

# Local Hamiltonians & Quantum Complexity

Daniel Nagaj

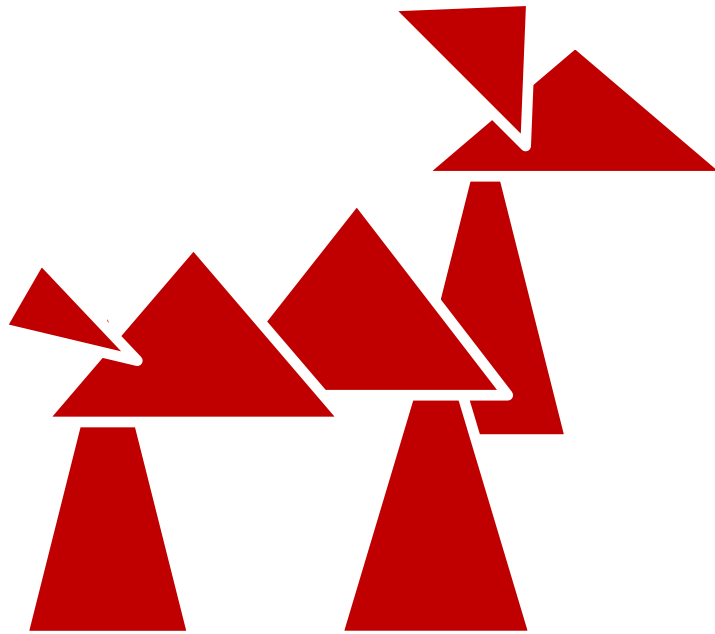


universität  
wien



2014 | 4 | 11

CS lunch @ USC



# Local Hamiltonians & Quantum Complexity

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universität  
wien



2014 | 4 | 11

CS lunch @ USC

# 1 Hamiltonians?

optimization & dynamics



# 2 complexity

checking (quantum) proofs



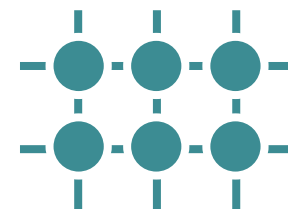
# 3 ground states?

how hard is it to find them: QMA



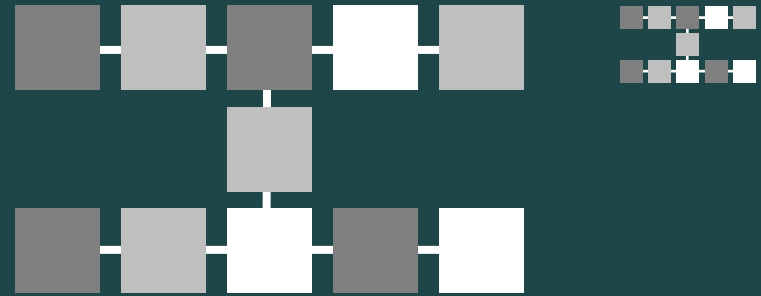
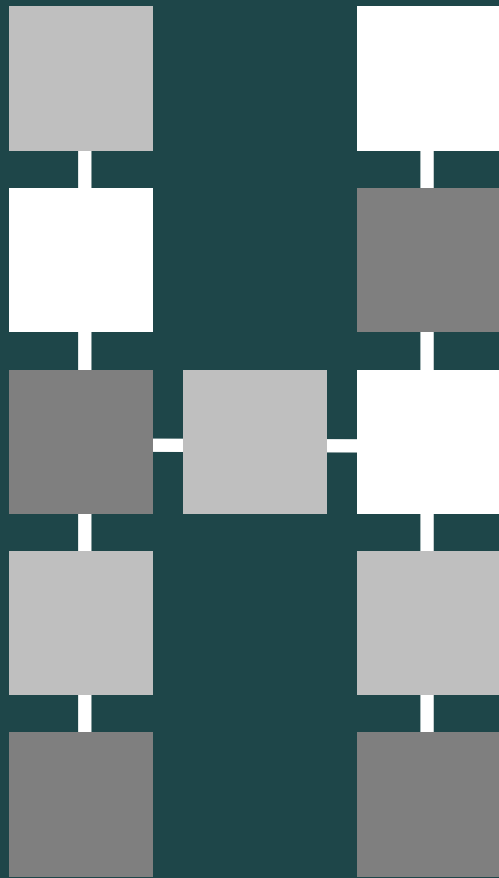
# 4 tensor networks

heuristics based on low entanglement

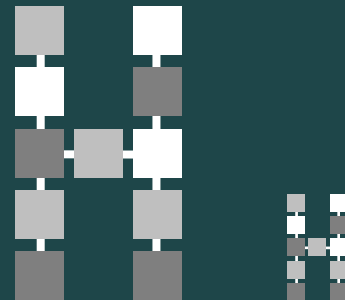


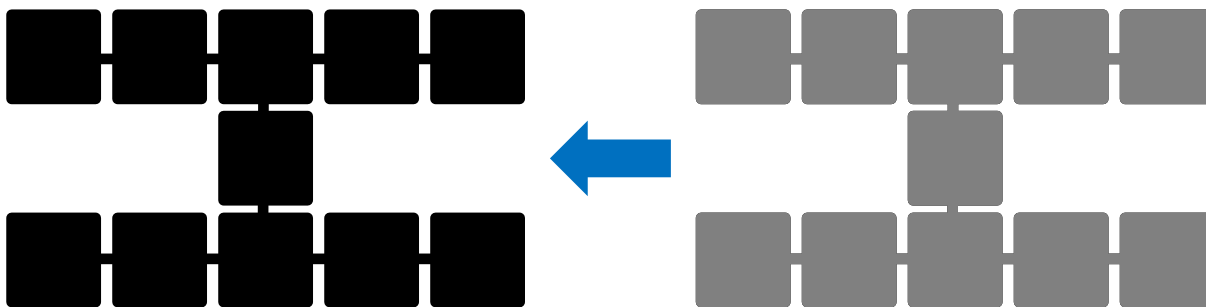
LH





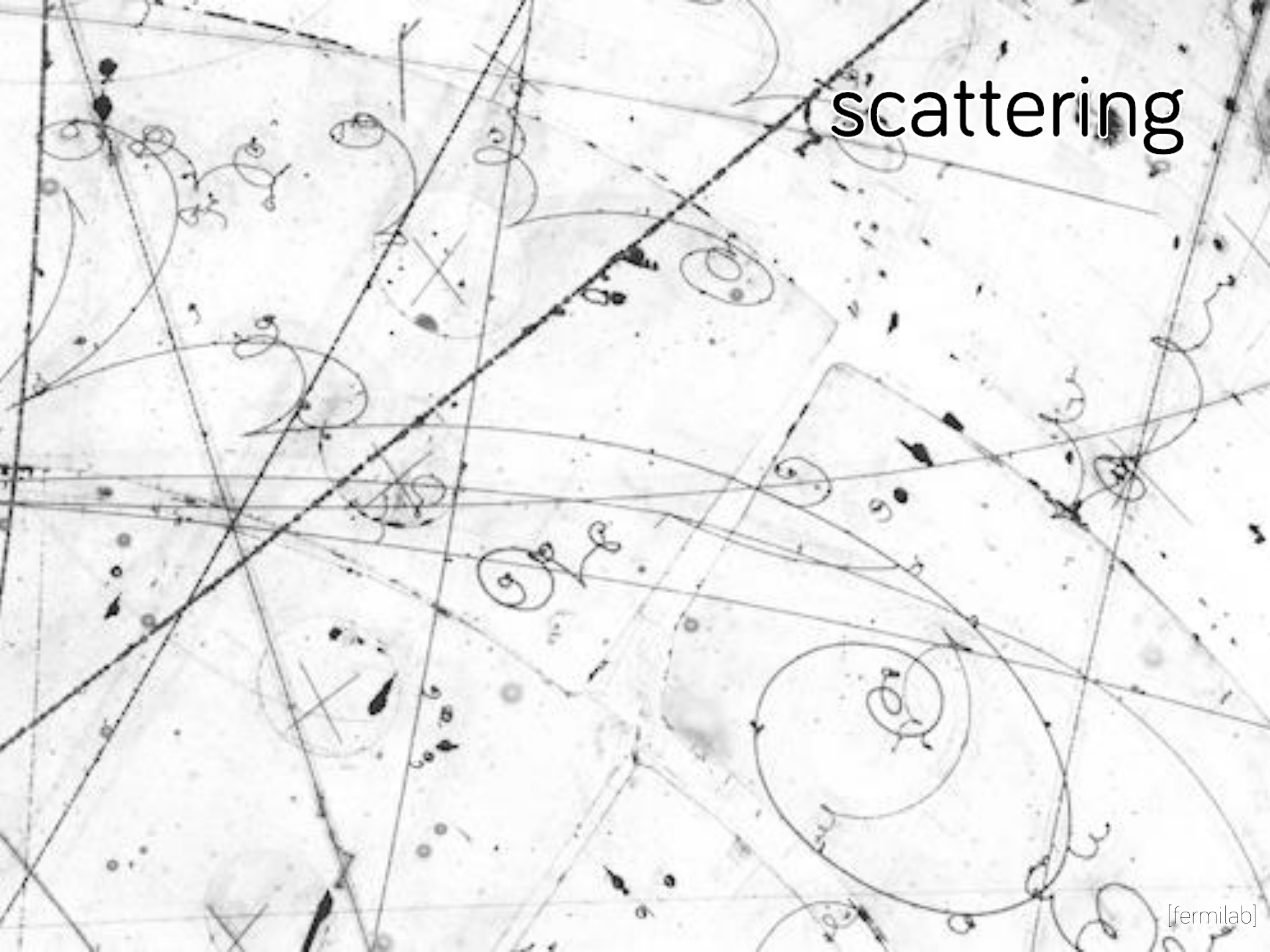
# Hamiltonians





dynamics  
 $i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$

scattering



# scattering

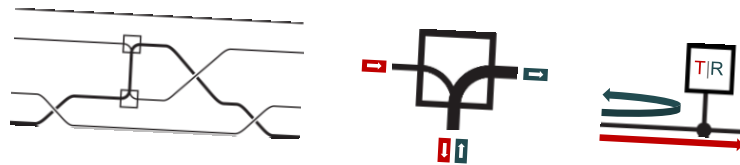
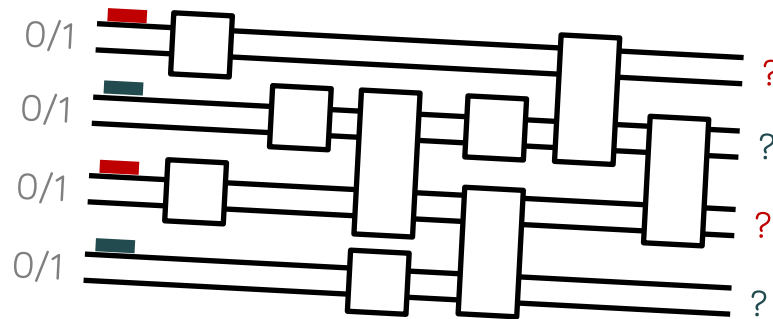
## Universal computation by multi-particle quantum walk

- dual-rail encoding  
N wavepackets

$$a_j^\dagger a_k + a_k^\dagger a_j$$

- CPHASE: interaction

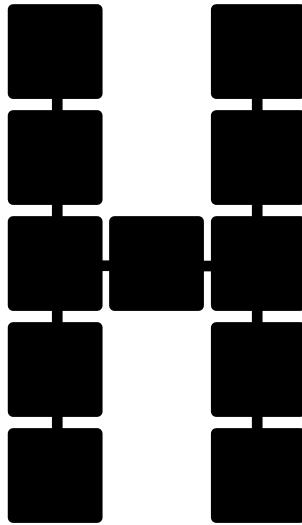
$$a_j^\dagger a_k^\dagger a_j a_k$$



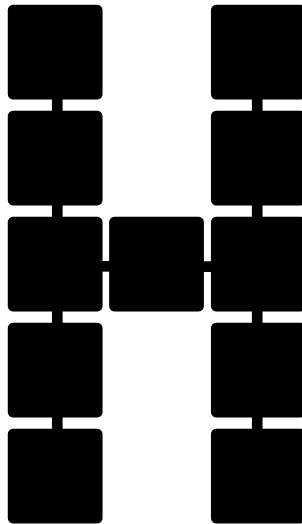
[Childs, Gosset, Webb, Science 339, 791 (2013)]



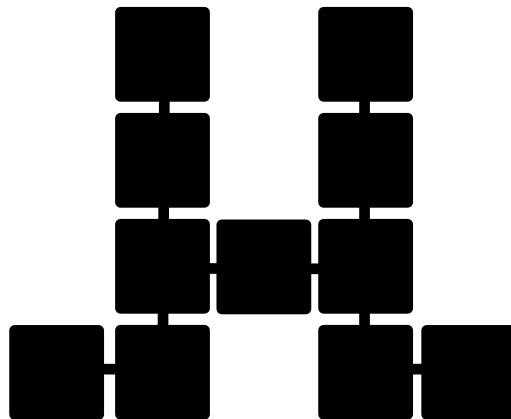
optimization



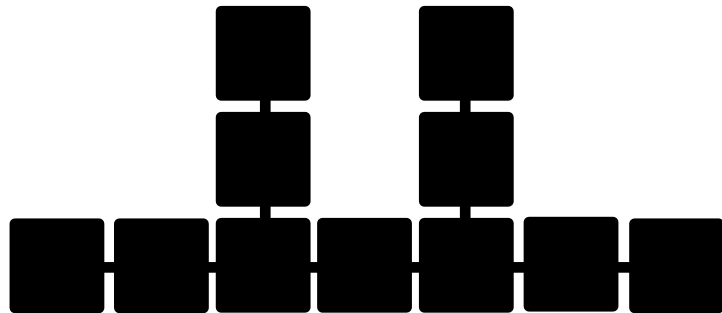
optimization



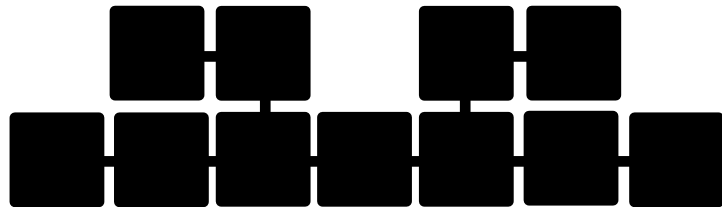
optimization



optimization

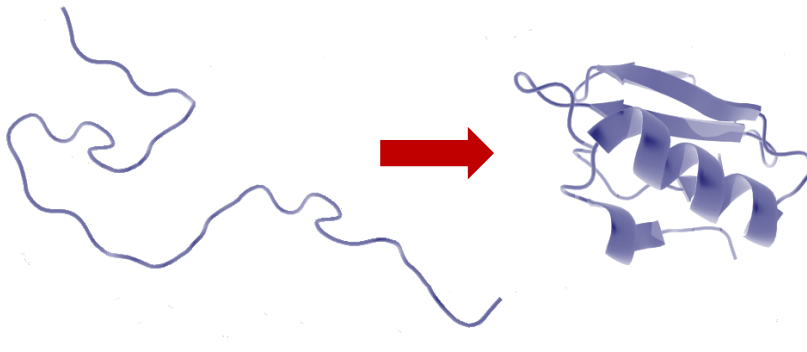


optimization

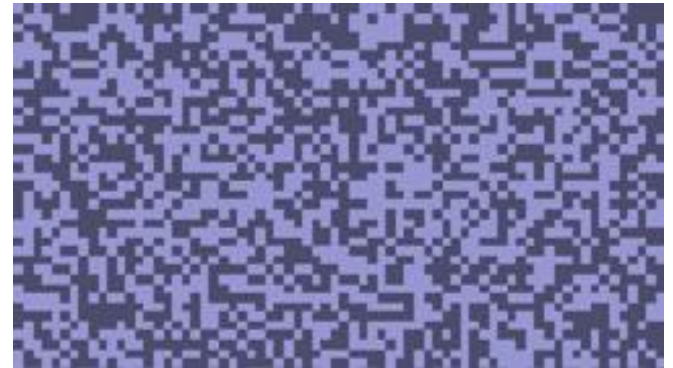


protein folding

spin glasses



[wikipedia]



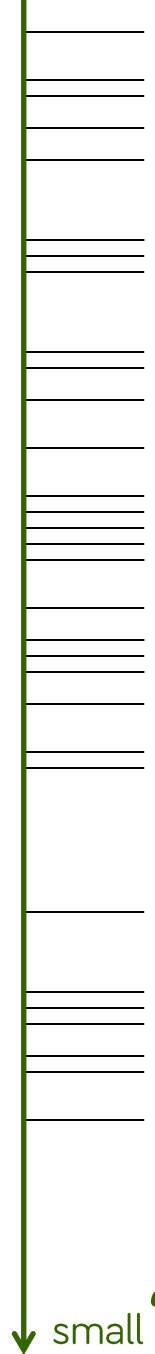
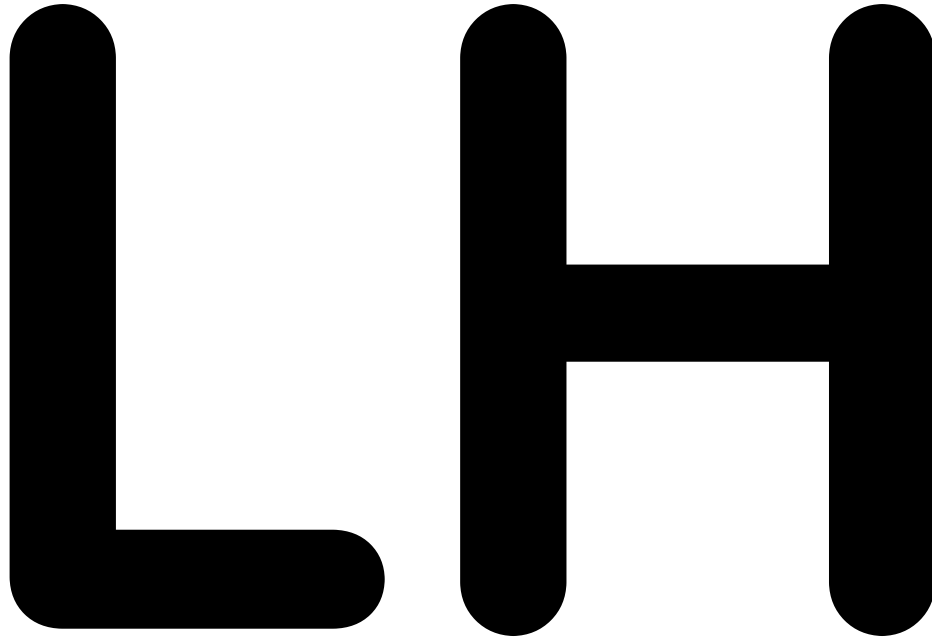
[uni-koeln.de]

local Hamiltonians

1

# Hamiltonians and their ground states

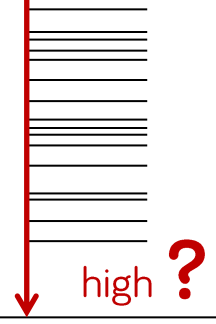
Is  
the  
ground  
state  
energy  
of a



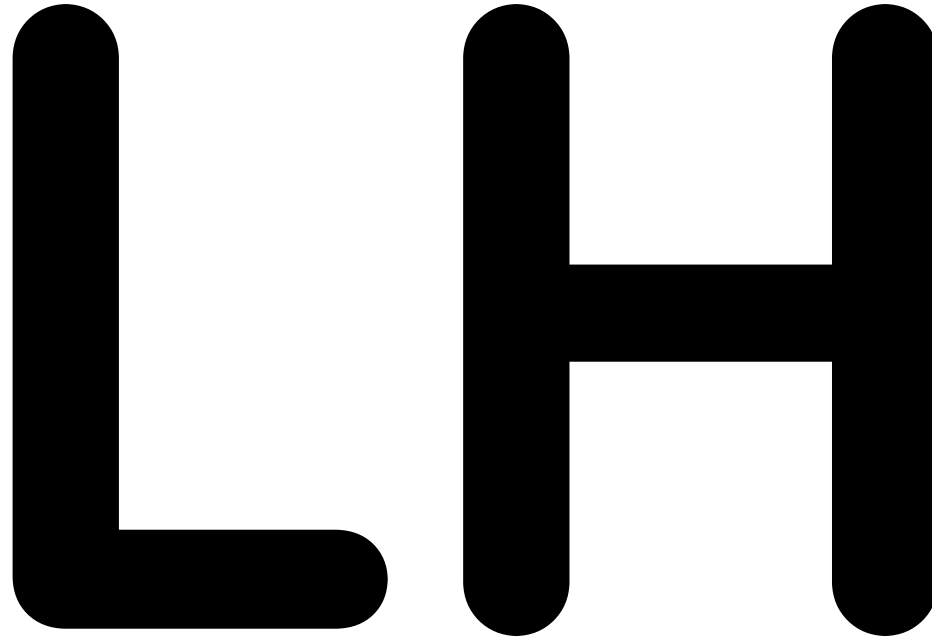
small?

1

# Hamiltonians and their ground states



Is  
the  
ground  
state  
energy  
of a

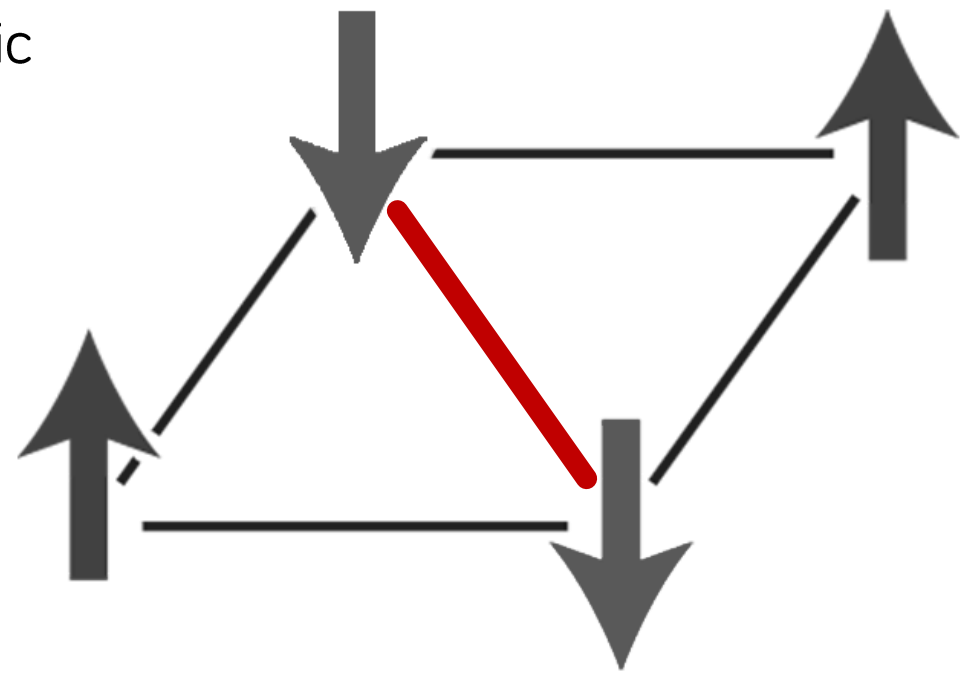




# 1 Frustrated systems

*Everybody can't be happy.*

antiferromagnetic  
spin glass



a global  
ground state

# HARD?

find & describe it?  
is it entangled?

frustrated

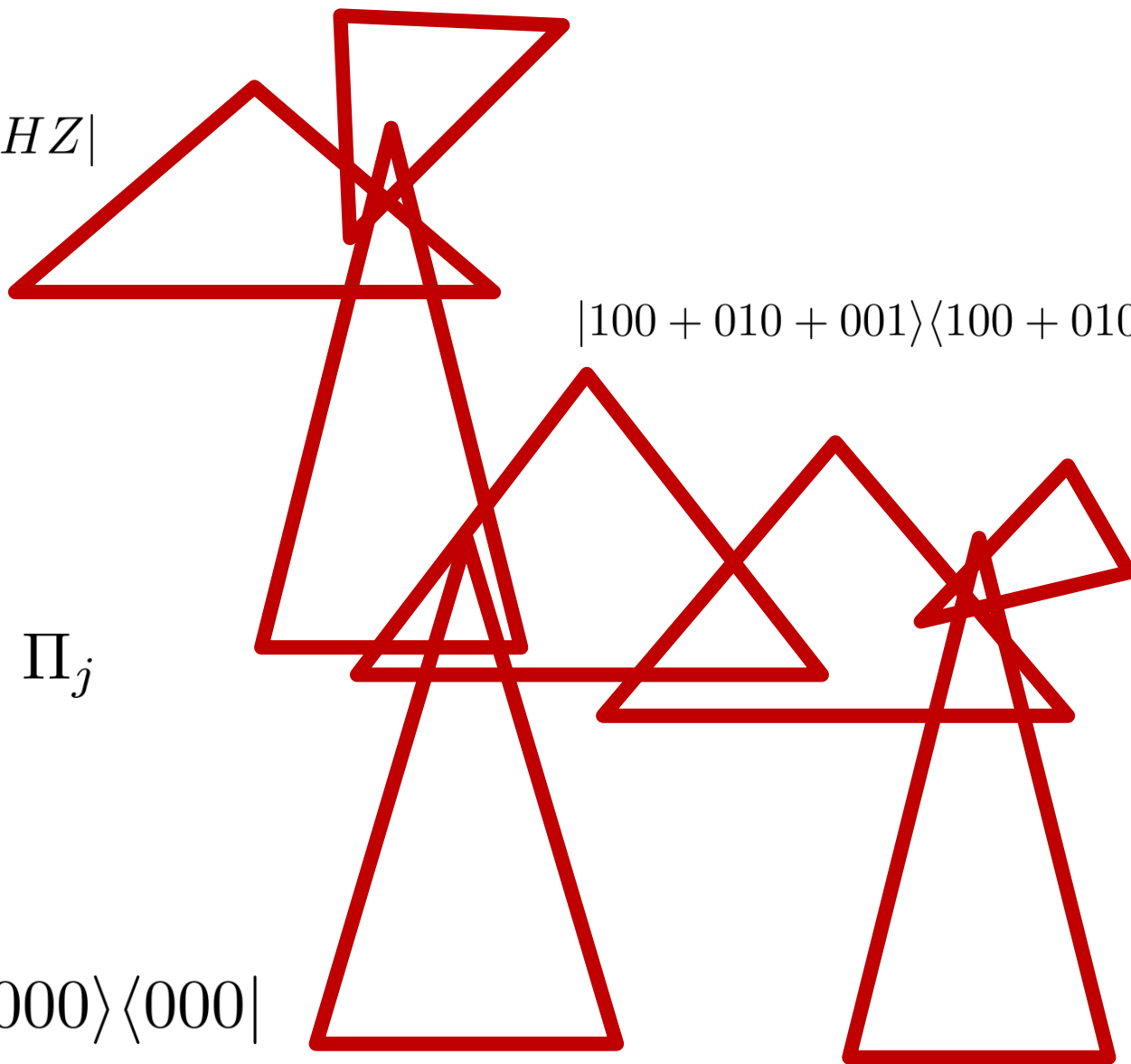
FRUST  
RATED

$|GHZ\rangle\langle GHZ|$

$|100 + 010 + 001\rangle\langle 100 + 010 + 001|$

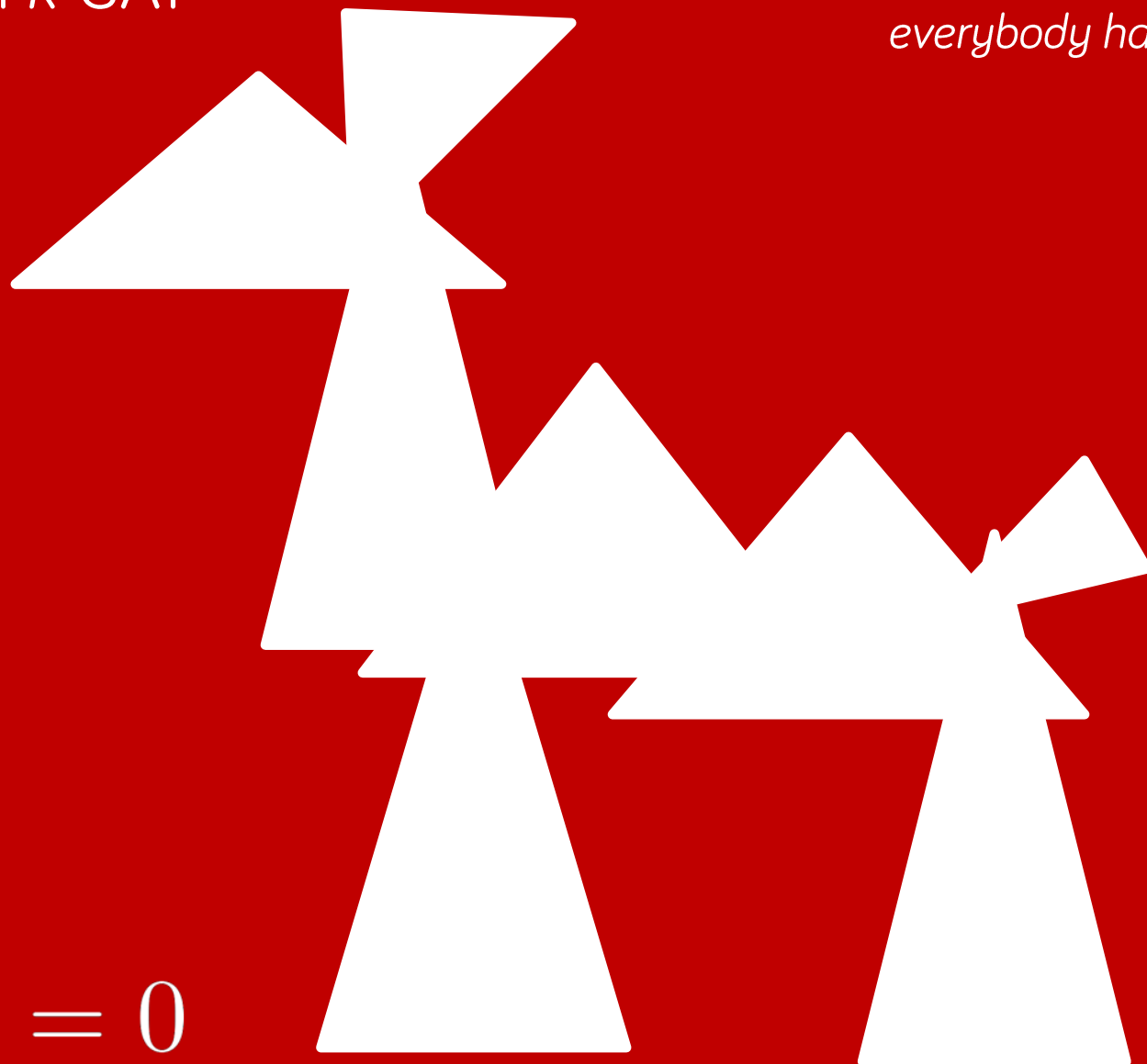
■  $k$ -local  
projectors  $\Pi_j$

$|000\rangle\langle 000|$



# Quantum $k$ -SAT

*Can we make everybody happy?*



$$\Pi^j |\psi\rangle = 0$$

# 1 Local ( $k$ -body) Hamiltonians

- optimization

QMA-completeness

$$H(t) = \sum_j H_j(t)$$

- dynamics

BQP universality

- local particle dimension



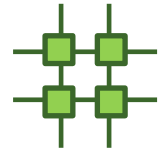
- interaction geometry



- time independence



- translational invariance



- promise gap, eigenvalue gap, energy  $\times$  time cost



how hard  
is this  
question



# We Can Do It!



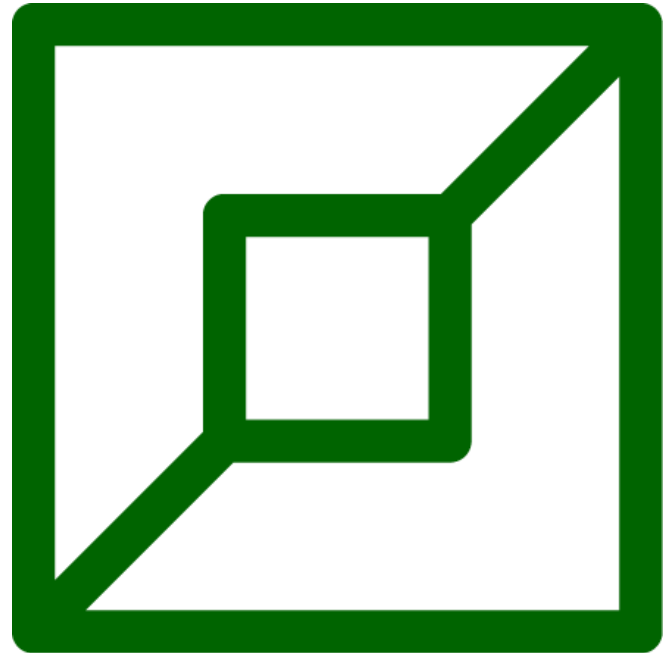
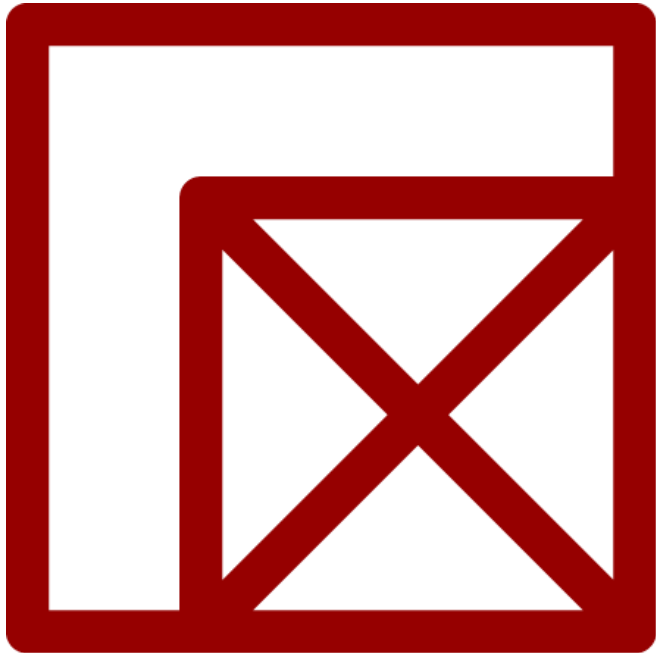
# P

[Howard Miller]

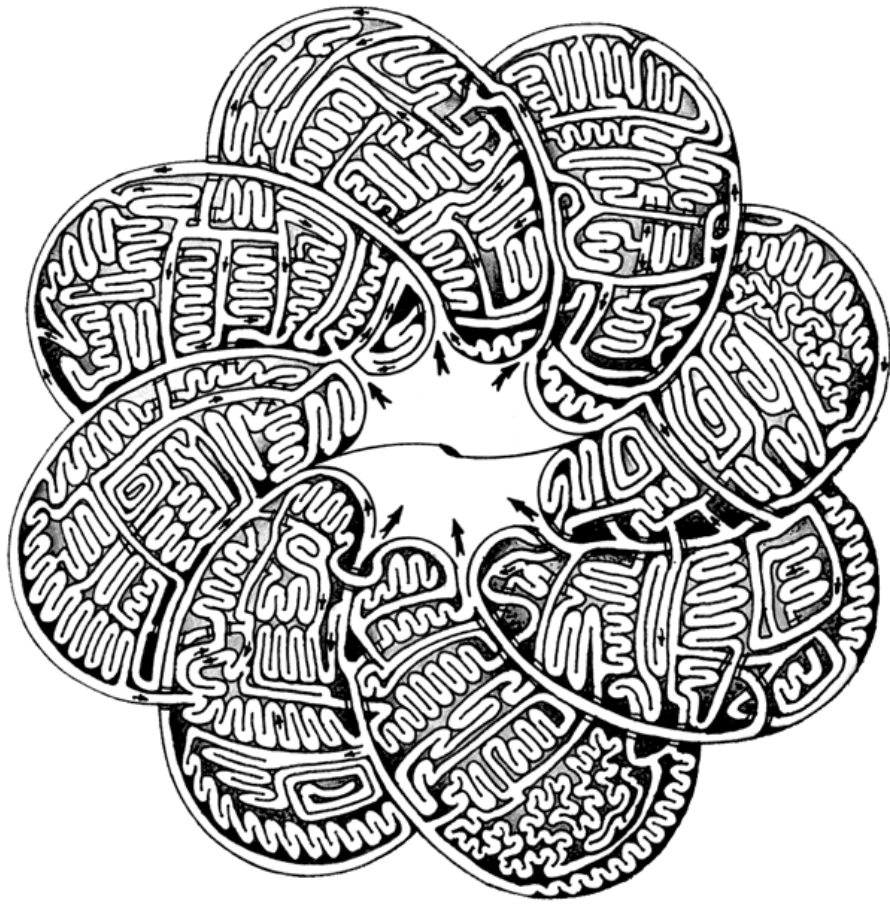
POST FEB. 15 TO FEB. 20



WAR PRODUCTION CO-ORDINATING COMMITTEE





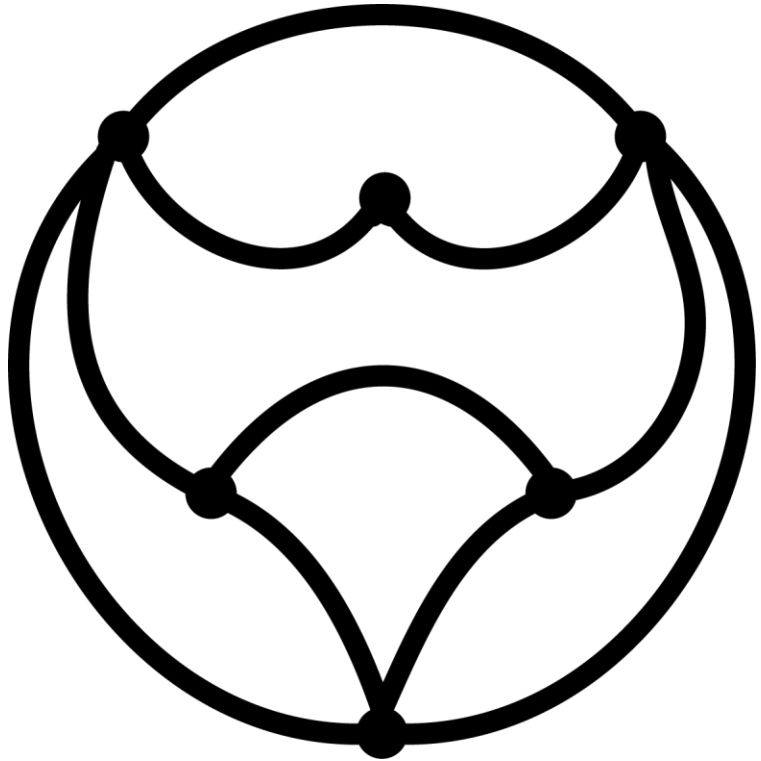
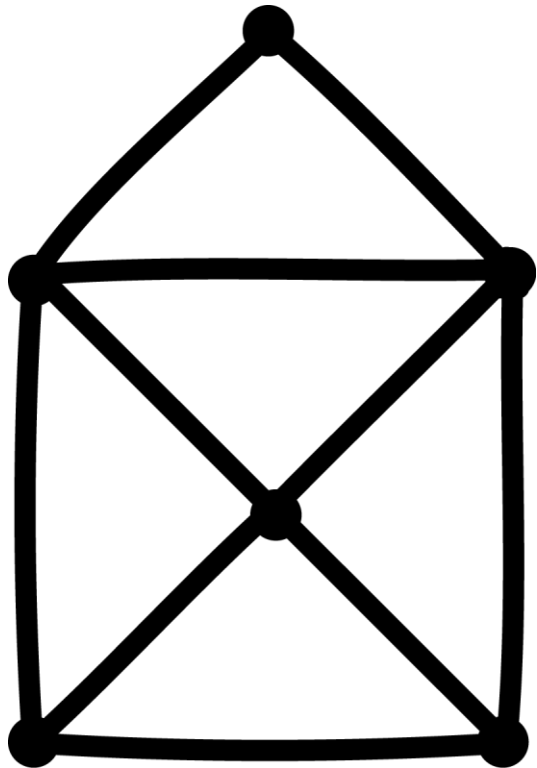


# NP

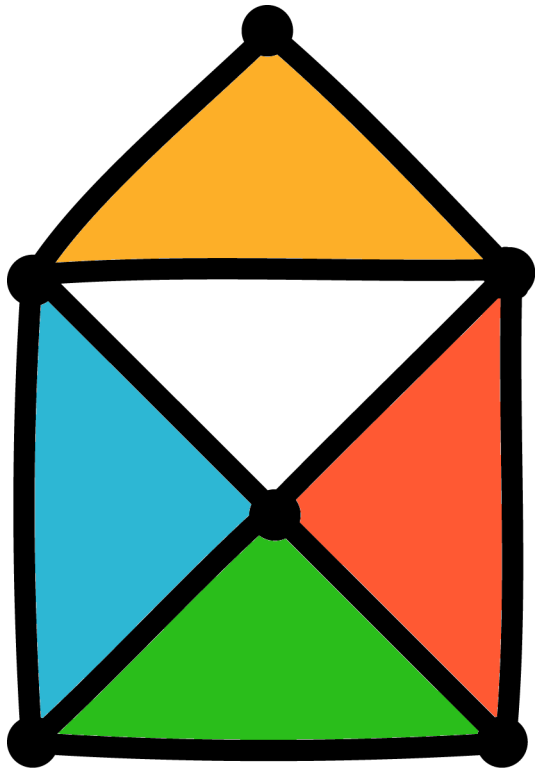


[maze: Andrew Bernhardt]  
[A+M: [primaryresources.co.uk](http://primaryresources.co.uk)]

## 2 A graph isomorphism puzzle



## 2 A graph isomorphism puzzle



2 A cryptarithmic puzzle

$$\begin{array}{r} \text{DID} \\ + \text{DINOS} \\ \hline \text{CROAK} \end{array}$$



2 A cryptarithmic puzzle

$$\begin{array}{r} \phantom{+} 595 \\ + 59842 \\ \hline 60437 \end{array}$$



## 2 The NP protocol

*Did dinosaurs exist?*



a proof

## 2 The NP protocol

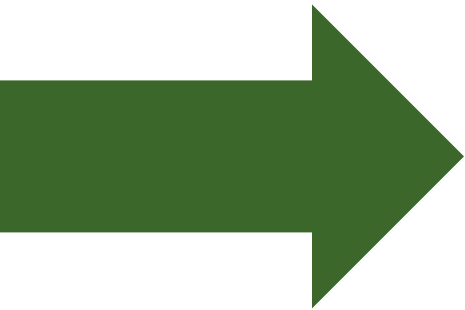
*Did dinosaurs exist?*



a witness

## 2 The class NP

*Yes/no questions, easy to verify solutions.*



a verification  
circuit

*from a uniform family*



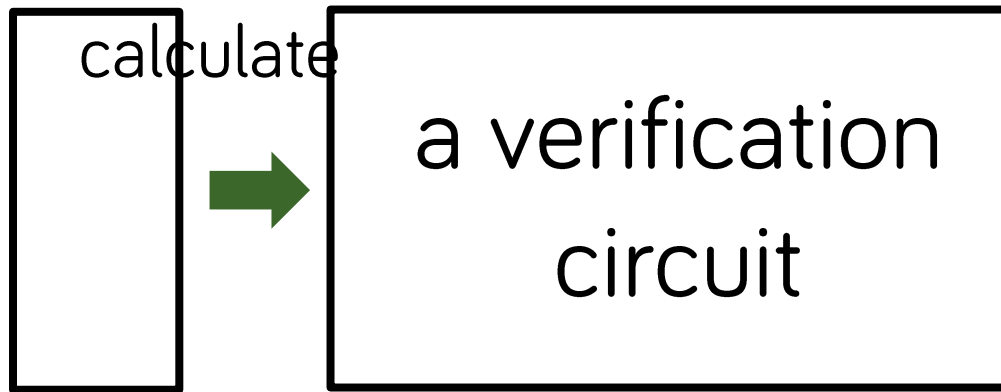
YES? Accept a good proof.

NO? Reject forgeries.



## 2 The class P

*Yes/no questions that we can answer.*

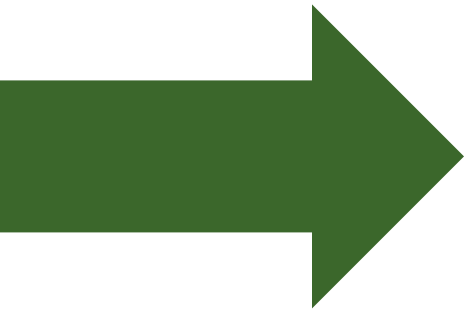


YES? Figure it out by yourself.

NO? Figure it out by yourself.

## 2 The class NP

*Yes/no questions, easy to verify solutions.*



a verification  
circuit

*from a uniform family*



YES? Accept a good proof.

NO? Reject any witness.

## 2 NP-hardness

The mother of them all.

- Can you solve this problem? You just solved all of NP.



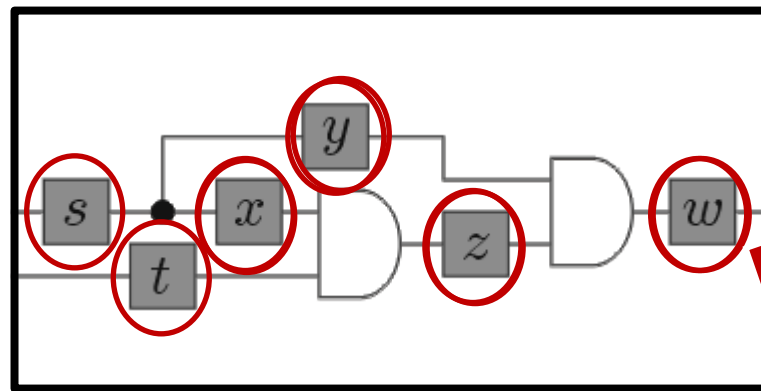
a verification  
circuit

Could this  
circuit ever  
output 1?

## 2 NP-hardness

The mother of them all.

- Can you solve this problem? You just solved all of NP.



Could this circuit ever output 1?

3-local conditions

$$(\dots \vee \dots \vee \dots) \wedge (\dots \vee \dots \vee \dots) \wedge \dots$$

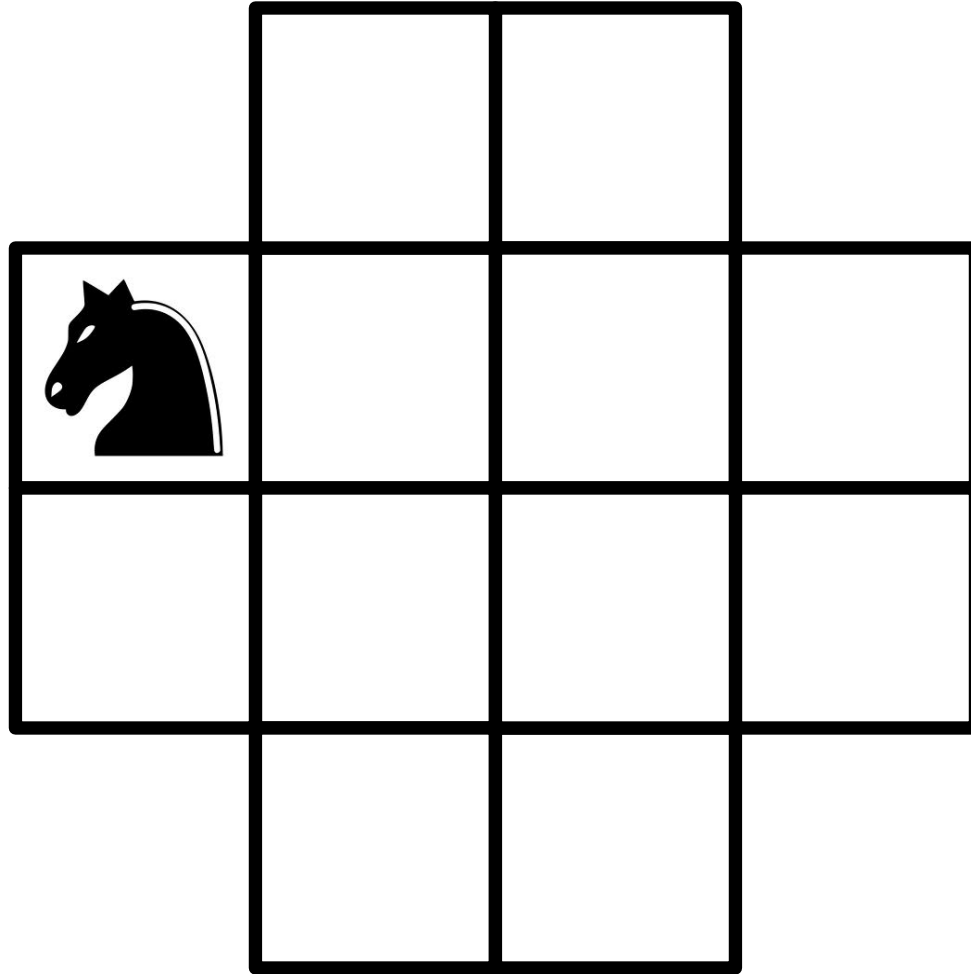
- 3-SAT is NP-hard.

in NP.

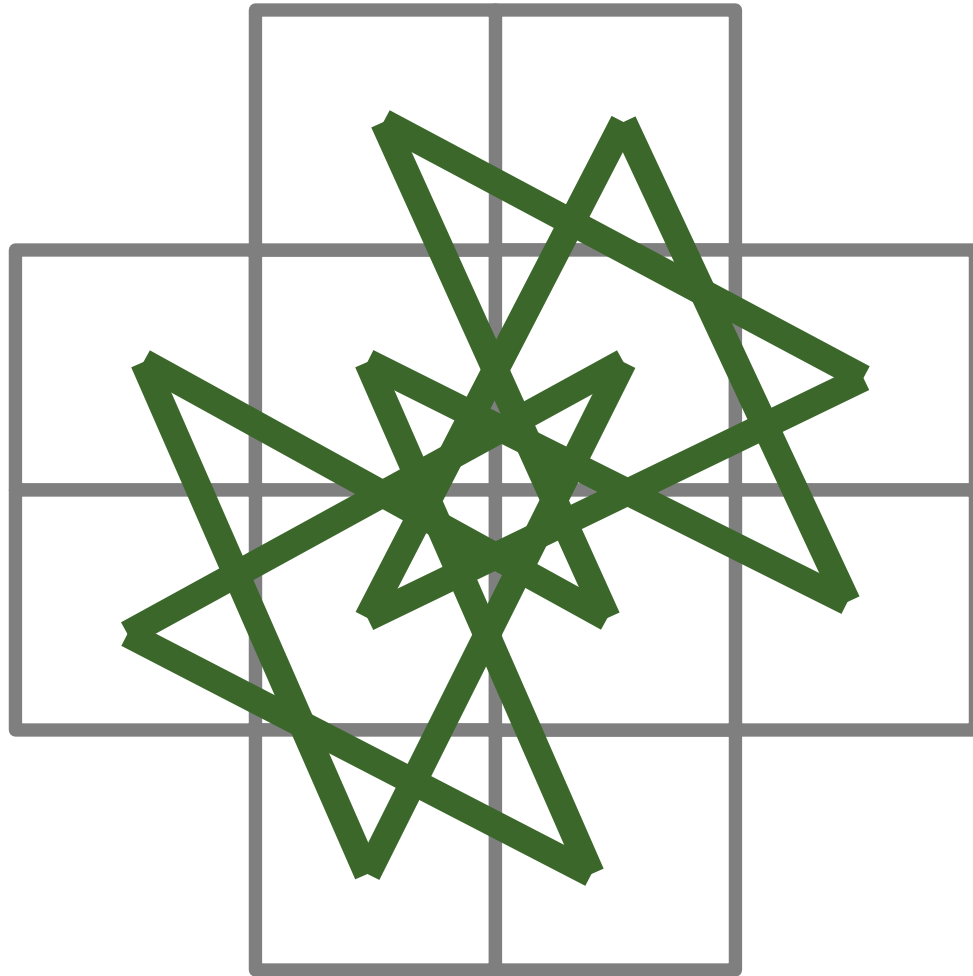
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NP-complete. [Cook, Levin]

## 2 Hamiltonian cycle (also NP-c)

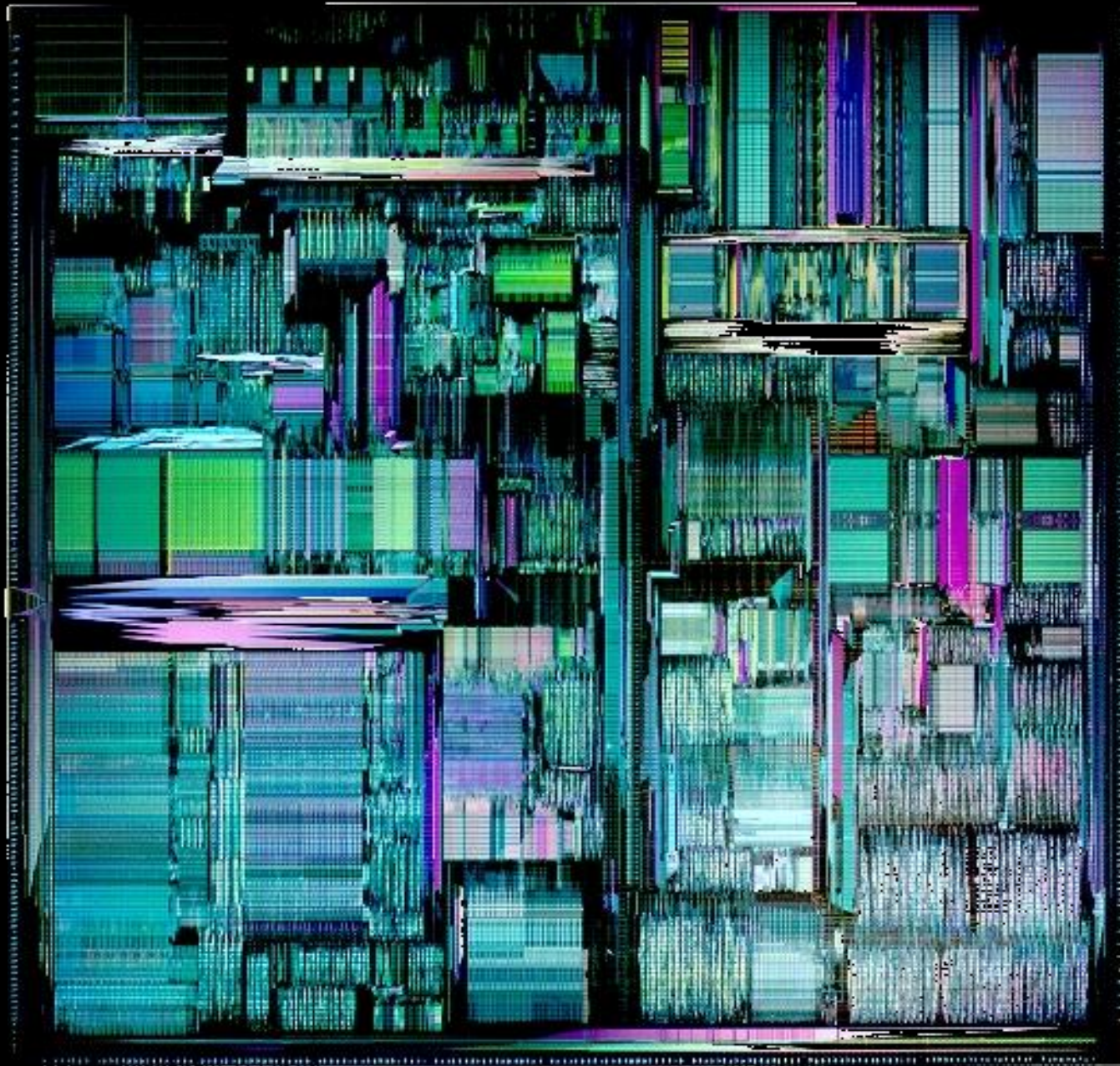


## 2 Hamiltonian cycle (also NP-c)



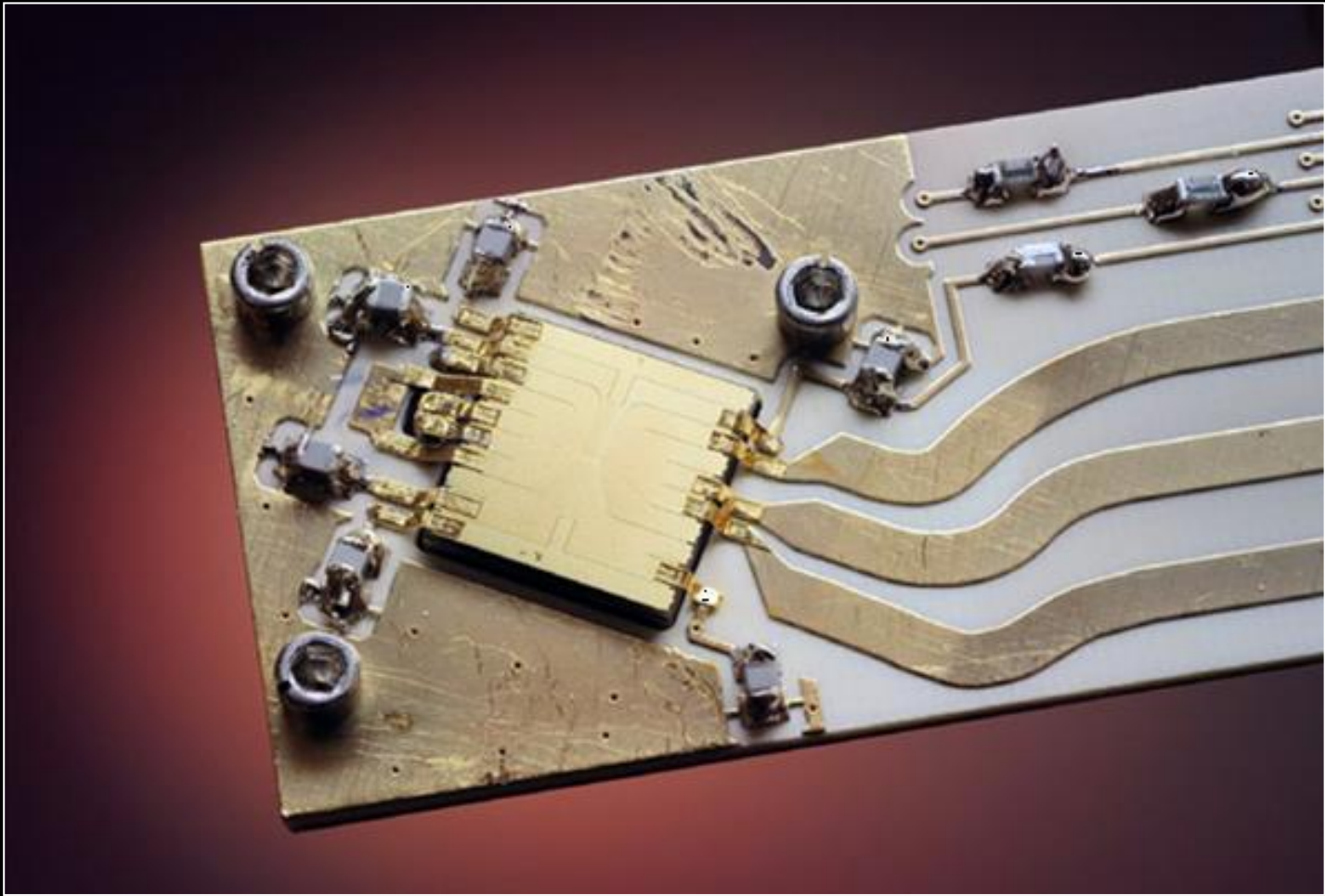


the puzzles  
of QMA



[1995 Pentium Pro





[NIST gold ion trap on aluminum-nitride backing, Y.Colombe/NIST]

DAVID

[wire



BOUNDED ERROR  
QUANTUM  
POLYNOMIAL TIME

[tha

## 2 The MA protocol

*Did dinosaurs exist?*





## 2 The MA protocol

*Did dinosaurs exist?*



## 2 The MA protocol

*Did dinosaurs exist?*

YES?  
Eager to be  
convinced.



[magnifying glass: hllllllal]

## 2 The MA protocol

*Recognizing fakes?*



## 2 The MA protocol

*Recognizing fakes?*

NO?  
Don't be  
fooled  
easily.





## 2 Probabilistic checks

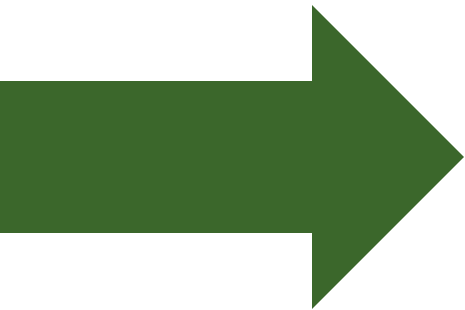
Sometimes reject  
a genuine proof?

Accept  
a fake?



# 1 The MA protocol

*Probabilistic checks.*



probabilistic  
verification

*from a uniform family*



YES? Accept a good proof with  $p > a$ .

NO? Probability of accepting  $p < b$ .



## 2 The QMA protocol

Quantum checks.



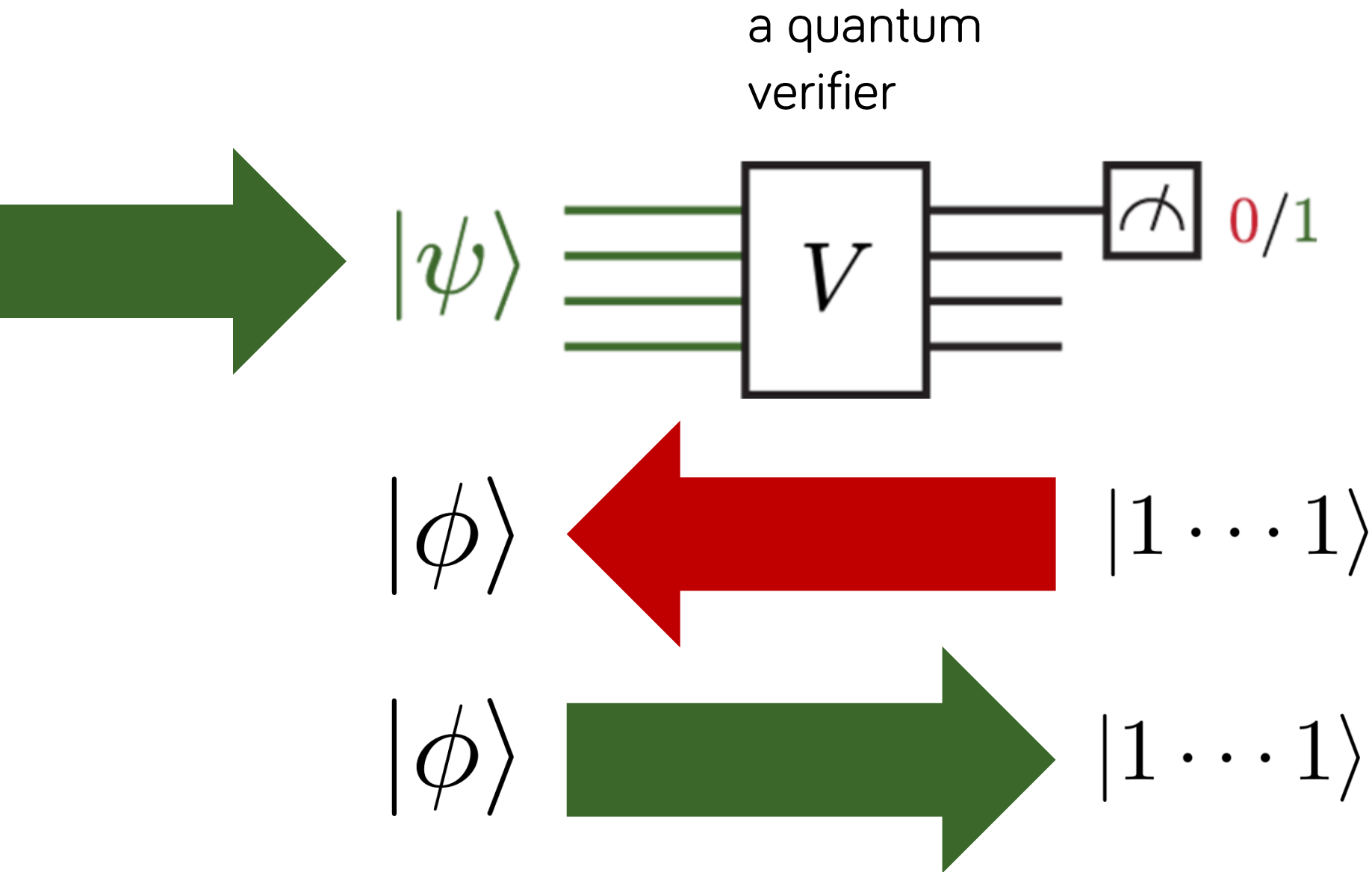
YES? Accept a good proof with  $p > a$ .

NO? Probability of accepting  $p < b$ .



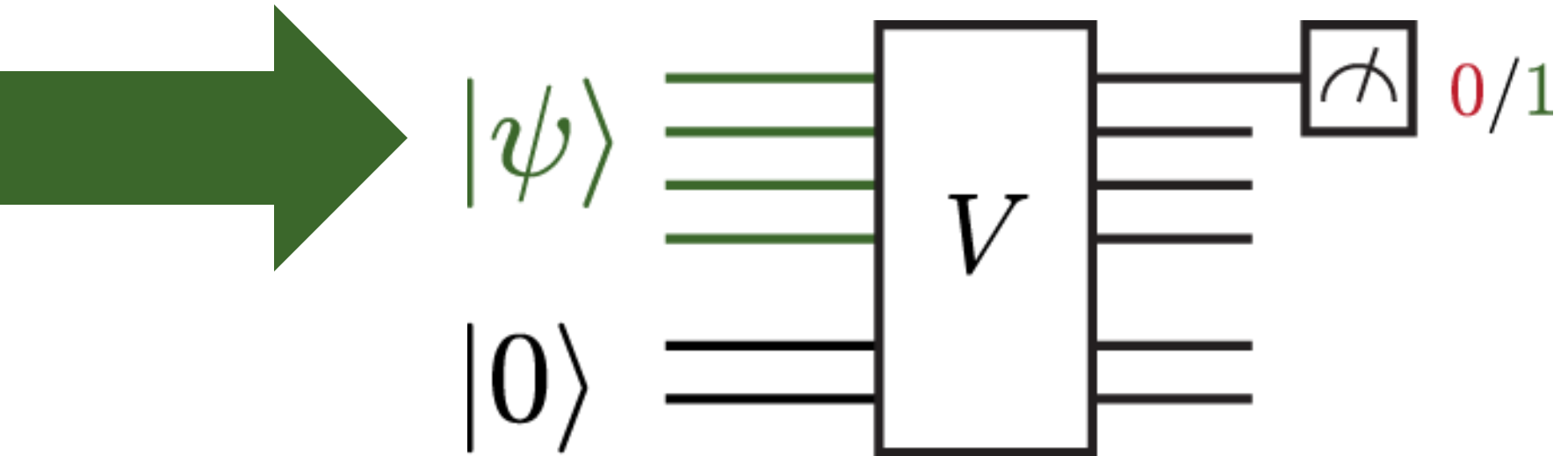
## 2 The QMA protocol

*This is too simple.*



## 2 The QMA protocol

*Ancillas are necessary.*

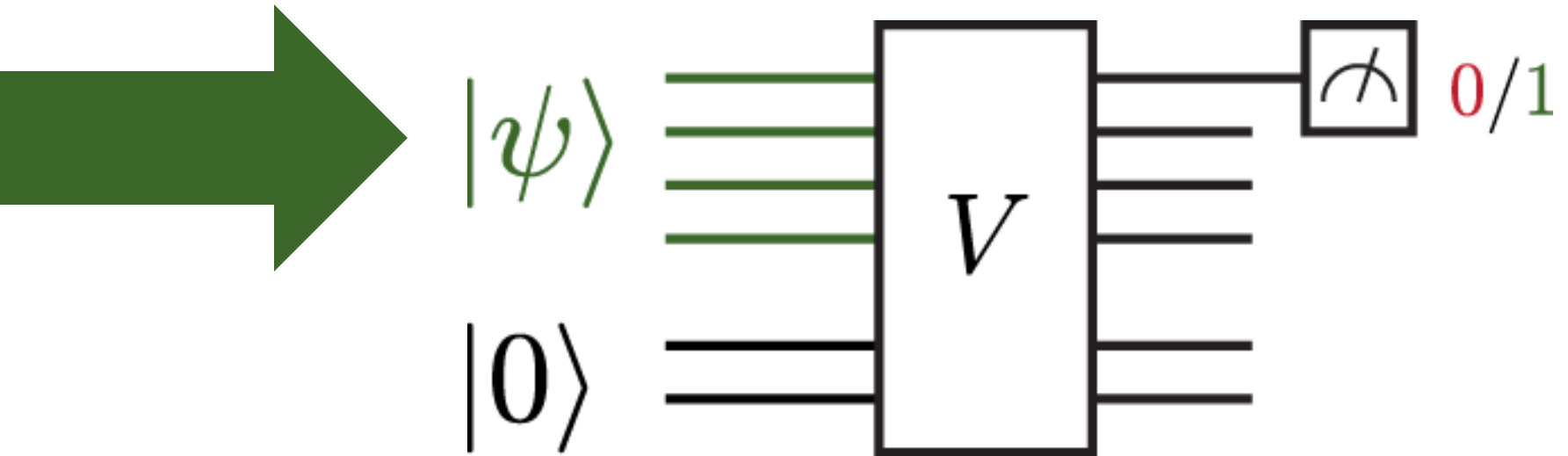


YES? Accept a good proof with  $p > a$ .

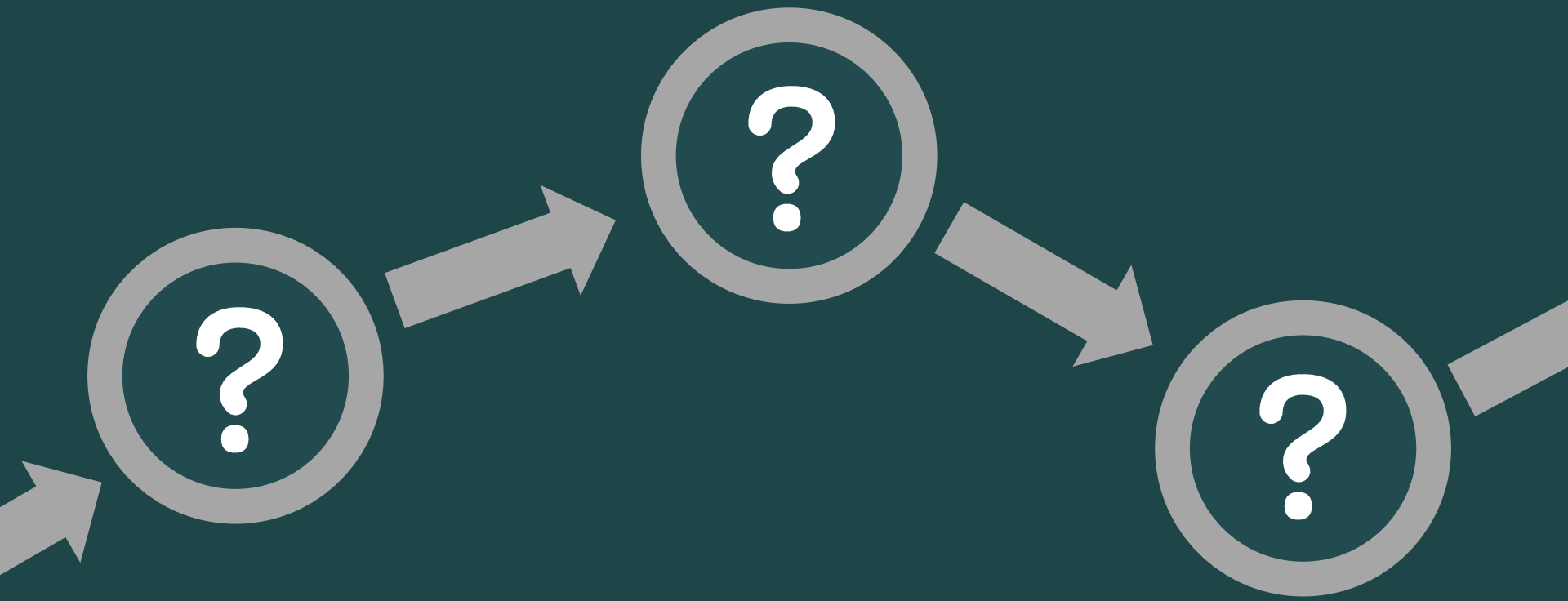
NO? Probability of accepting  $p < b$ .



## 2 A QMA-hard question

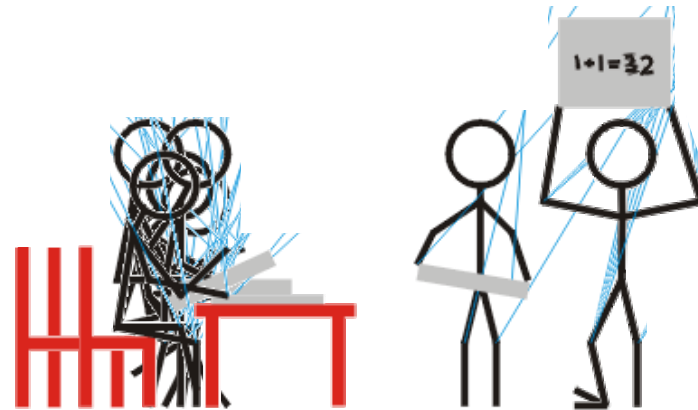


Could we feed this quantum verifier something that likely outputs 1?



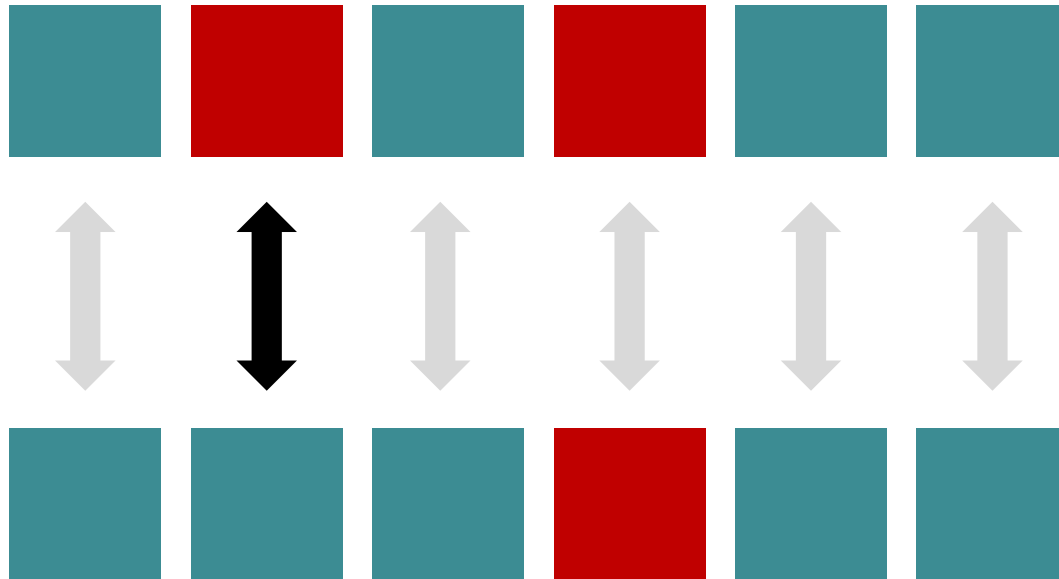
the history state  
ground

## 2 Snapshots of a computation

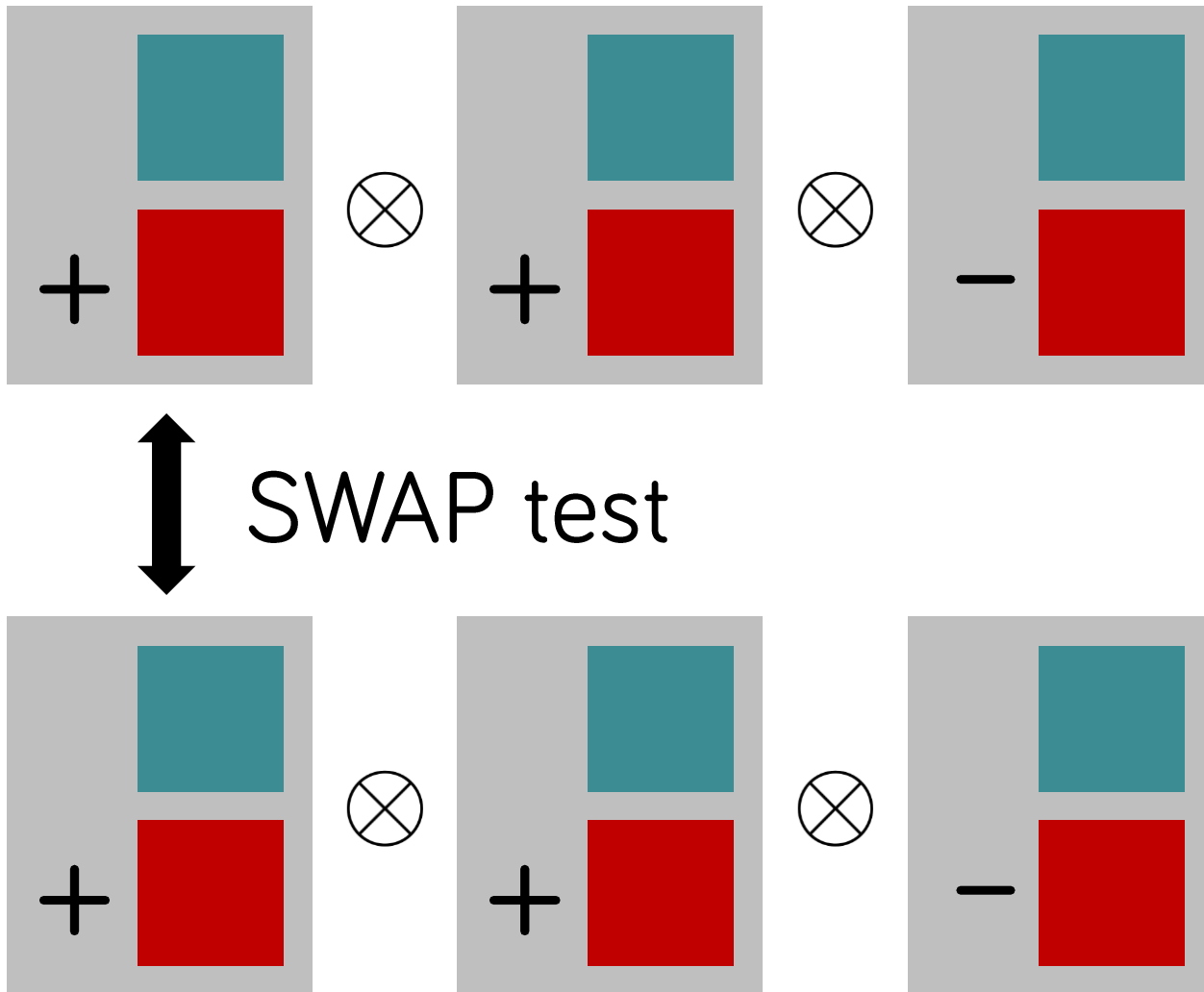




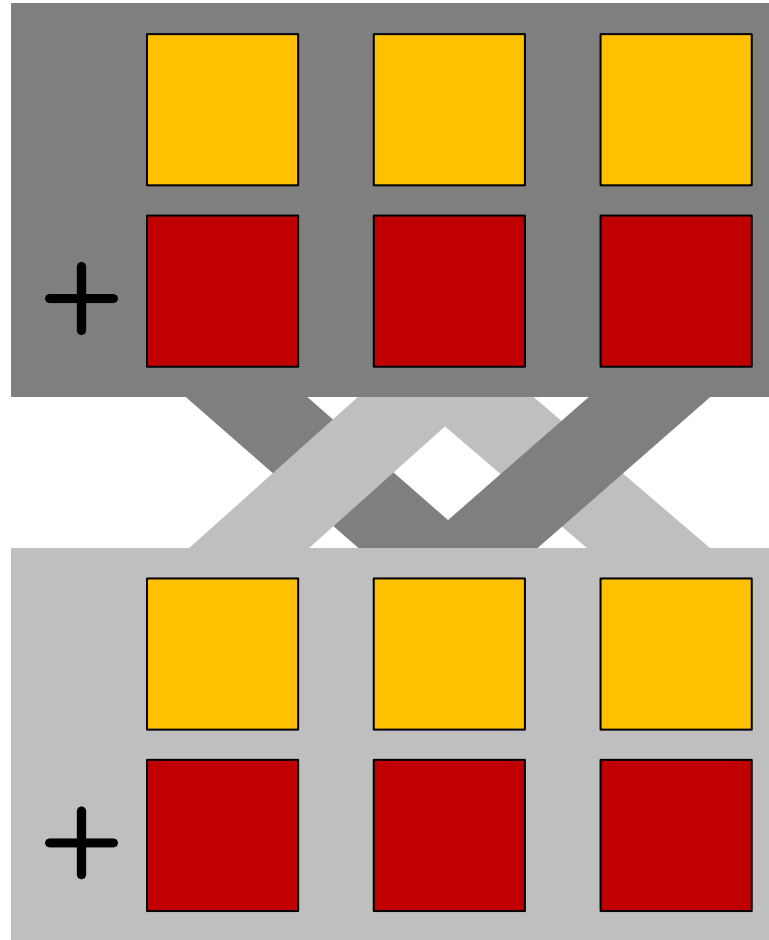
Locally comparing **strings**.



Locally comparing product states.

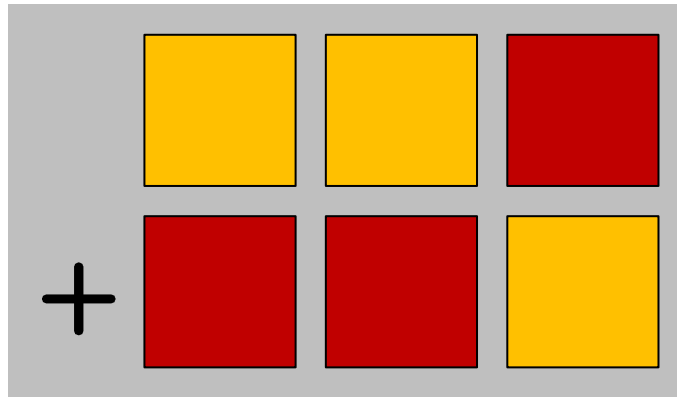


Locally comparing entangled states?



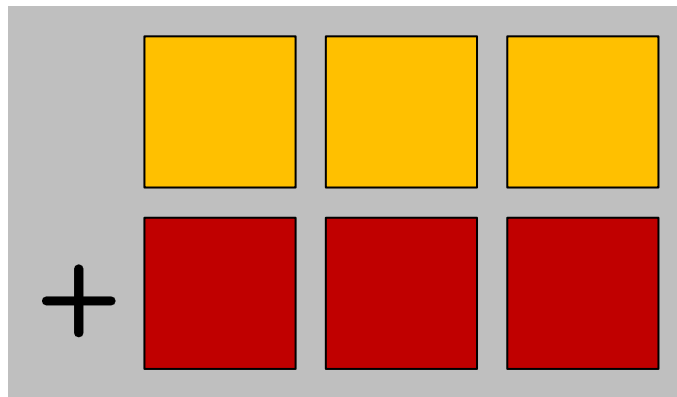
UGH!

## 2 Labeling the data



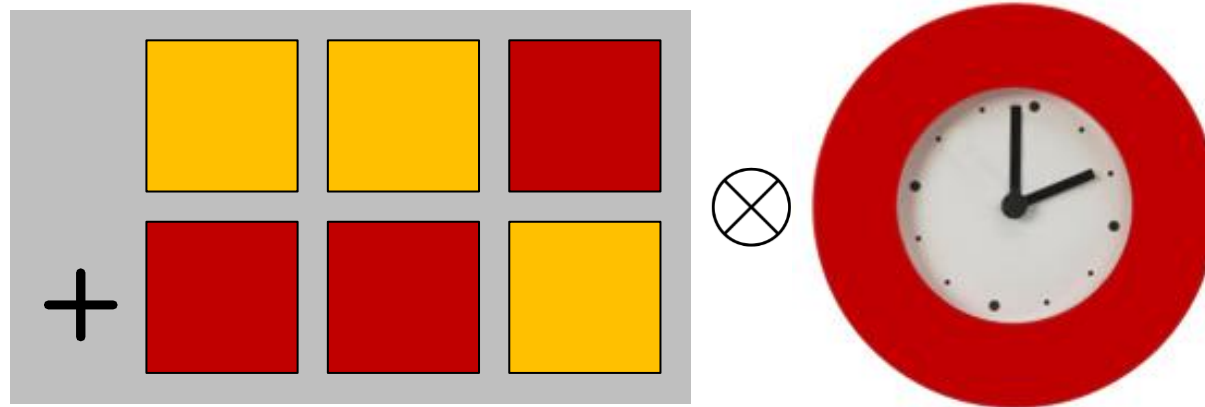
$U^\dagger$     $U$

Hard to compare directly (locally).

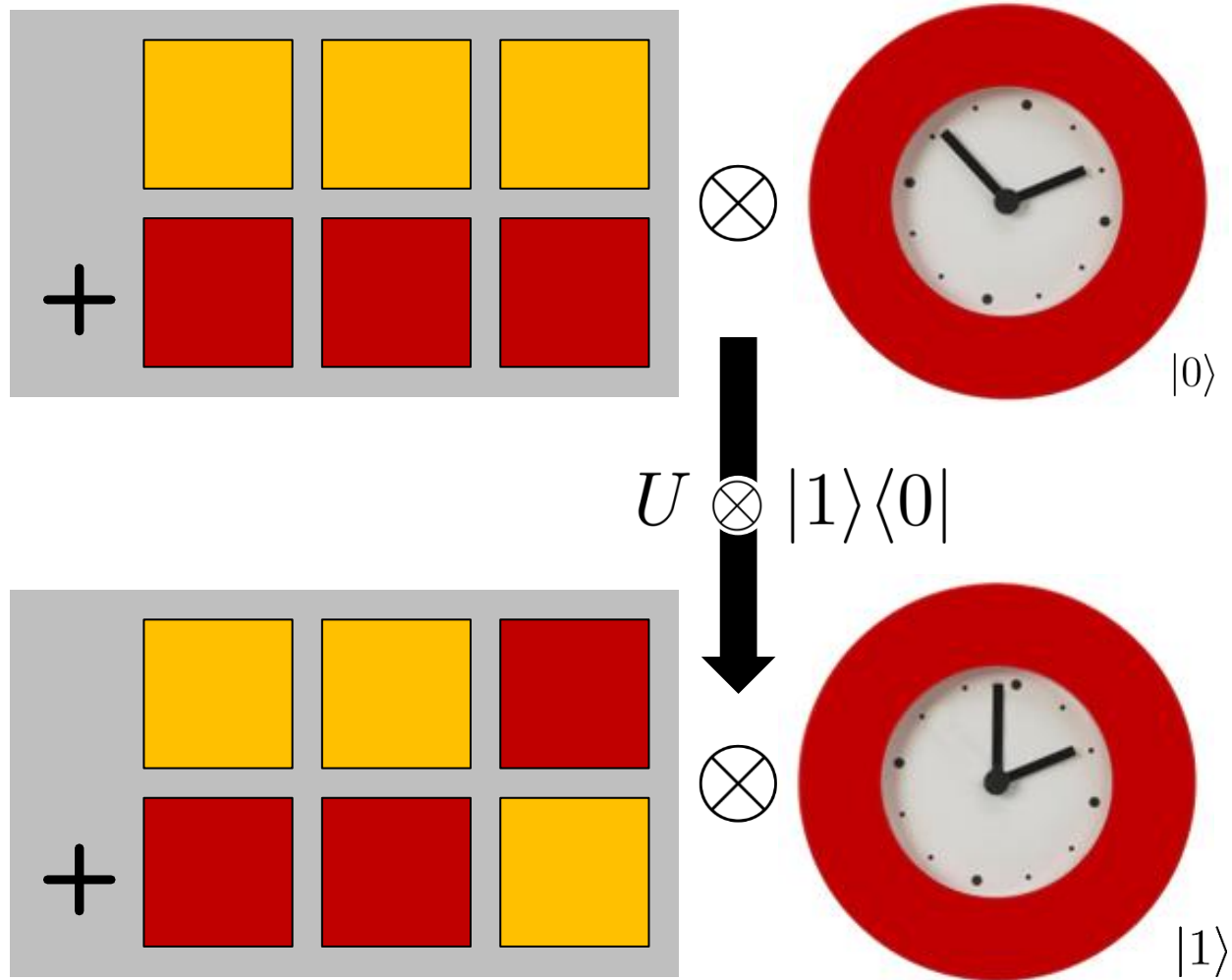


*a clock*

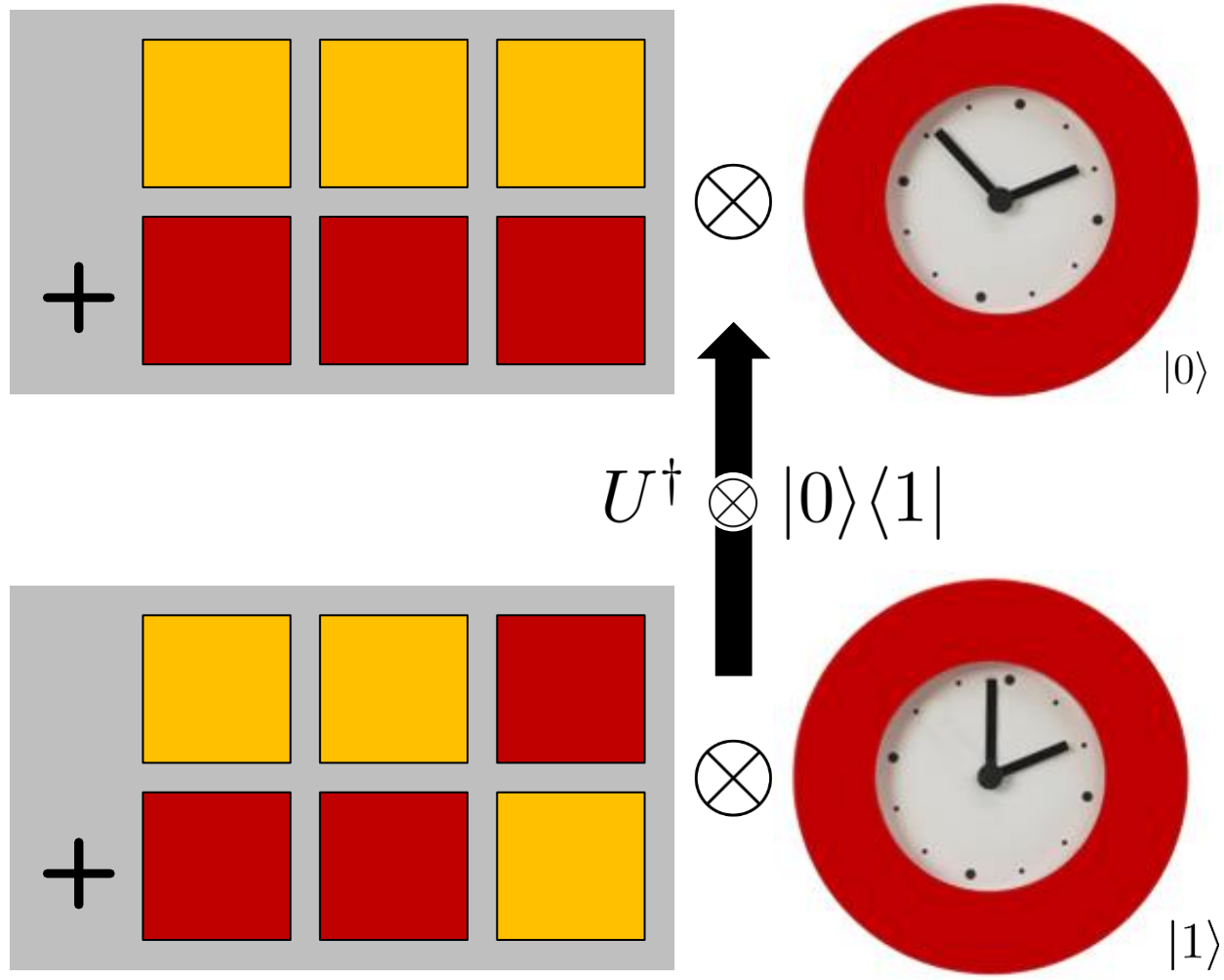
## 2 Labeling the data



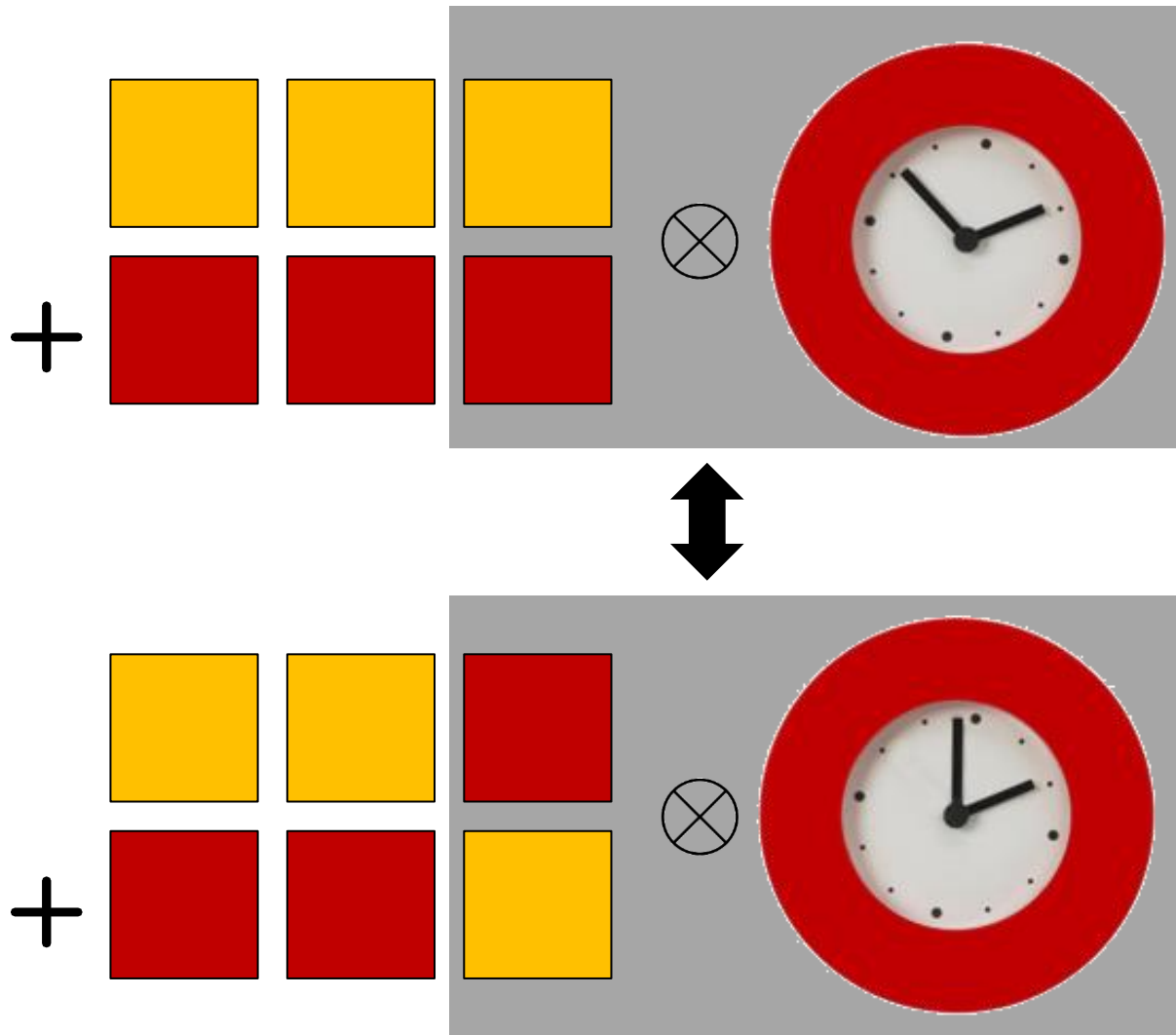
## 2 The data & the clock



## 2 The data & the clock



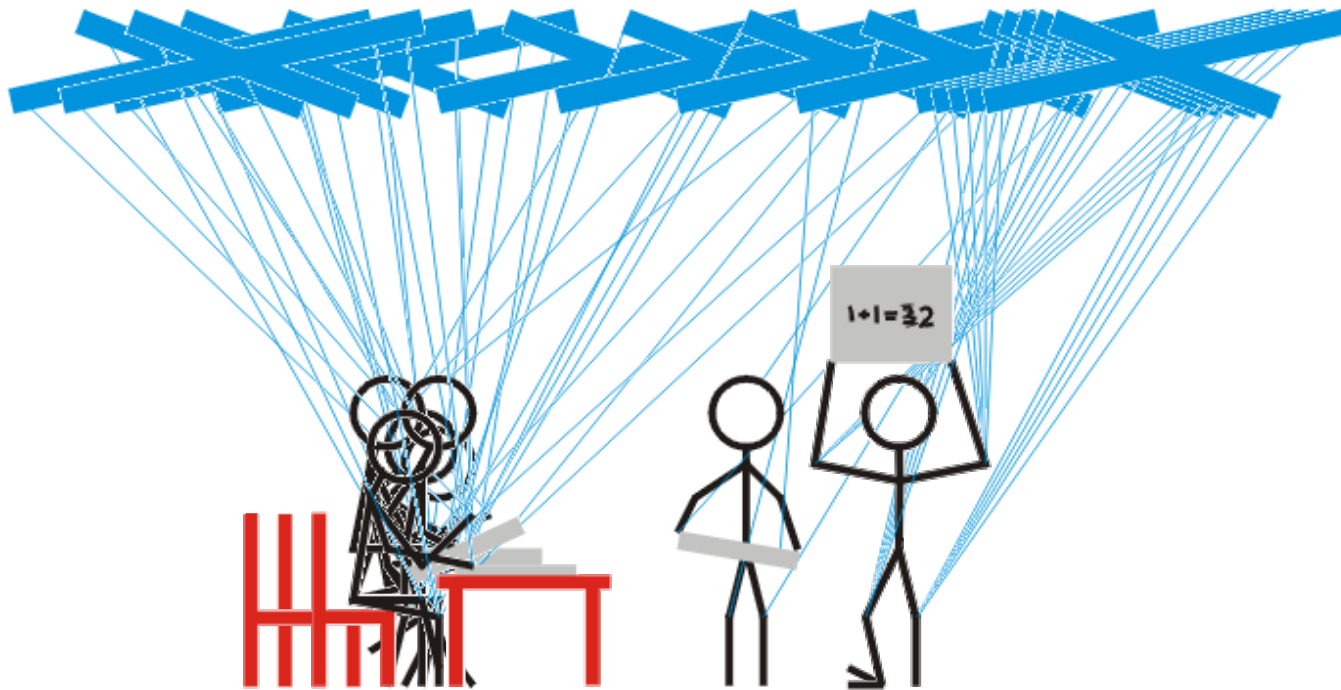
## 2 The data & the clock: locally comparing related states



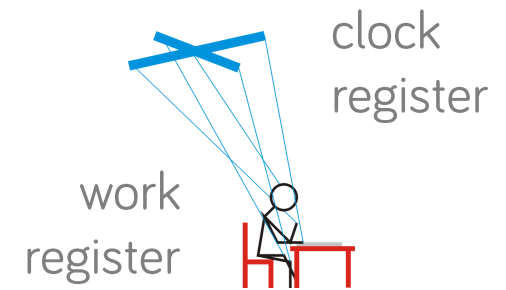


## 2 Feynman's computer

The data & a pointer.



## 2 Feynman's (Hamiltonian) computer

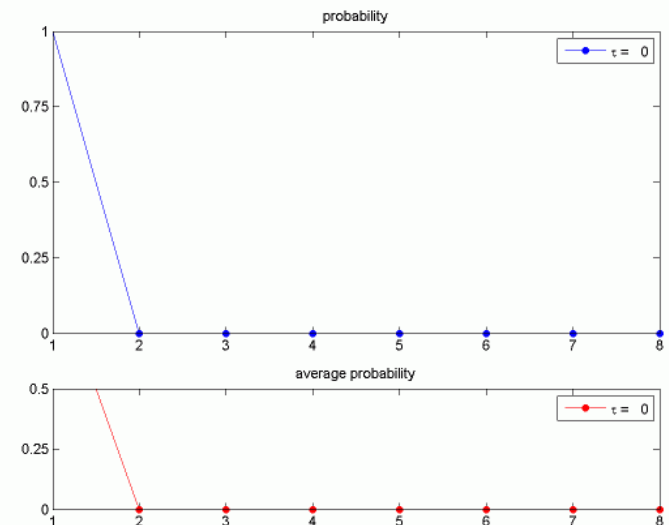


### ■ The Hamiltonian

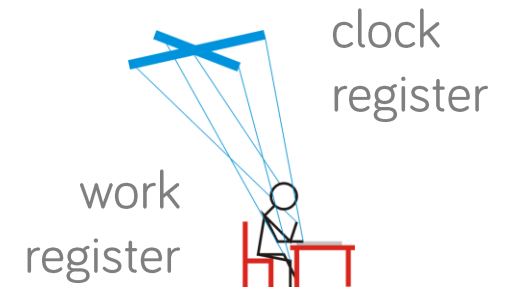
$$H_F = - \sum_{t=1}^L \left( U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

### ■ A quantum walk on a "line"

$$\begin{aligned} & |\varphi_0\rangle \otimes |0\rangle_c \\ & U_1 |\varphi_0\rangle \otimes |1\rangle_c \\ & U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c \\ & U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c \end{aligned}$$



## 2 Feynman's (Hamiltonian) computer

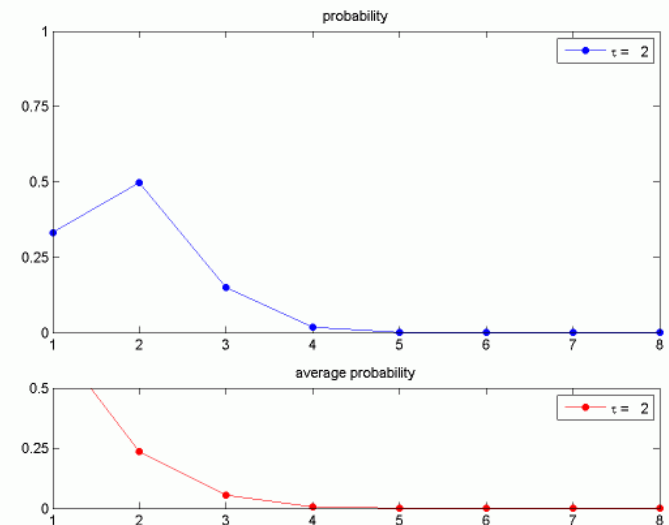


### ■ The Hamiltonian

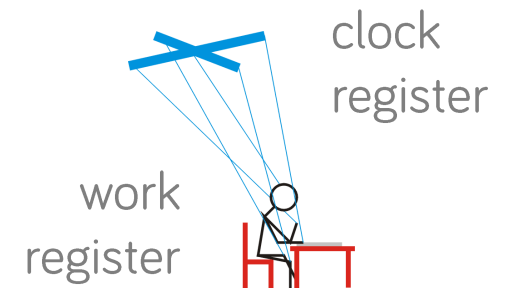
$$H_F = - \sum_{t=1}^L \left( U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

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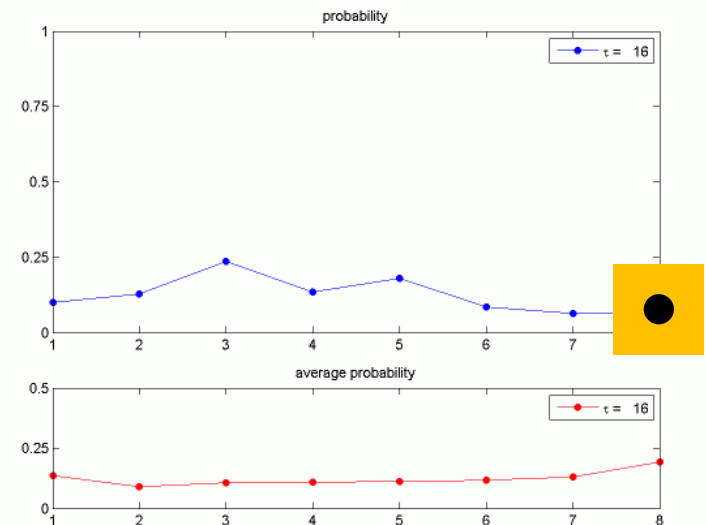
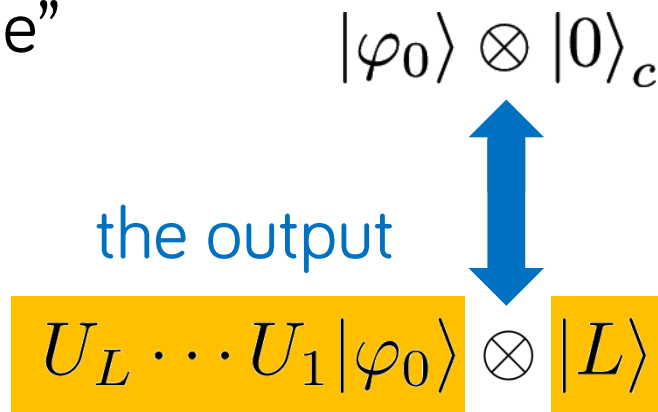
## 2 Feynman's (Hamiltonian) computer



### ■ The Hamiltonian

$$H_F = - \sum_{t=1}^L \left( U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

### ■ A quantum walk on a "line"



## 2 Hamiltonian QC

It's a walk.

- Feynman's Hamiltonian

$$H_F = - \sum_{t=1}^L \left( U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

- The “line” of states

$$\begin{array}{l} |\varphi_0\rangle \otimes |0\rangle_c \\ U_1 |\varphi_0\rangle \otimes |1\rangle_c \\ U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c \\ U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c \\ U_4 U_3 U_2 U_1 |\varphi_0\rangle \otimes |4\rangle_c \end{array} \quad H_F = - \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The eigenvectors: combinations of plane waves

## 2 Hamiltonian QC

It's a walk.

- Feynman's Hamiltonian

$$H_F = - \sum_{t=1}^L \left( U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

- The “line” of states

a possibility: wrap around a circle

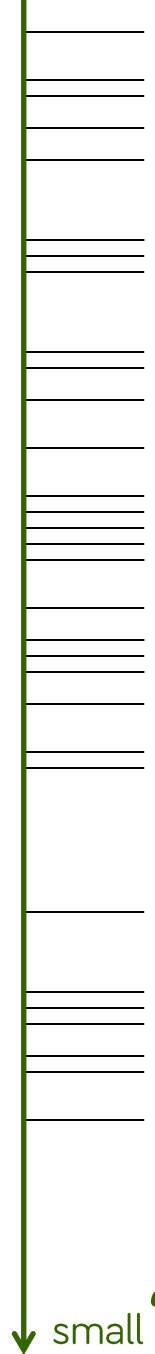
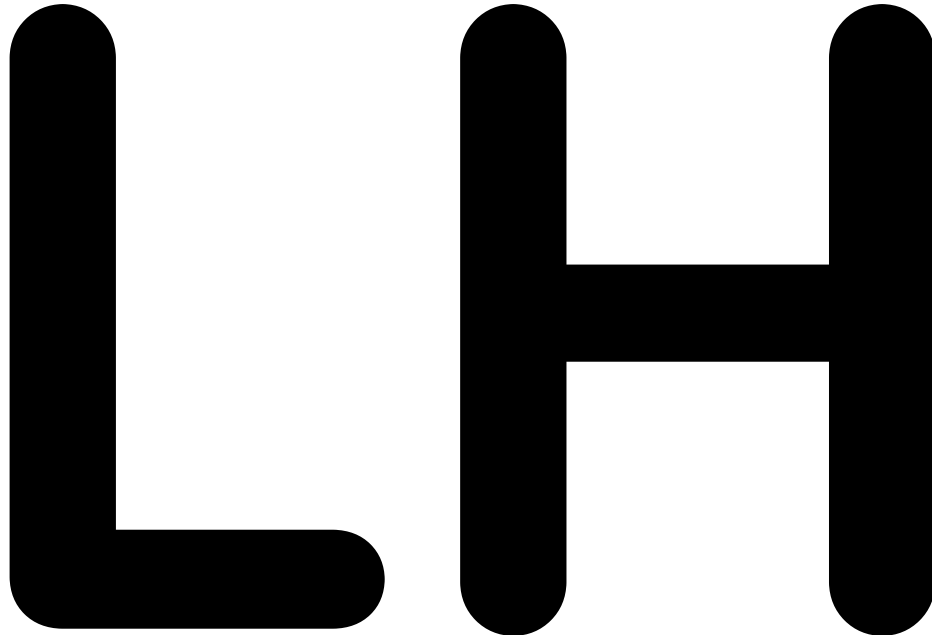
$$\begin{array}{l} |\varphi_0\rangle \otimes |0\rangle_c \\ U_1 |\varphi_0\rangle \otimes |1\rangle_c \\ U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c \\ U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c \\ U_4 U_3 U_2 U_1 |\varphi_0\rangle \otimes |4\rangle_c \end{array} \quad H_F = - \begin{bmatrix} 0 & 1 & 0 & 0 & \mathbf{1} \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ \mathbf{1} & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The eigenvectors: combinations of plane waves

1

# Hamiltonians and their ground states

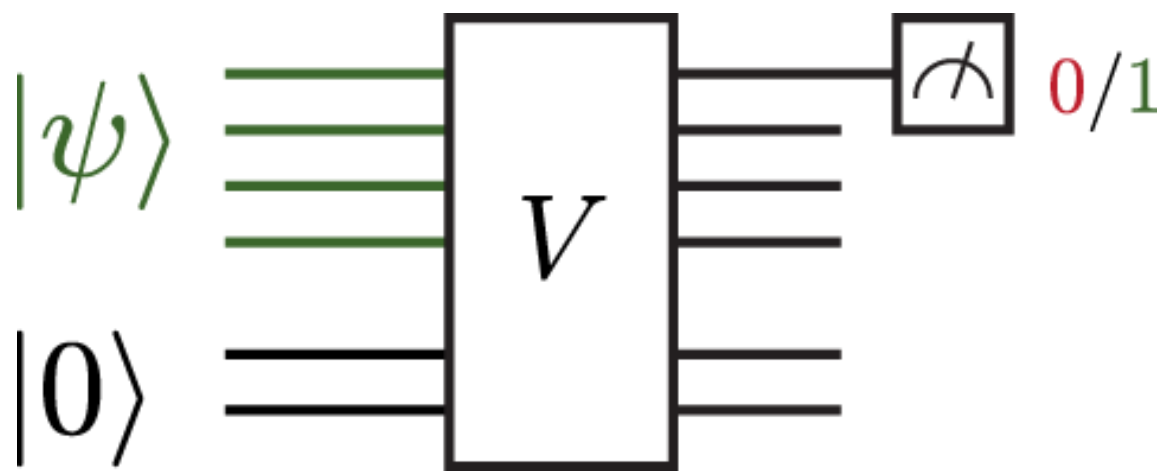
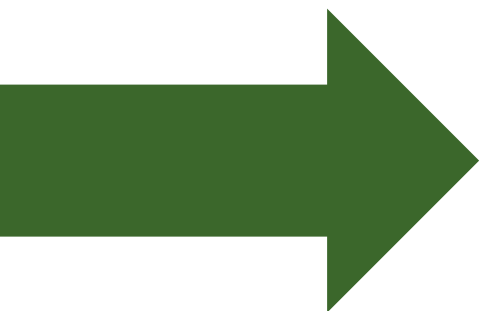
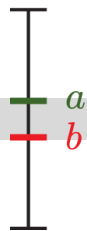
Is  
the  
ground  
state  
energy  
of a



small?

# 1 The QMA protocol

YES? Accept a good proof with  $p > a$ .  
the promise  
NO? Probability of accepting  $p < b$ .

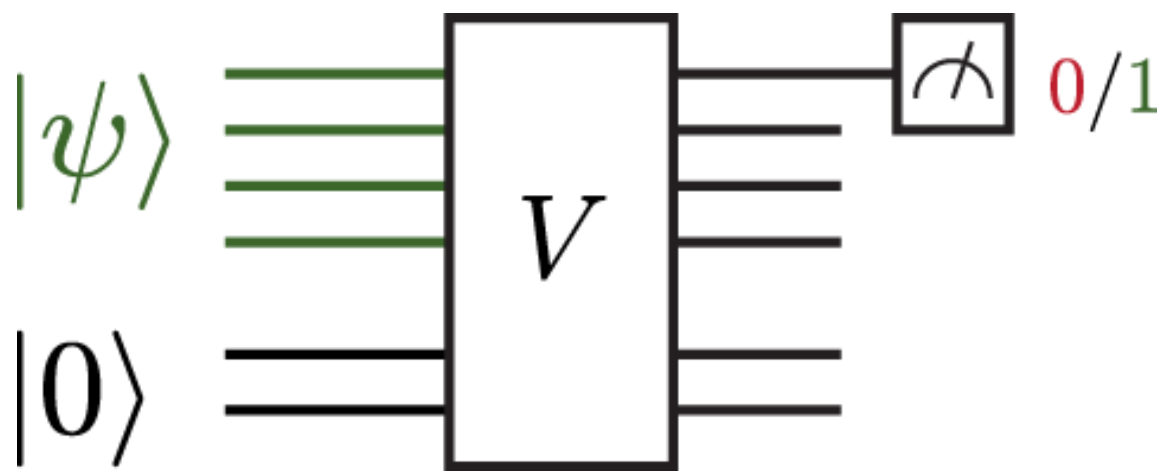
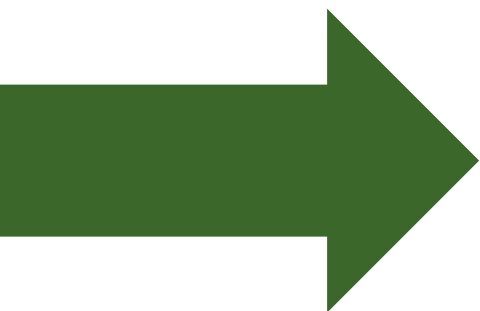
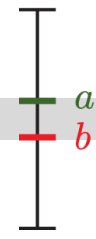


- Is there an acceptable quantum witness?
- Is some local Hamiltonian (nearly) frustration-free?



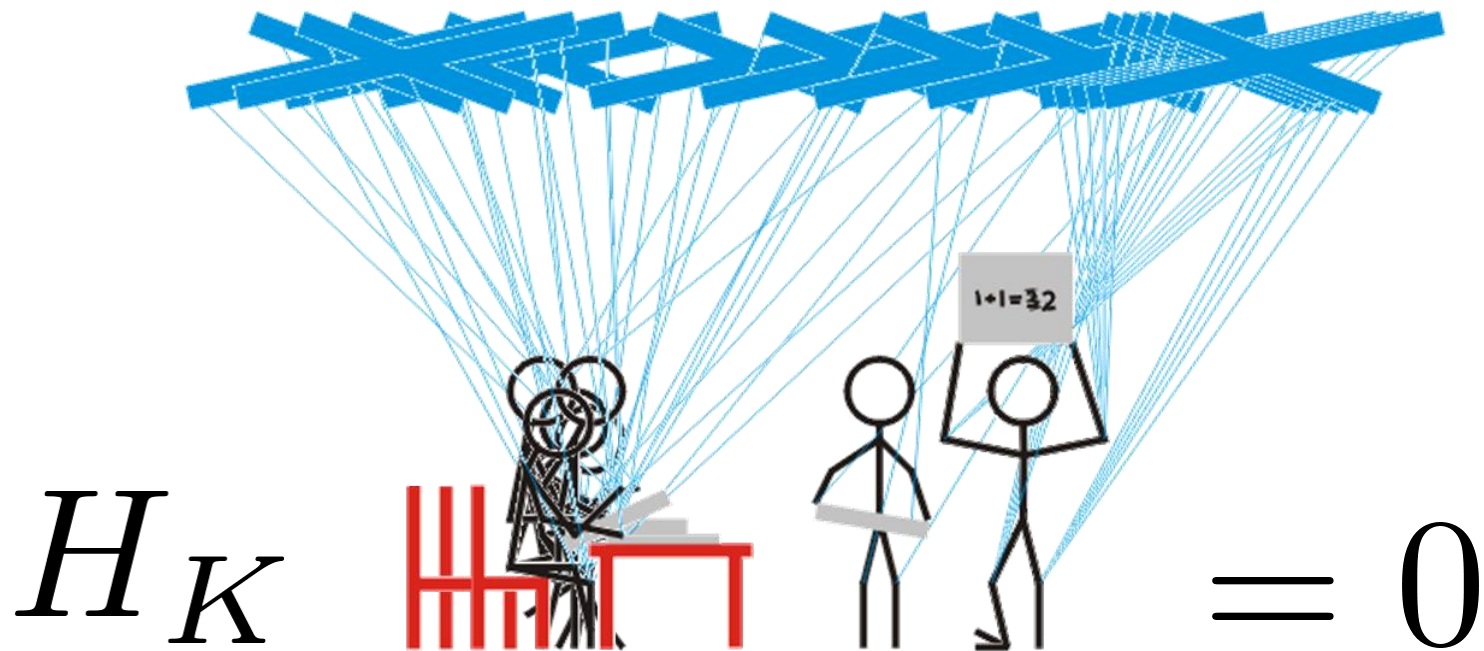
# 1 The QMA protocol

YES? Accept a good proof with  $p > a$ .  
the promise  
NO? Probability of accepting  $p < b$ .



- Is there an acceptable quantum witness?
- Does some local Hamiltonian have a low ground energy?

## 2 The history state: a ground state



$$H_K = 0$$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$

$\underbrace{\hspace{10em}}_{U_t \cdots U_1 |\varphi_0\rangle}$

## 2 The history state is a ground state

Local Hamiltonian

*k-local*  
*c-o-n-d-i-t-i-o-n-s*

clock encoding  
state progression  
initialization

$$|\dots 000 \dots 0\rangle \otimes |0\rangle$$

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$

output

$$|\dots 1\rangle \otimes |T\rangle$$



## 2 Checking proper computation

*Antisymmetry checks.*

- uniform superpositions: zero-energy eigenstates

$$H_t = \frac{1}{2} \left( |t+1\rangle\langle t+1| + |t\rangle\langle t| \right) - \frac{1}{2} \left( U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^\dagger \otimes |t\rangle\langle t+1| \right)$$

*Feynman's Hamiltonian*

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

*a projector*

$$|\varphi_t\rangle \otimes |t\rangle$$
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

*a nice basis*

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$



## 2 Checking proper computation

*Antisymmetry checks.*

$$\sum_{t=1}^L H_t = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

*positive  
semidefinite*

$$\begin{aligned} &|\varphi_t\rangle \otimes |t\rangle \\ &|\varphi_{t+1}\rangle \otimes |t+1\rangle \end{aligned}$$

*a nice basis*

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^T |\varphi_t\rangle \otimes |t\rangle$$



## 2 Checking proper computation

*Antisymmetry checks.*

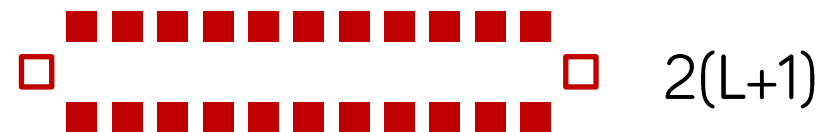
$$\sum_{t=1}^L H_t = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

*positive semidefinite*

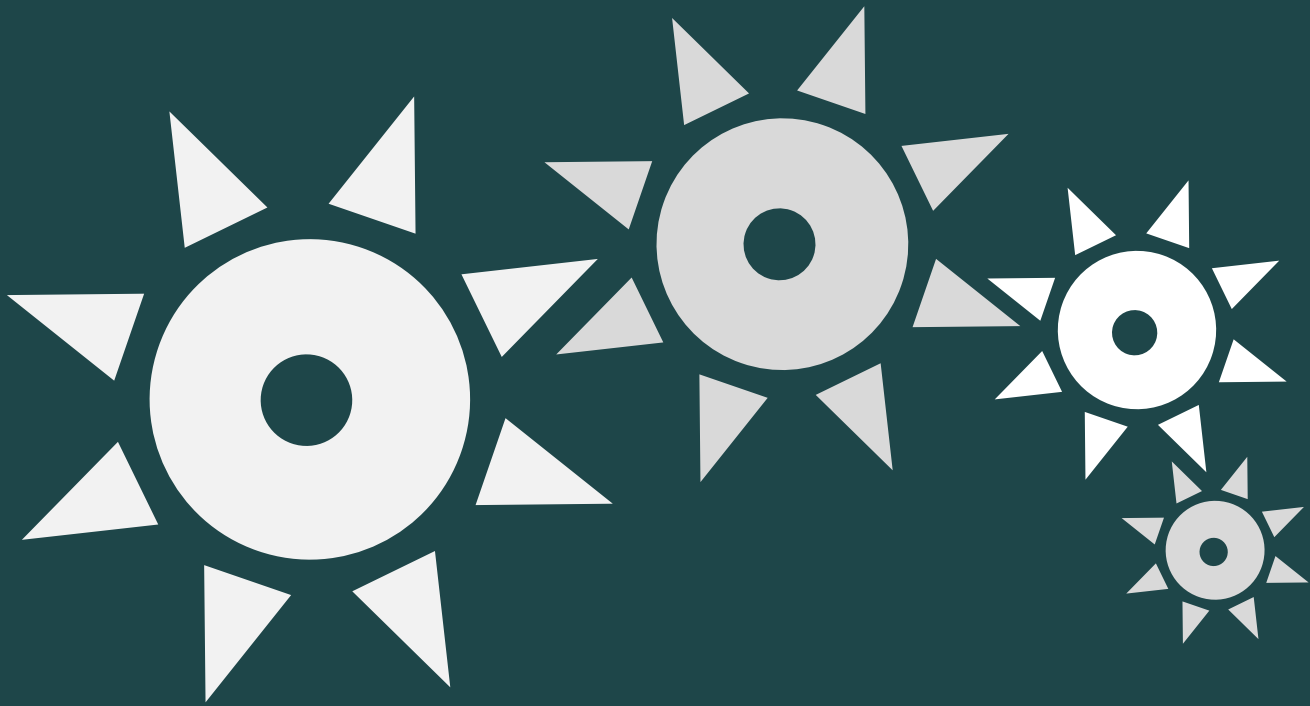
$$\sum_t e^{-ipt} |\varphi_t\rangle \otimes |t\rangle$$

*local?*

*eigenvectors:  
combinations of  
plane waves*



an  $L^{-2}$  eigenvalue gap



a clock workshop

### 3 Constructing local clocks

- the pulse





### 3 Constructing local clocks

- the pulse



transitions: 2-local

- joining the states  
by projectors



### 3 Constructing local clocks

- the pulse



transitions: 2-local

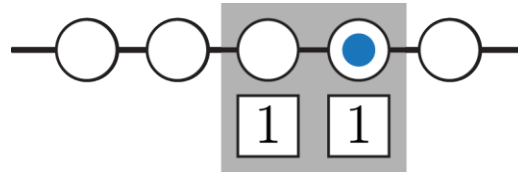
$$\begin{array}{c} 00100 \\ \downarrow \\ +00010 \end{array}$$

- joining the states  
by projectors

$$|01 - 10\rangle\langle 01 - 10|$$

### 3 Constructing local clocks

- the pulse



transitions: 2-local  
2-qubit gates: 4-local

interaction  
with the data

- joining the states

by projectors  $|01 - 10\rangle\langle 01 - 10|$

### 3 Constructing local clocks

- the pulse



transitions: 2-local  
2-qubit gates: 4-local

00000

a “dead” state

Initialization!

- joining the states  
by projectors

$$|01 - 10\rangle\langle 01 - 10|$$

### 3 Constructing local clocks

- the domain wall 

$$\begin{aligned} |t\rangle &= |1\rangle \\ &= |10000\rangle \end{aligned}$$

- 2-local terms  
“compatible” with  
**11...1100...00**



$$|01\rangle\langle 01|$$

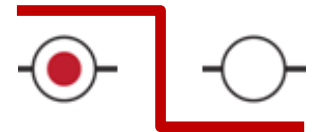
### 3 Constructing local clocks

- the domain wall  transitions: 3-local

$$\begin{aligned}
 |t\rangle &= |\mathbf{3}\rangle \\
 &= |11000\rangle
 \end{aligned}$$

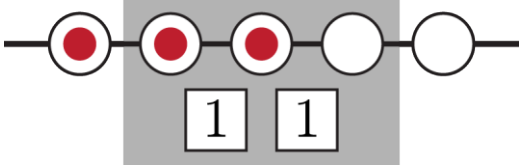
- joining states by transitions?  $|100 - 110\rangle\langle 100 - 110|$

- enforce a domain wall: fix the ends



- the ground state  $\cdots + |2\rangle + |3\rangle + \cdots$

### 3 Constructing local clocks

- the domain wall  transitions: 3-local  
2-qubit gates: 5-local
- interacting with work (data) qubits

$$H_t = \frac{1}{2} (|t+1\rangle\langle t+1| + |t\rangle\langle t|) - \frac{1}{2} \left( U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^\dagger \otimes |t\rangle\langle t+1| \right)$$

5-local

● YES

ground state



- NO

ground state

● NO



lower bound on the  
ground state energy

good  
clock  
states

... 01 ...  
bad  
clock  
states

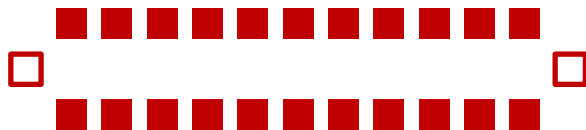
history states

non-uniform  
superpositions

# history states



a polynomially small gap



$$\Delta = O(L^{-2})$$



history states

well

badly

initialized history states

well

initialized histories

accepted  
states



well

initialized histories

accepted  
states

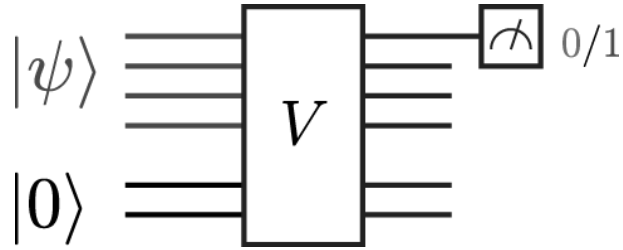
$$H_A + H_B$$

$$\lambda_0 \geq \sin^2 \frac{\vartheta}{2} \times \min(\Delta_A, \Delta_B)$$

$\uparrow$   $L^{-2}$   $\uparrow$   $L^{-1}$

# 3-LH and QMA verification

[N., Mozes 07]



# LH

**NO**

$V$  is unlikely to accept anything ( $\epsilon$ )

lowest eigenvalue

$$\geq \frac{c(1 - \sqrt{\epsilon})}{L^2}$$



promise gap  $L^{-2}$

(needs  $\epsilon=L^{-1}$ )

**YES**

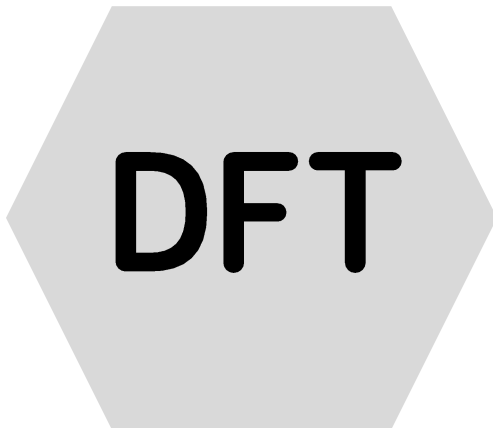
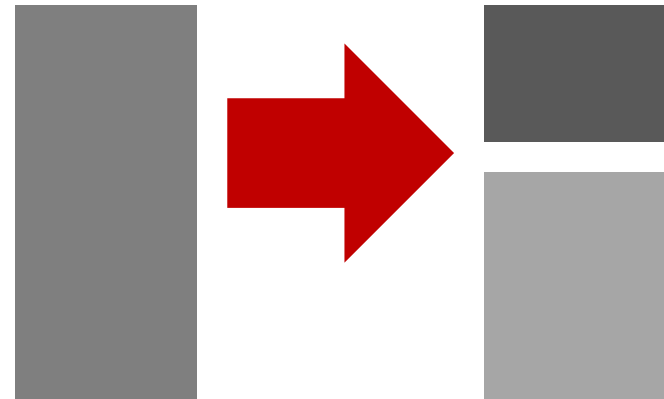
a well accepted proof  $(1-\epsilon)$

the history state

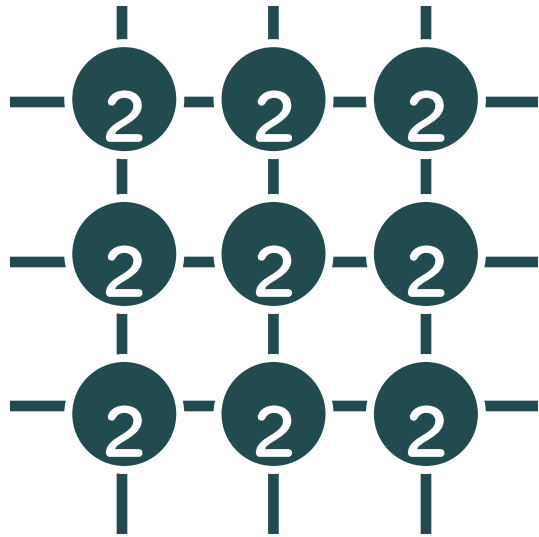
$$\leq \frac{\epsilon}{L + 1}$$

## 2 Other QMA-complete problems

[Bookatz '13]

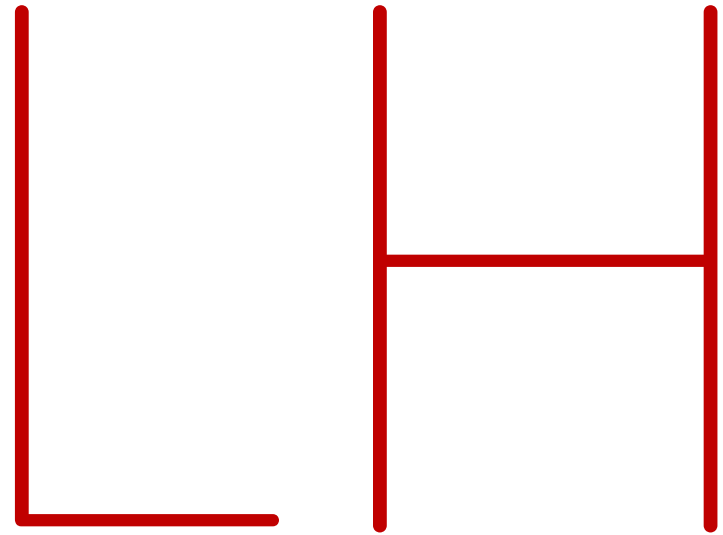


## 2 2-local Hamiltonian is QMA complete

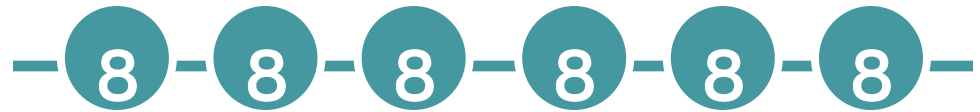


[Oliveira, Terhal '04]

a global minimum



$$\sum H_{j k}$$

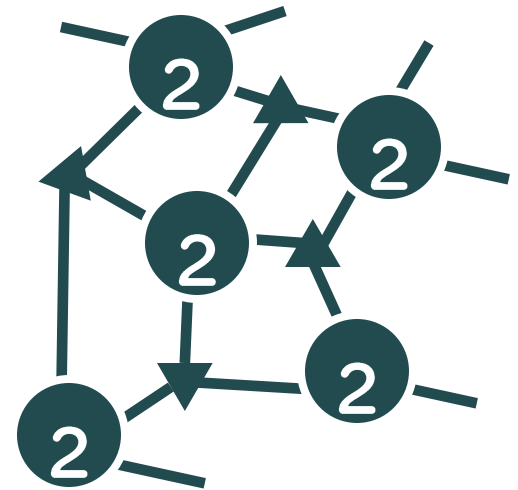


[Hallgren, N., Narayanaswami '13]

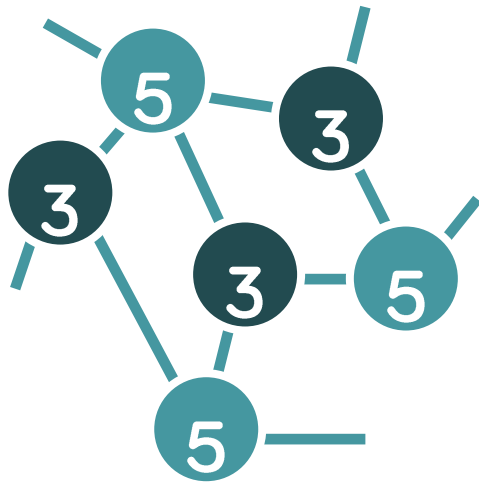
## 2 QMA<sub>1</sub>-complete problems



[N. '08]



[Gosset, N. '13]



[Eldar, Regev '08]

unfrustrated  
qSAT

# projections & gadgets

### 3 From a 5-local to a 3-local clock [Kempe, Regev]

■ the domain wall  transitions: 1-local

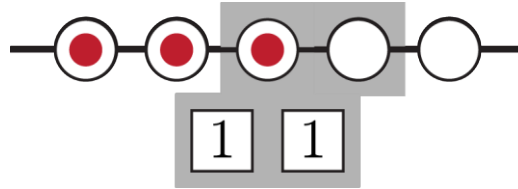
■ transitions  $|1\rangle\langle 0| + |0\rangle\langle 1|$

■ punish mistimed transitions?

$$|01\rangle\langle 01|$$

### 3 From a 5-local to a 3-local clock [Kempe, Regev]

- the domain wall



transitions: 1-local  
2-qubit gates: 3-local

- transitions

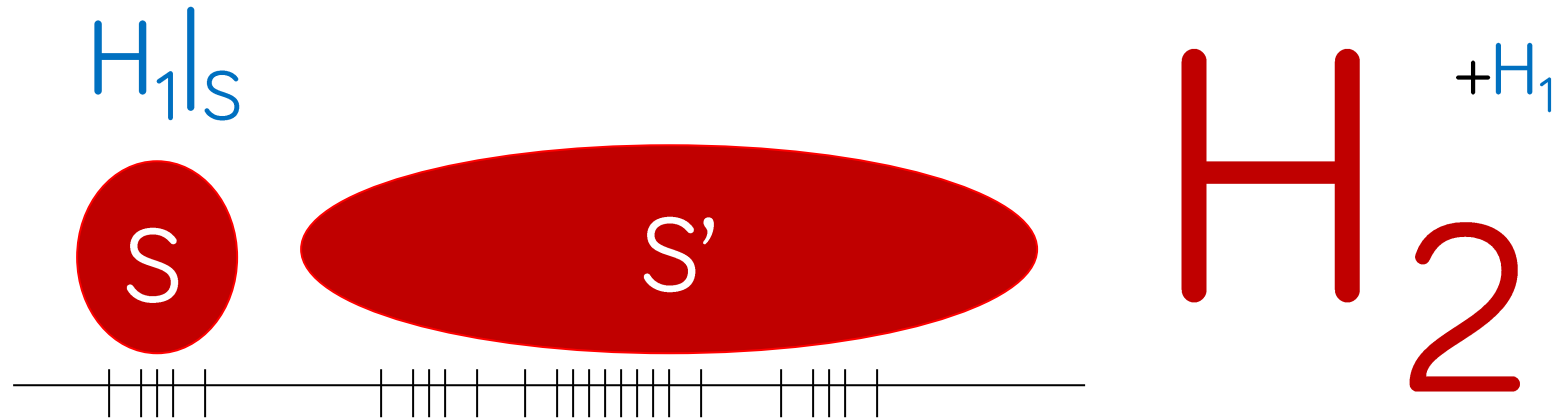
$$|10\rangle\langle 10|_{1,2} + |10\rangle\langle 10|_{2,3} - X_2$$

- punish mistimed transitions

$$|01\rangle\langle 01|$$



## 4 The projection lemma: a useful tool for proving gaps

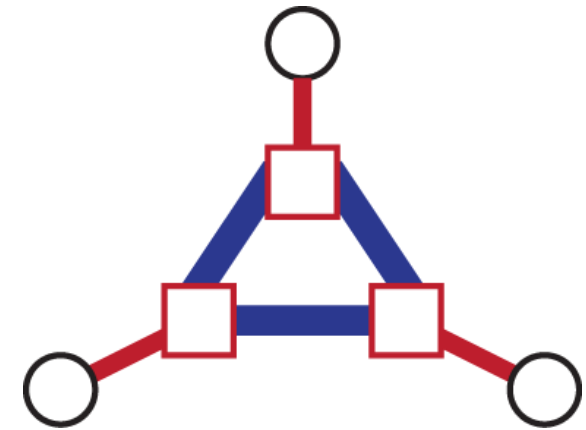
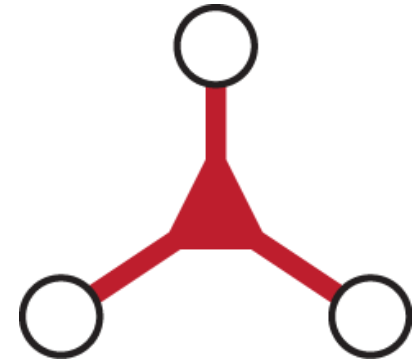
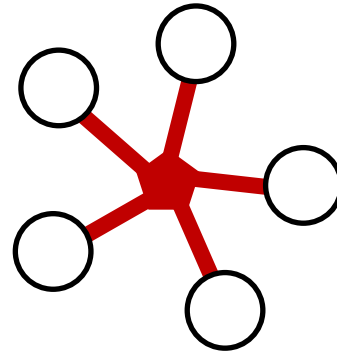
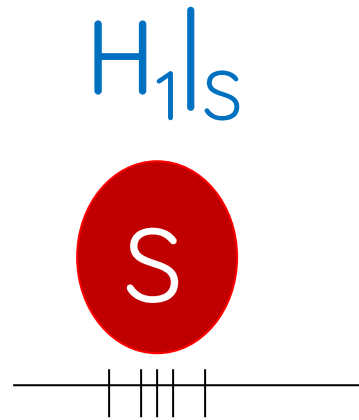


- a HIGH energy penalty for “illegal” states?
- the low energy states live near the “legal” subspace

## 4 The projection lemma in action

$$\begin{array}{c} H_1 |s\rangle \\ \text{S} \\ \text{-----} \\ | \quad | \quad | \quad | \quad | \end{array} \quad |1\rangle\langle 0| + |0\rangle\langle 1| \quad \begin{array}{l} |1110000\rangle \\ |1111000\rangle \\ |1111100\rangle \end{array}$$

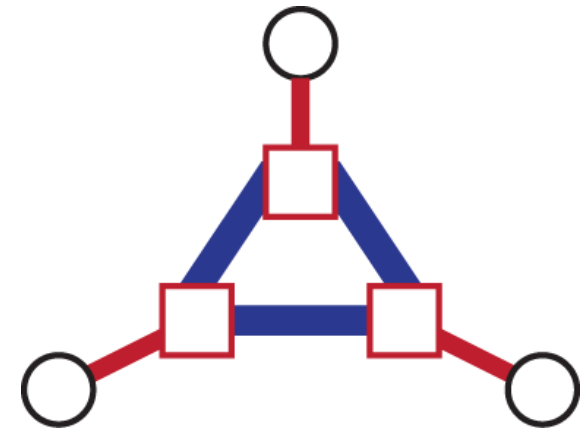
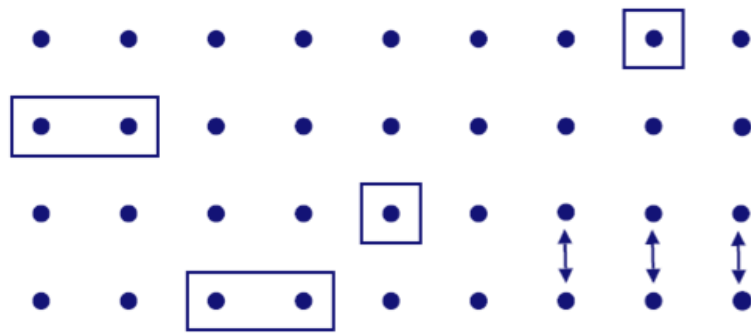
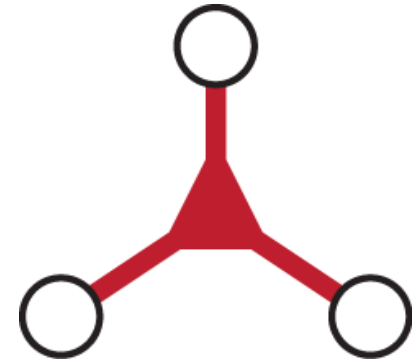
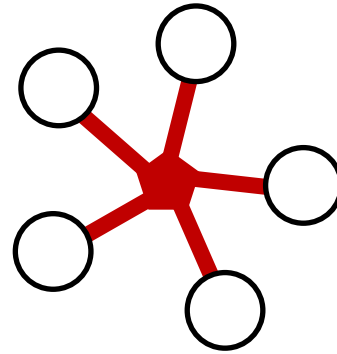
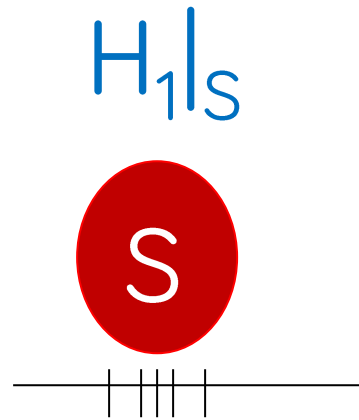
## 4 The projection lemma: a useful tool for proving gaps



[Kempe, Kitaev, Regev '03]

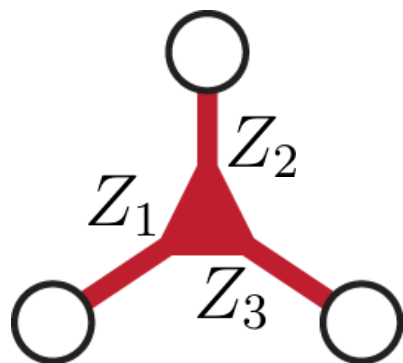
- 3-LH that works well in the “good clock subspace”
- 2-LH from effective interactions

# 4 The projection lemma: a useful tool for proving gaps



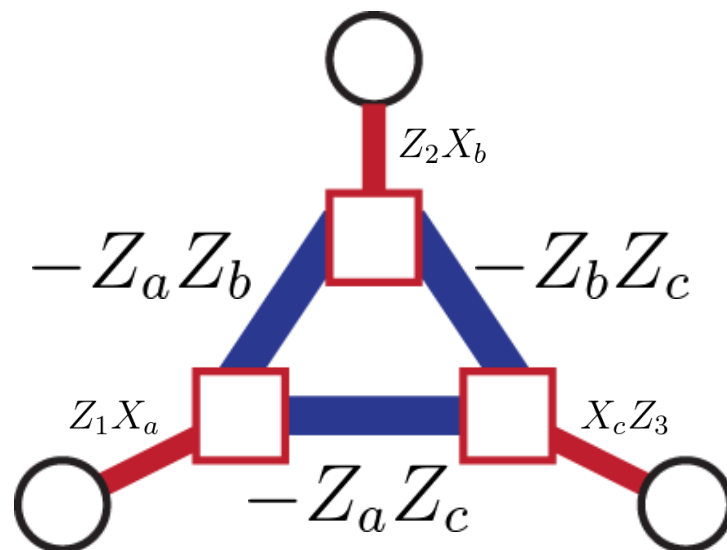
- 2-loc. H. in 2D [Oliveira, Terhal '05]

### 3 Further decreasing locality: a “3 from 2” gadget



- strongly coupled ancillas (a new energy scale)
- perturbation theory

$$G'(z) = (z\mathbb{I} - H')^{-1}$$

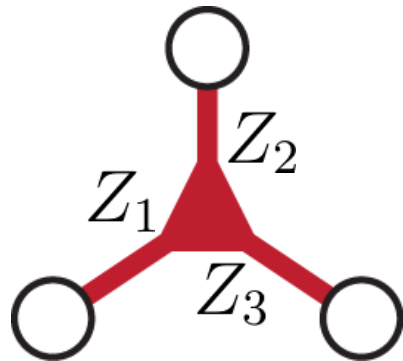


$$H' = H + V$$

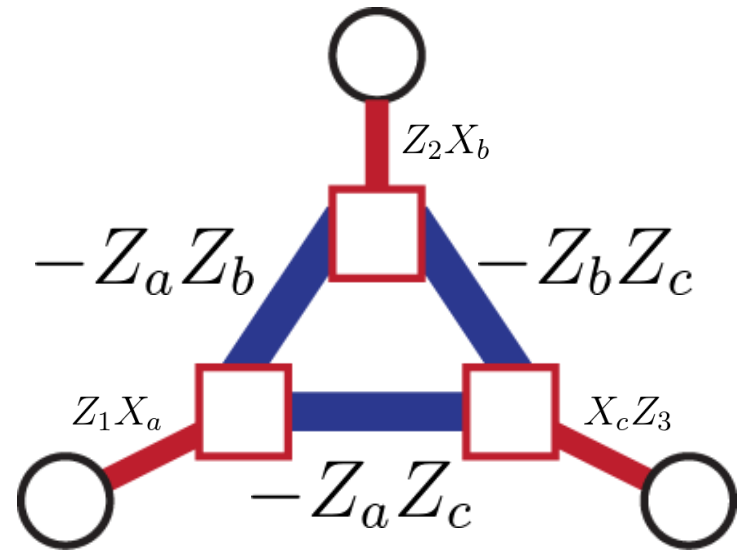
$\|H\| \gg \|V\|$

$$S = \text{span} \{ |000\rangle, |111\rangle \}$$

### 3 Further decreasing locality: a “3 from 2” gadget



- strongly coupled ancillas (a new energy scale)
- perturbation theory gives us an effective Hamiltonian



$$H' = H + V$$

$\|H\| \gg \|V\|$

$$S = \text{span} \{ |000\rangle, |111\rangle \}$$

$$V|_S$$

projection  
lemma

$$V^2|_S$$

unwanted  
(subtract)

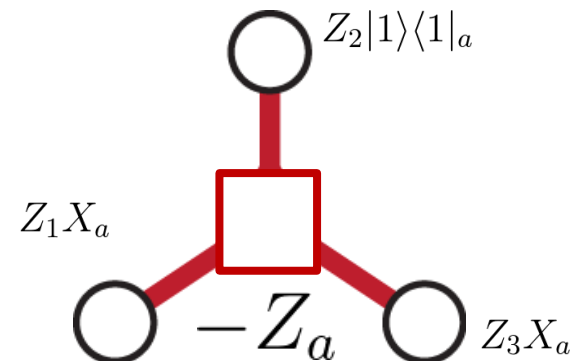
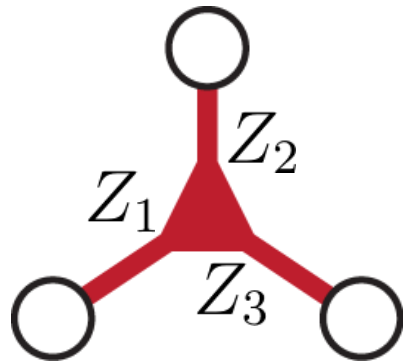
$$V^3|_S$$

the effective  
3-local term

[Kempe, Kitaev, Regev '03]

### 3 STRONG local fields, OK interactions

[Cao et al., 1311.2555]



- strongly bound a single ancilla still needs strong interactions
- perturbation theory gives us an effective Hamiltonian

$$S = \{|0\rangle\}$$

$$H' = H + V$$

$$\|H\| \gg \|V\|$$

$$V|_S$$

projection  
lemma

$$V^2|_S$$

unwanted  
(subtract)

$$V^3|_S$$

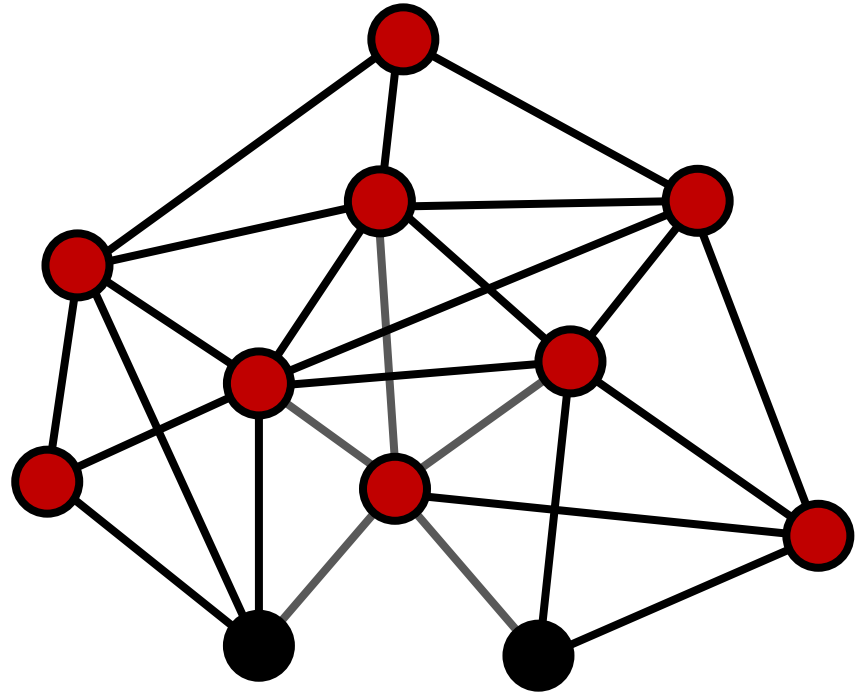
the effective  
3-local term

special cases (Z-basis)  
exact gadgets!

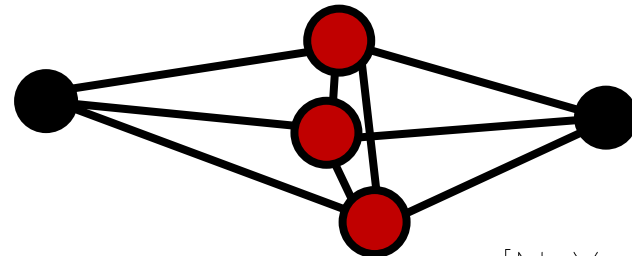
[Biamonte 0801.3800]

### 3 “Strengthening”, intermediary gadgets?

- classically easy: copy



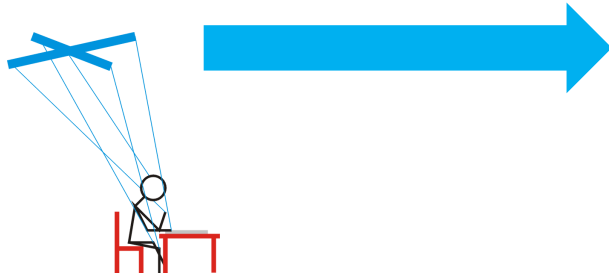
- quantumly?



[N., Yudong Cao]

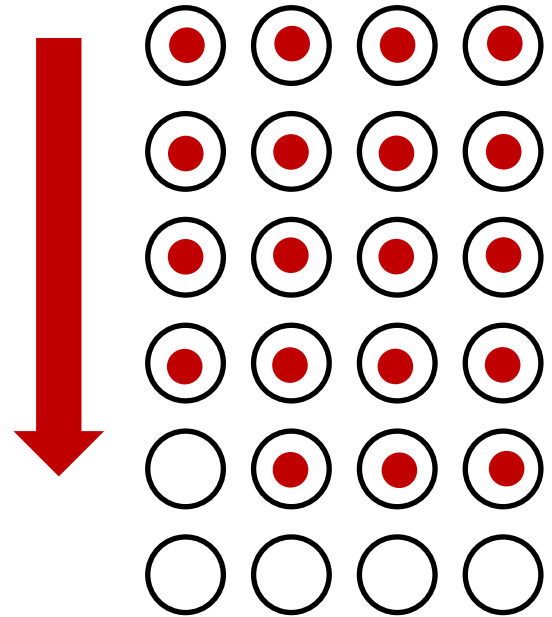


clock/work registers



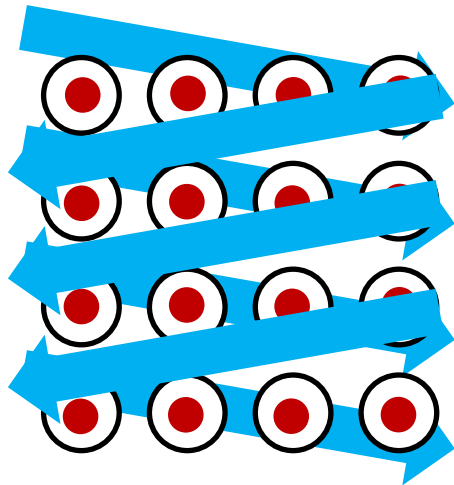
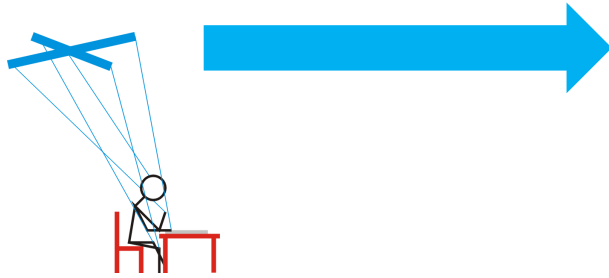
[Kempe, Kitaev, Regev]

a geometric clock



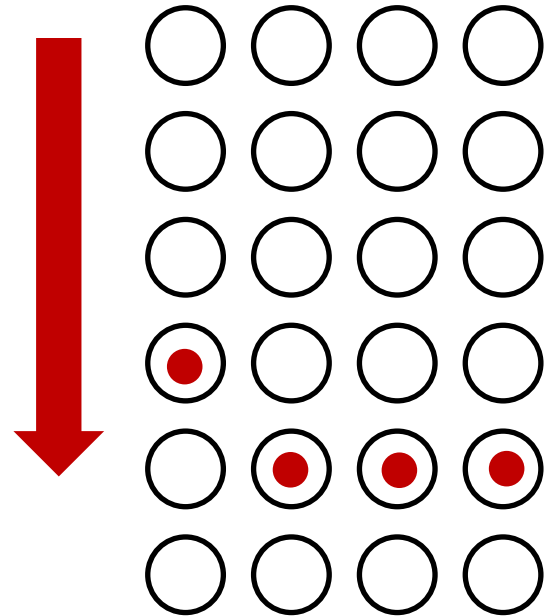
[Mizel] [Aharonov+]

clock/work registers



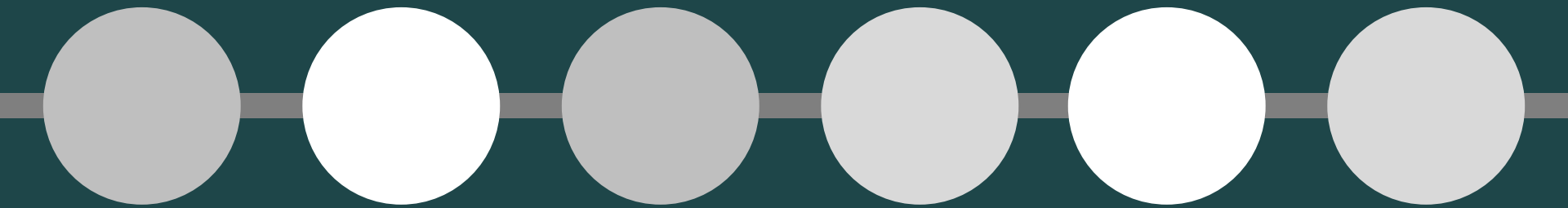
geometric locality

a geometric clock

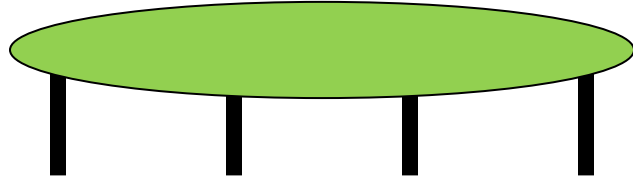


moving data on a line

l-H-a-m-i-l-t-o-n-i-a-n-s-i-n-1-D-L-o-c-a-l-H-a-m-i-l-t-o-n-i-a-n-s-i-n-1-D-L-o-c-a-l-H-a-m

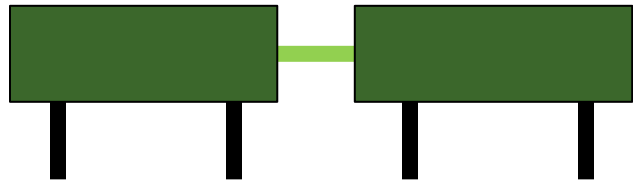


## 4 Matrix Product States



$$\sum_{s,t,u,v=0}^1 c_{stuv} |stuv\rangle$$

- Schmidt decomposition

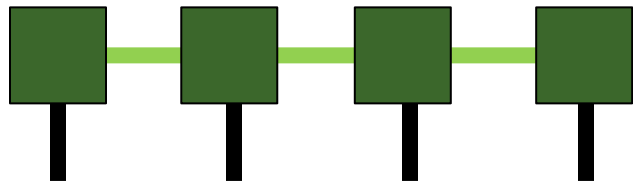


$$\sum_{b=1}^{\chi} Q_b^{st} R_b^{uv}$$

- many decompositions



a local description



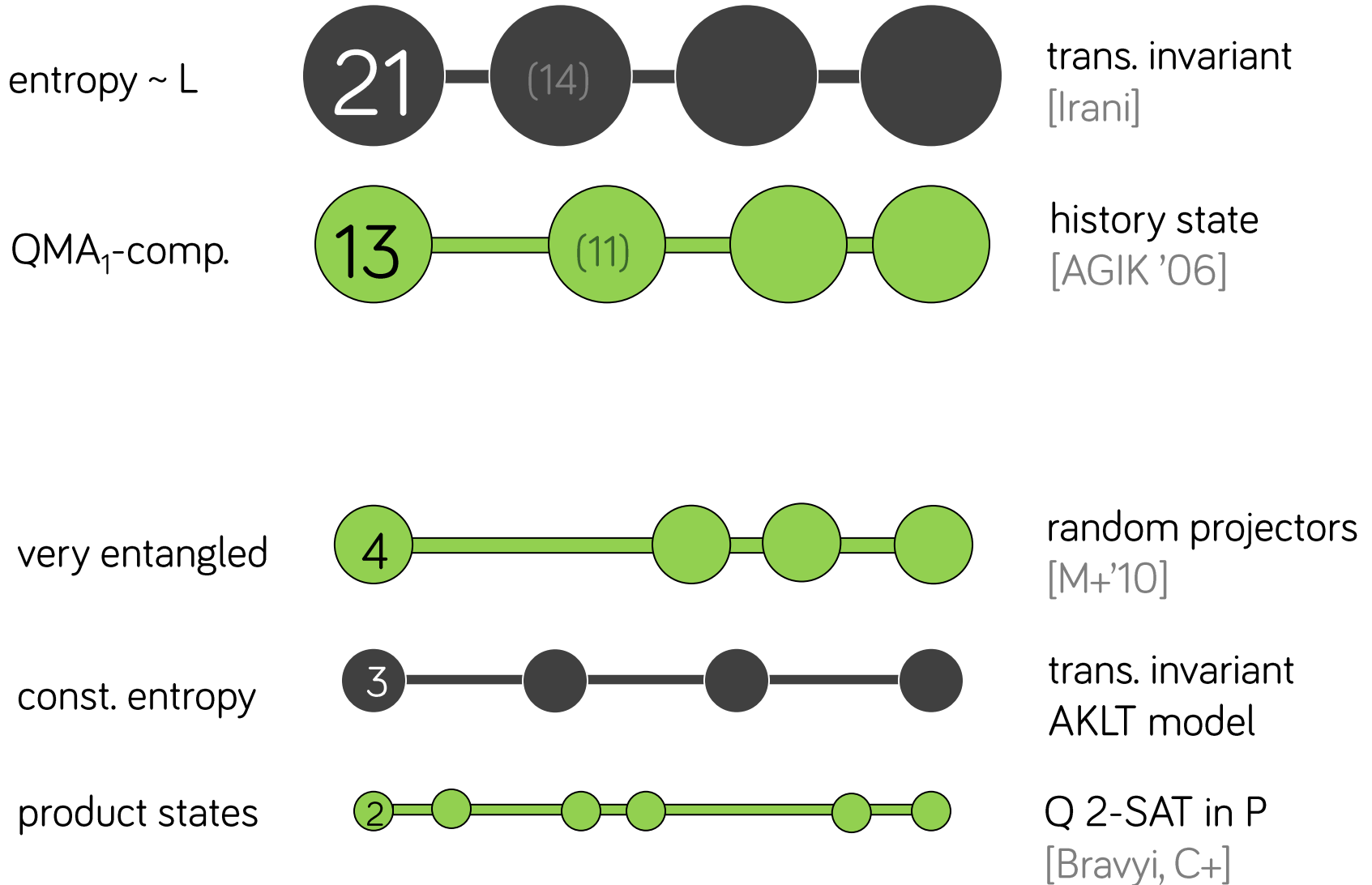
$$\sum_{b=1}^{\chi} \sum_{a=1}^{\chi} A_a^s B_{ab}^t \sum_{c=1}^{\chi} C_{bc}^u D_c^v$$

- low entanglement ansatz, local optimization, easy manipulation

# 4 Ground states in 1D

How hard is it to find/describe them?

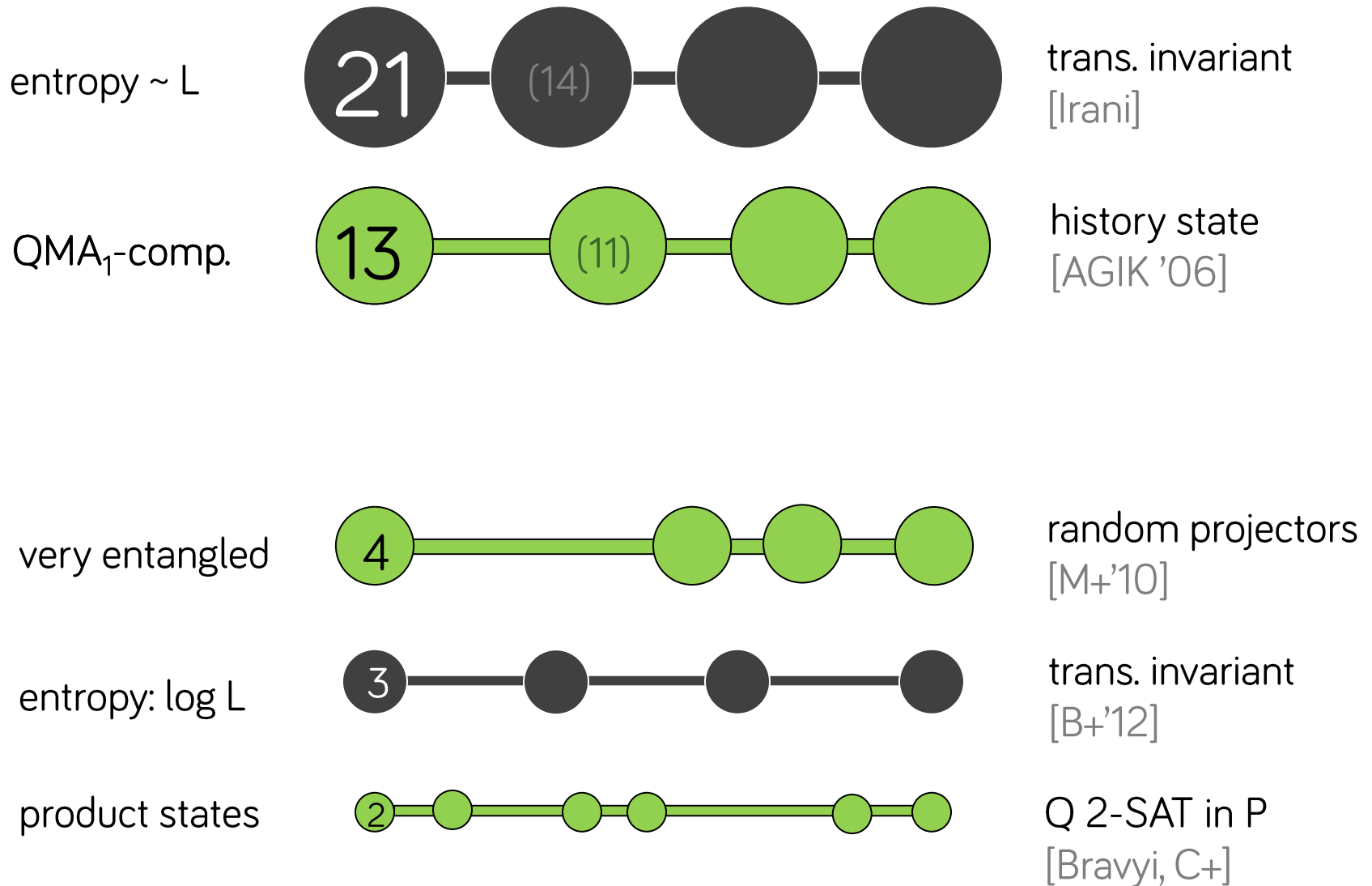
constant gap: OK [Landau+ '13]



# 4 Ground states in 1D

How hard is it to find/describe them?

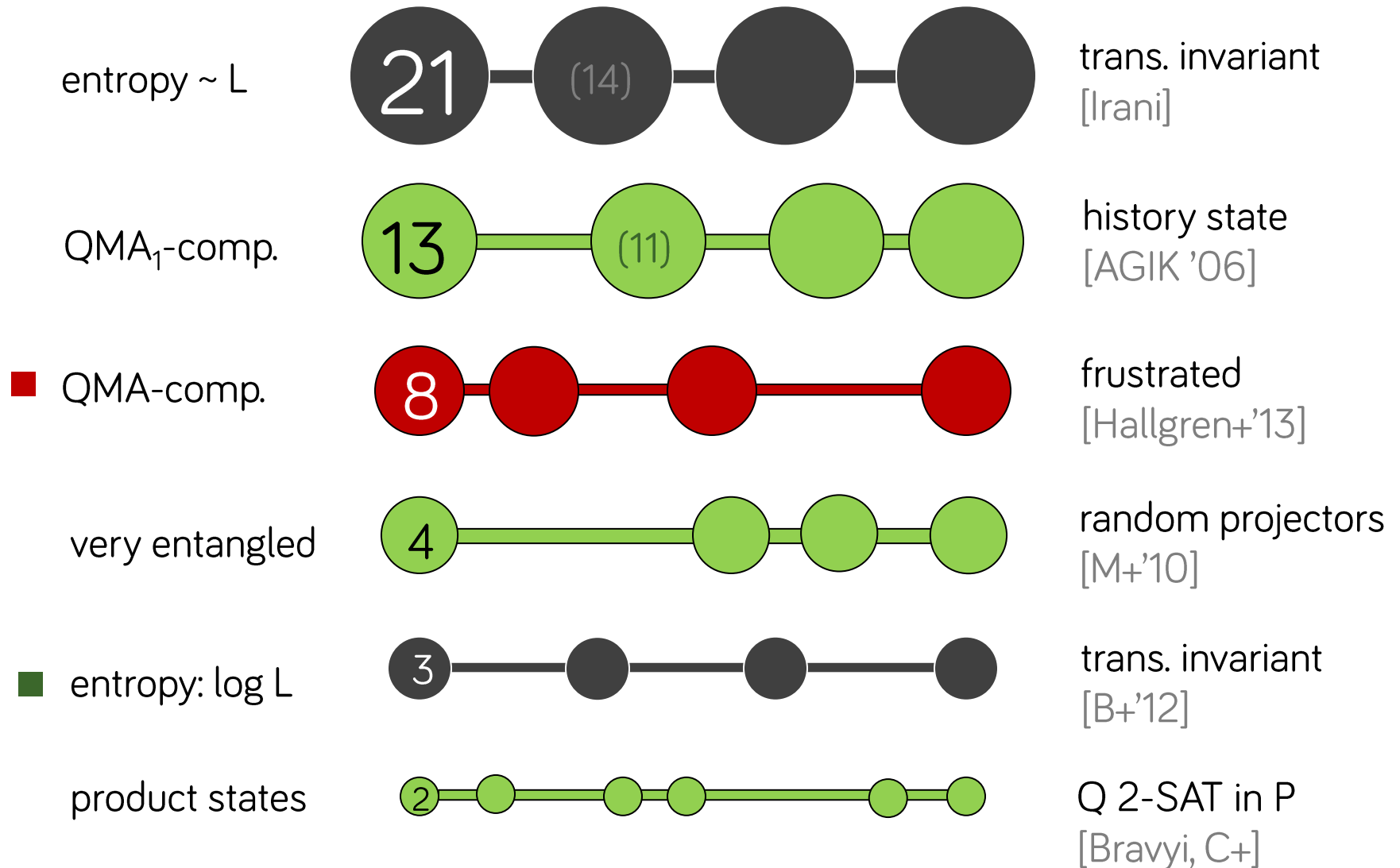
constant gap: OK [Landau+ '13]



# 4 Ground states in 1D

How hard is it to find/describe them?

constant gap: OK [Landau+ '13]



1

# Hamiltonians?

optimization & dynamics



2

# complexity

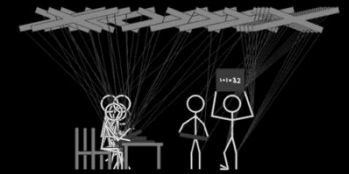
checking (quantum) proofs



3

# ground states?

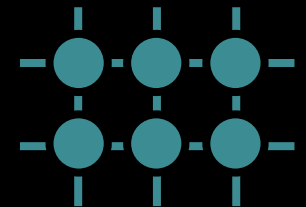
how hard is it to find them: QMA



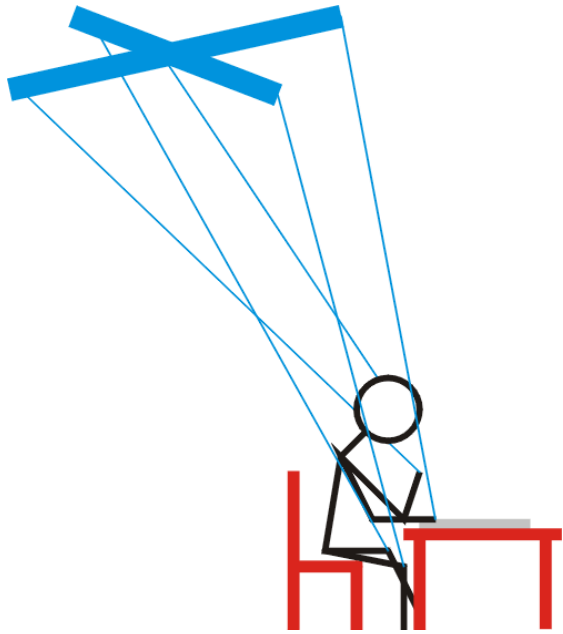
4

# tensor networks

heuristics based on low entanglement







# Local Hamiltonians & Quantum Complexity

Daniel Nagaj



universität  
wien



2014 | 4 | 11

CS lunch @ USC