

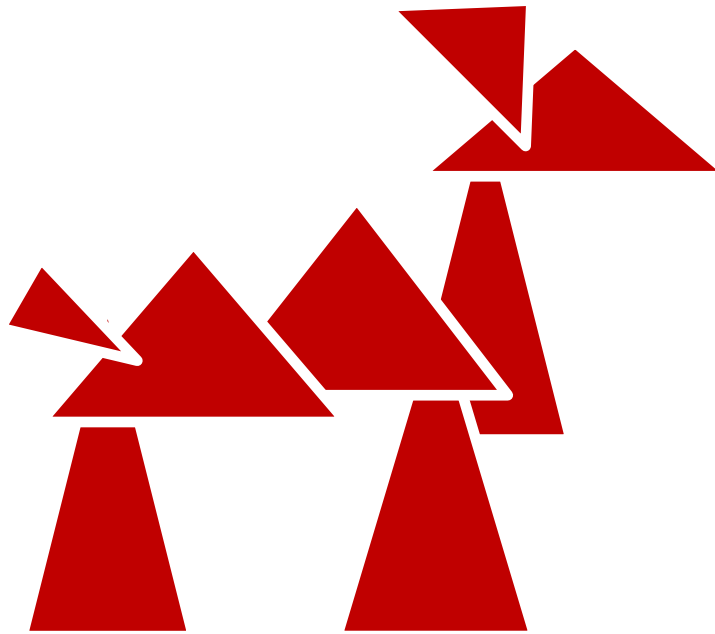
Local Hamiltonians & Quantum Complexity

Daniel Nagaj



2014 | 6 | 30

MPI PKS



Local Hamiltonians & Quantum Complexity

Daniel Nagaj



universität
wien

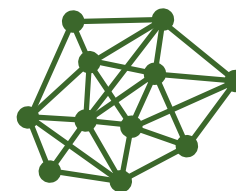


2014 | 6 | 30

MPI PKS

1 Hamiltonians?

optimization & dynamics



2 complexity

checking (quantum) proofs



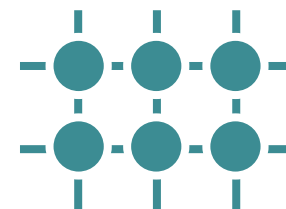
3 ground states?

how hard is it to find them: QMA



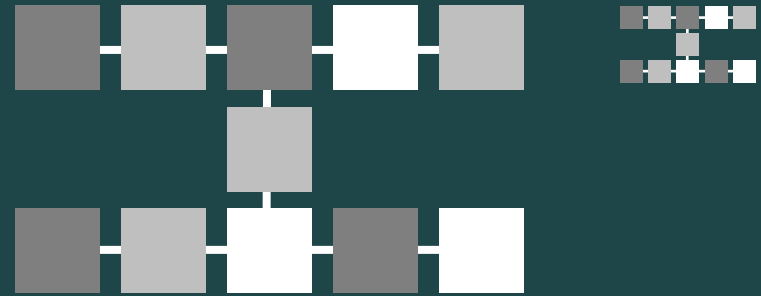
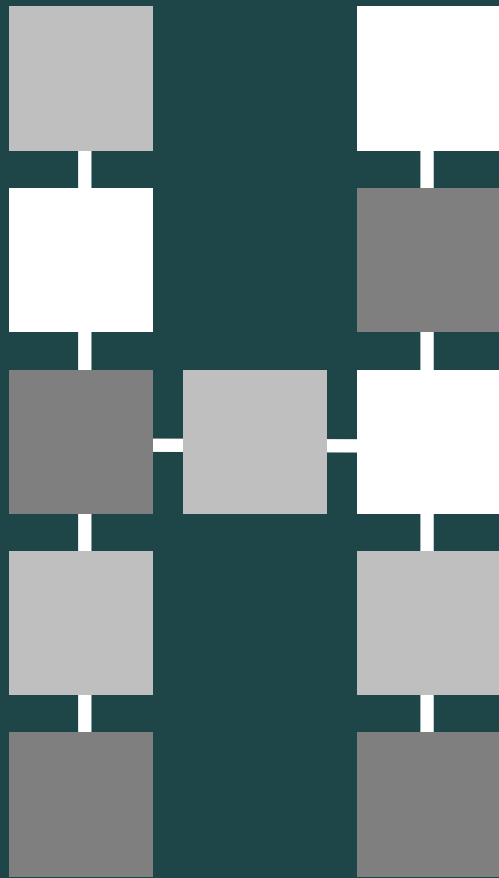
4 tensor networks

heuristics based on low entanglement

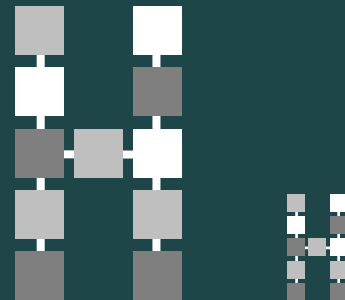


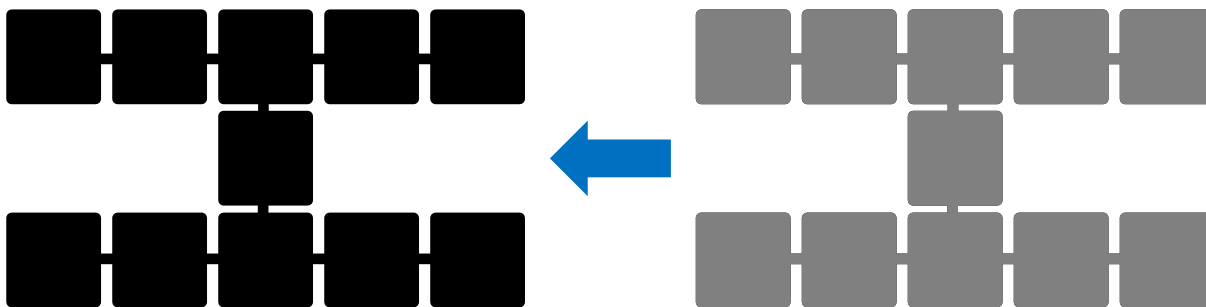
LH





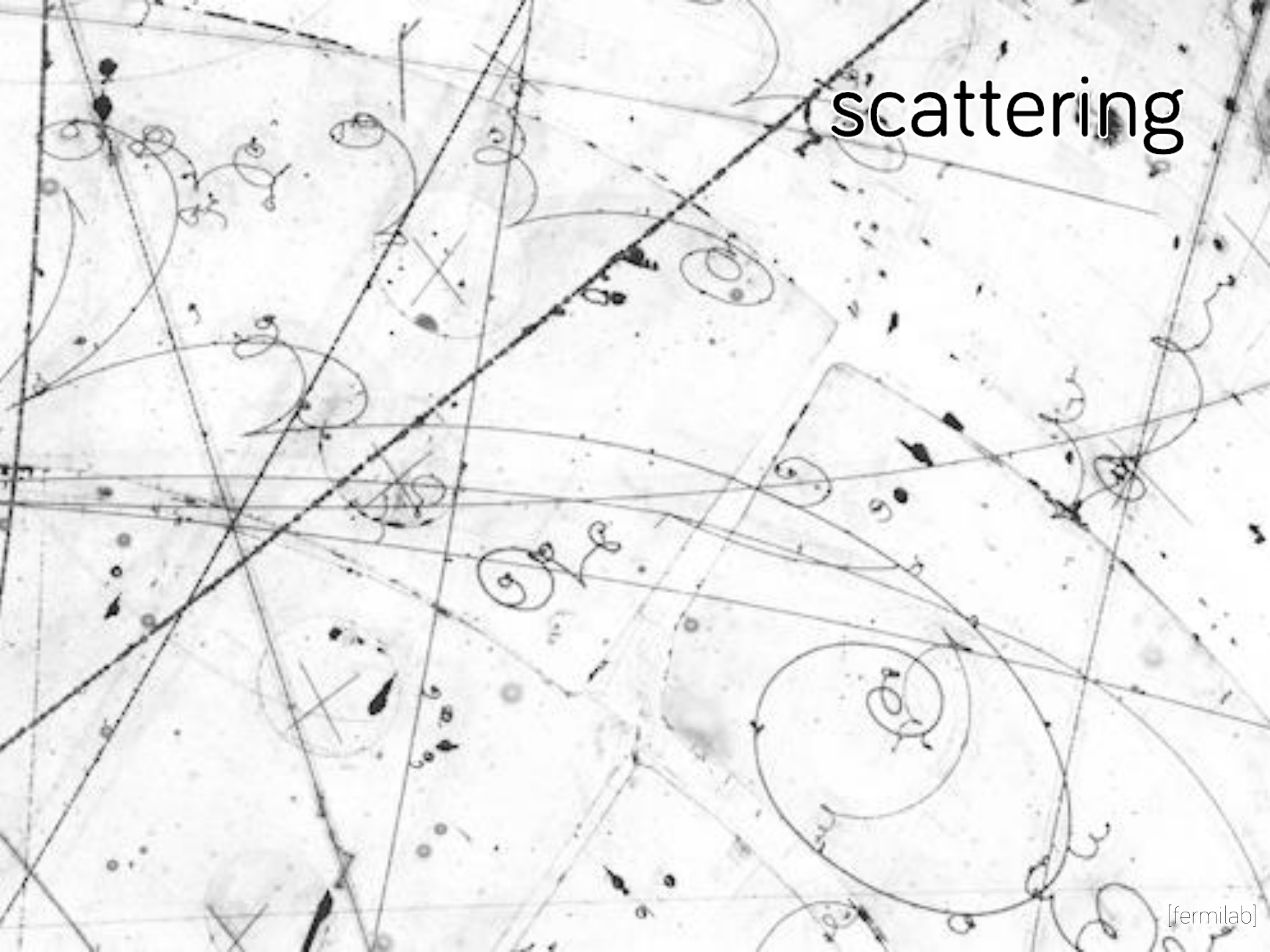
Hamiltonians





dynamics
 $i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$

scattering



scattering

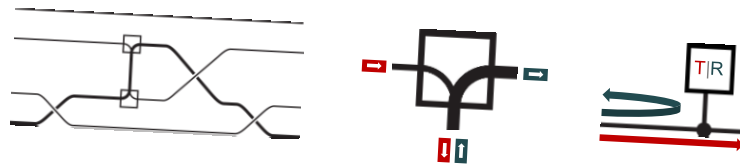
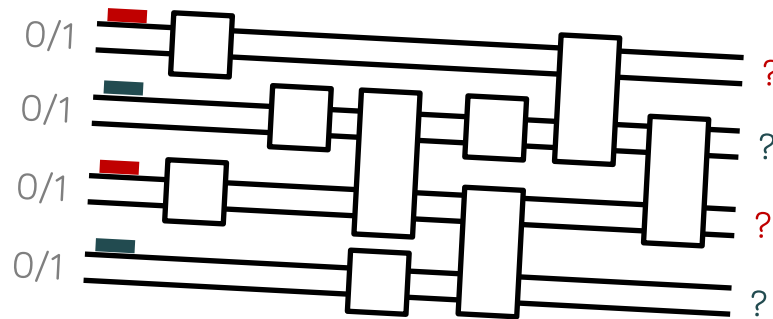
Universal computation by multi-particle quantum walk

- dual-rail encoding
N wavepackets

$$a_j^\dagger a_k + a_k^\dagger a_j$$

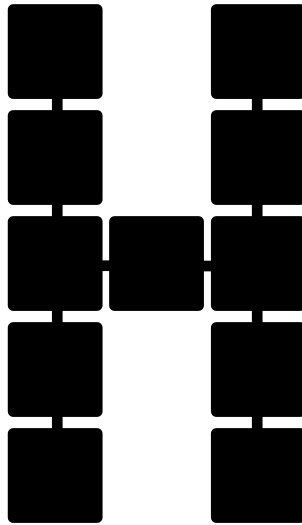
- CPHASE: interaction

$$a_j^\dagger a_k^\dagger a_j a_k$$

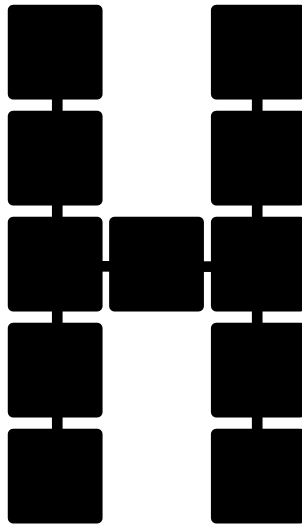


[Childs, Gosset, Webb, Science 339, 791 (2013)]

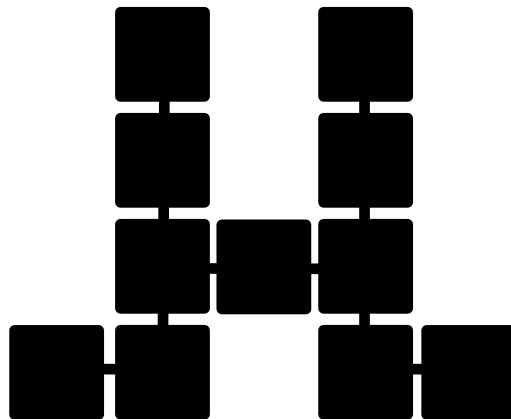
optimization



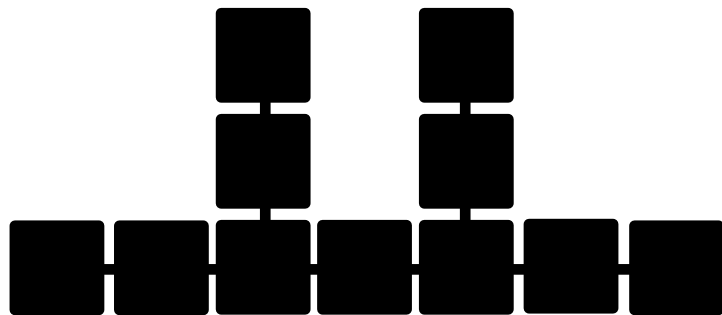
optimization



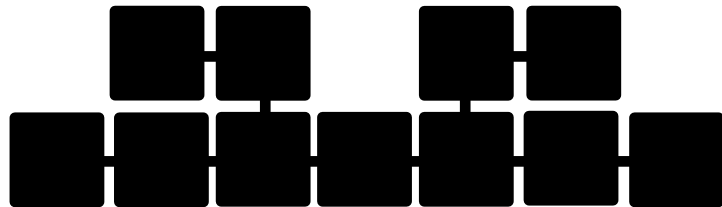
optimization



optimization

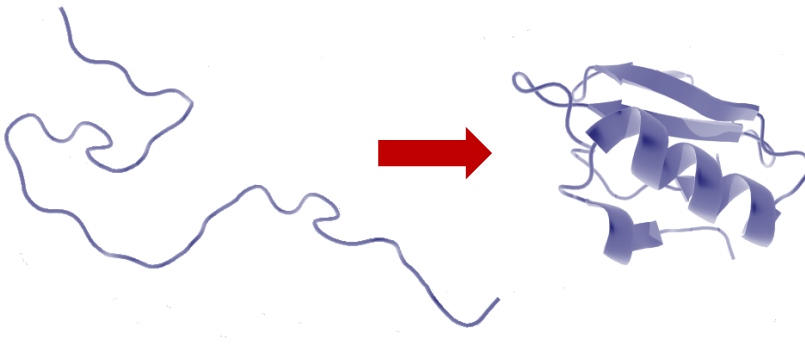


optimization

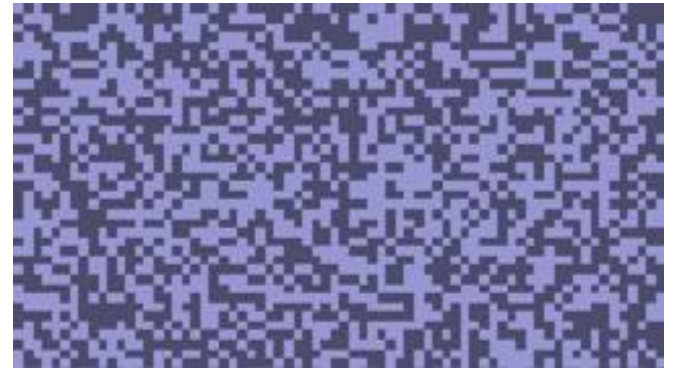


protein folding

spin glasses



[wikipedia]



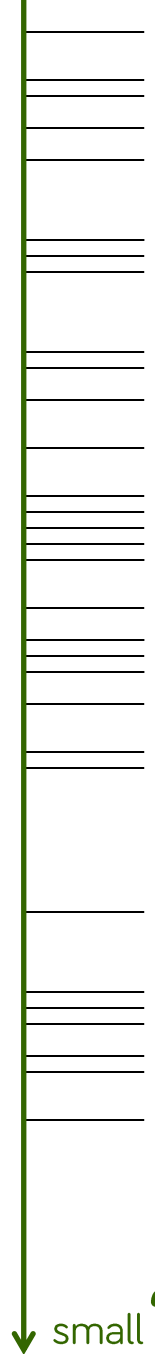
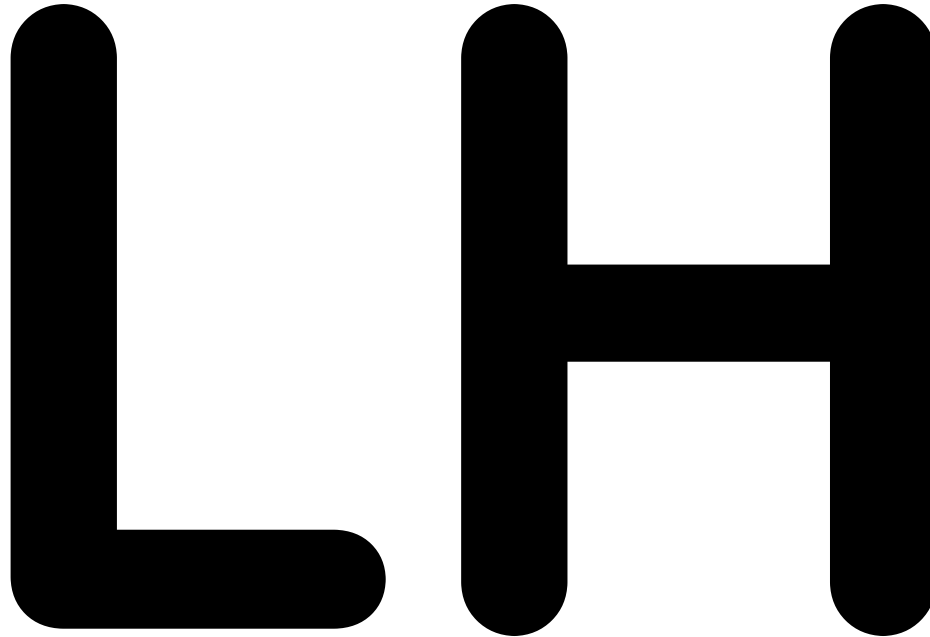
[uni-koeln.de]

local Hamiltonians

1

Hamiltonians and their ground states

Is
the
ground
state
energy
of a



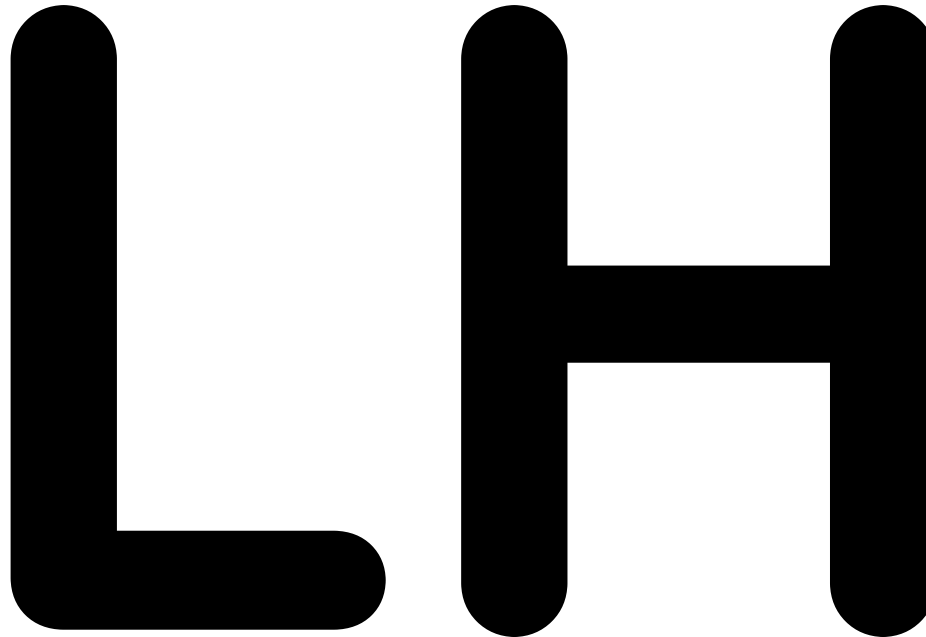
↓ small ?

1

Hamiltonians and their ground states



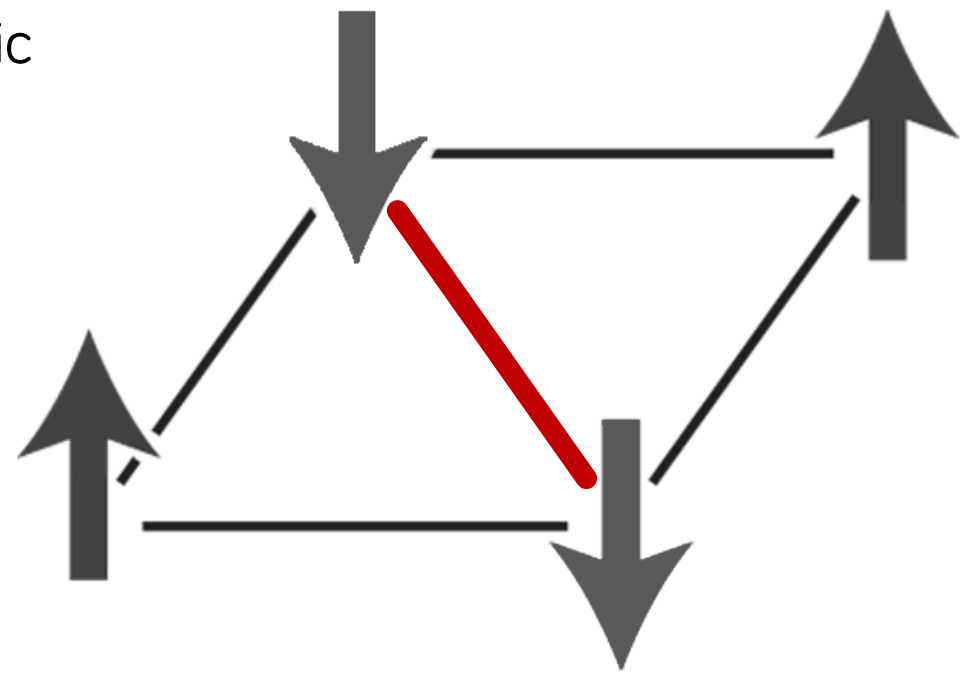
Is
the
ground
state
energy
of a



1 Frustrated systems

Everybody can't be happy.

antiferromagnetic
spin glass



a global
ground state

HARD?

find & describe it?
is it entangled?

frustrated

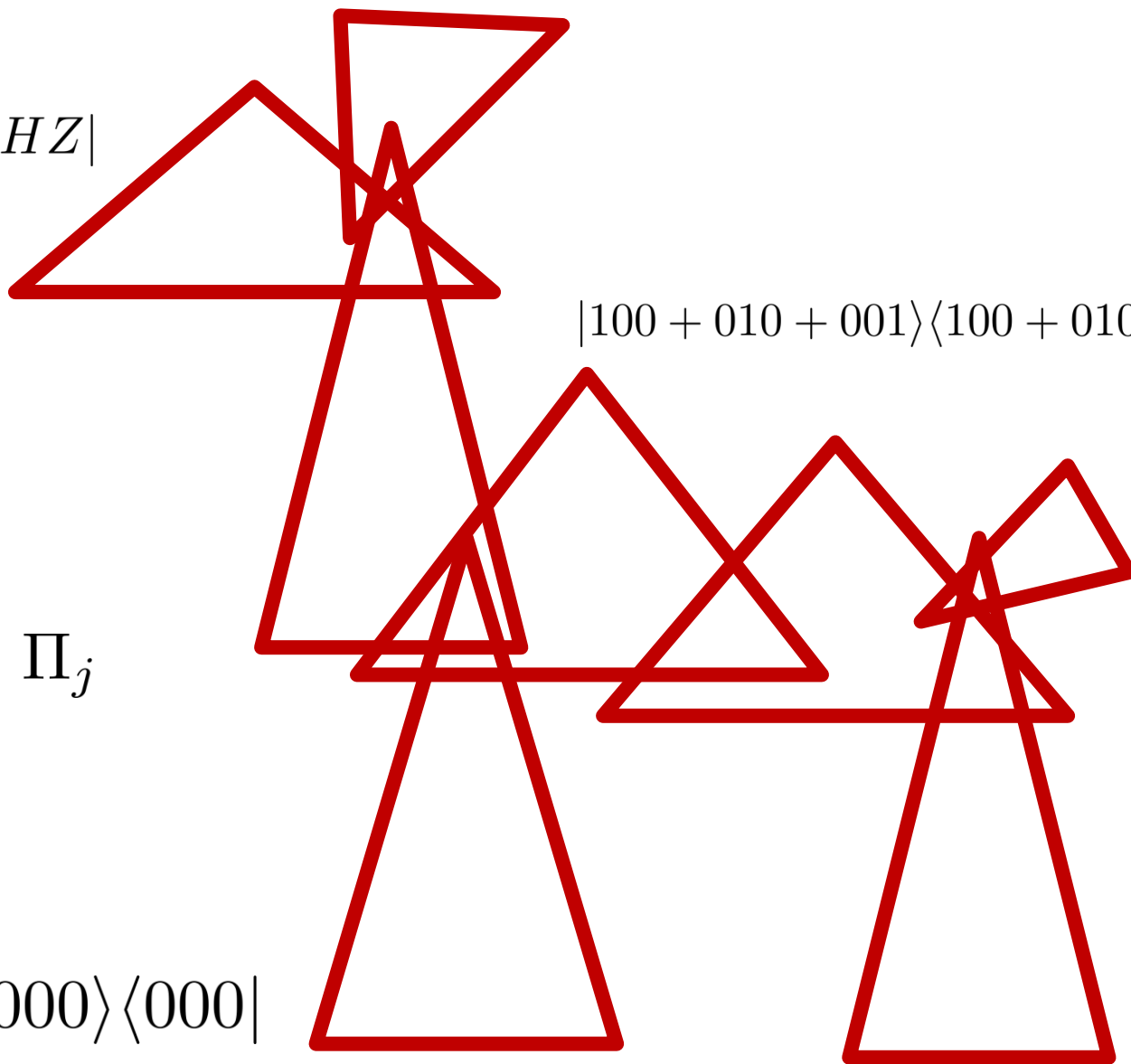
FRUST
RATED

$|GHZ\rangle\langle GHZ|$

$|100 + 010 + 001\rangle\langle 100 + 010 + 001|$

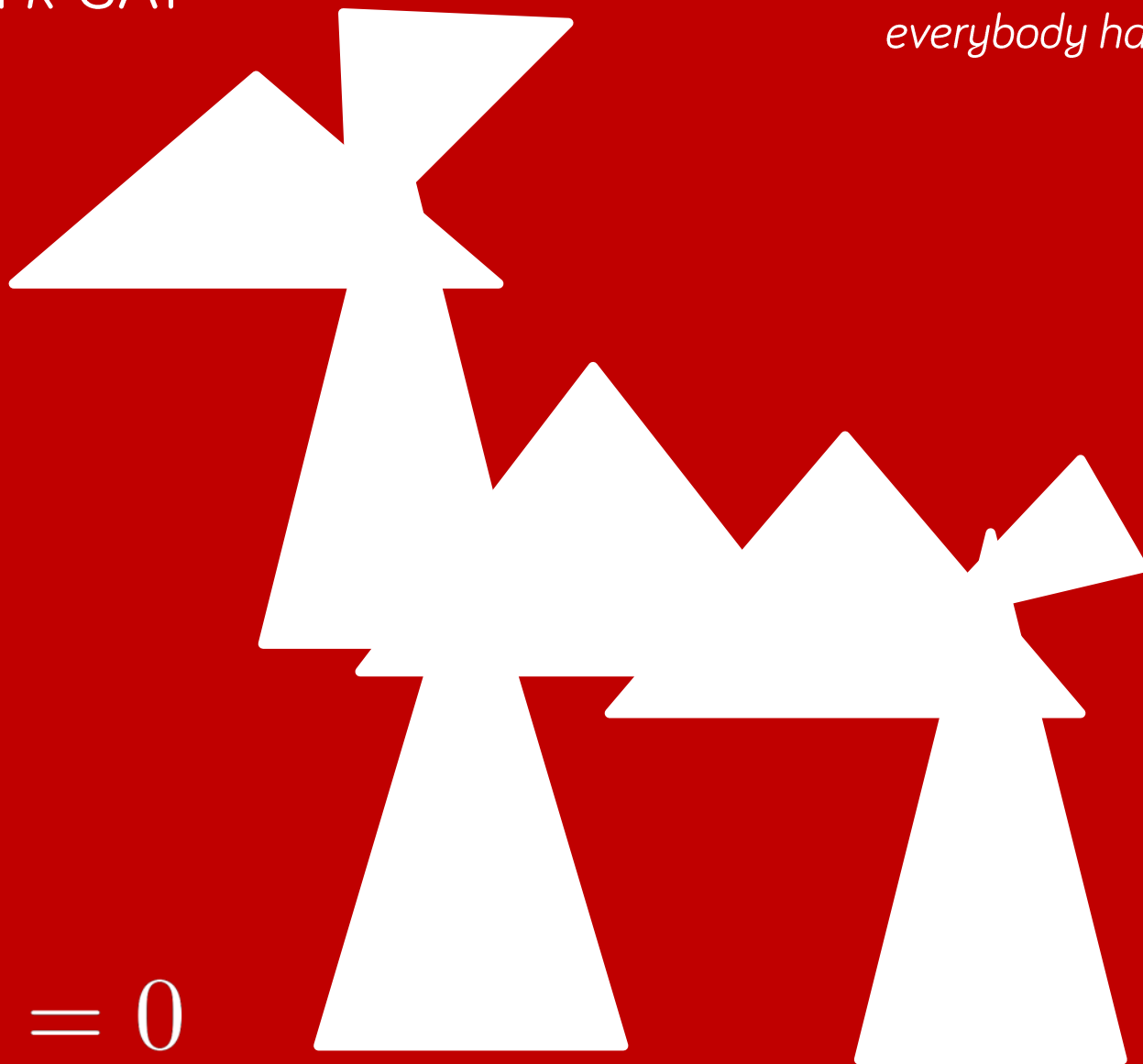
■ k -local
projectors Π_j

$|000\rangle\langle 000|$



Quantum k -SAT

Can we make everybody happy?



$$\Pi^j |\psi\rangle = 0$$

1 Local (k -body) Hamiltonians

- optimization

QMA-completeness

$$H(t) = \sum_j H_j(t)$$

- dynamics

BQP universality

- local particle dimension



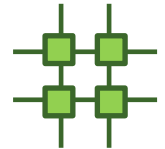
- interaction geometry



- time independence



- translational invariance



- promise gap, eigenvalue gap, energy \times time cost



how hard
is this
question



We Can Do It!



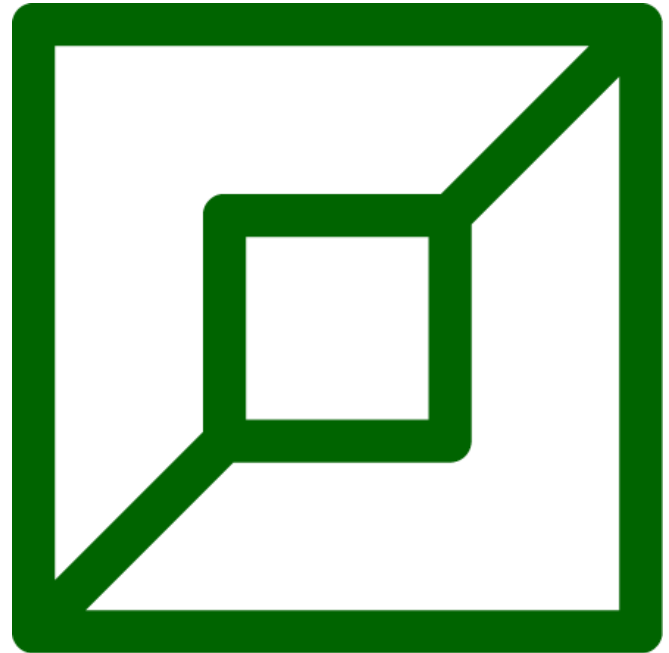
P

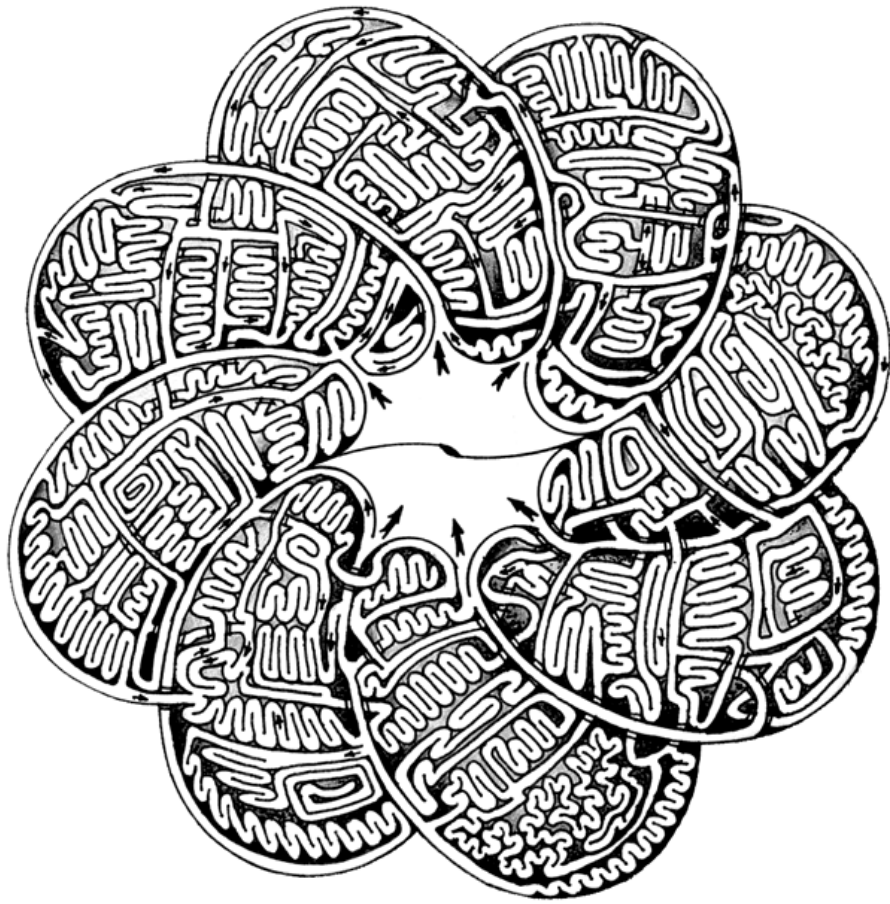
[Howard Miller]

POST FEB. 15 TO FEB. 20



WAR PRODUCTION CO-ORDINATING COMMITTEE



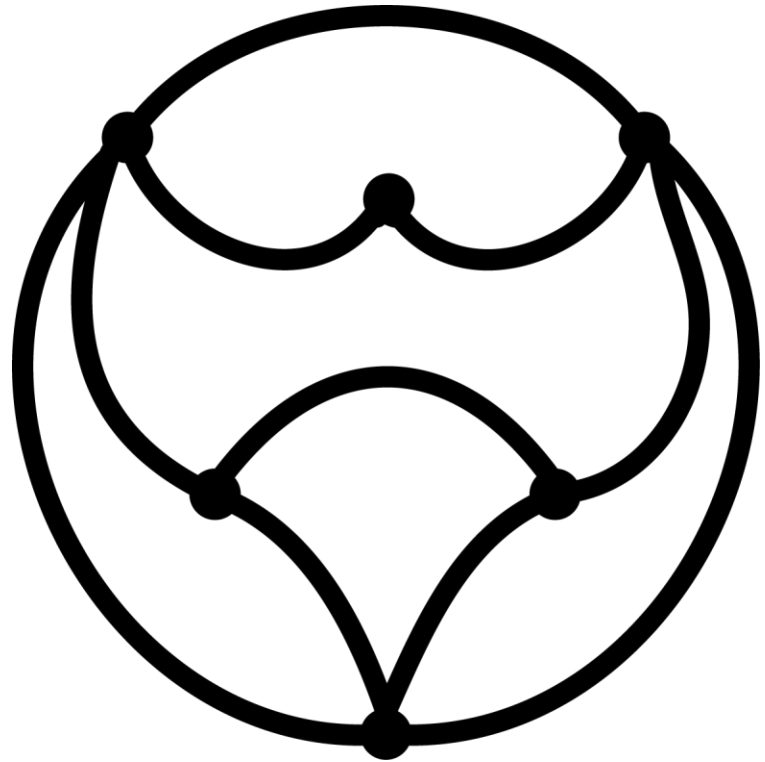
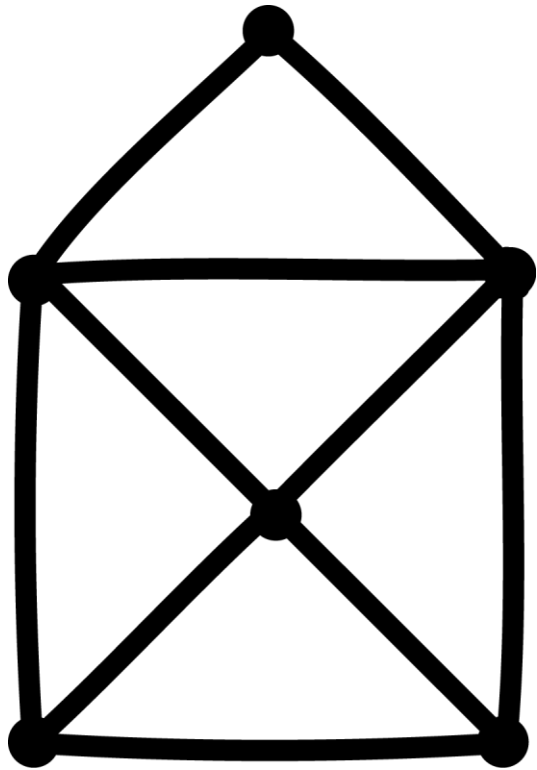


NP

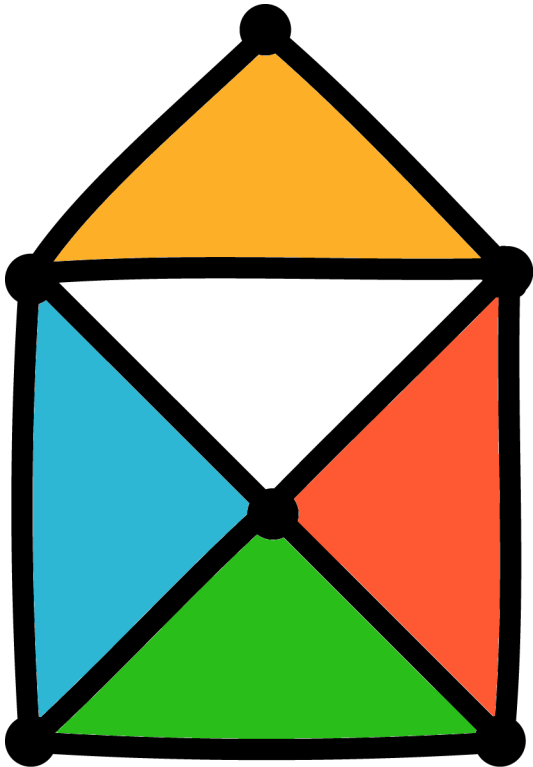


[maze: Andrew Bernhardt]
[A+M: primaryresources.co.uk]

2 A graph isomorphism puzzle



2 A graph isomorphism puzzle



2 A cryptarithmic puzzle

$$\begin{array}{r} \text{DID} \\ + \text{DINOS} \\ \hline \text{CROAK} \end{array}$$



2 A cryptarithmic puzzle

$$\begin{array}{r} 595 \\ + 59842 \\ \hline 60437 \end{array}$$



2 The NP protocol

Did dinosaurs exist?



a proof

2 The NP protocol

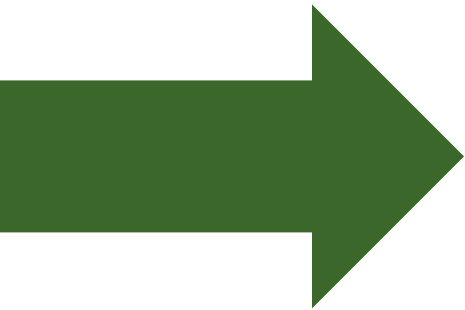
Did dinosaurs exist?



a witness

2 The class NP

Yes/no questions, easy to verify solutions.



a verification
circuit

from a uniform family

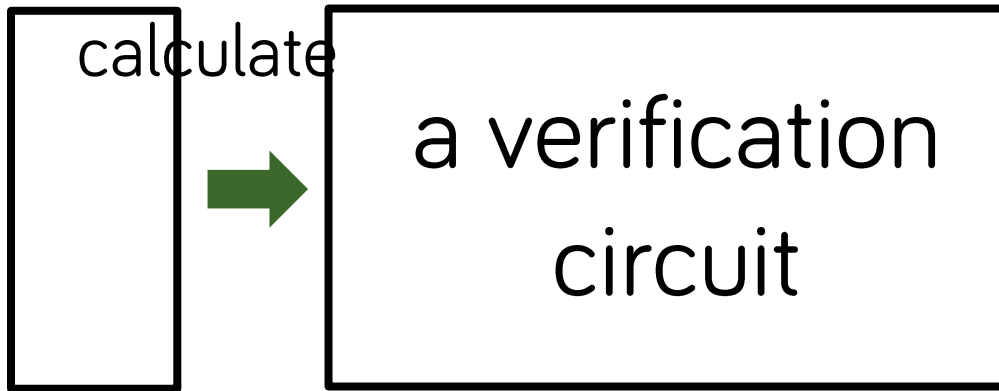


YES? Accept a good proof.

NO? Reject forgeries.

2 The class P

Yes/no questions that we can answer.

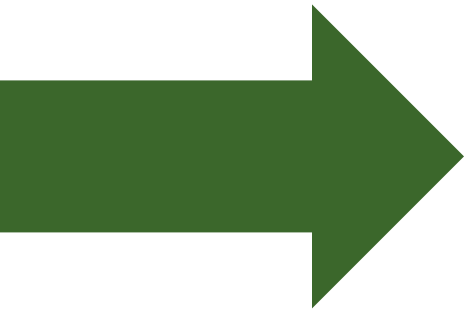


YES? Figure it out by yourself.

NO? Figure it out by yourself.

2 The class NP

Yes/no questions, easy to verify solutions.



a verification
circuit

from a uniform family



YES? Accept a good proof.

NO? Reject any witness.

2 NP-hardness

The mother of them all.

- Can you solve this problem? You just solved all of NP.



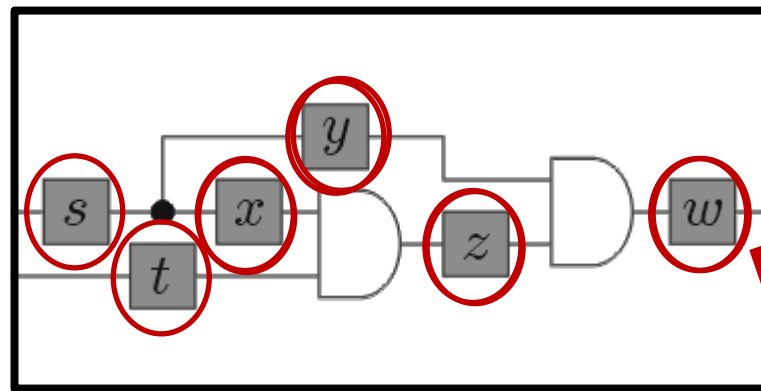
a verification
circuit

Could this
circuit ever
output 1?

2 NP-hardness

The mother of them all.

- Can you solve this problem? You just solved all of NP.



Could this circuit ever output 1?

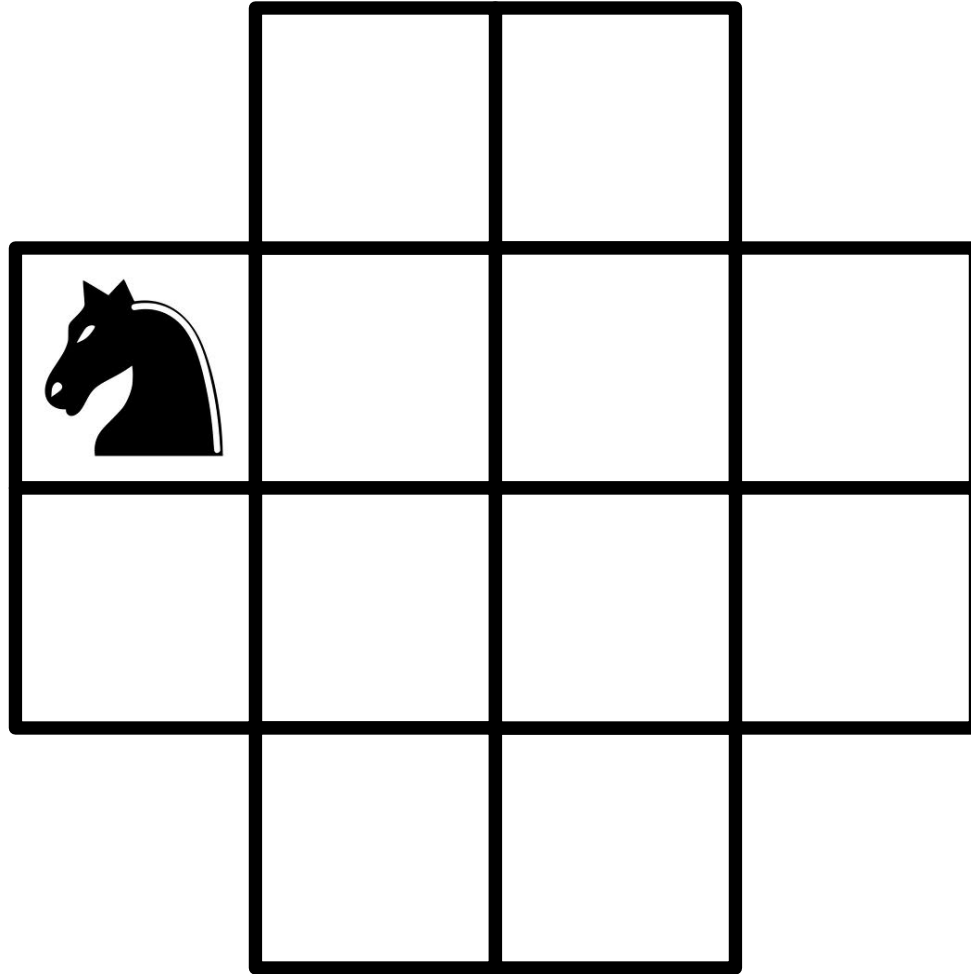
3-local conditions

$$(\dots \vee \dots \vee \dots) \wedge (\dots \vee \dots \vee \dots) \wedge \dots$$

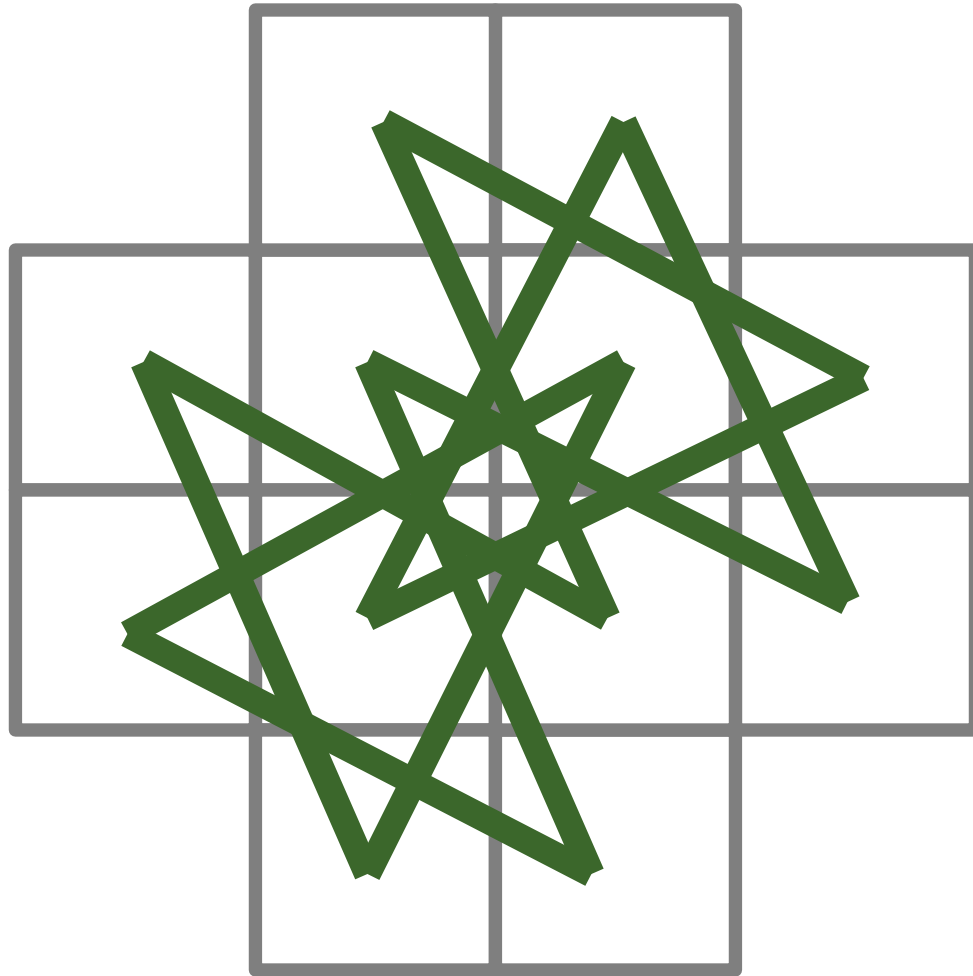
- 3-SAT is NP-hard.
in NP.

NP-complete. [Cook, Levin]

2 Hamiltonian cycle (also NP-c)

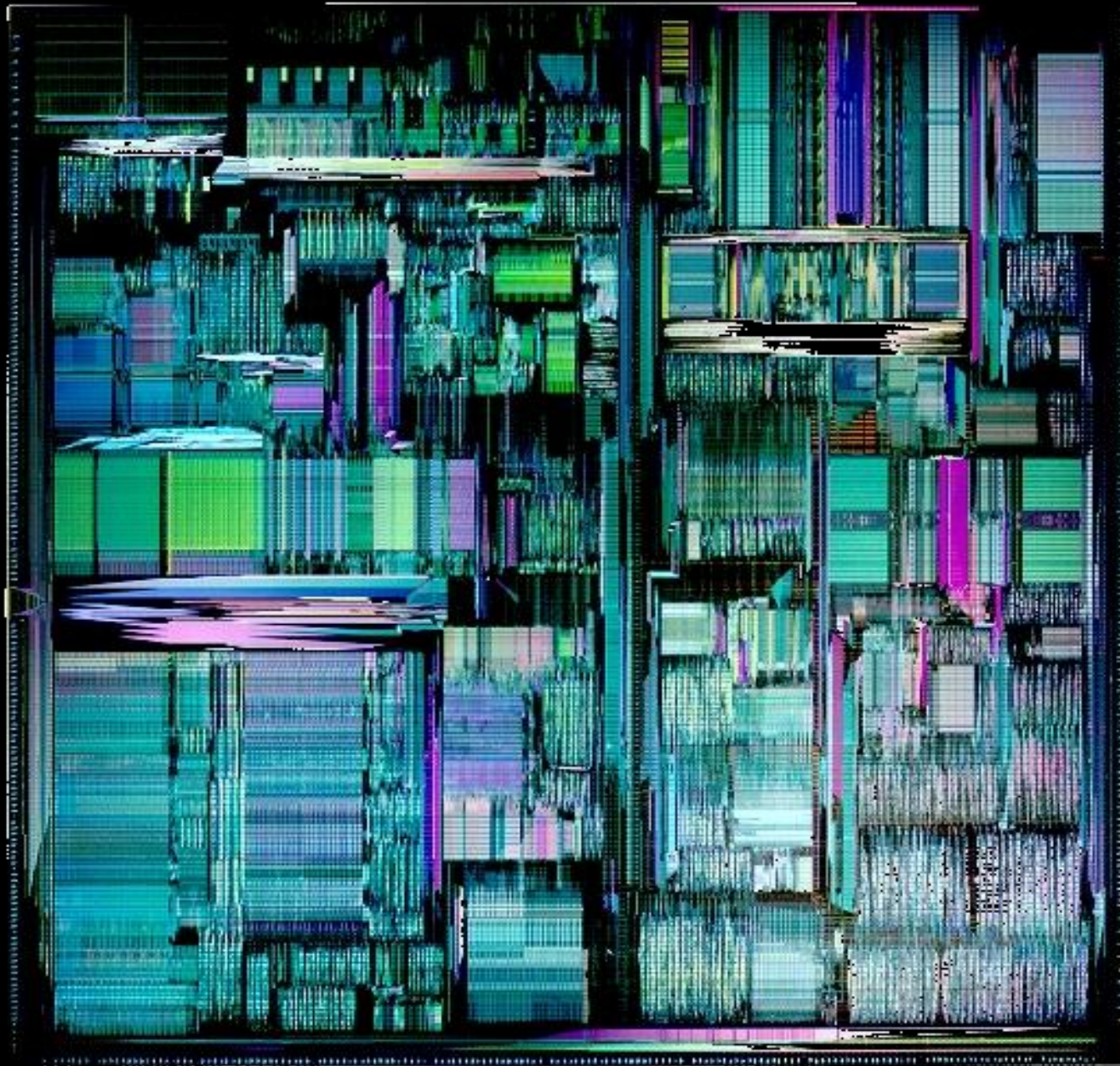


2 Hamiltonian cycle (also NP-c)

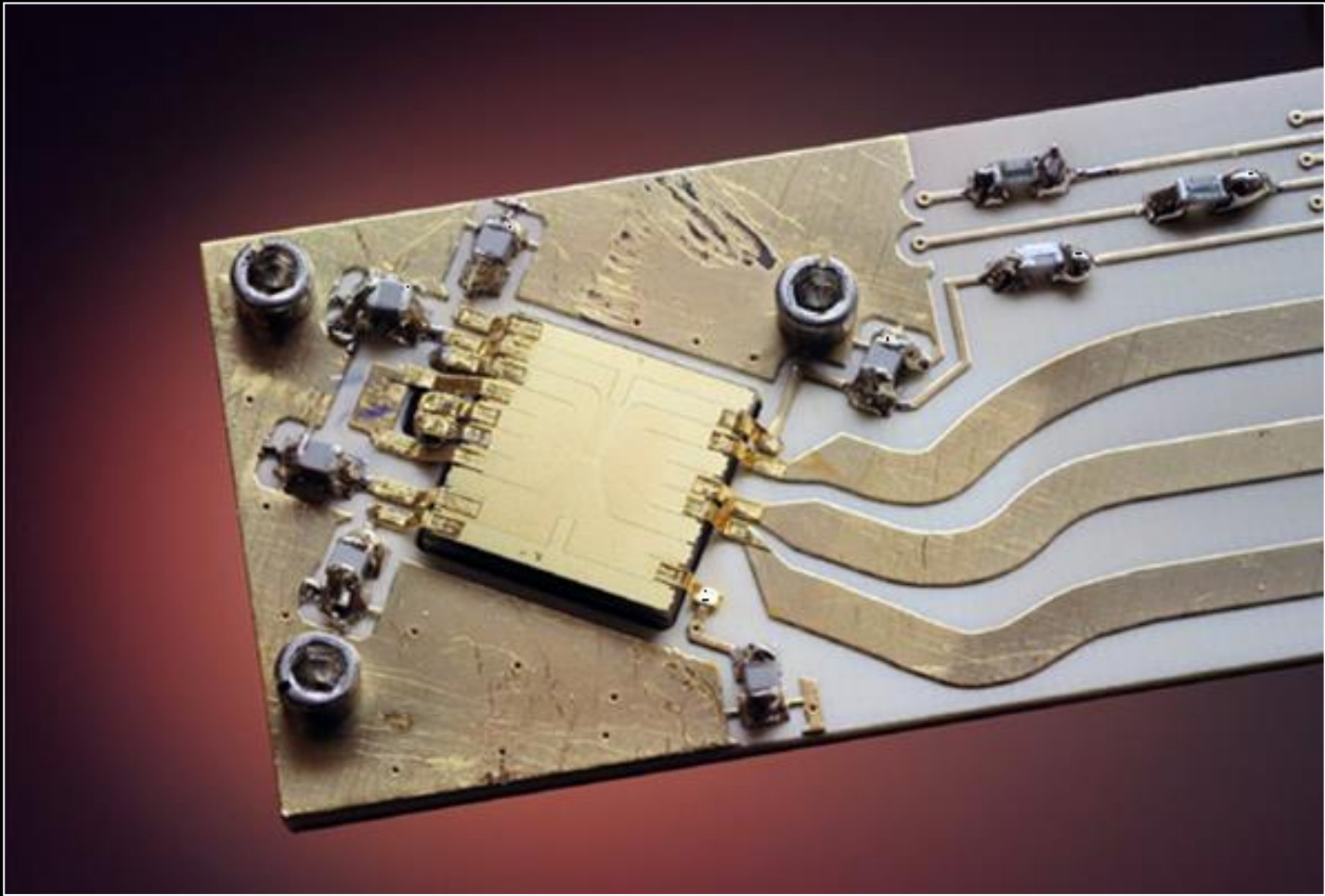




the puzzles of QMA



[1995 Pentium Pro



[NIST gold ion trap on aluminum-nitride backing, Y.Colombe/NIST]

DAVID

[wire



BOUNDED ERROR
QUANTUM
POLYNOMIAL TIME

[tha

2 The MA protocol

Did dinosaurs exist?



2 The MA protocol

Did dinosaurs exist?



2 The MA protocol

Did dinosaurs exist?

YES?
Eager to be
convinced.



[magnifying glass: hllllllal]

2 The MA protocol

Recognizing fakes?



2 The MA protocol

Recognizing fakes?

NO?
Don't be
fooled
easily.



2 Probabilistic checks

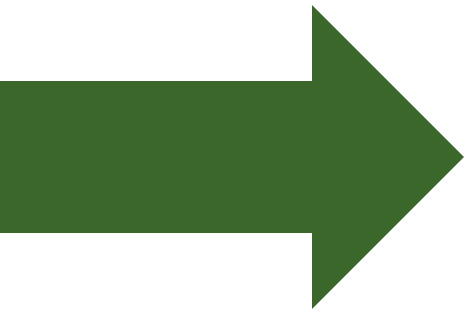
Sometimes reject
a genuine proof?

Accept
a fake?



1 The MA protocol

Probabilistic checks.



probabilistic
verification

from a uniform family



YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.



2 The QMA protocol

Quantum checks.



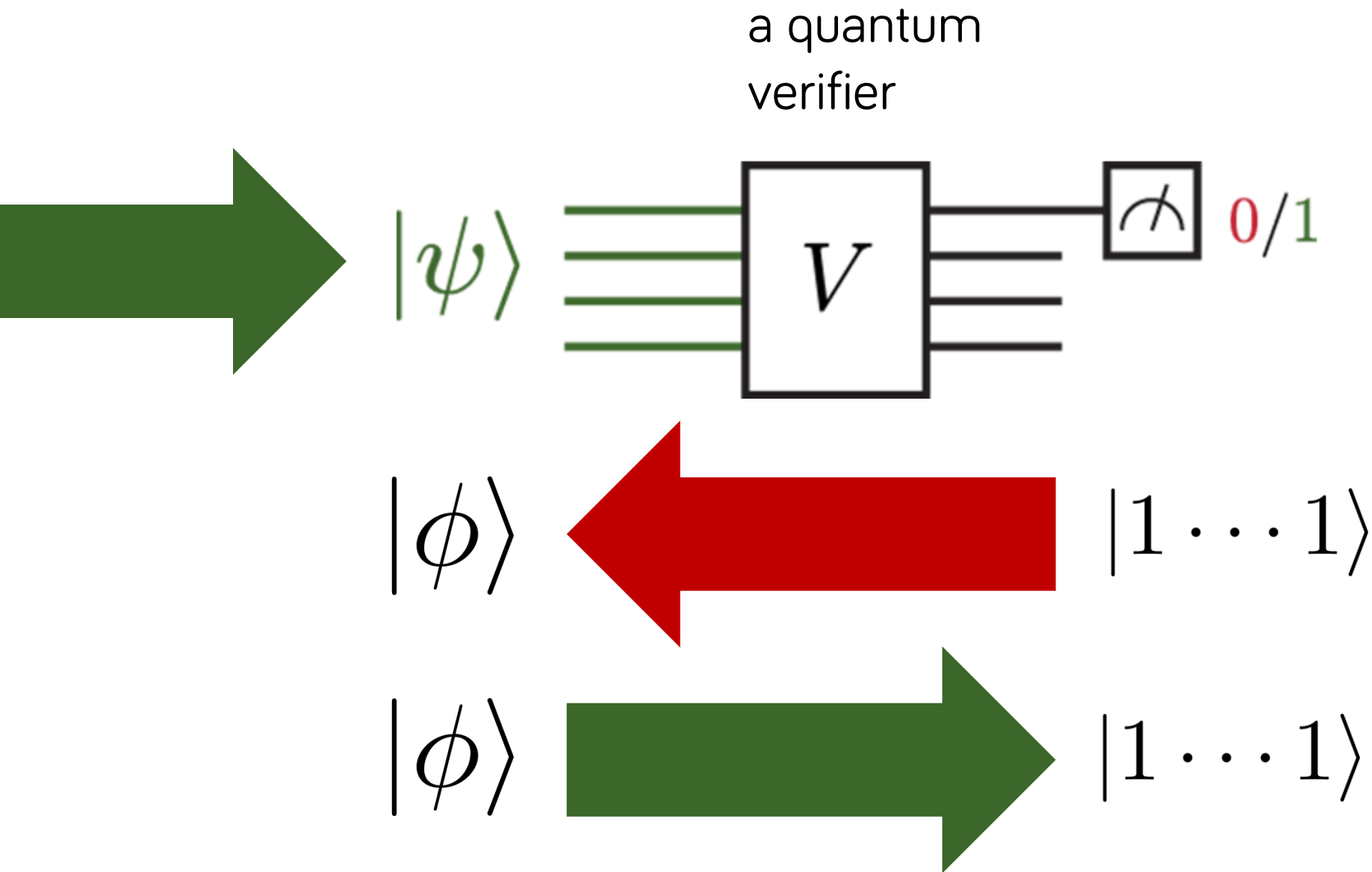
YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.



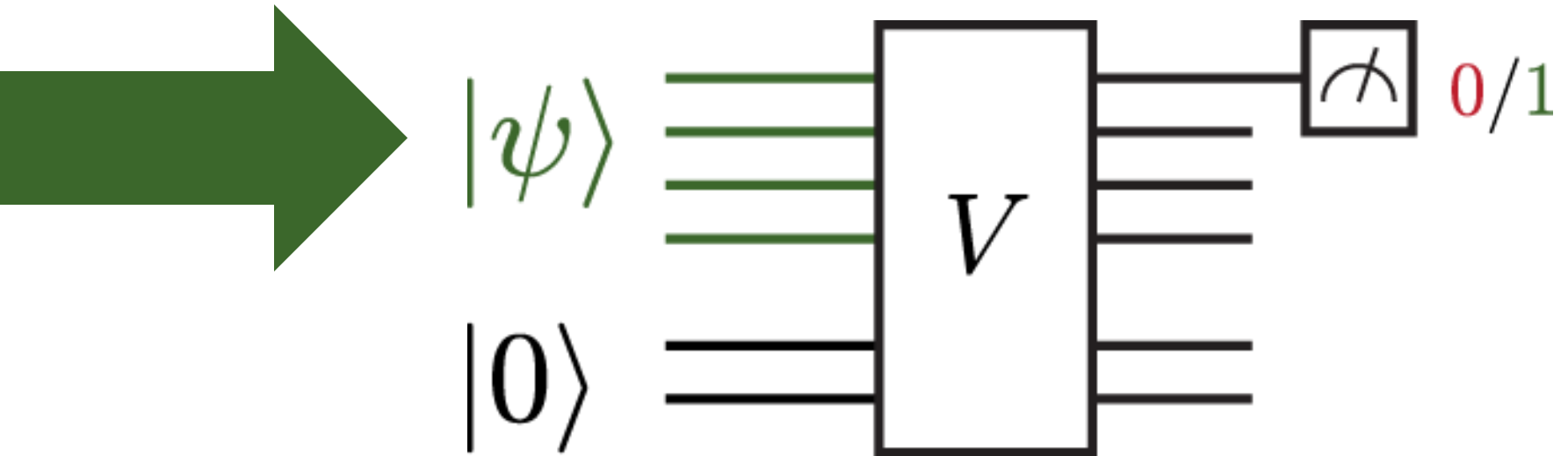
2 The QMA protocol

This is too simple.



2 The QMA protocol

Ancillas are necessary.

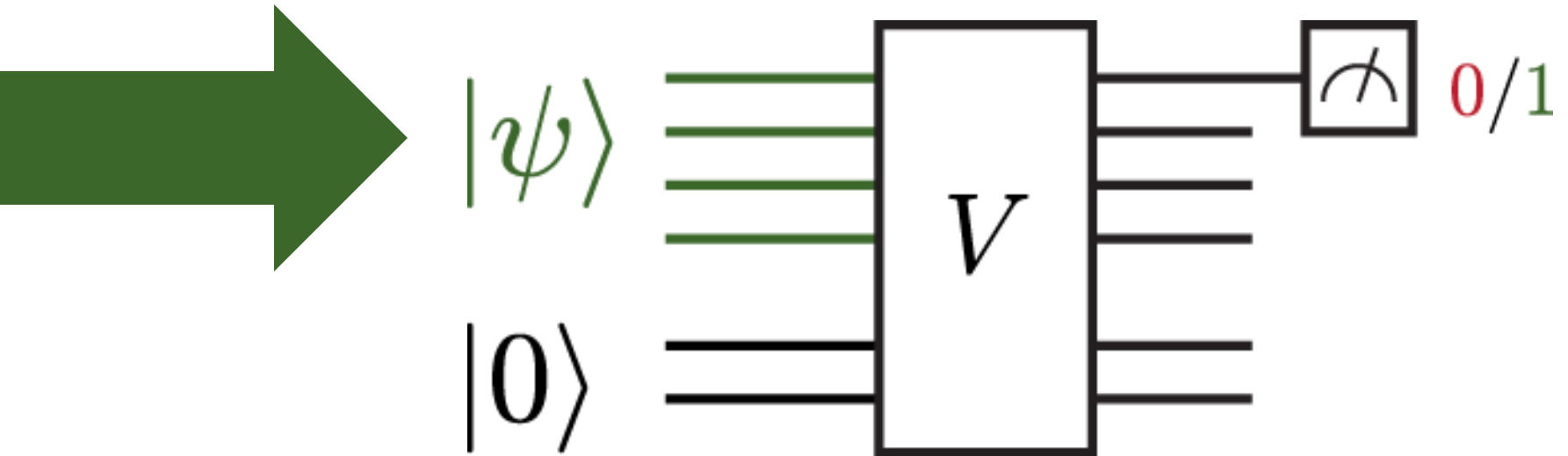


YES? Accept a good proof with $p > a$.

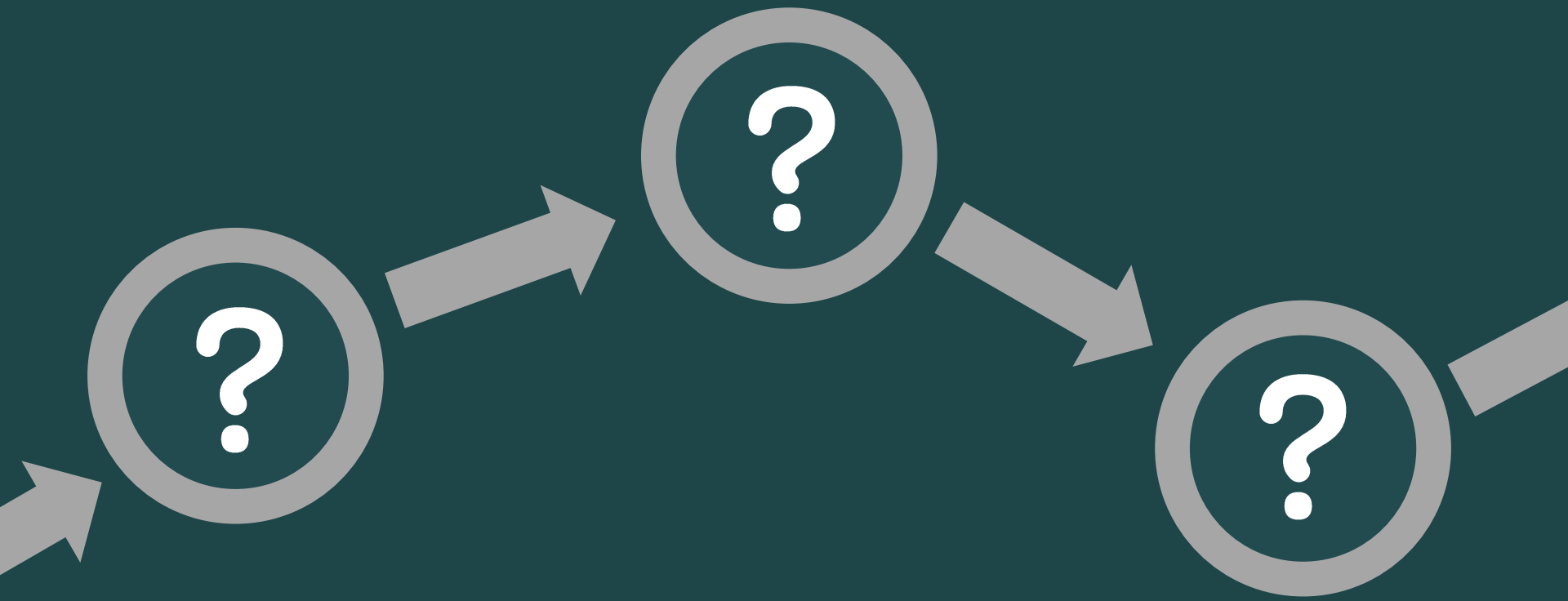
NO? Probability of accepting $p < b$.



2 A QMA-hard question

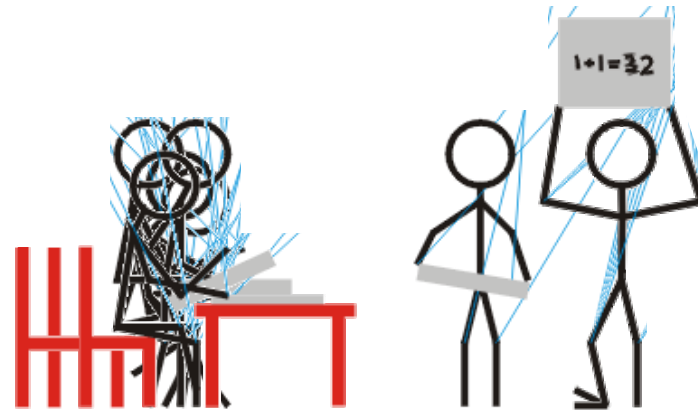


Could we feed this quantum verifier something that likely outputs 1?

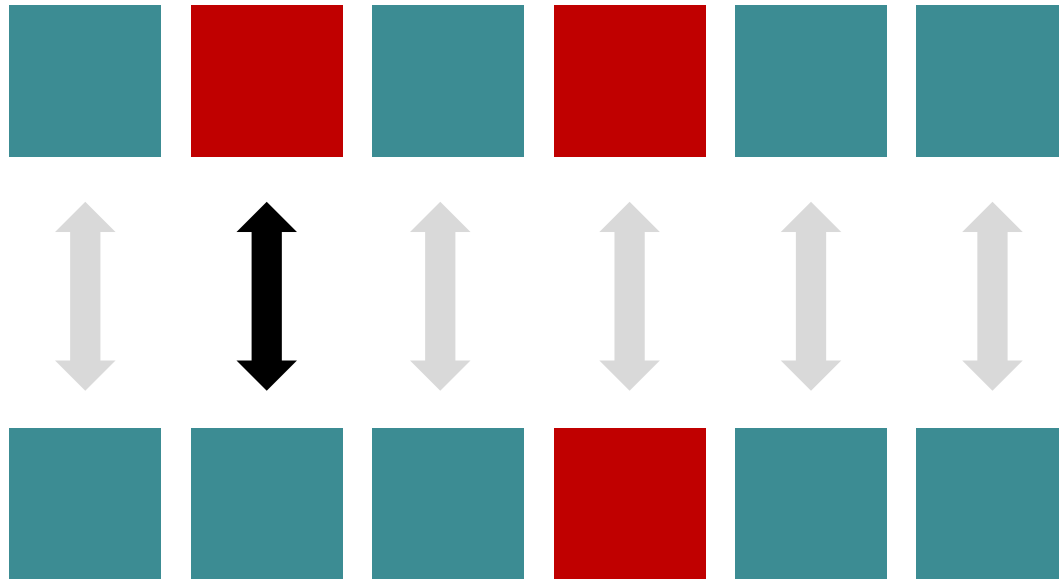


the history state
ground

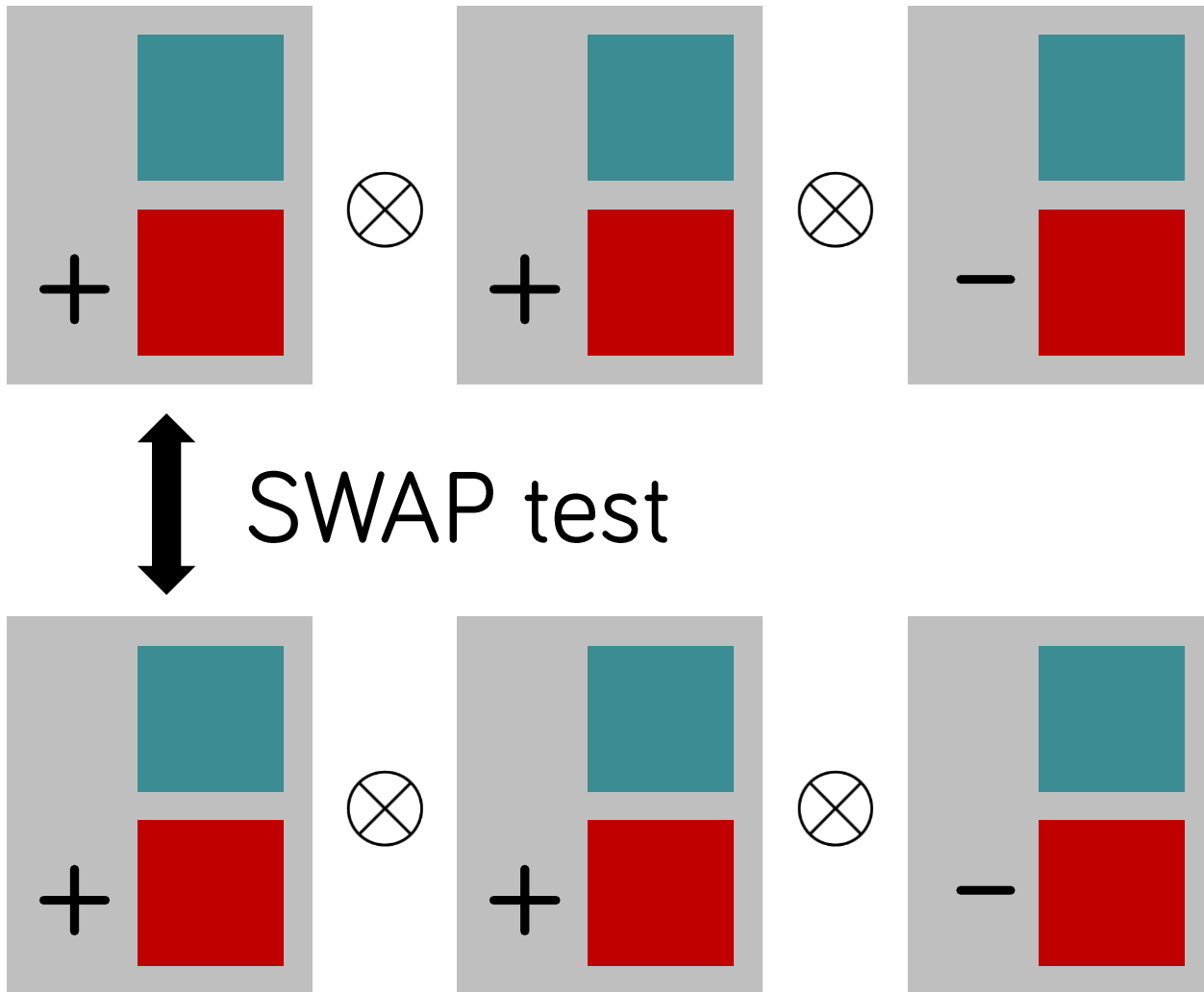
2 Snapshots of a computation



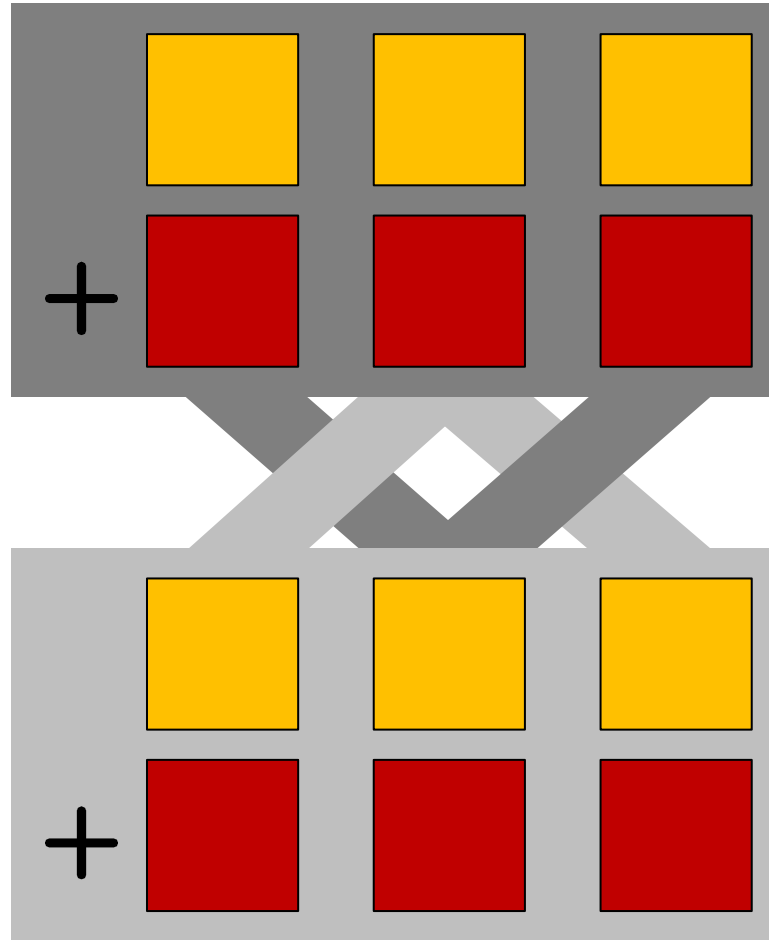
Locally comparing **strings**.



Locally comparing product states.

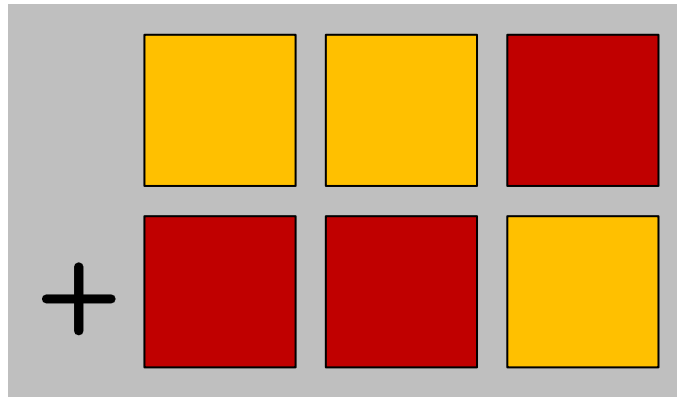


Locally comparing entangled states?



UGH!

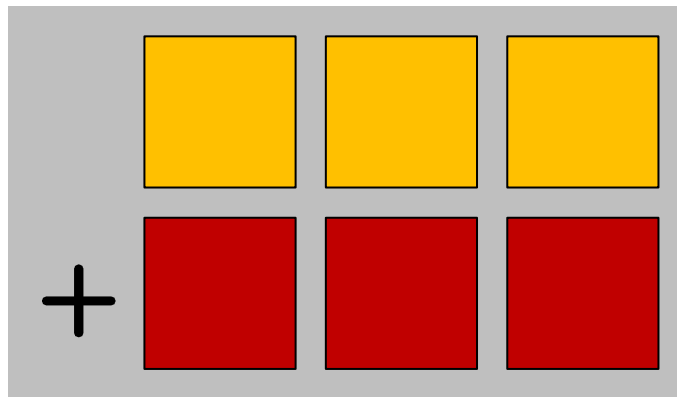
2 Labeling the data



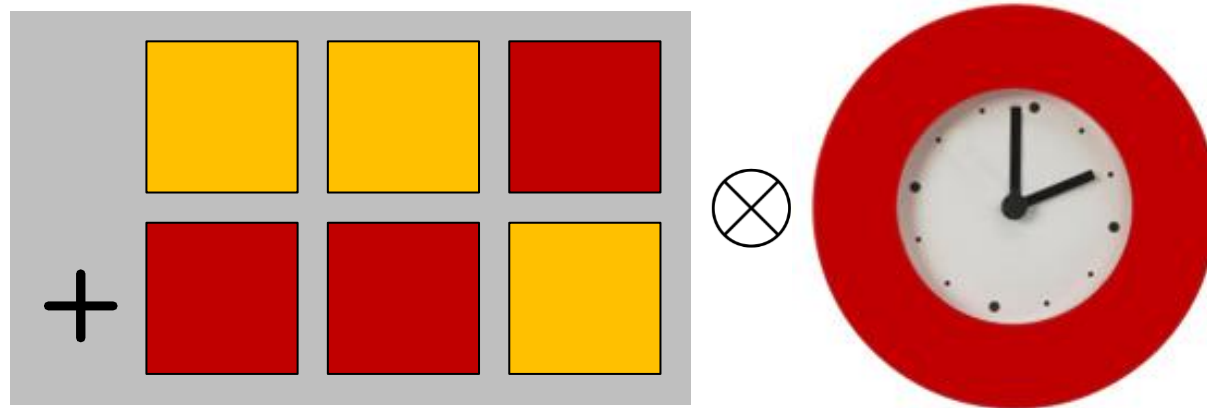
Hard to compare directly (locally).

U^\dagger   U

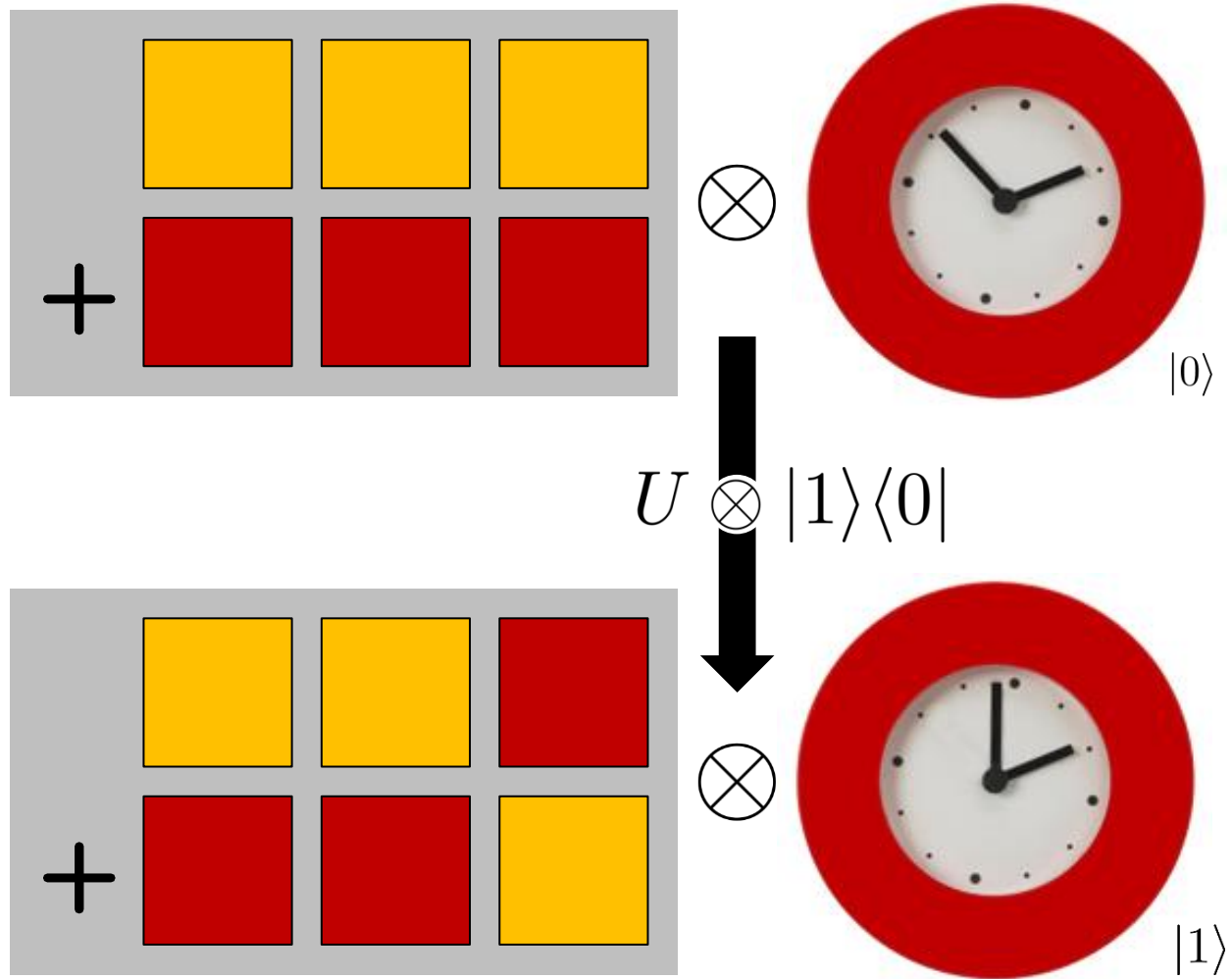
a clock



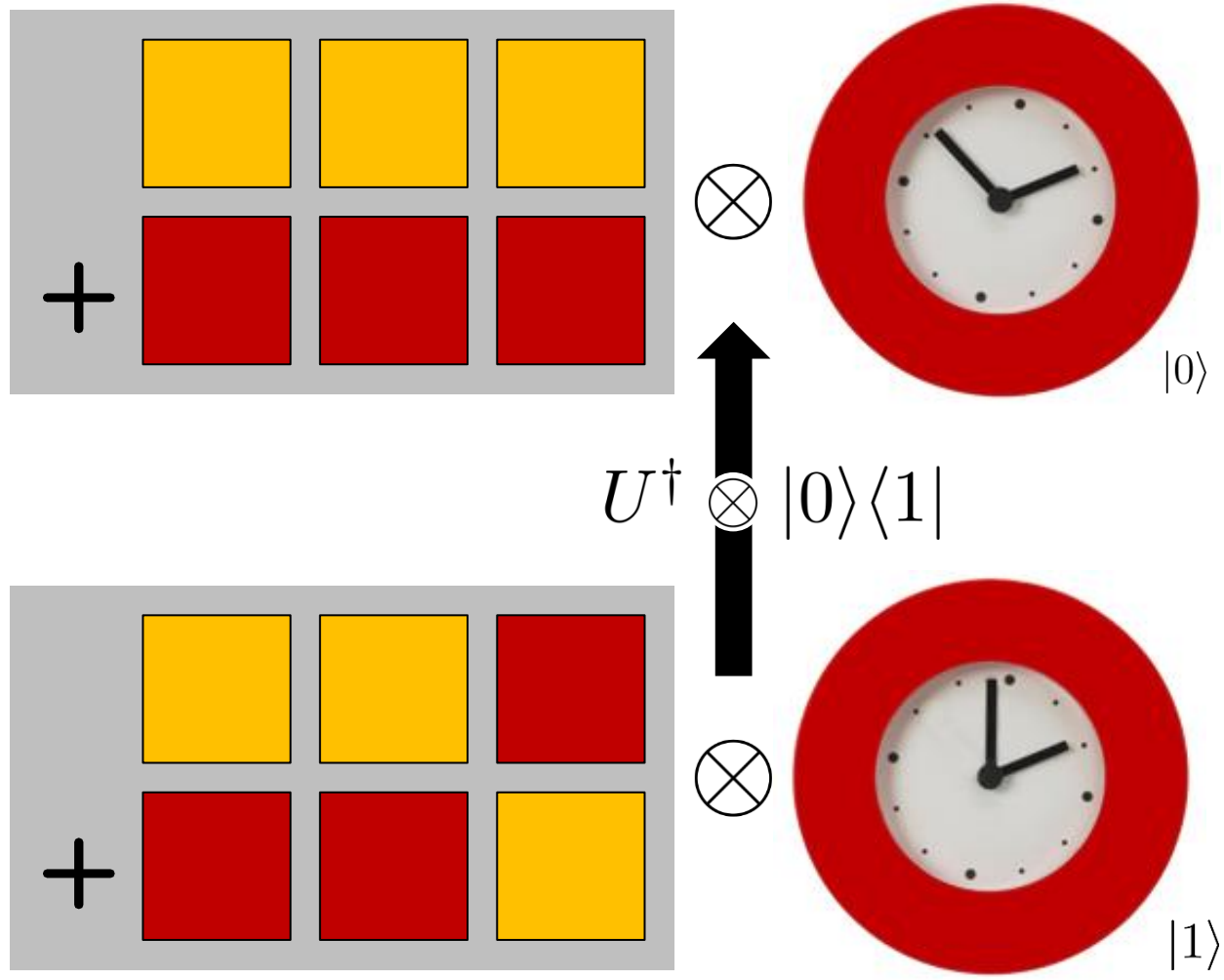
2 Labeling the data



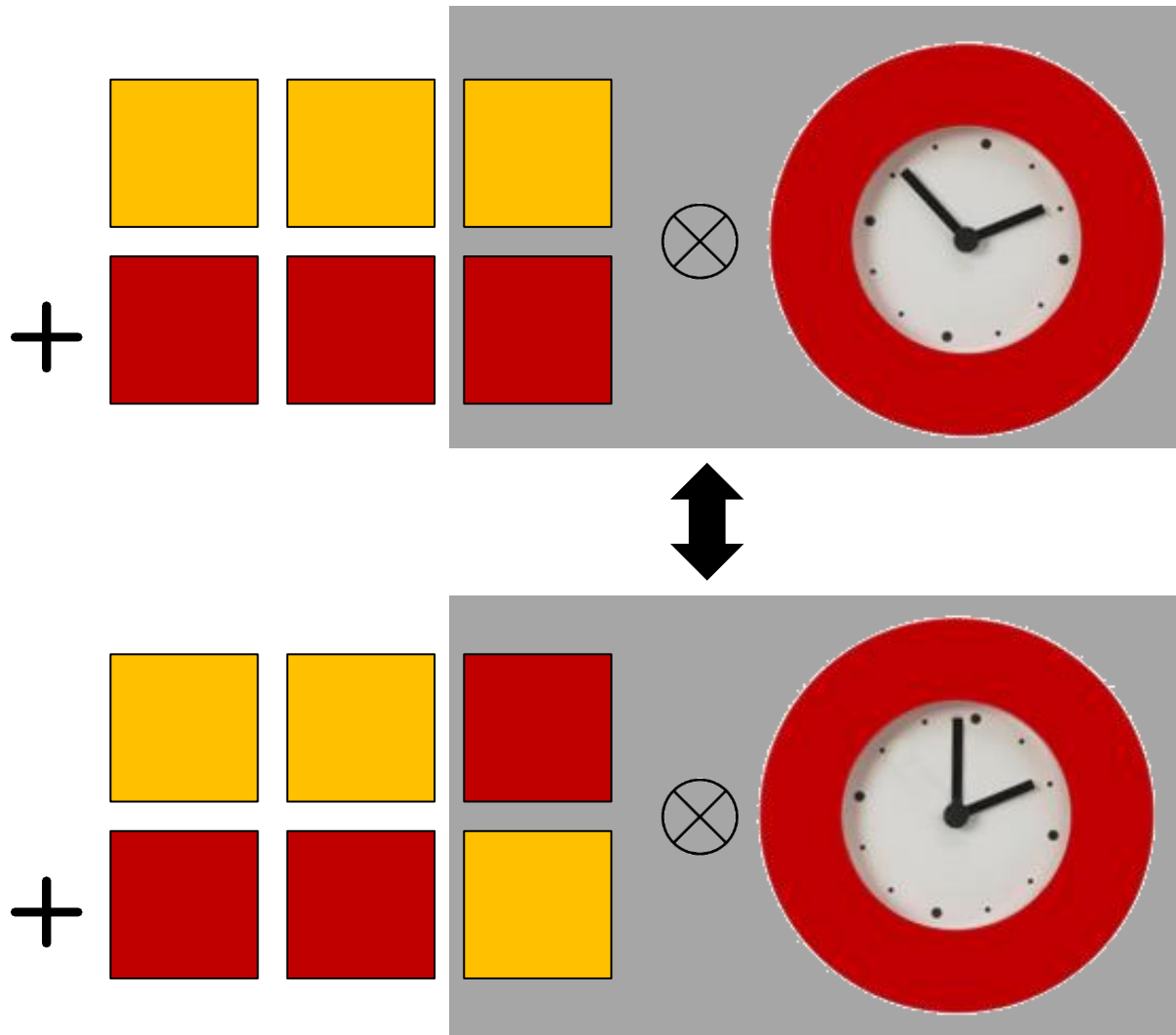
2 The data & the clock



2 The data & the clock

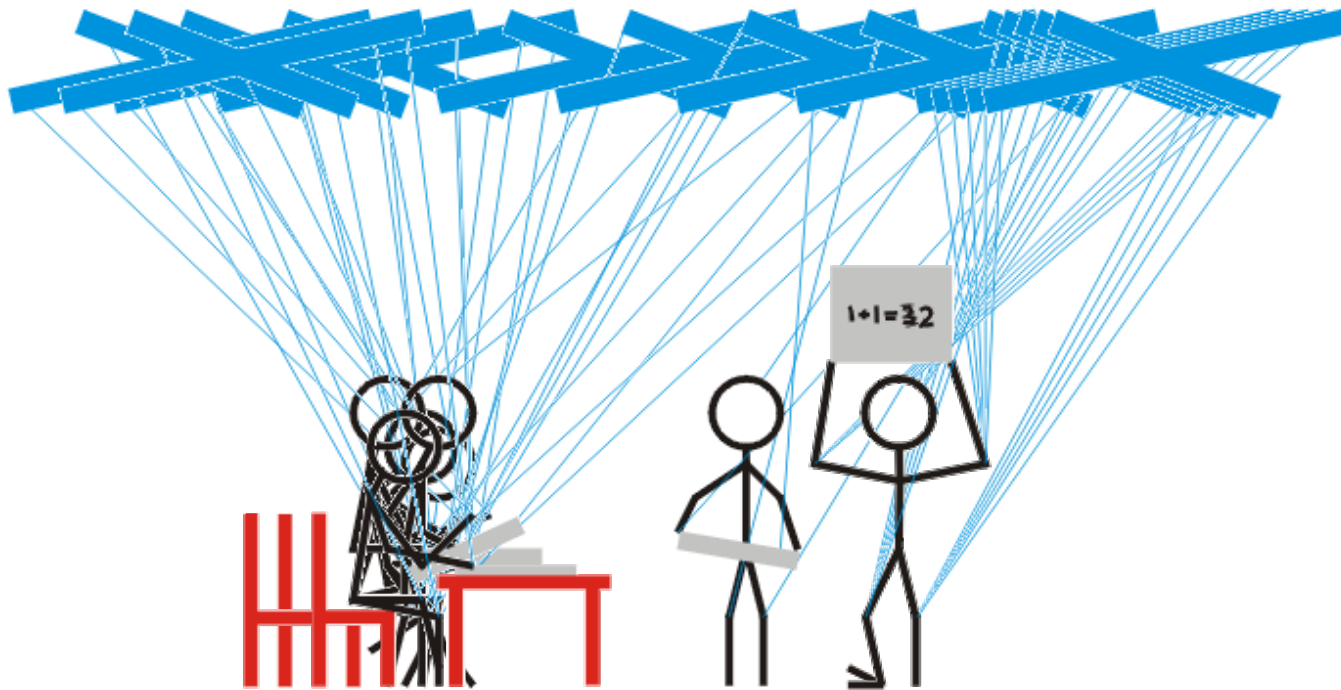


2 The data & the clock: locally comparing related states

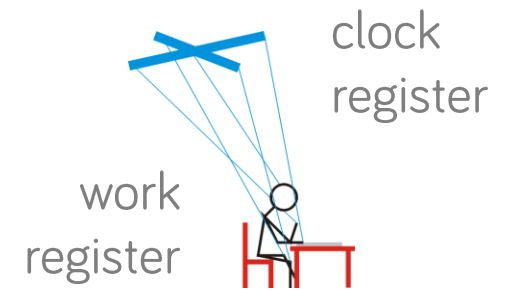


2 Feynman's computer

The data & a pointer.



2 Feynman's (Hamiltonian) computer

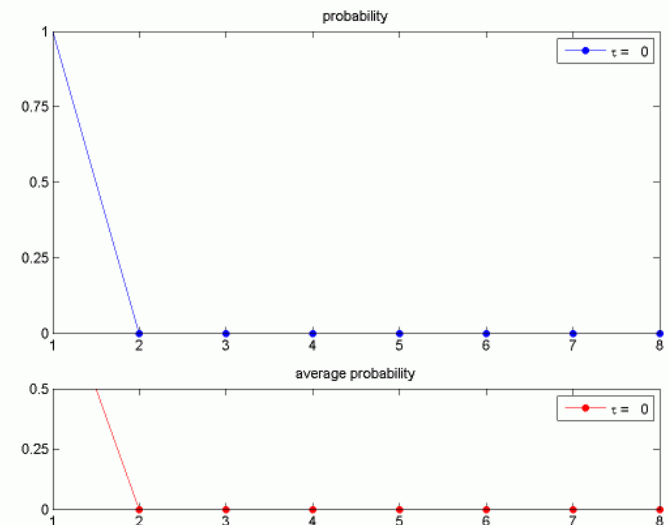


■ The Hamiltonian

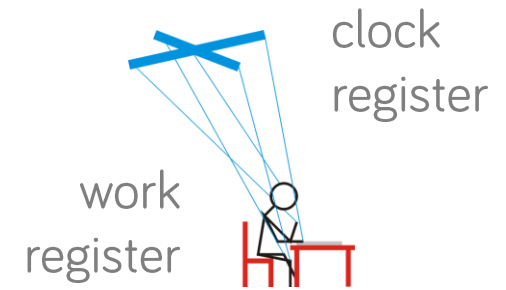
$$H_F = - \sum_{t=1}^L \left(U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

■ A quantum walk on a "line"

$$\begin{aligned} & |\varphi_0\rangle \otimes |0\rangle_c \\ & U_1 |\varphi_0\rangle \otimes |1\rangle_c \\ & U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c \\ & U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c \end{aligned}$$



2 Feynman's (Hamiltonian) computer

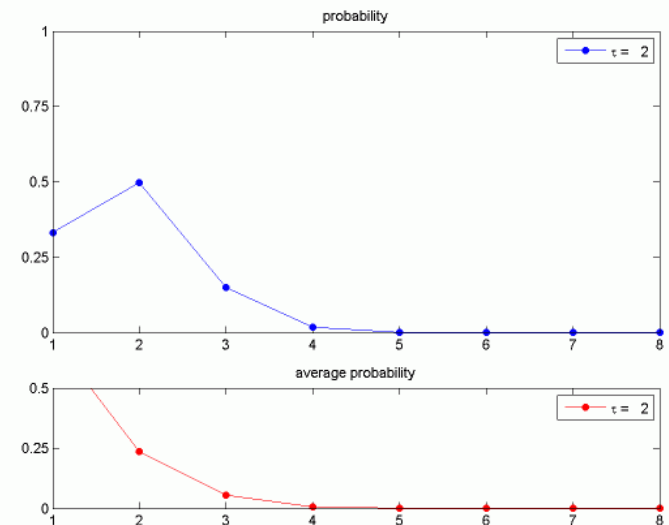


■ The Hamiltonian

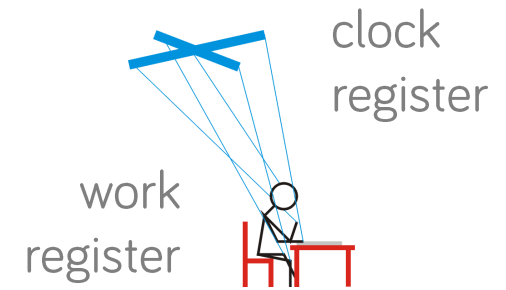
$$H_F = - \sum_{t=1}^L \left(U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

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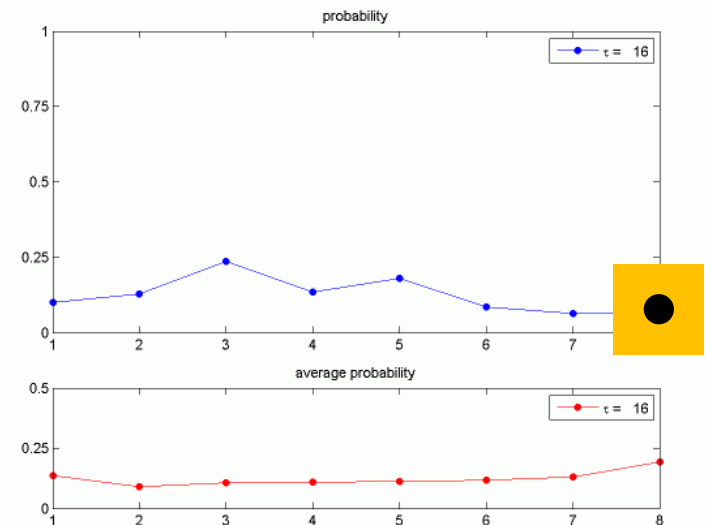
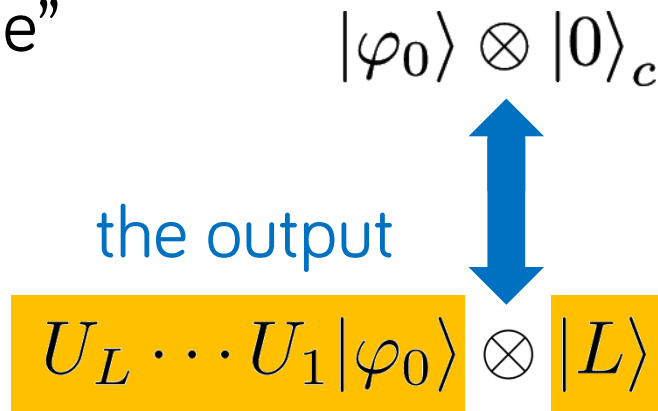
2 Feynman's (Hamiltonian) computer



■ The Hamiltonian

$$H_F = - \sum_{t=1}^L \left(U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

■ A quantum walk on a "line"



2 Hamiltonian QC

It's a walk.

- Feynman's Hamiltonian

$$H_F = - \sum_{t=1}^L \left(U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

- The “line” of states

$$\begin{array}{l} |\varphi_0\rangle \otimes |0\rangle_c \\ U_1 |\varphi_0\rangle \otimes |1\rangle_c \\ U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c \\ U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c \\ U_4 U_3 U_2 U_1 |\varphi_0\rangle \otimes |4\rangle_c \end{array} \quad H_F = - \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The eigenvectors: combinations of plane waves

2 Hamiltonian QC

It's a walk.

- Feynman's Hamiltonian

$$H_F = - \sum_{t=1}^L \left(U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

- The “line” of states

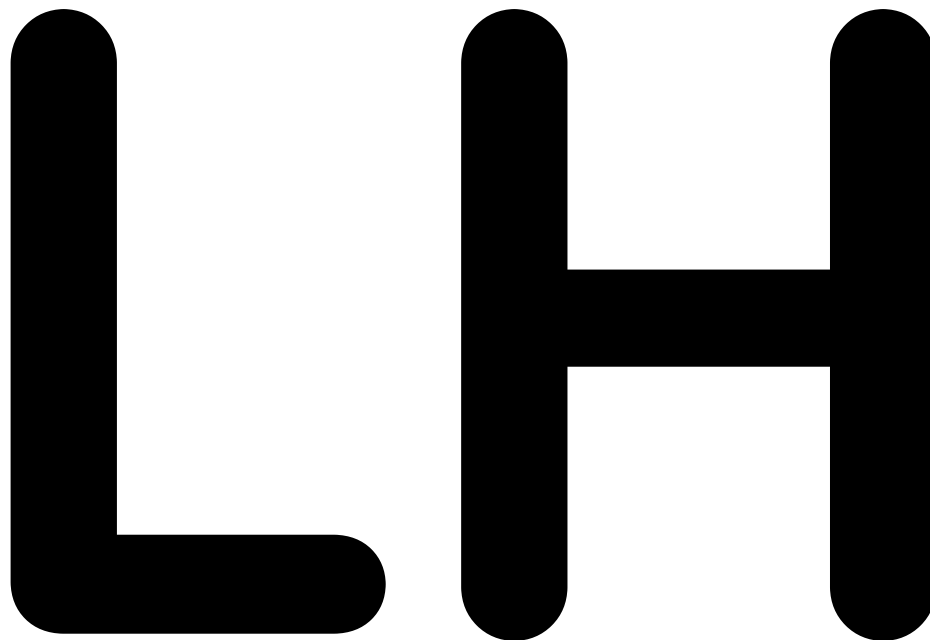
a possibility: wrap around a circle

$$\begin{array}{l} |\varphi_0\rangle \otimes |0\rangle_c \\ U_1 |\varphi_0\rangle \otimes |1\rangle_c \\ U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c \\ U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c \\ U_4 U_3 U_2 U_1 |\varphi_0\rangle \otimes |4\rangle_c \end{array} \quad H_F = - \begin{bmatrix} 0 & 1 & 0 & 0 & \mathbf{1} \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ \mathbf{1} & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The eigenvectors: combinations of plane waves

2 Hamiltonians and their ground states

Is
the
ground
state
energy
of a



↓ small ?

2 The QMA protocol

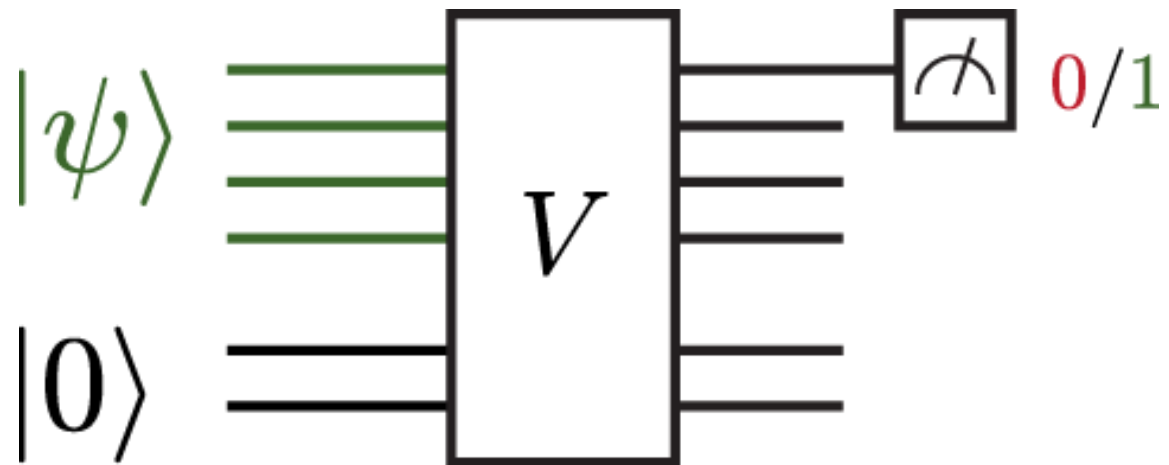
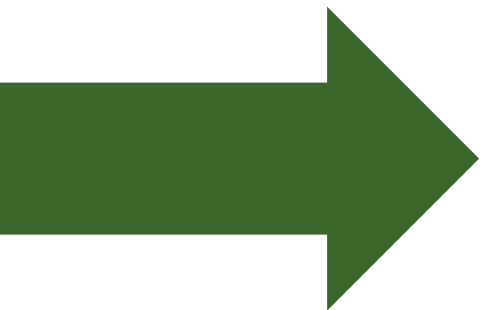
YES?

Accept a good proof with $p > a$.

↓ the promise

NO?

Probability of accepting $p < b$.



- Is there an acceptable quantum witness?
- Is some local Hamiltonian (nearly) frustration-free?

2 The QMA protocol

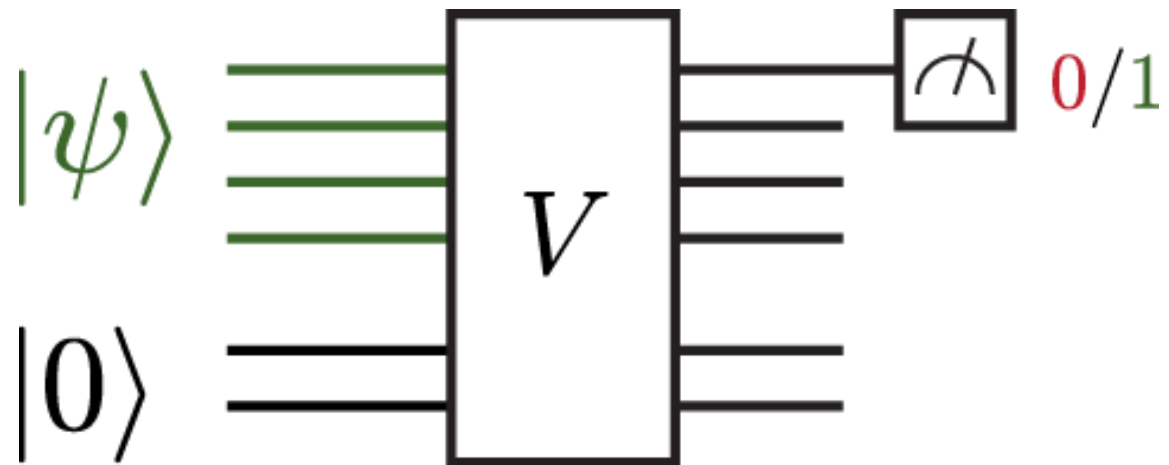
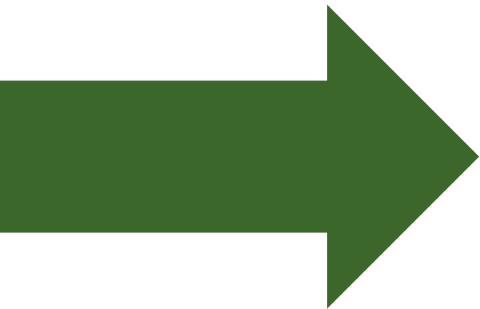
YES?

↓ the promise

NO?

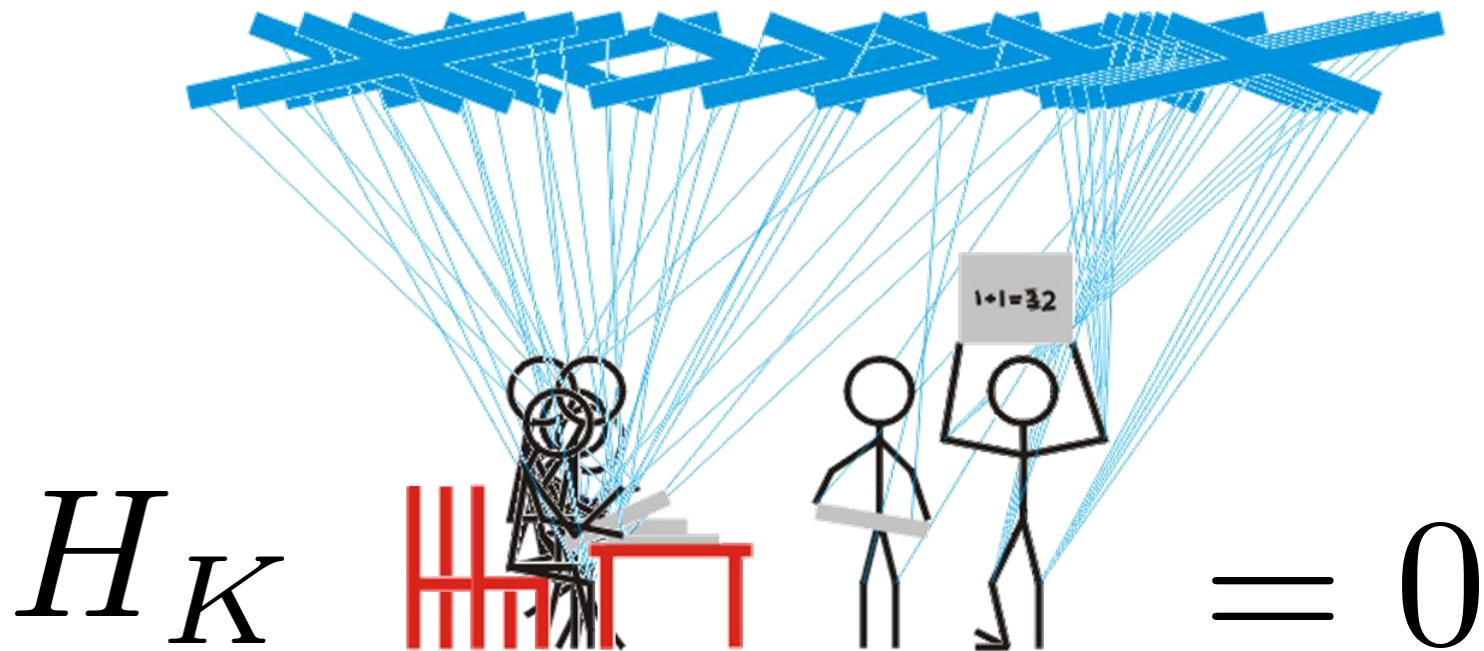
Accept a good proof with $p > a$.

Probability of accepting $p < b$.



- Is there an acceptable quantum witness?
- Is the ground energy of a local Hamiltonian *really* low?

2 The history state: a ground state



$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$

$\underbrace{\hspace{10em}}_{U_t \cdots U_1 |\varphi_0\rangle}$

2 The history state is a ground state

Local Hamiltonian

k-local
c-o-n-d-i-t-i-o-n-s

clock encoding
state progression
initialization

$$|\dots 000 \dots 0\rangle \otimes |0\rangle$$

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$

output

$$|\dots 1\rangle \otimes |T\rangle$$



2 Checking proper computation

Antisymmetry checks.

- uniform superpositions: zero-energy eigenstates

$$H_t = \frac{1}{2} (|t+1\rangle\langle t+1| + |t\rangle\langle t|) - \frac{1}{2} (U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^\dagger \otimes |t\rangle\langle t+1|)$$

Feynman's Hamiltonian

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

a projector

$$|\varphi_t\rangle \otimes |t\rangle$$
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$



2 Checking proper computation

Antisymmetry checks.

$$\sum_{t=1}^L H_t = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

*positive
semidefinite*

$$\begin{aligned} &|\varphi_t\rangle \otimes |t\rangle \\ &|\varphi_{t+1}\rangle \otimes |t+1\rangle \end{aligned}$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$



2 Checking proper computation

Antisymmetry checks.

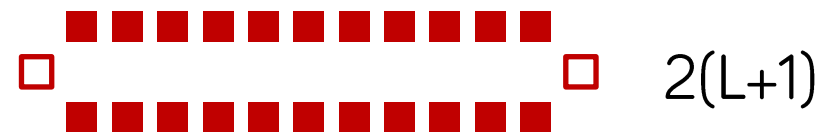
$$\sum_{t=1}^L H_t = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

positive semidefinite

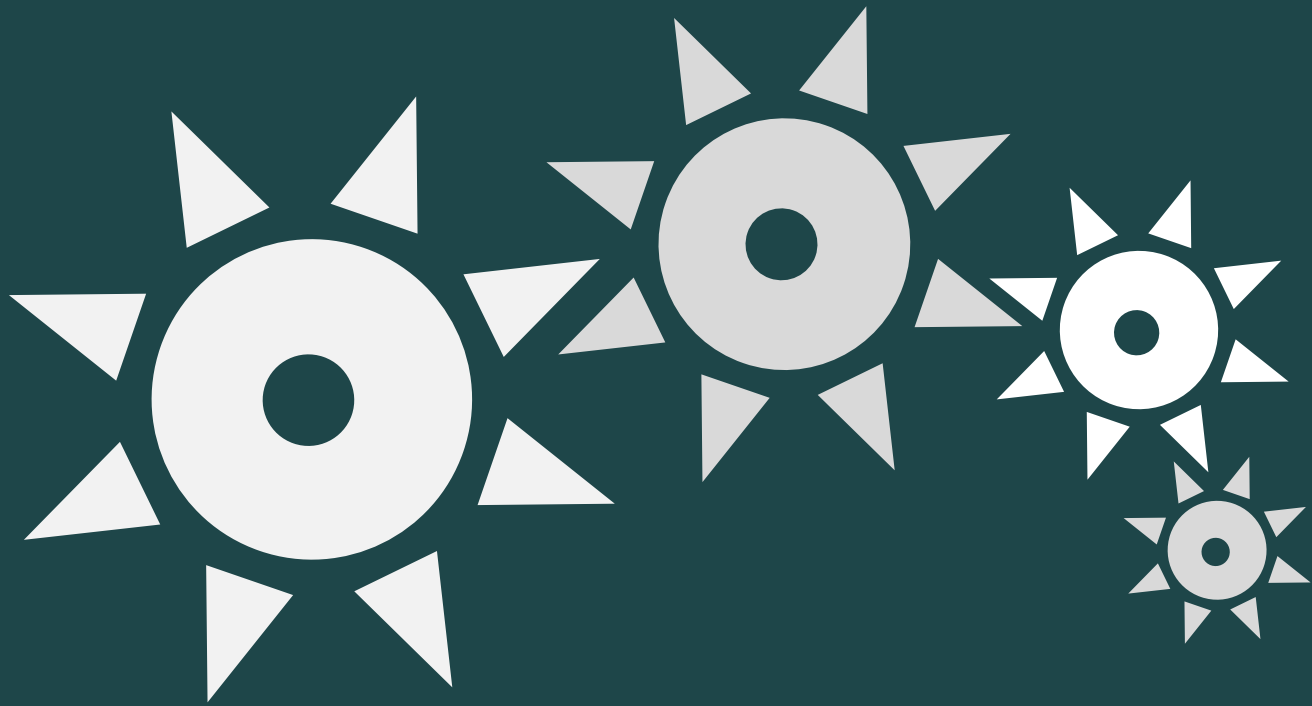
$$\sum_t e^{-ipt} |\varphi_t\rangle \otimes |t\rangle$$

local?

*eigenvectors:
combinations of
plane waves*



an L^{-2} eigenvalue gap



a clock workshop

3 Constructing local clocks

- the pulse



3 Constructing local clocks

- the pulse



transitions: 2-local

- joining the states
by projectors



3 Constructing local clocks

- the pulse



transitions: 2-local

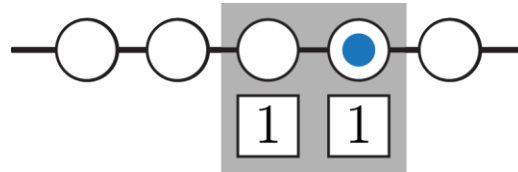
$$\begin{array}{c} 00100 \\ \downarrow \\ +00010 \end{array}$$

- joining the states
by projectors

$$|01 - 10\rangle\langle 01 - 10|$$

3 Constructing local clocks

- the pulse



transitions: 2-local
2-qubit gates: 4-local

interaction
with the data

- joining the states

by projectors $|01 - 10\rangle\langle 01 - 10|$

3 Constructing local clocks

- the pulse



transitions: 2-local
2-qubit gates: 4-local

00000

a “dead” state

Initialization!

- joining the states
by projectors

$$|01 - 10\rangle\langle 01 - 10|$$

3 Constructing local clocks

- the domain wall 

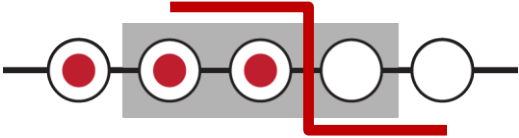
$$\begin{aligned} |t\rangle &= |1\rangle \\ &= |10000\rangle \end{aligned}$$

- 2-local terms
“compatible” with
11...1100...00



$$|01\rangle\langle 01|$$

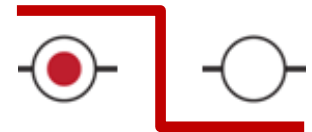
3 Constructing local clocks

- the domain wall  transitions: 3-local

$$\begin{aligned}
 |t\rangle &= |\mathbf{3}\rangle \\
 &= |11000\rangle
 \end{aligned}$$

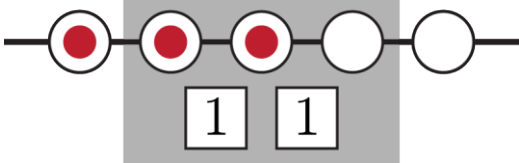
- joining states by transitions? $|100 - 110\rangle\langle 100 - 110|$

- enforce a domain wall: fix the ends



- the ground state $\cdots + |2\rangle + |3\rangle + \cdots$

3 Constructing local clocks

- the domain wall  transitions: 3-local
2-qubit gates: 5-local
- interacting with work (data) qubits

$$H_t = \frac{1}{2} (|t+1\rangle\langle t+1| + |t\rangle\langle t|) - \frac{1}{2} \left(U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^\dagger \otimes |t\rangle\langle t+1| \right)$$

5-local

● YES

ground state

- NO

ground state

● NO



lower bound on the
ground state energy

good
clock
states

... 01 ...
bad
clock
states

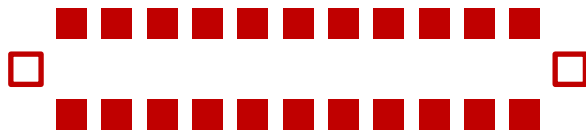
history states

non-uniform
superpositions

history states



a polynomially small gap



$$\Delta = O(L^{-2})$$



history states

well

badly

initialized history states

well

initialized histories

accepted
states

well

initialized histories

accepted
states

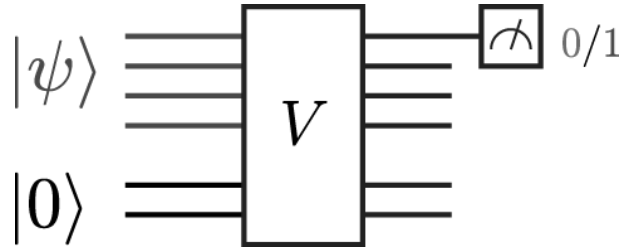
$$H_A + H_B$$

$$\lambda_0 \geq \sin^2 \frac{\vartheta}{2} \times \min(\Delta_A, \Delta_B)$$

\uparrow L^{-2} \uparrow L^{-1}

3-LH and QMA verification

[N., Mozes 07]



LH

NO

V is unlikely to accept anything (ϵ)

lowest eigenvalue

$$\geq \frac{c(1 - \sqrt{\epsilon})}{L^2}$$



promise gap L^{-2}

(needs $\epsilon=L^{-1}$)

YES

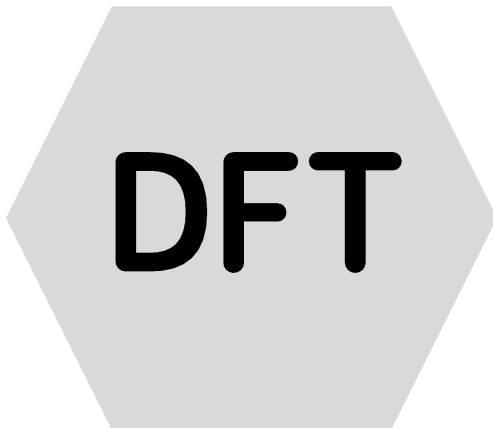
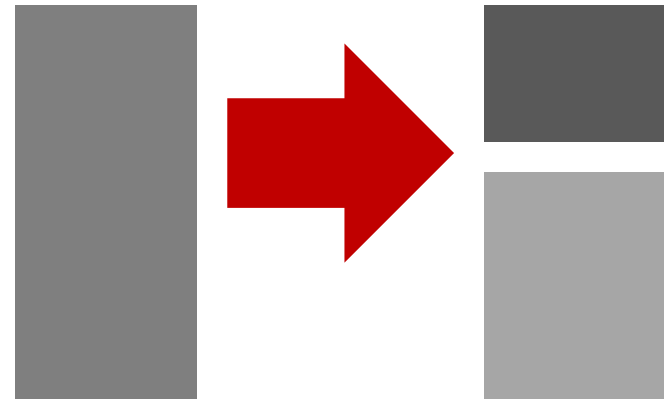
a well accepted proof $(1-\epsilon)$

the history state

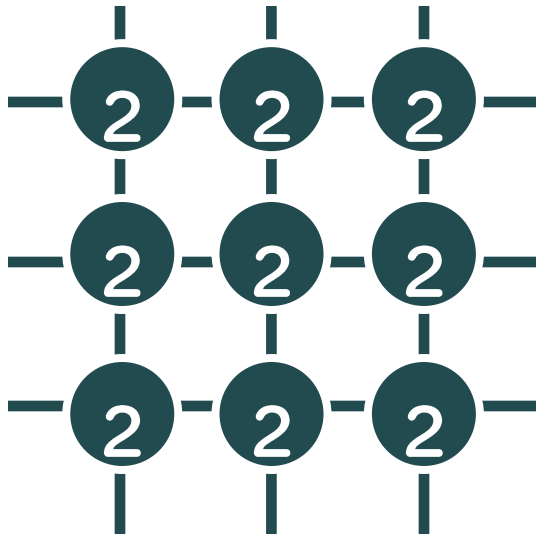
$$\leq \frac{\epsilon}{L+1}$$

2 Other QMA-complete problems

[Bookatz '13]

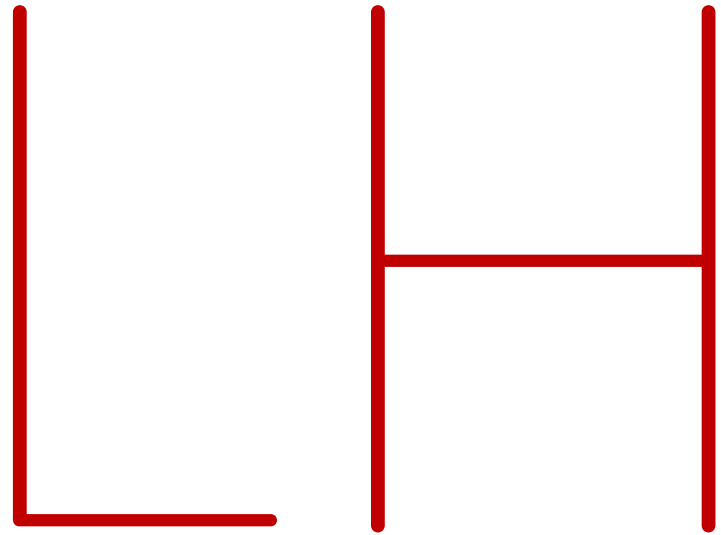


2 2-local Hamiltonian is QMA complete

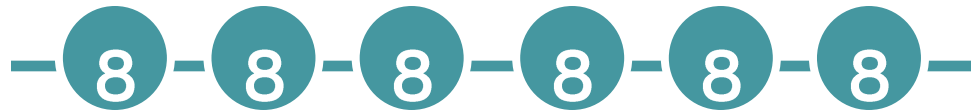


[Oliveira, Terhal '04]

a global minimum



$$\sum H_{jk}$$

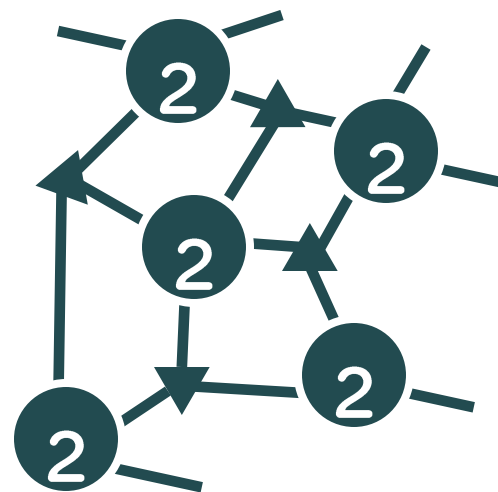


[Hallgren, N., Narayanaswami '13]

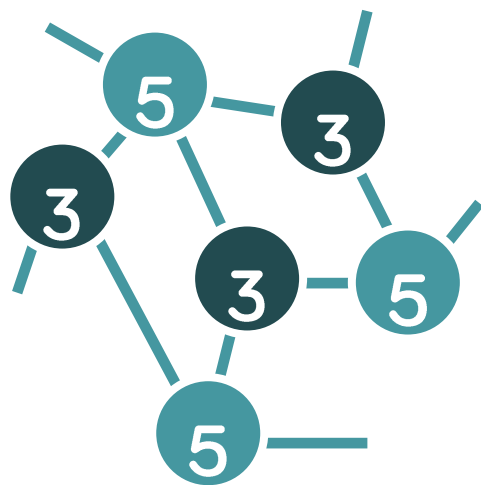
2 QMA₁-complete problems



[N. '08]



[Gosset, N. '13]



[Eldar, Regev '08]

unfrustrated
qSAT

The background features a dark teal color on the left side, transitioning into a lighter teal on the right. A prominent diagonal line runs from the bottom-left towards the top-right, dividing the space. There are also horizontal and vertical bands of varying shades of teal, creating a layered, geometric effect.

projections & gadgets

3 From a 5-local to a 3-local clock [Kempe, Regev]

■ the domain wall  transitions: 1-local

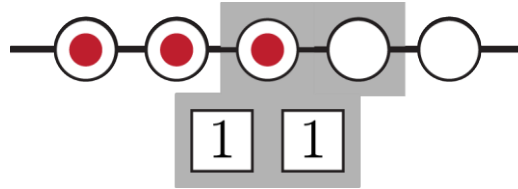
■ transitions $|1\rangle\langle 0| + |0\rangle\langle 1|$

■ punish mistimed transitions?

$$|01\rangle\langle 01|$$

3 From a 5-local to a 3-local clock [Kempe, Regev]

- the domain wall



transitions: 1-local
2-qubit gates: 3-local

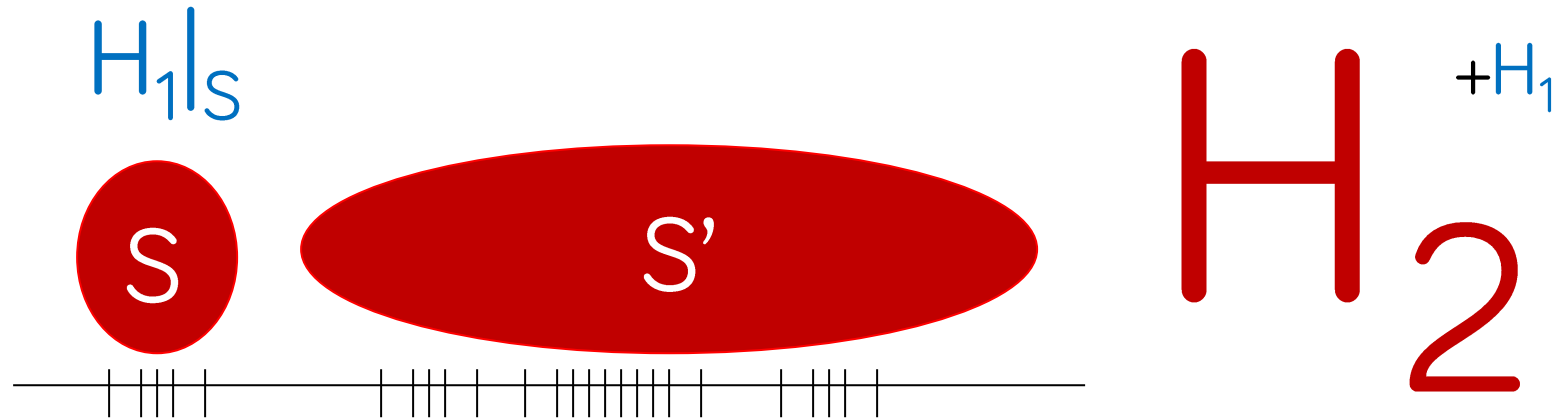
- transitions

$$|10\rangle\langle 10|_{1,2} + |10\rangle\langle 10|_{2,3} - X_2$$

- punish mistimed transitions

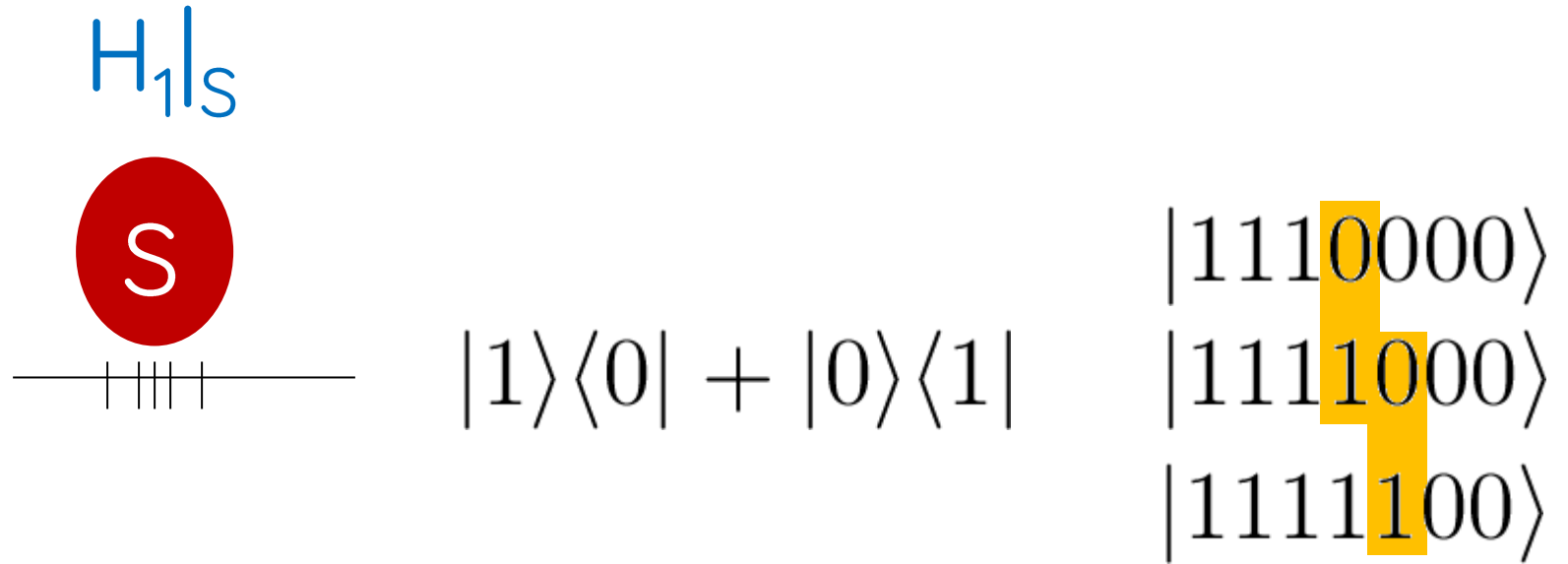
$$|01\rangle\langle 01|$$

4 The projection lemma: a useful tool for proving gaps

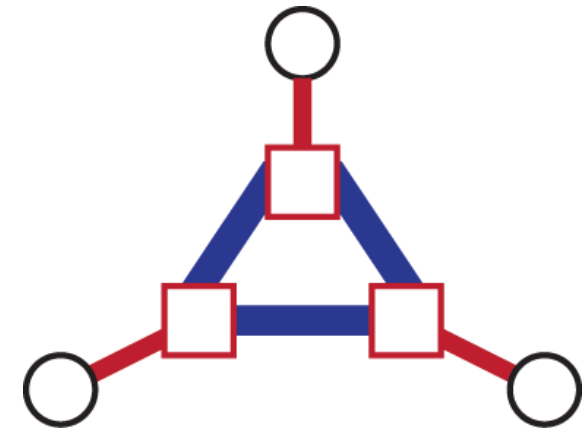
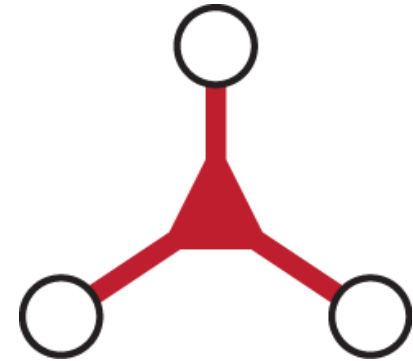
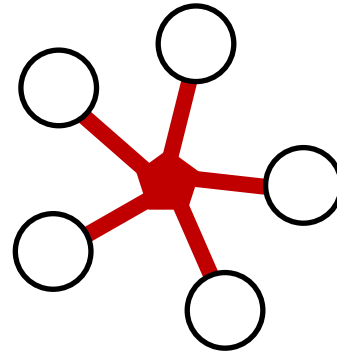
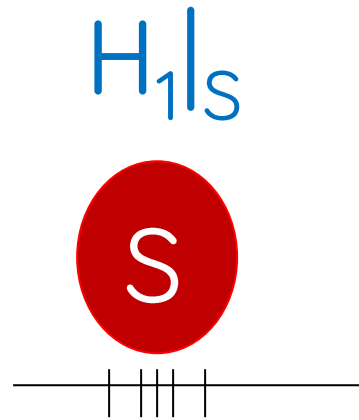


- a HIGH energy penalty for “illegal” states?
- the low energy states live near the “legal” subspace

4 The projection lemma in action



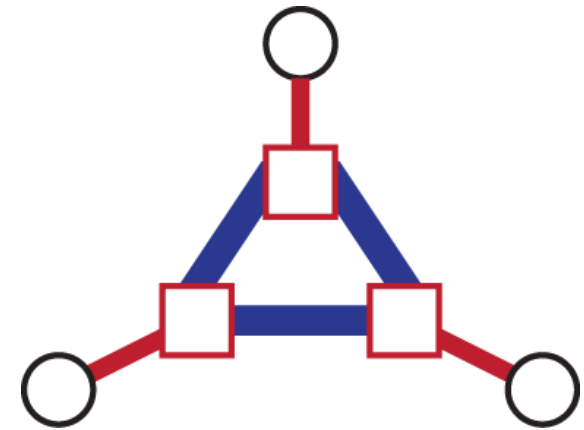
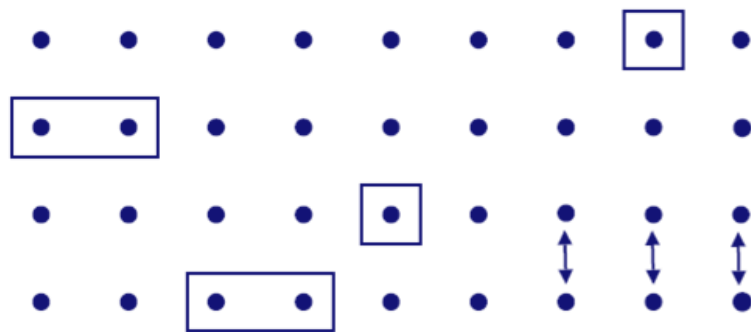
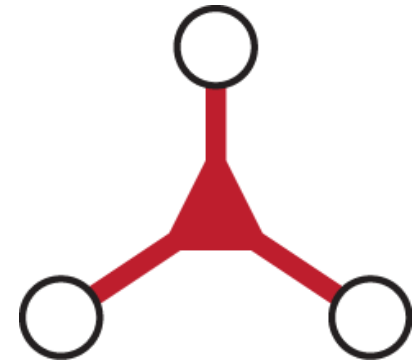
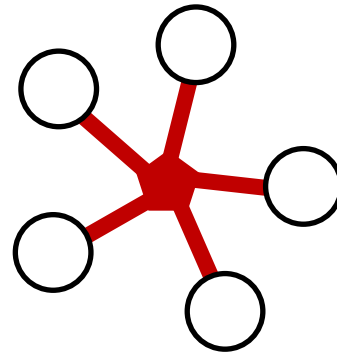
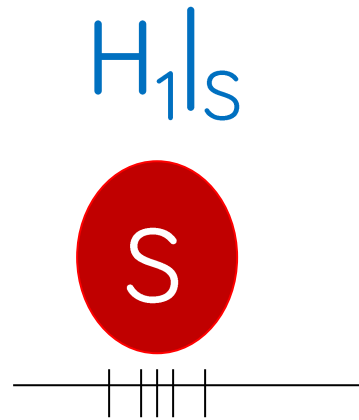
4 The projection lemma: a useful tool for proving gaps



- 3-LH that works well in the “good clock subspace”
- 2-LH from effective interactions

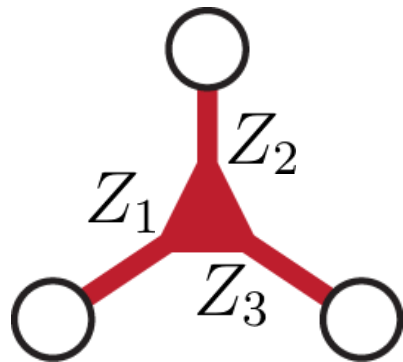
[Kempe, Kitaev, Regev '03]

4 The projection lemma: a useful tool for proving gaps



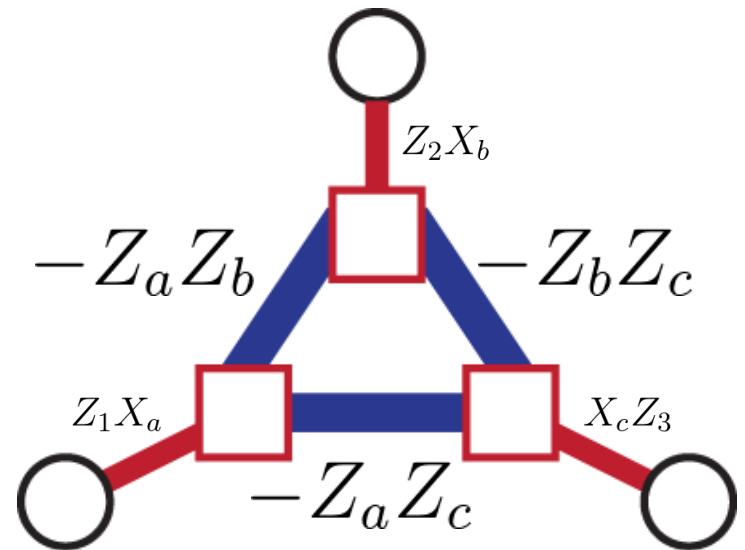
- 2-loc. H. in 2D [Oliveira, Terhal '05]

3 Further decreasing locality: a “3 from 2” gadget



- strongly coupled ancillas (a new energy scale)
- perturbation theory

$$G'(z) = (z\mathbb{I} - H')^{-1}$$

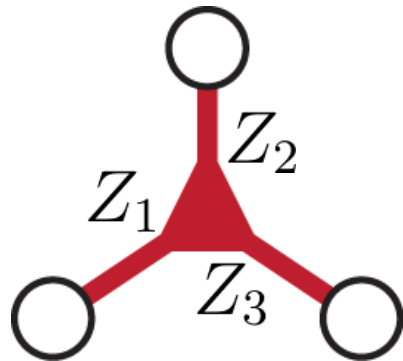


$$H' = H + V$$

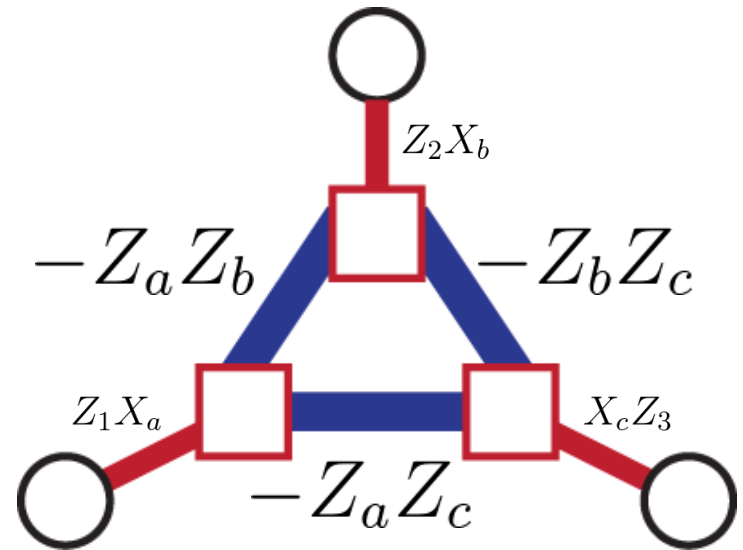
$\|H\| \gg \|V\|$

$$S = \text{span} \{ |000\rangle, |111\rangle \}$$

3 Further decreasing locality: a “3 from 2” gadget



- strongly coupled ancillas (a new energy scale)
- perturbation theory gives us an effective Hamiltonian



$$H' = H + V$$

$\|H\| \gg \|V\|$

$$S = \text{span} \{ |000\rangle, |111\rangle \}$$

$$V|_S$$

projection
lemma

$$V^2|_S$$

unwanted
(subtract)

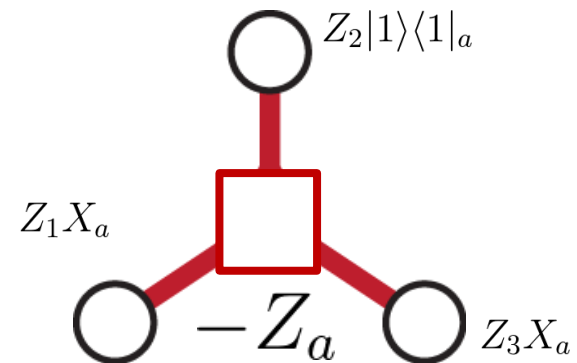
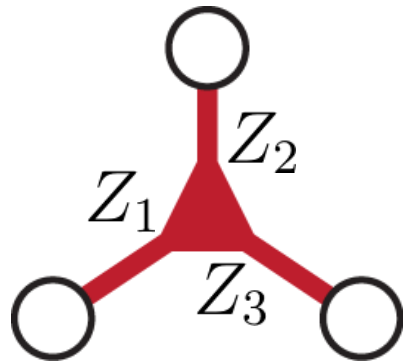
$$V^3|_S$$

the effective
3-local term

[Kempe, Kitaev, Regev '03]

3 STRONG local fields, OK interactions

[Cao et al., 1311.2555]



- strongly bound a single ancilla still needs strong interactions
- perturbation theory gives us an effective Hamiltonian

$$S = \{|0\rangle\}$$

$$H' = H + V$$

$$\|H\| \gg \|V\|$$

$$V|_S$$

projection
lemma

$$V^2|_S$$

unwanted
(subtract)

$$V^3|_S$$

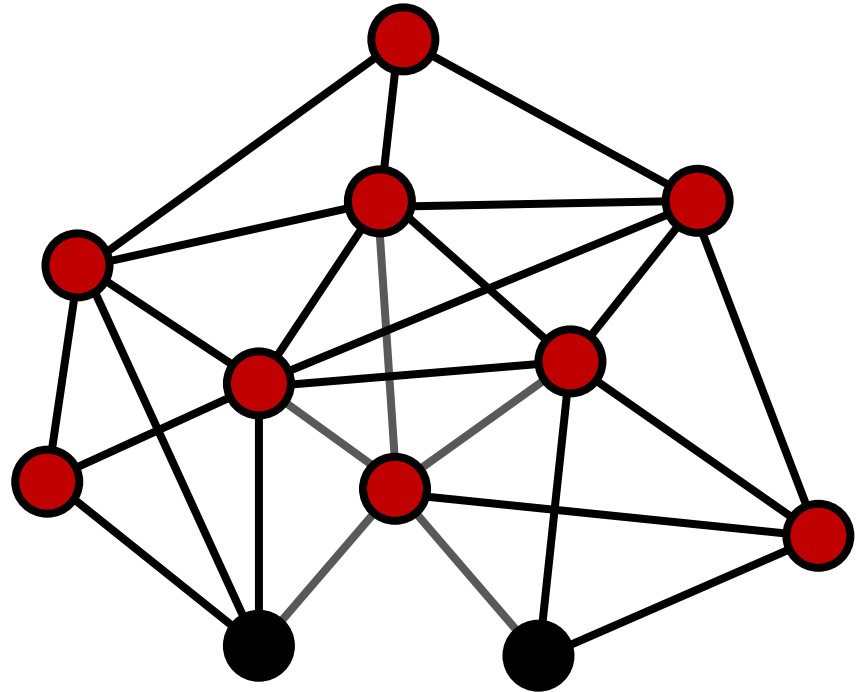
the effective
3-local term

special cases (Z-basis)
exact gadgets!

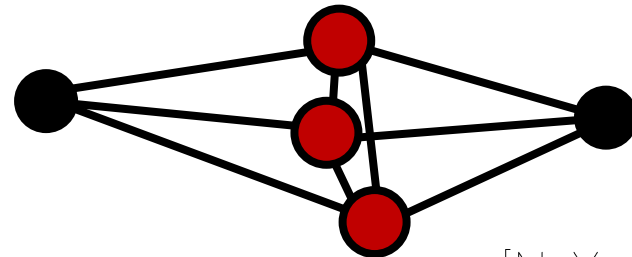
[Biamonte 0801.3800]

3 “Strengthening”, intermediary gadgets?

- classically easy: copy

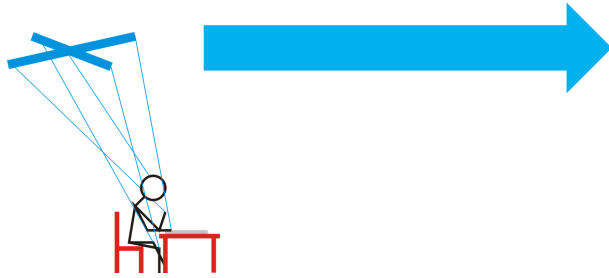


- quantumly?



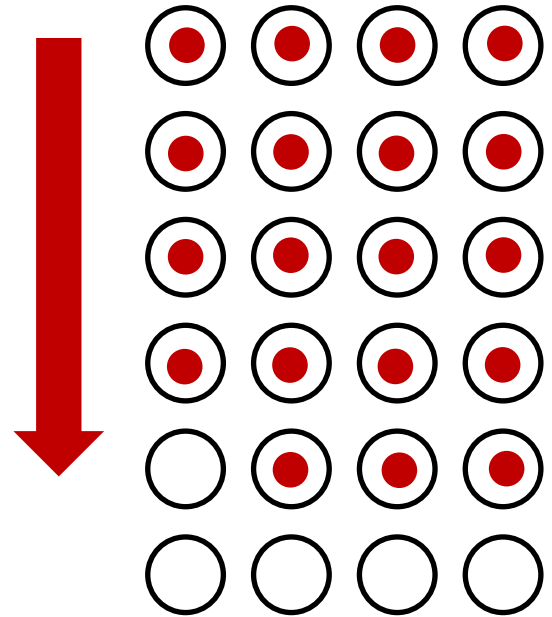
[N., Yudong Cao]

clock/work registers



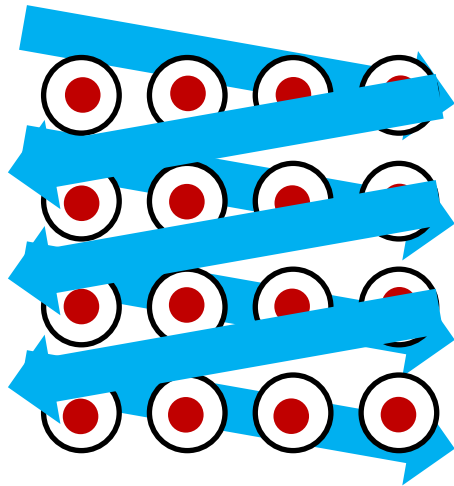
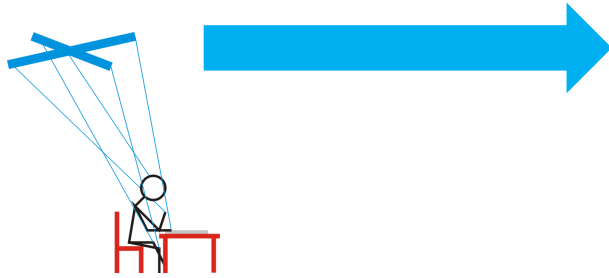
[Kempe, Kitaev, Regev]

a geometric clock



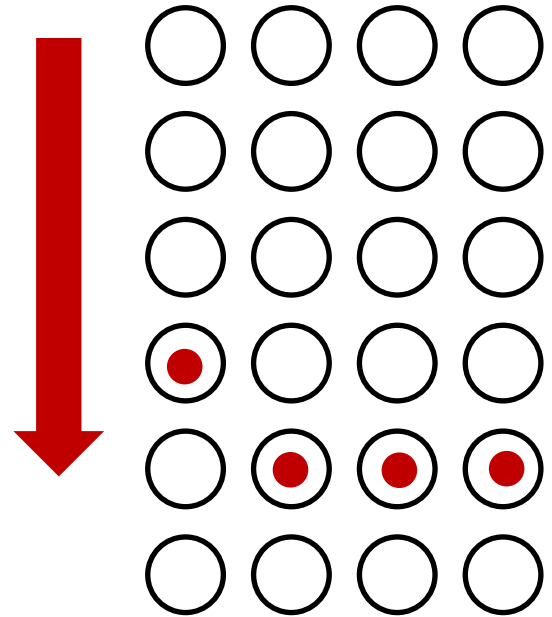
[Mizel] [Aharonov+]

clock/work registers



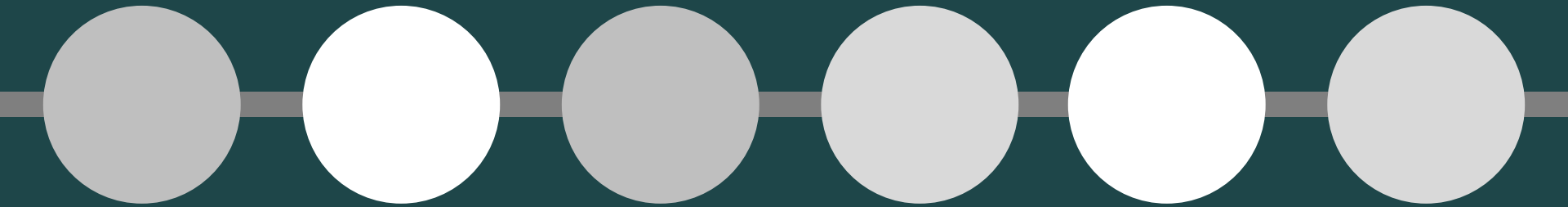
geometric locality

a geometric clock

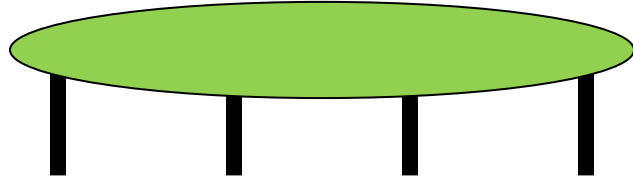


moving data on a line

l-H-a-m-i-l-t-o-n-i-a-n-s-i-n-1-D-L-o-c-a-l-H-a-m-i-l-t-o-n-i-a-n-s-i-n-1-D-L-o-c-a-l-H-a-m

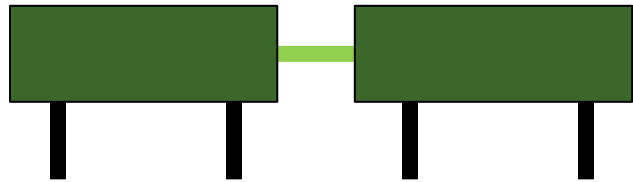


4 Matrix Product States



$$\sum_{s,t,u,v=0}^1 c_{stuv} |stuv\rangle$$

- Schmidt decomposition

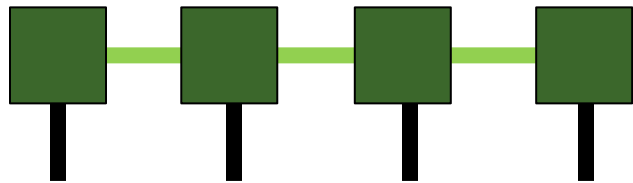


$$\sum_{b=1}^{\chi} Q_b^{st} R_b^{uv}$$

- many decompositions



a local description



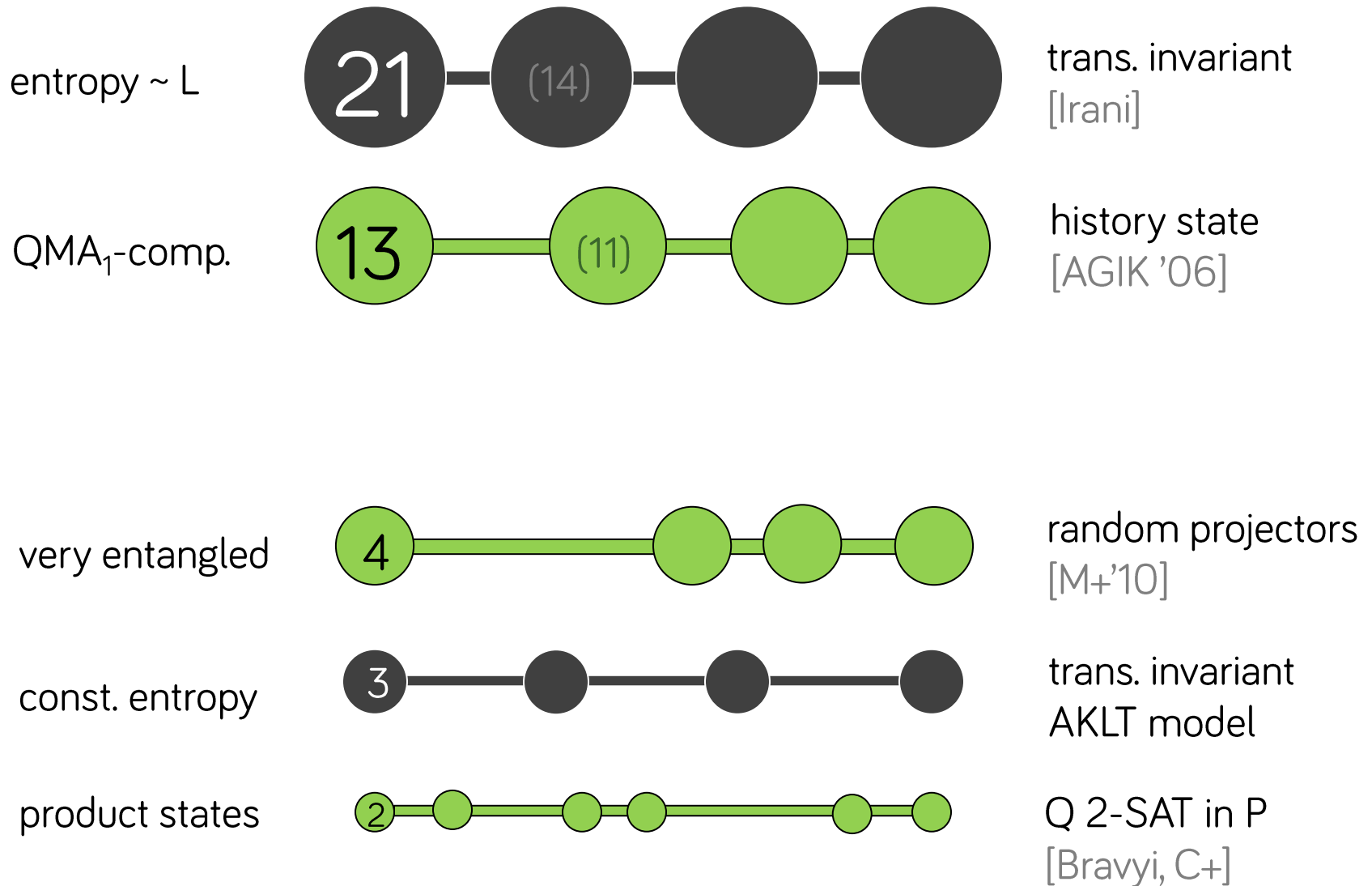
$$\sum_{b=1}^{\chi} \sum_{a=1}^{\chi} A_a^s B_{ab}^t \sum_{c=1}^{\chi} C_{bc}^u D_c^v$$

- low entanglement ansatz, local optimization, easy manipulation

4 Ground states in 1D

How hard is it to find/describe them?

constant gap: OK [Landau+ '13]

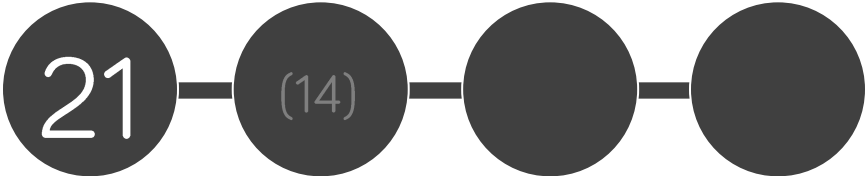


4 Ground states in 1D

How hard is it to find/describe them?

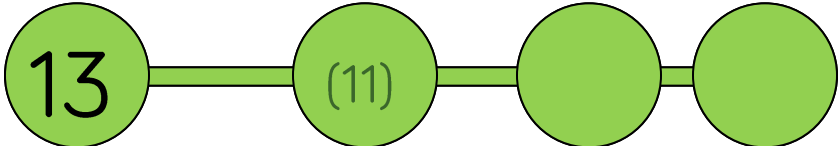
constant gap: OK [Landau+ '13]

entropy $\sim L$



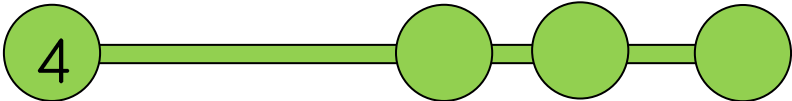
trans. invariant [Irani]

QMA_1 -comp.



history state [AGIK '06]

very entangled




random projectors [M+'10]

■ entropy: $\log L$



trans. invariant [B+'12]

product states

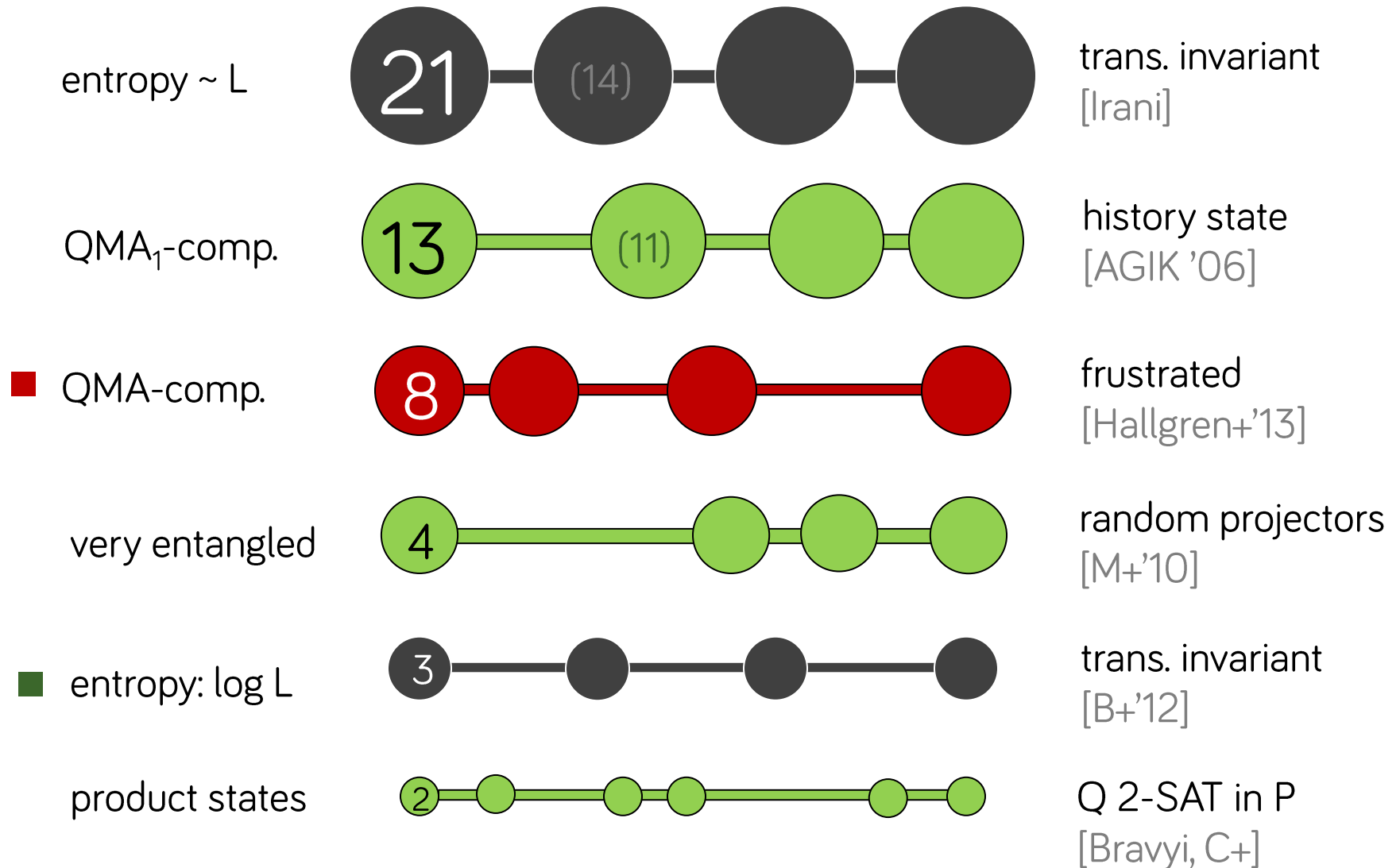


Q 2-SAT in P [Bravyi, C+]

4 Ground states in 1D

How hard is it to find/describe them?

constant gap: OK [Landau+ '13]



1

Hamiltonians?

optimization & dynamics



2

complexity

checking (quantum) proofs



3

ground states?

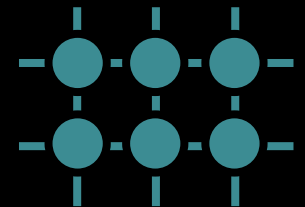
how hard is it to find them: QMA

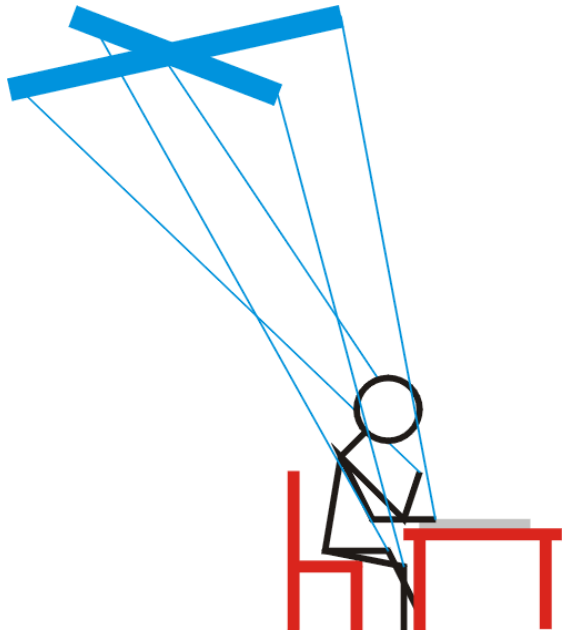


4

tensor networks

heuristics based on low entanglement





Local Hamiltonians & Quantum Complexity

Daniel Nagaj



2014 | 6 | 30

MPI PKS