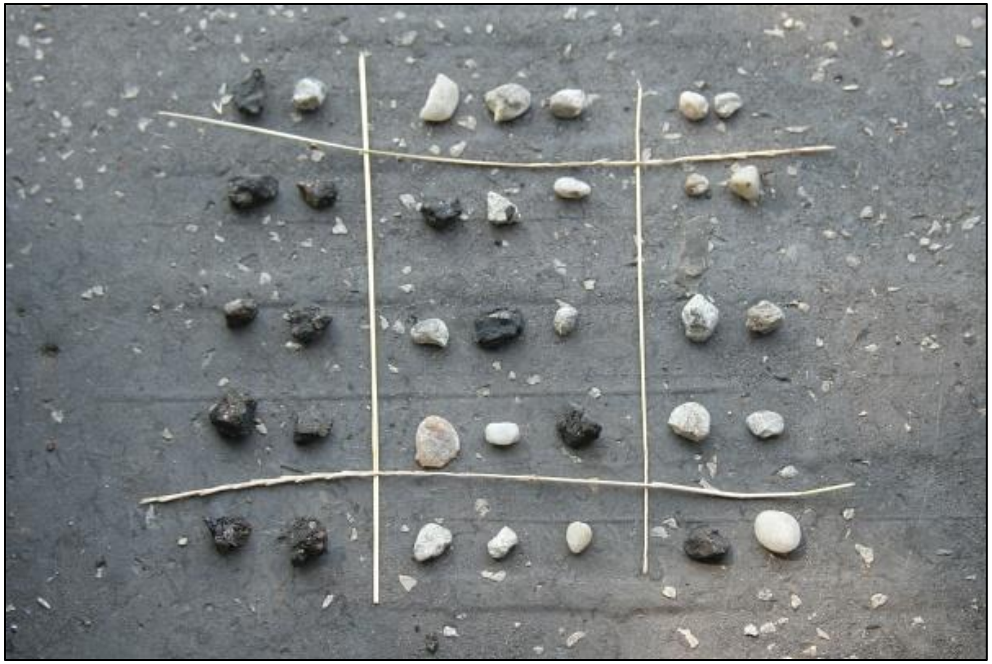
can we really scale up this stuff?

5 **the limits**

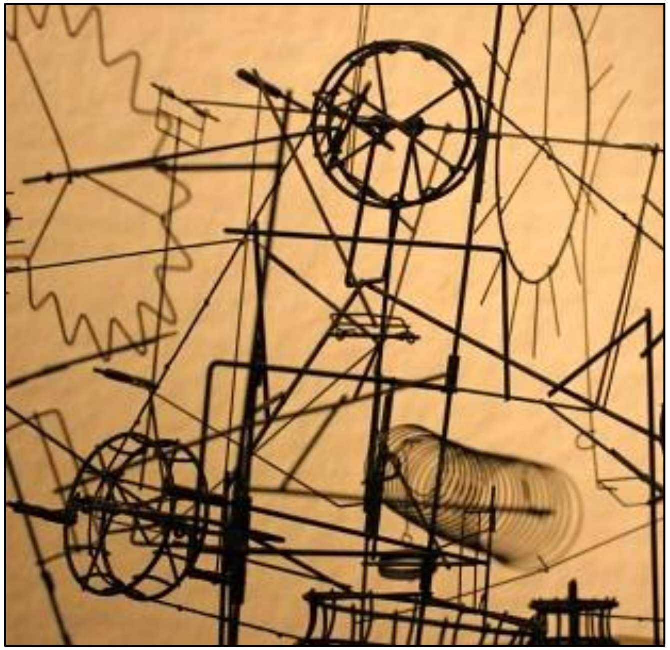
complexity & limits of q. computing



What/how does nature allow us to compute?



Won't it quickly break down?



Exact computation with imprecise elements in a noisy environment?

1 Quantum computation & qubits

- qubits instead of bits

  statest in a Hilbert space

$$|\varphi\rangle = c_0|0\rangle + c_1|1\rangle = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

- time evolution

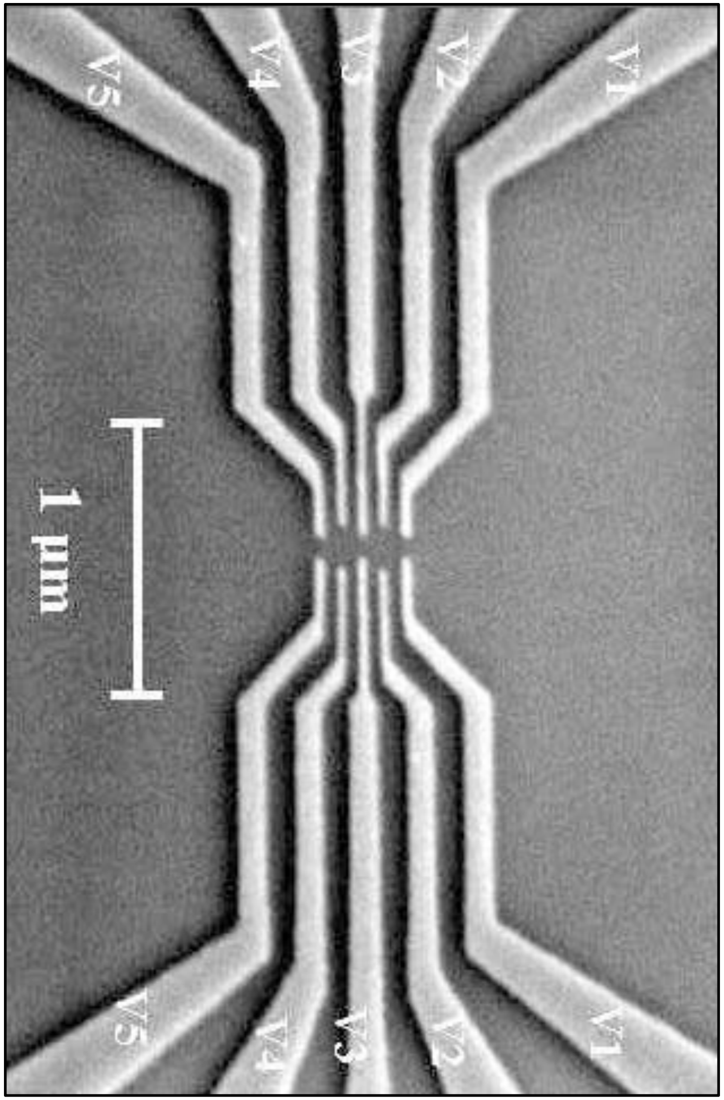
  Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

  unitarity

$$|\psi(t)\rangle = U_{t,0}|\psi(0)\rangle$$

- a final measurement



[a quantum dot, Purdue University]

1 Quantum computation & qubits

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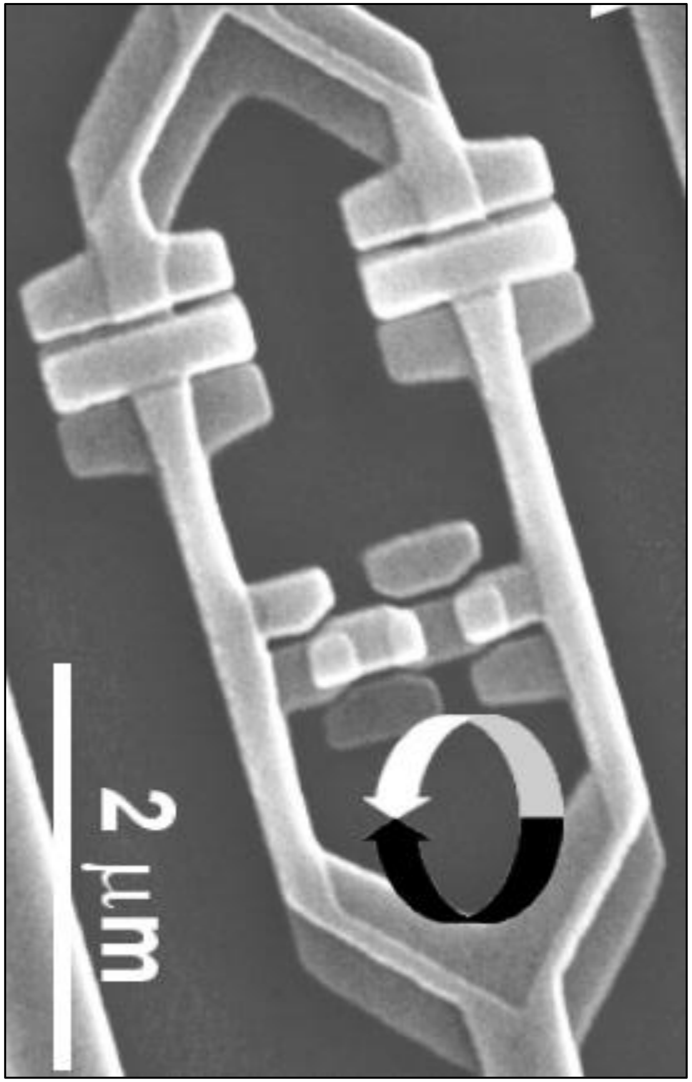
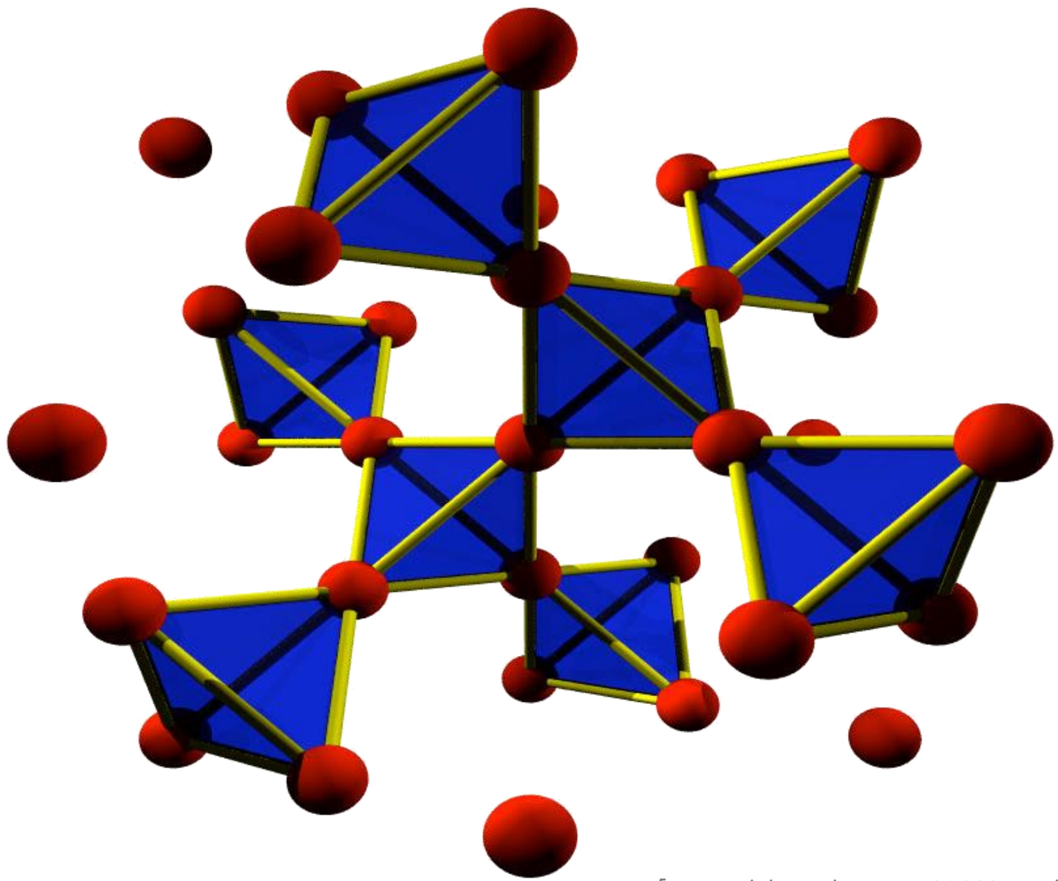
- a final measurement



[a superconducting flux qubit, Florida State Uni.]

1 Quantum computation & qubits

- *N* qubits



[pyrochlore lattice, U Waterloo]

$$2^N$$

ground state?

evolution?

control?

1 Quantum circuits

- single-qubit operations

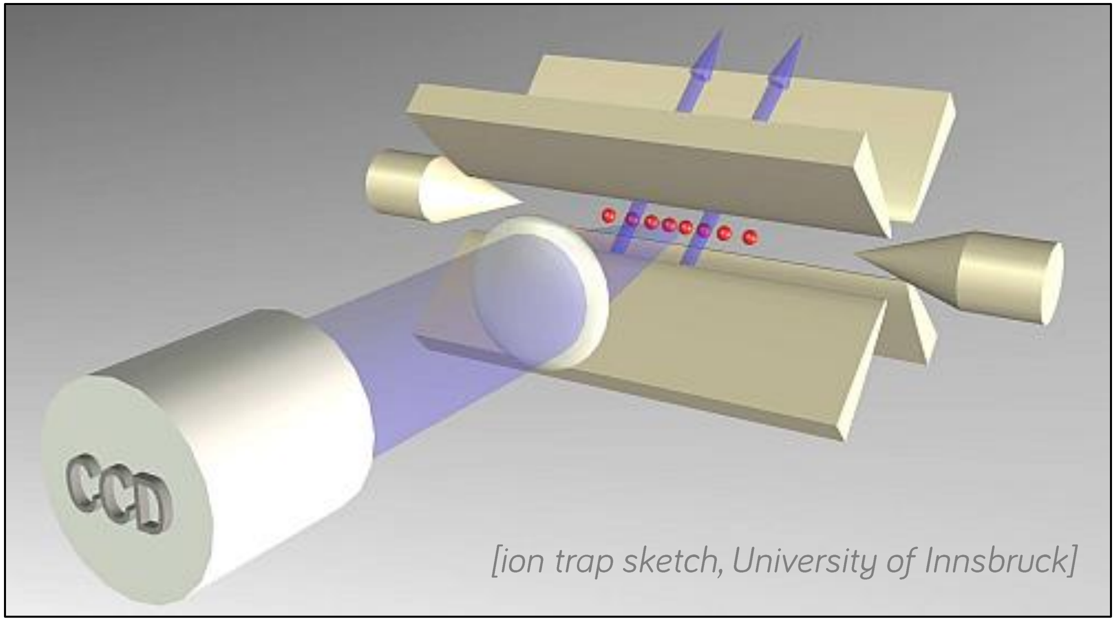
$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

controlled 2-qubit gates

$$\begin{aligned} \text{CNOT} &= |0\rangle\langle 0|_1 \otimes \mathbb{I}_2 \\ &+ |1\rangle\langle 1|_1 \otimes \sigma_2^x \end{aligned}$$

- output
  Z-basis
  measurements

- reality:
  decoherence
  imprecise control



[ion trap sketch, University of Innsbruck]

1 DiVincenzo criteria for quantum computation

- well-defined qubits

$$|0\rangle \quad |1\rangle$$

- (pure-state) initialization

$$|000\cdots 0\rangle$$

- universal gate set

$$R_x^{\varphi}, R_Z^{\varphi}, \mathrm{CNOT}$$

- comp. basis measurement

$$|0\rangle\langle 0|, |1\rangle\langle 1|$$

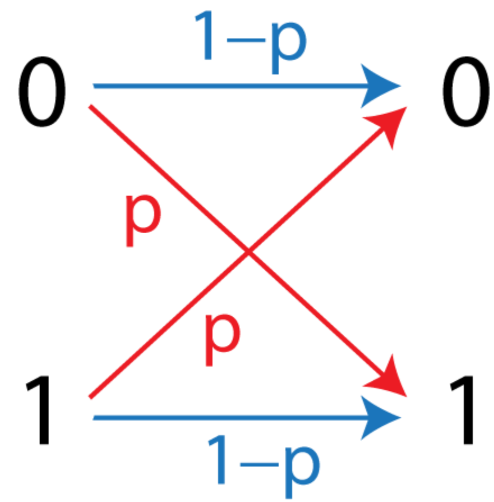
- long coherence times

$$\left(|0\rangle + |1\rangle\right)/\sqrt{2}$$

+ scalability
+ (flying qubits)

1 Simple (classical) error correction: repetition

- a bit-flip error



- redundant information

$$0 \longrightarrow 000$$
$$1 \longrightarrow 111$$

- majority voting

$0 \leftarrow 000, 001, 010, 100$

$1 \leftarrow 011, 101, 110, 111$

- post-correction error probability

$$3p^2(1-p) + p^3 = O(p^2)$$

1 A quantum no-go: QM is linear … no-cloning

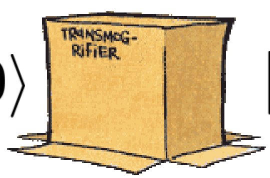
- we can copy orthogonal (classical) states

$$|0\rangle \qquad |1\rangle \qquad \frac{|0\rangle+|1\rangle}{\sqrt{2}} \qquad \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

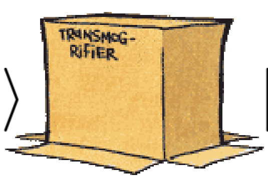
- non-orthogonal states?

$$|0\rangle \qquad \frac{|0\rangle+|1\rangle}{\sqrt{2}}$$

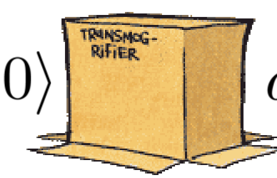
- let’s have a cloning machine

$|0\rangle\,|0\rangle$ 

$|0\rangle\,|0\rangle$

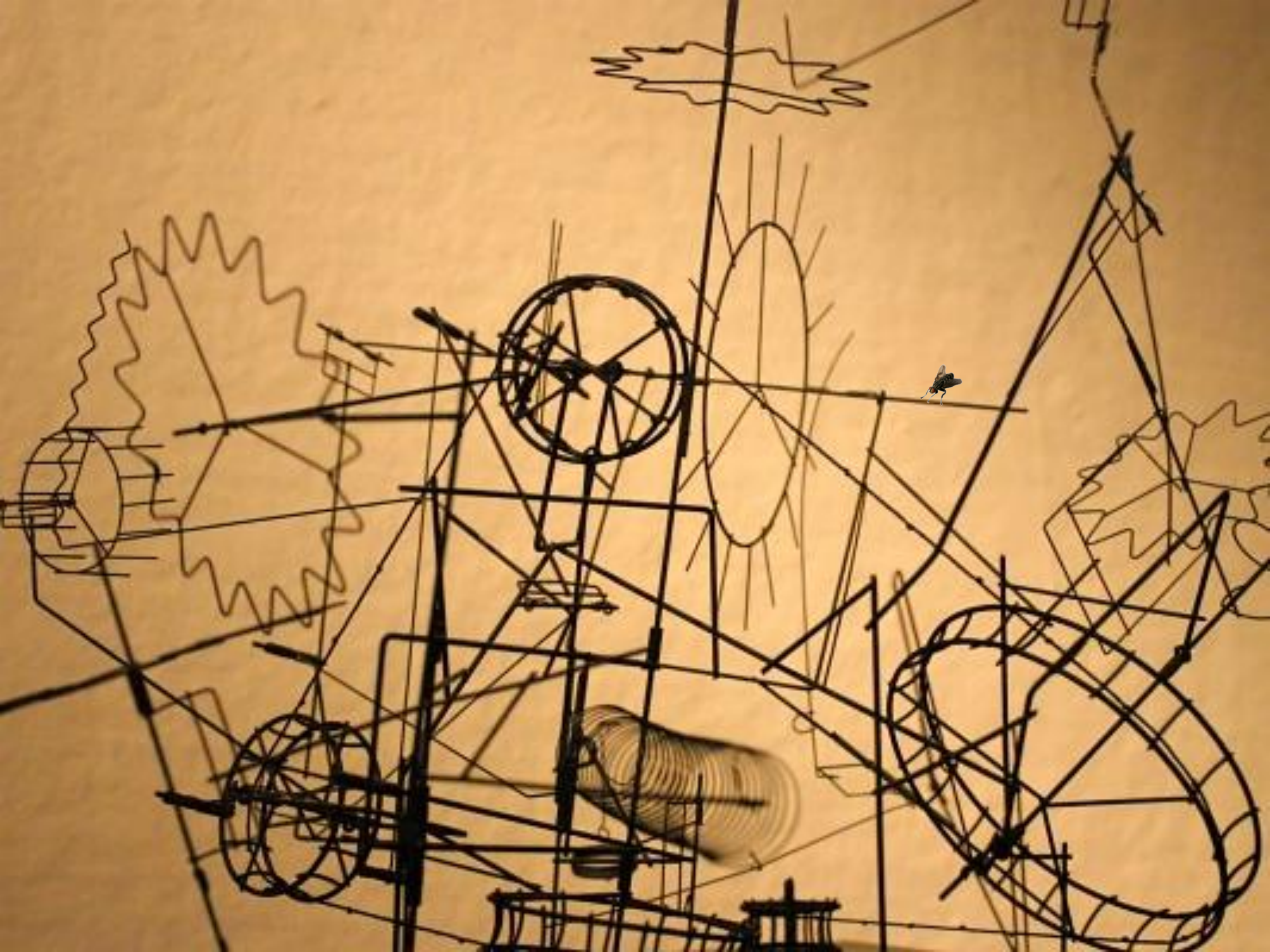
$|1\rangle\,|0\rangle$ 

$|1\rangle\,|1\rangle$

$(a\,|0\rangle + b\,|1\rangle)\,|0\rangle$ 

$a\,|00\rangle + b\,|11\rangle$

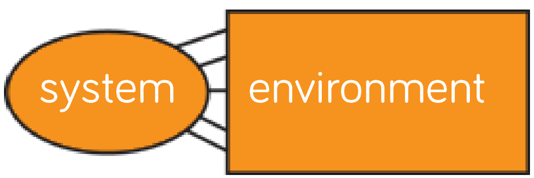
It doesn’t work!



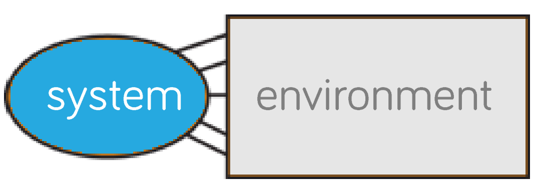
2 Quantum computing & decoherence

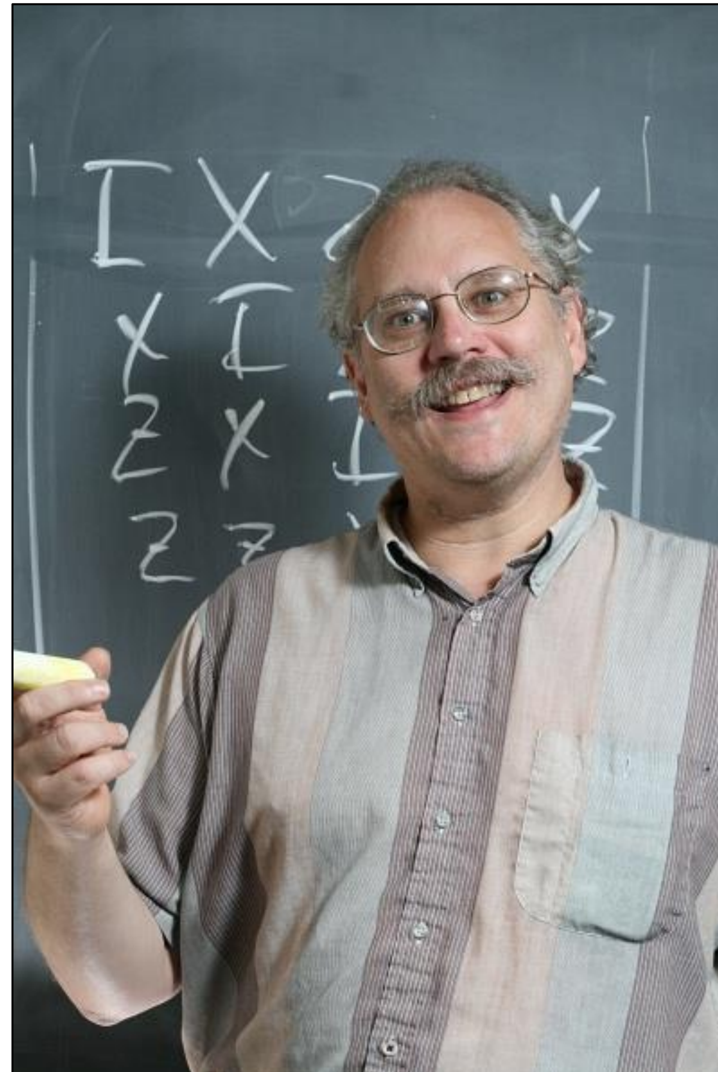
- a perfect computer from faulty parts?

$$|\varphi_t\rangle = U_t U_{t-1} \ldots U_2 U_1 |\varphi_0\rangle$$



ρ  $\sum_i E_i \rho E_i^\dagger$





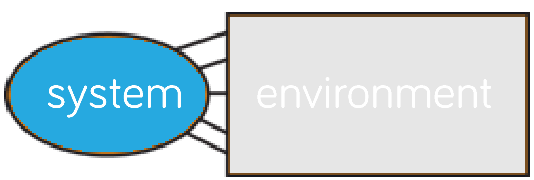
2 Quantum computing & decoherence

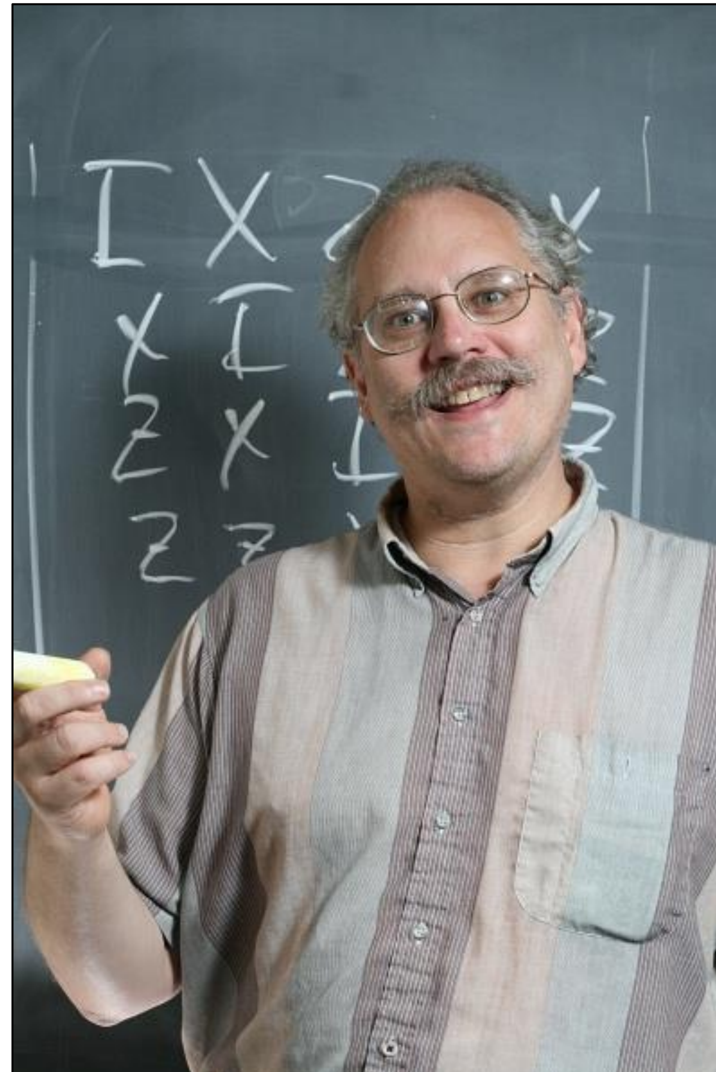
- a perfect computer
  from faulty parts?

$$|\varphi_t\rangle = U_t U_{t-1} \ldots U_2 U_1 \, |\varphi_0\rangle$$

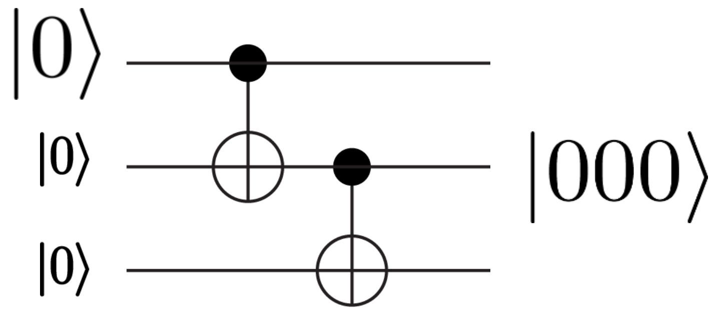
- error correction codes
  [CSS: Calderbank, Shor, Steane]

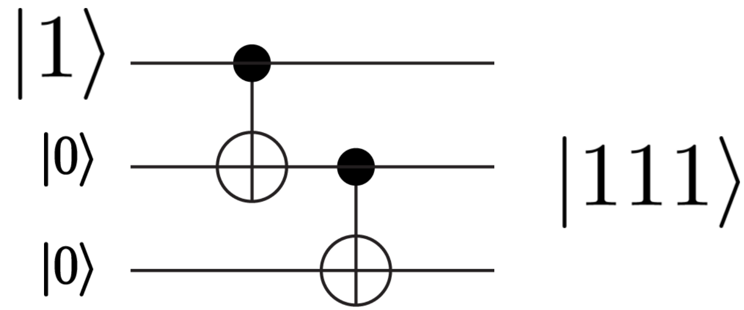
$$\rho \longrightarrow \sum_i E_i \rho E_i^\dagger$$

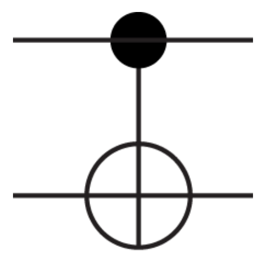




2 The quantum bit-flip code

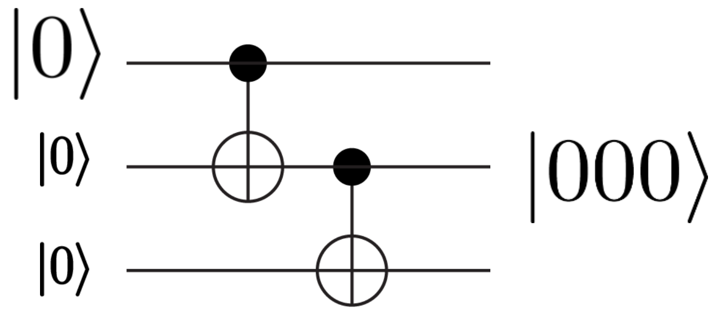


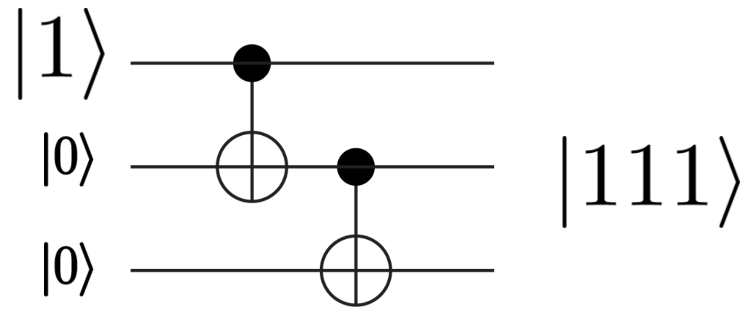


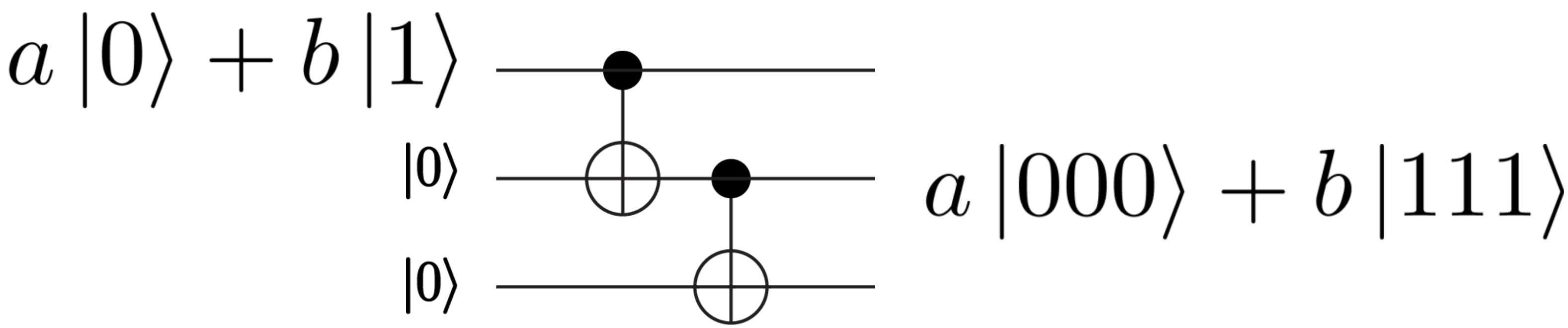


$$
\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |01\rangle \\
|10\rangle &\rightarrow |11\rangle \\
|11\rangle &\rightarrow |10\rangle
\end{aligned}
$$

2 The quantum bit-flip code







2 The quantum bit-flip code

$$a\,|000\rangle + b\,|111\rangle$$

$$a\,|000\rangle + b\,|111\rangle$$

$$a\,|100\rangle + b\,|011\rangle$$

$$a\,|010\rangle + b\,|101\rangle$$

$$a\,|001\rangle + b\,|110\rangle$$

$$a\,|110\rangle + b\,|001\rangle$$

$$a\,|101\rangle + b\,|010\rangle$$

$$a\,|011\rangle + b\,|100\rangle$$

$$a\,|111\rangle + b\,|000\rangle$$

- how to detect what happened without disturbing the data?
- are there unitaries that leave the code alone?

2 The quantum bit-flip code

- measure: Z_1Z_2 & Z_1Z_3
- nothing: I
  errors: X_1, X_2, X_3

let's repair it … how?

Z_1Z_2	Z_1Z_3	
+	+	$a\|000\rangle + b\|111\rangle$
−	−	$a\|100\rangle + b\|011\rangle$
−	+	$a\|010\rangle + b\|101\rangle$
+	−	$a\|001\rangle + b\|110\rangle$

$\downarrow X_3$

$$a|000\rangle + b|111\rangle$$

2 The quantum bit-flip code

- measure: Z_1Z_2 & Z_1Z_3
- nothing: I
  errors: X_1, X_2, X_3

Z_1Z_2	Z_1Z_3	corrected
$+$	$+$	$a\|000\rangle + b\|111\rangle$
$-$	$-$	$a\|100\rangle + b\|011\rangle$
$-$	$+$	$a\|010\rangle + b\|101\rangle$
$+$	$-$	$a\|001\rangle + b\|110\rangle$

$a|110\rangle + b|001\rangle$
$a|101\rangle + b|010\rangle$
$a|011\rangle + b|100\rangle$
$a|111\rangle + b|000\rangle$

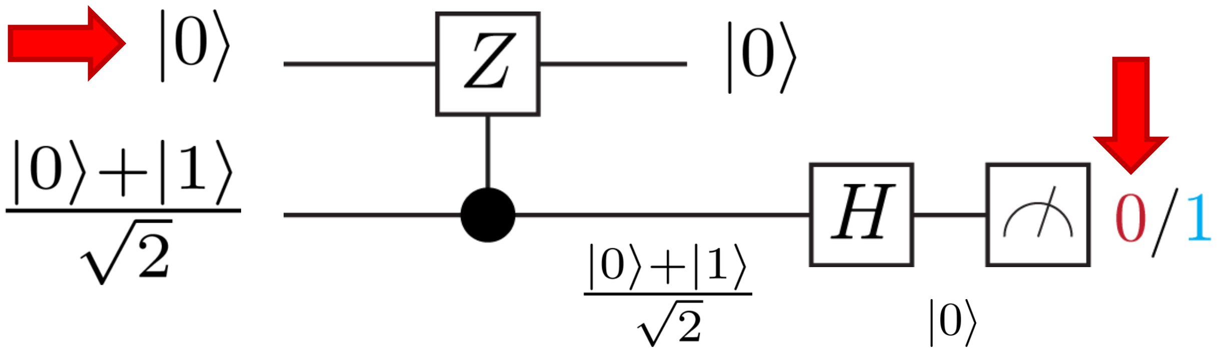
messed up

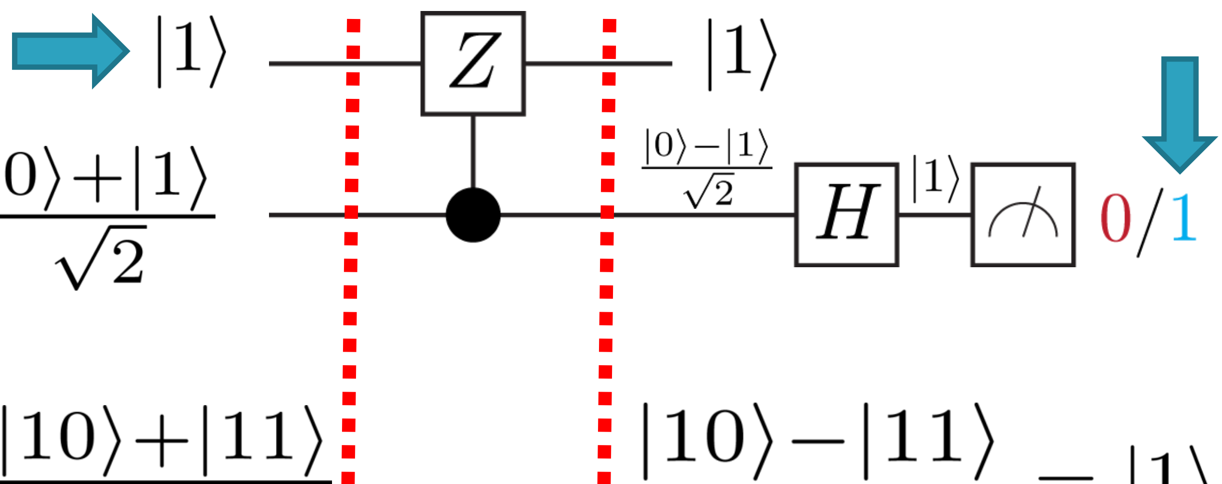
- post-correction error probability

$$3p^2(1-p) + p^3 = O(p^2)$$

- measuring Z_1Z_2 without destroying the state?

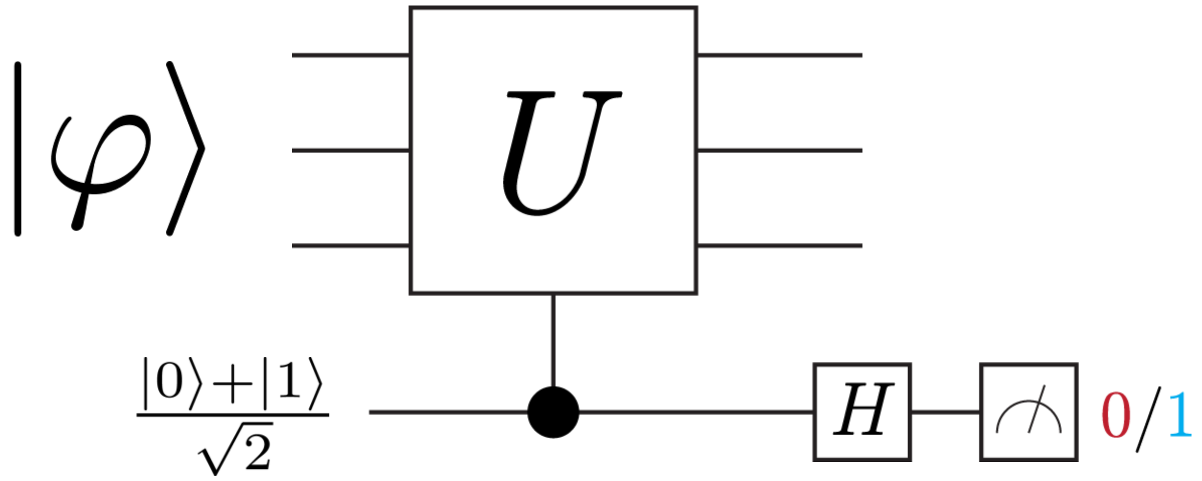
2 Measuring in the eigenbasis of the operator Z





$$\frac{|10\rangle+|11\rangle}{\sqrt{2}} \qquad \frac{|10\rangle-|11\rangle}{\sqrt{2}} = |1\rangle \otimes \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

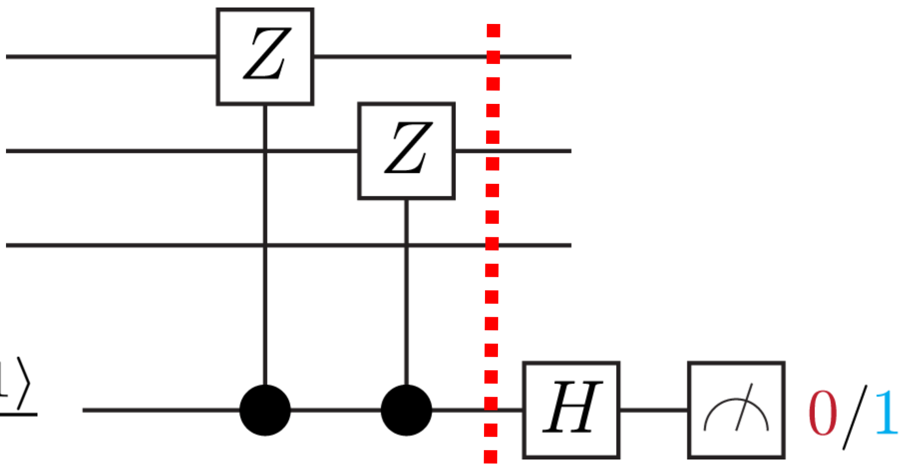
2 Measuring in the eigenbasis of an operator U



2 Measuring in the eigenbasis of the operator Z_1Z_2

$$|\varphi\rangle = a\,|00\alpha\rangle + b\,|11\beta\rangle + c\,|01\gamma\rangle + d\,|10\delta\rangle$$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

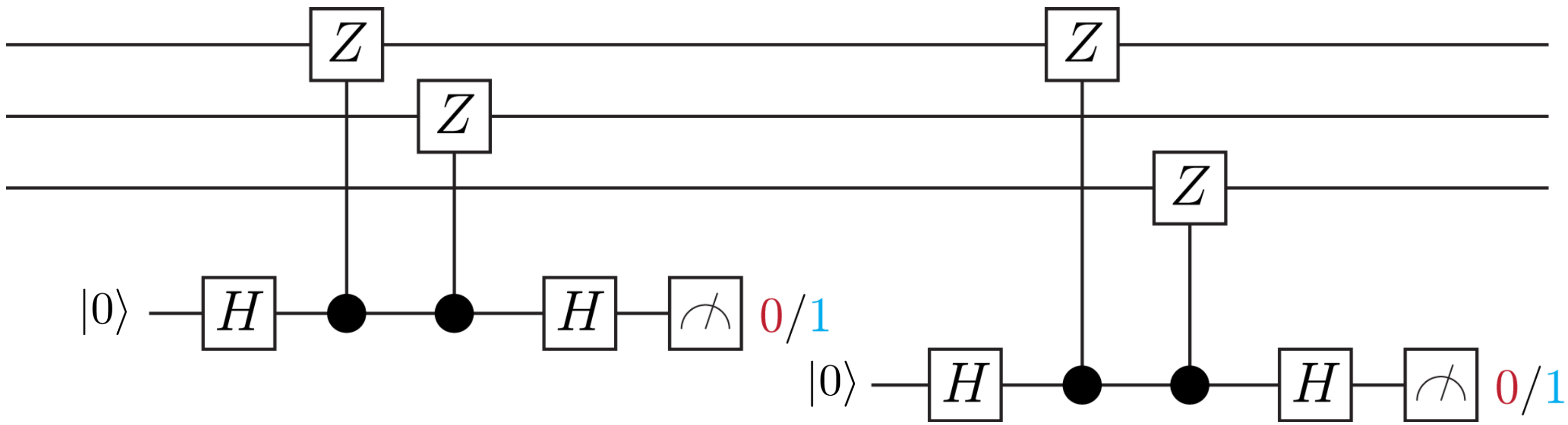


$$(a\,|00\alpha\rangle + b\,|11\beta\rangle) \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$
$$+ (c\,|01\gamma\rangle + d\,|10\delta\rangle) \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

- a projective measurement in the eigenbasis of Z_1Z_2

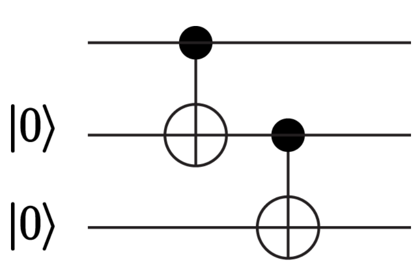
2 The quantum bit-flip code

- measure the error, not the data ...

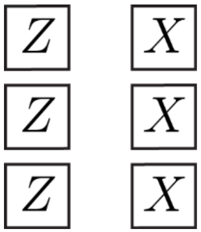


- project into ZZ eigenstates ... enforce a scenario ... repair

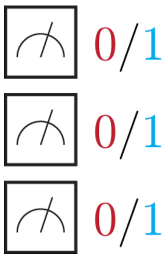
- encoding



operations



decoding



2 Shor's 9-qubit code

- repair 1 bit flip and/or 1 phase flip ...

$$|0\rangle \Rightarrow \frac{(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)}{2\sqrt{2}}$$

$$|1\rangle \Rightarrow \frac{(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)}{2\sqrt{2}}$$

- bit-flip detection (X_k) $Z_1Z_2, Z_1Z_3, Z_4Z_5, Z_4Z_6, Z_7Z_8, Z_7Z_9$
- phase-flip detection (Z_k) $X_1X_2X_3X_4X_5X_6, X_4X_5X_6X_7X_8X_9$
- 1-qubit Pauli errors
  can be decomposed into
  bit/phase flips: I_k, X_k, Z_k, Y_k ($= iZ_kX_k$)

2 Shor's 9-qubit code

$$a|0\rangle + b|1\rangle$$

- repair 1 bit flip and/or 1 phase flip …

$$a\,\frac{(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)}{2\sqrt{2}}$$

$$+b\,\frac{(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)}{2\sqrt{2}}$$

- bit-flip detection (Z_k) Z_1Z_2, Z_1Z_3, Z_4Z_5, Z_4Z_6, Z_7Z_8, Z_7Z_9
- phase-flip detection (X_k) $X_1X_2X_3X_4X_5X_6$, $X_4X_5X_6X_7X_8X_9$
- 1-qubit Pauli errors
  can be decomposed into
  bit/phase flips: I_k, X_k, Z_k, Y_k ($= iZ_kX_k$)

2 Shor's 9-qubit code

- repair 1 bit flip and/or 1 phase flip ...

$$a\frac{i(|010\rangle-|101\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)}{2\sqrt{2}}$$

$$+b\frac{i(|010\rangle+|101\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)}{2\sqrt{2}}$$

- bit-flip detection (Z_k) Z_1Z_2, Z_1Z_3, Z_4Z_5, Z_4Z_6, Z_7Z_8, Z_7Z_9
- phase-flip detection (X_k) $X_1X_2X_3X_4X_5X_6$, $X_4X_5X_6X_7X_8X_9$
- 1-qubit Pauli errors can be decomposed into bit/phase flips: I_k, X_k, Z_k, Y_k ($= iZ_kX_k$)

2 Shor's 9-qubit code

- repair 1 bit flip and/or 1 phase flip ...

$$|0\rangle \Rightarrow \frac{(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)}{2\sqrt{2}}$$

$$|1\rangle \Rightarrow \frac{(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)}{2\sqrt{2}}$$

- bit-flip detection (Z_k) $Z_1Z_2, Z_1Z_3, Z_4Z_5, Z_4Z_6, Z_7Z_8, Z_7Z_9$
- phase-flip detection (X_k) $X_1X_2X_3X_4X_5X_6, X_4X_5X_6X_7X_8X_9$

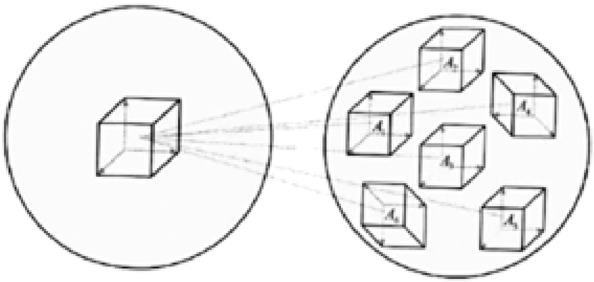
- **repair any 1-qubit error**
  (error discretization)

$$\rho \longrightarrow \sum_i E_i \rho E_i^\dagger$$

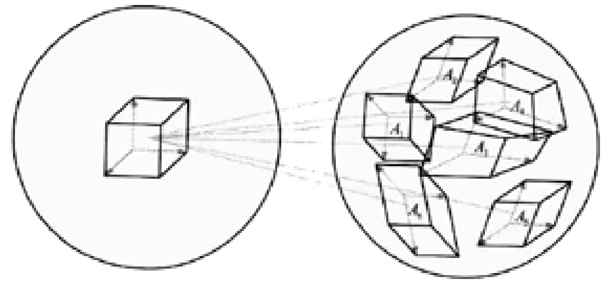
3 Stabilizer codes

- a group of *n*-*k* stabilizers
  (don't change the code, detect errors)

$$S = \langle g_1, g_2, \ldots, g_{n-k} \rangle$$



encoded



after error

[Nielsen & Chuang]

- a Pauli error up to weight 2*t* anticommutes with at least one of the stabilizers

$$\langle x| E(|y\rangle) |x\rangle = \langle x| E_i |y\rangle \langle y| E_i |x\rangle = 0$$

... no codeword overlap after the error

- *k* logical qubits in *n* physical ones, repair up to *t* errors

3 The 5-qubit code

[Knill et al., PRL 86, 5811 (2001)]

- stabilizer & operations

M_1	σ_x	σ_z	σ_z	σ_x	I
M_2	I	σ_x	σ_z	σ_z	σ_x
M_3	σ_x	I	σ_x	σ_z	σ_z
M_4	σ_z	σ_x	I	σ_x	σ_z
$\overline{X}$	σ_x	σ_x	σ_x	σ_x	σ_x
$\overline{Z}$	σ_z	σ_z	σ_z	σ_z	σ_z

n = 5, k = 1, t = 1

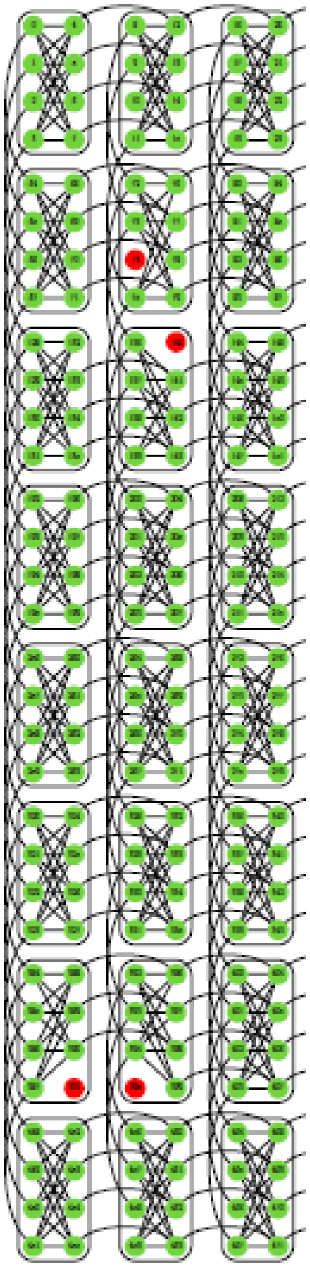
possible 1-qubit errors: 1 + 5 × 3 = 16

- codewords

$$\begin{aligned}|\overline{0}\rangle = \; & |00000\rangle + |10010\rangle + |01001\rangle + |10100\rangle \\ & + |01010\rangle - |11011\rangle - |00110\rangle - |11000\rangle \\ & - |11101\rangle - |00011\rangle - |11110\rangle - |01111\rangle \\ & - |10001\rangle - |01100\rangle - |10111\rangle + |00101\rangle\end{aligned}$$

$$\begin{aligned}|\overline{1}\rangle = \; & |11111\rangle + |01101\rangle + |10110\rangle + |01011\rangle \\ & + |10101\rangle - |00100\rangle - |11001\rangle - |00111\rangle \\ & - |00010\rangle - |11100\rangle - |00001\rangle - |10000\rangle \\ & - |01110\rangle - |10011\rangle - |01000\rangle + |11010\rangle\end{aligned}$$

4 stabilizers detect 16 possibilities



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Defining and detecting quantum speedup

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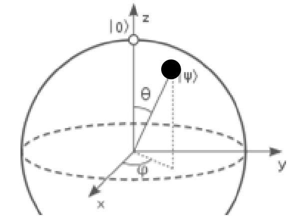
ABSTRACT | EDITOR'S SUMMARY

The development of small-scale quantum devices raises the question of how to fairly assess and detect quantum speedup. Here, we show how to define and measure quantum speedup and how to avoid pitfalls that might mask or fake such a speedup. We illustrate our discussion with data from tests run on a D-Wave Two device with up to 503 qubits. By using random spin glass instances as a benchmark, we found no evidence of quantum speedup when the entire data set is considered and obtained inconclusive results when comparing subsets of instances on an instance-by-instance basis. Our results do not rule out the possibility of speedup for other classes of problems and illustrate the subtle nature of the quantum speedup question.

1 **we need a qubit**

well, what can we do with it?

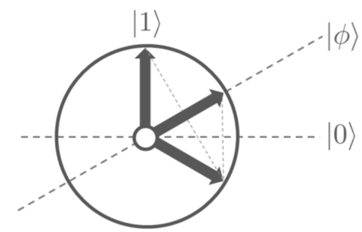


2 **EPR pairs**

and tricky 2-qubit protocols

3 **the algorithms**

that make quantum computing tick

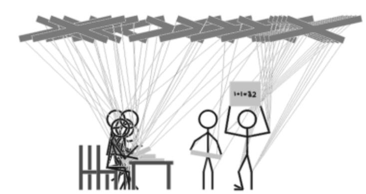


4 **error correction**

can we really scale up this stuff?

5 **the limits**

complexity & limits of q. computing



5 Quantum Info conclusions & discussion

- What's the point?
- Where are the problems?
- How are we doing?
- Let's have lunch!