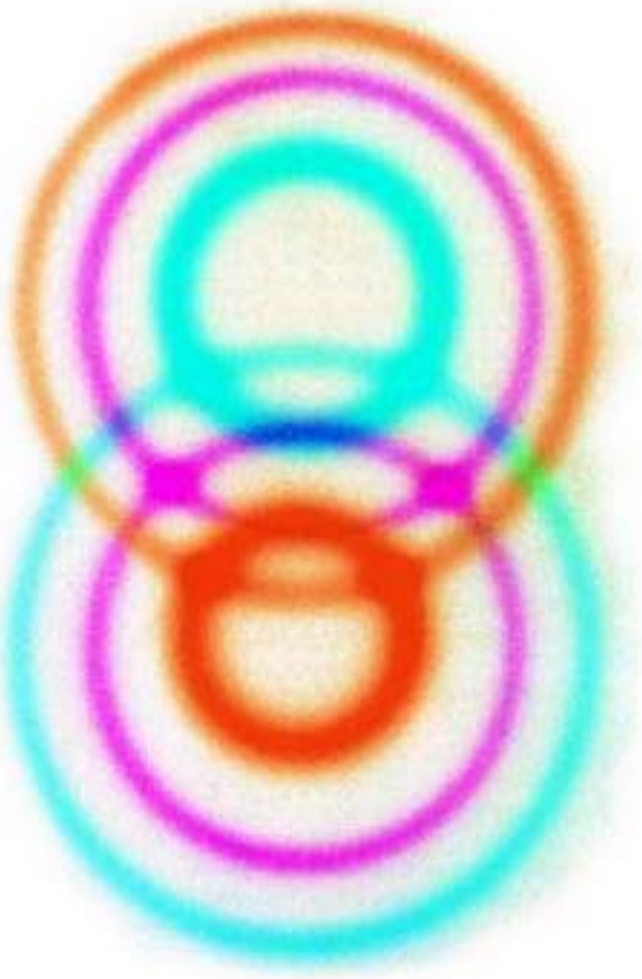


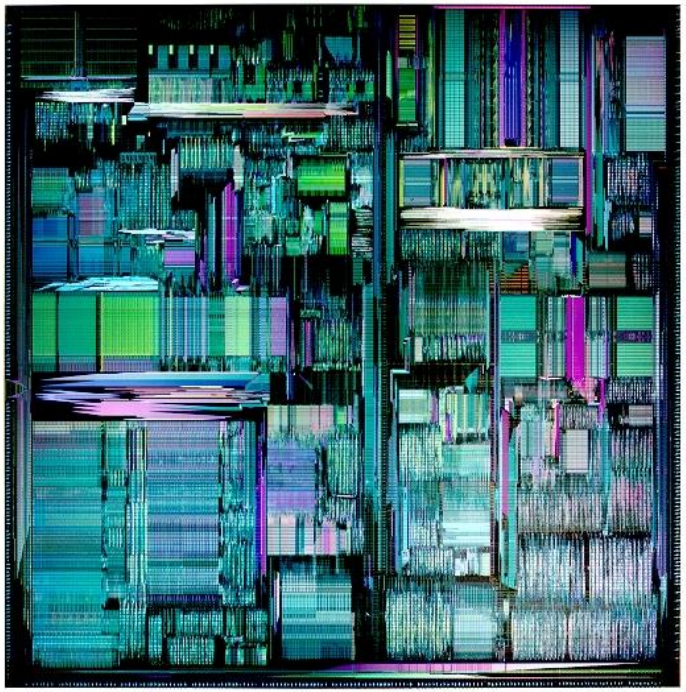
[qubit-ulm.com]

Introduction to **Quantum Computation**

ICTP-VAST-APCTP winter school
Hanoi, 12/2013

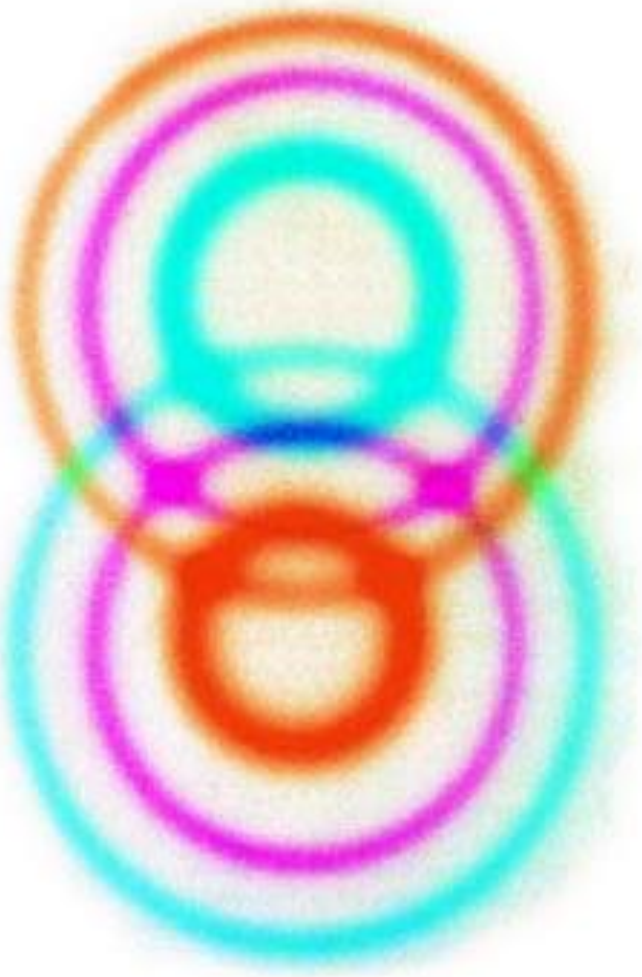
Daniel Nagaj

universität wien



[1995 Pentium Pro, www.tayloredge.com/museum]

What kinds of things does nature allow us to compute?



[qubit-ulm.com]

What kinds of things would nature allow us to compute

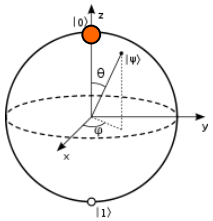
if we could utilize the power of quantum mechanics?

“Because nature is not classical, dammit, and if you want to make a simulation of nature – you’d better make it quantum mechanical!”

R. P. Feynman

1 **we need a qubit**

but what can one do with it?

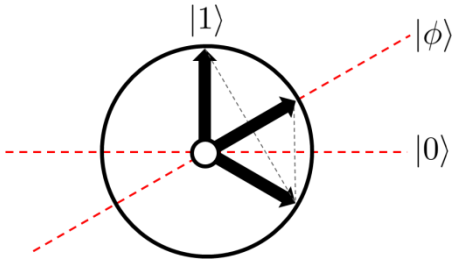


2 **EPR pairs**

give us cool 2-qubit protocols

3 **the algorithms**

that make quantum computing tick



4 **error correction**

can we really scale up this stuff?

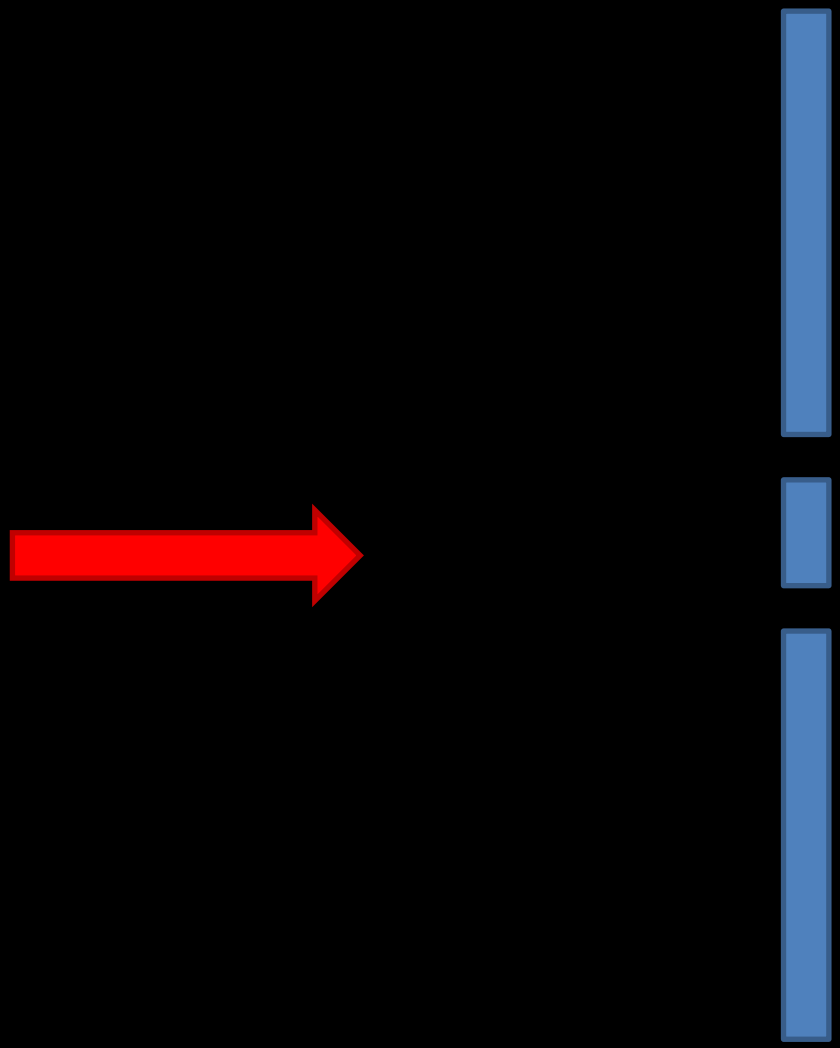
two slits and light

two slits and a laser

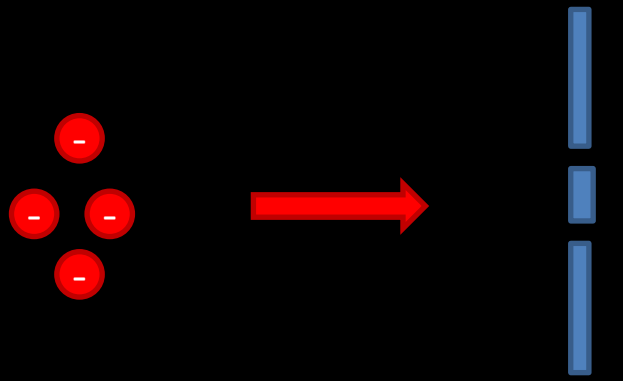
coherent
light

interference

superposition



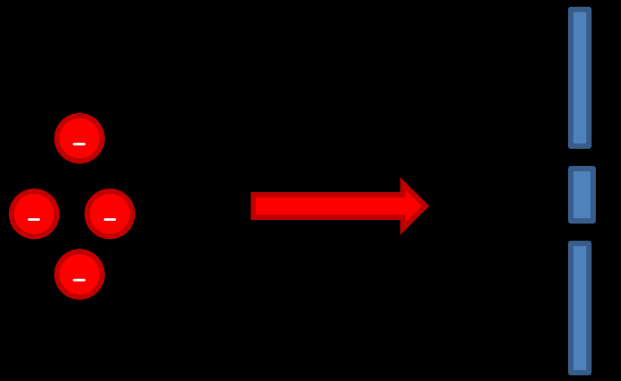
two slits and electrons

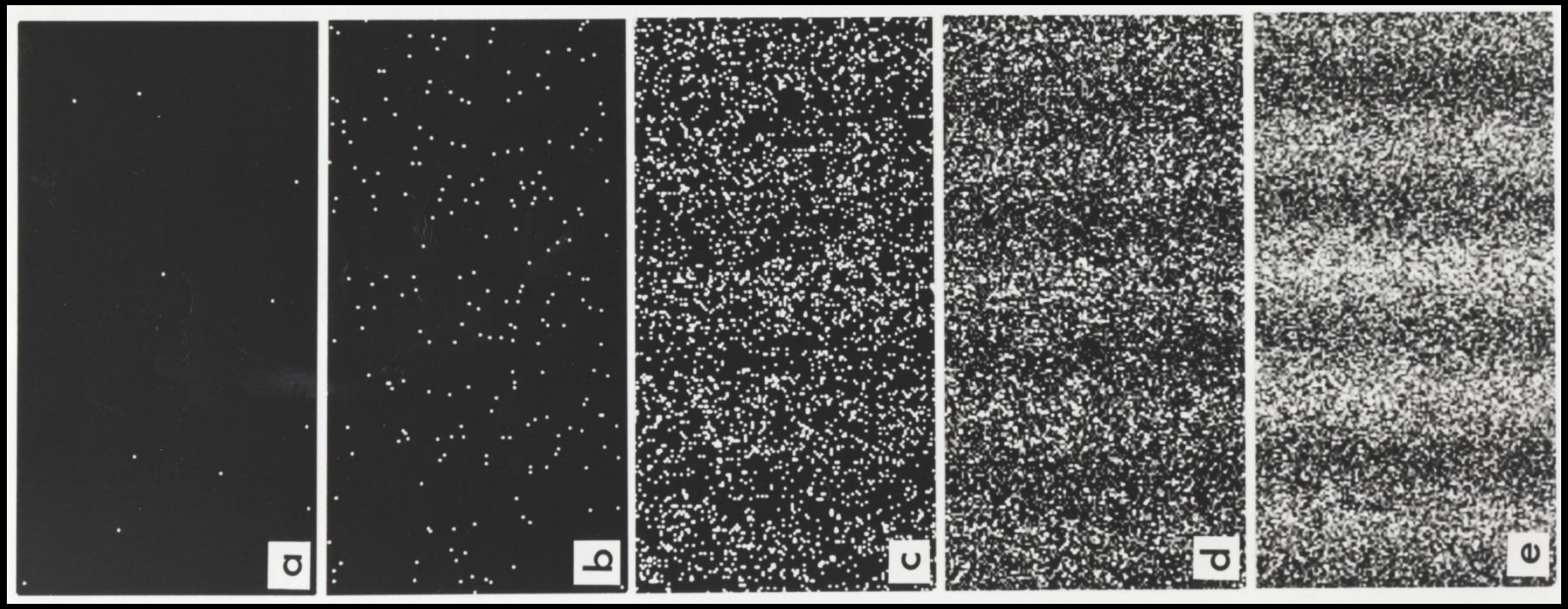


two slits and electrons

interference

superposition

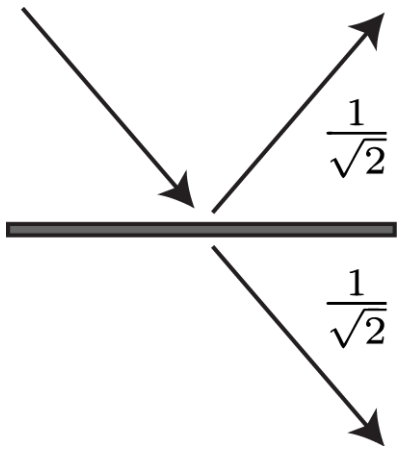


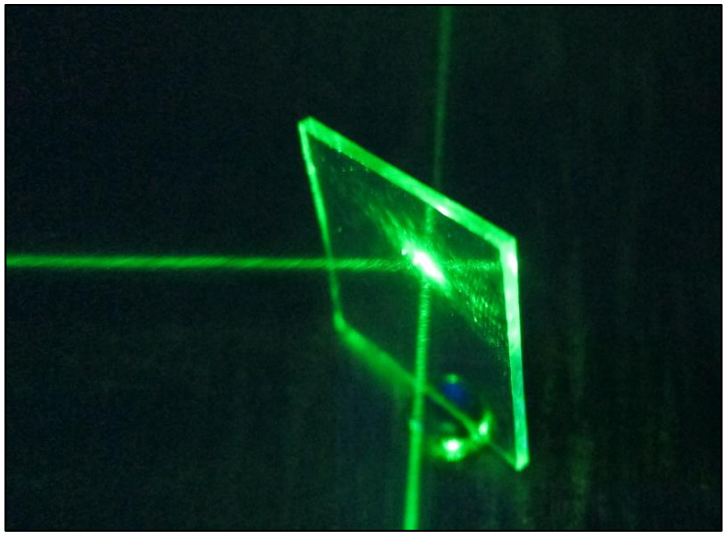


SCHRÖDINGER'S CAT IS

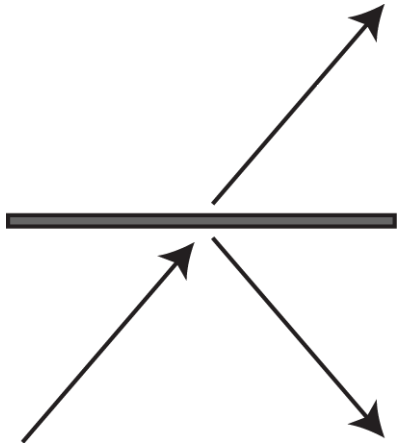
1 Beampslitters and superpositions

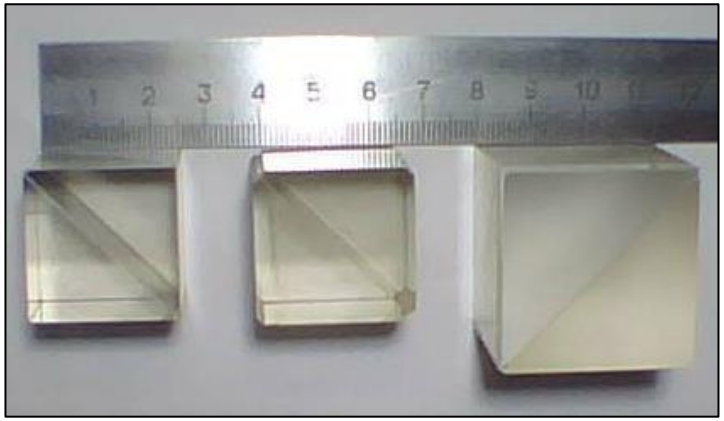
A single photon & a beamsplitter.





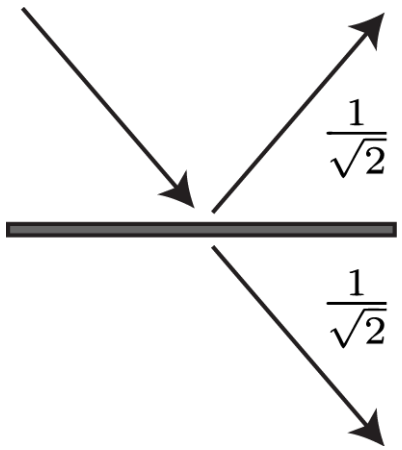
[wikipedia.org]





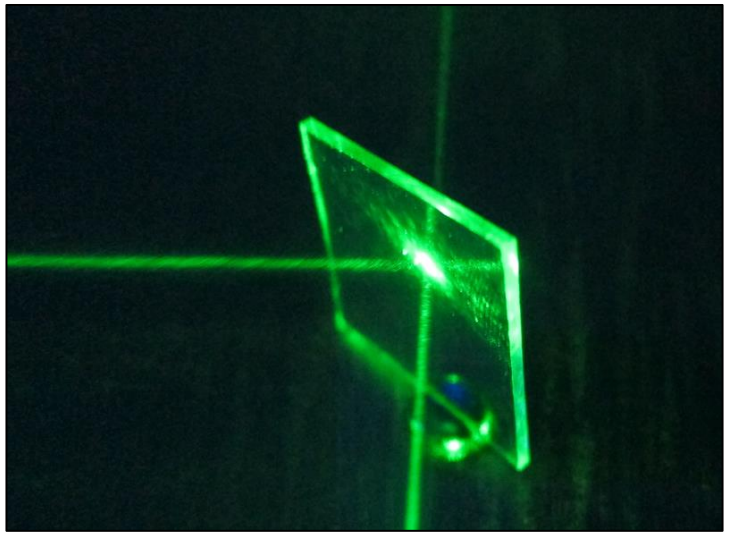
1 Beampslitters and superpositions

A single photon & a beamsplitter.

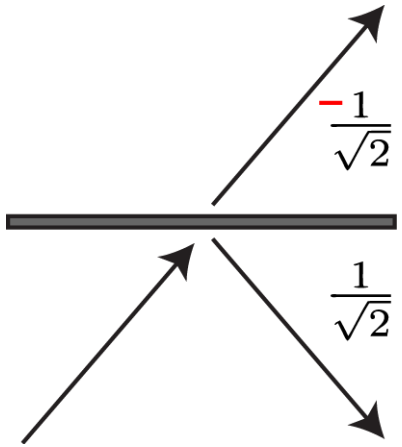


amplitude

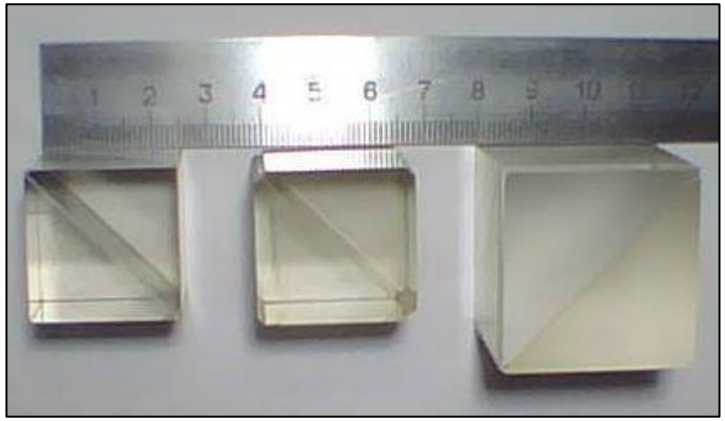
amplitude



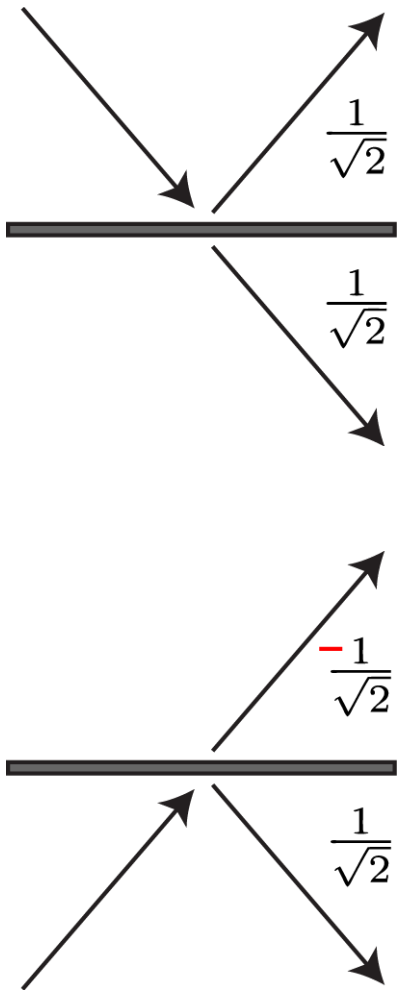
[wikipedia.org]

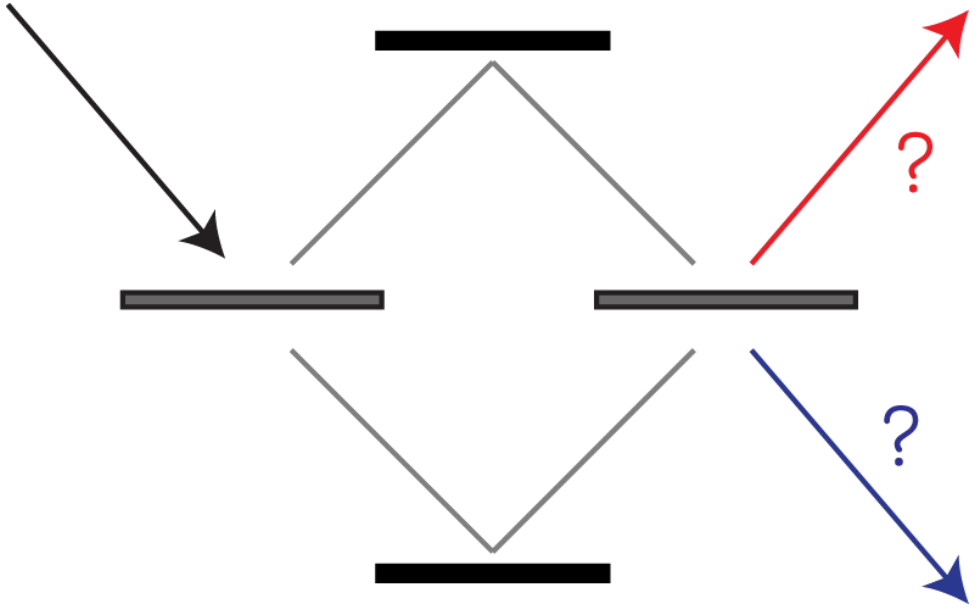


an extra phase!



1 Beampslitters and superpositions of light waves



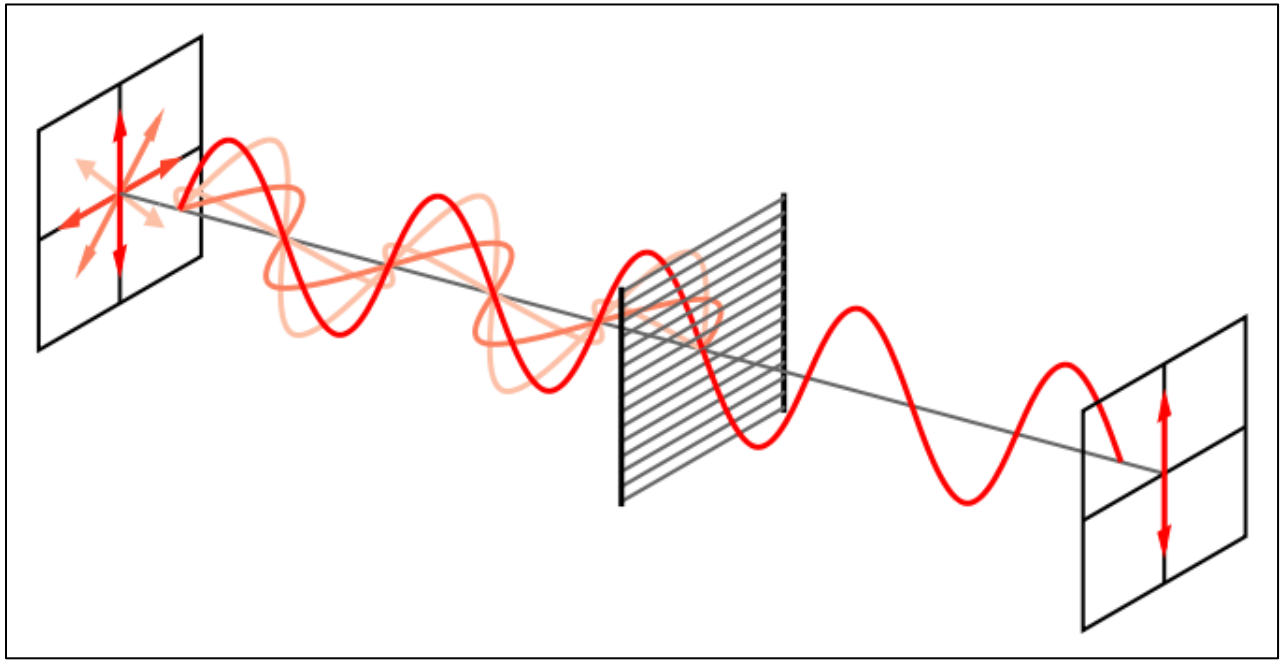


no filter

with a filter

1 Polarizing filters: looking through sunglasses

nonpolarized light: E in all directions

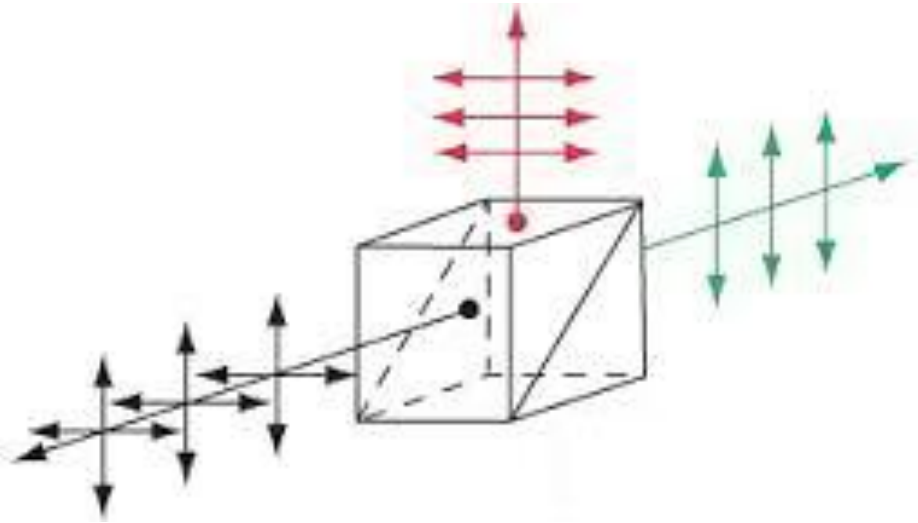


linear polarizer: let only one direction of E through

- a half-destructive measurement: pass/or not

1 Another option: polarizing beamsplitters

Horizontally polarized light goes one way, vertically polarized the other way.



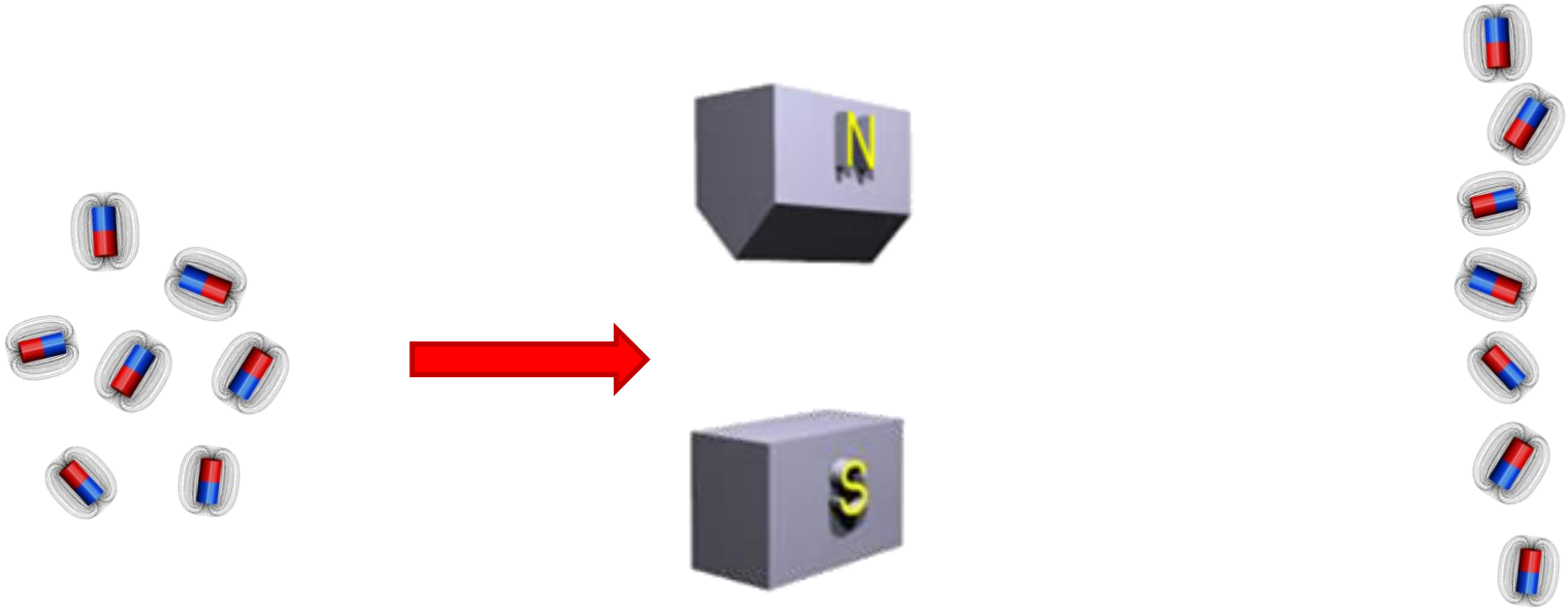
- What if we rotate the basis?

1 Do electrons contain small magnets?

Discovering the electron's spin.

inhomogeneous mag. field & magnets

just what we would expect



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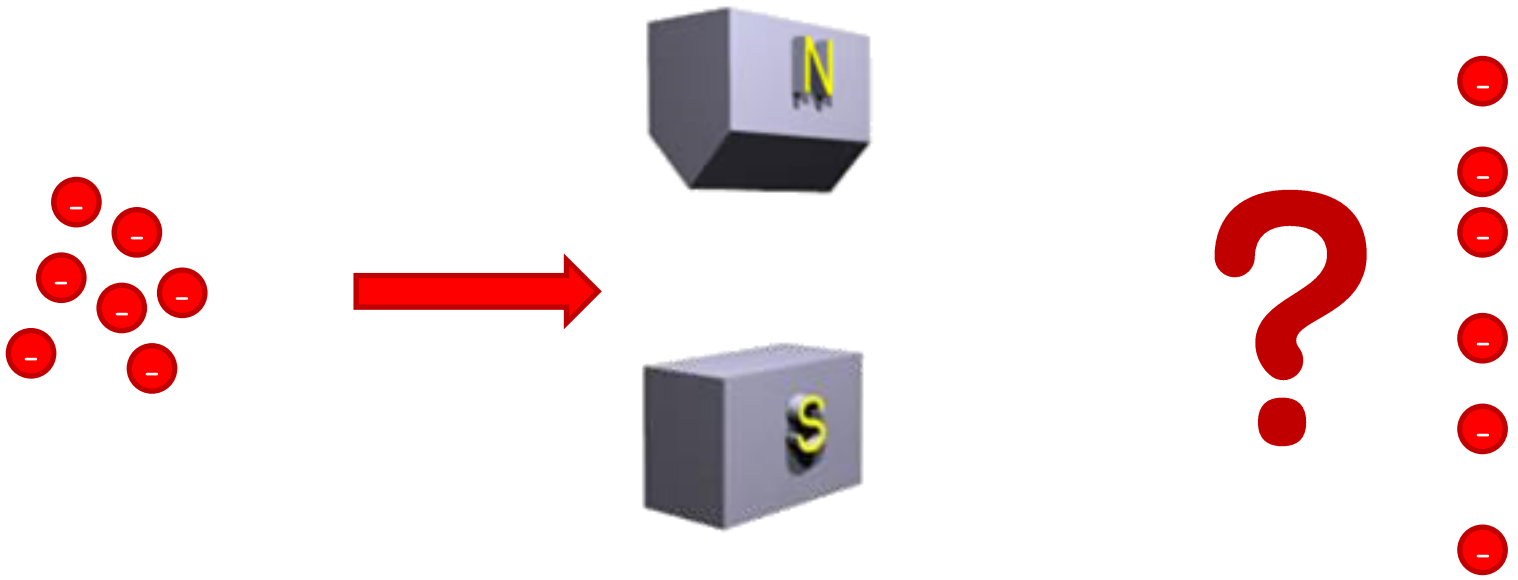
inhomogeneous mag. field & electrons

the Stern-Gerlach experiment (1922)



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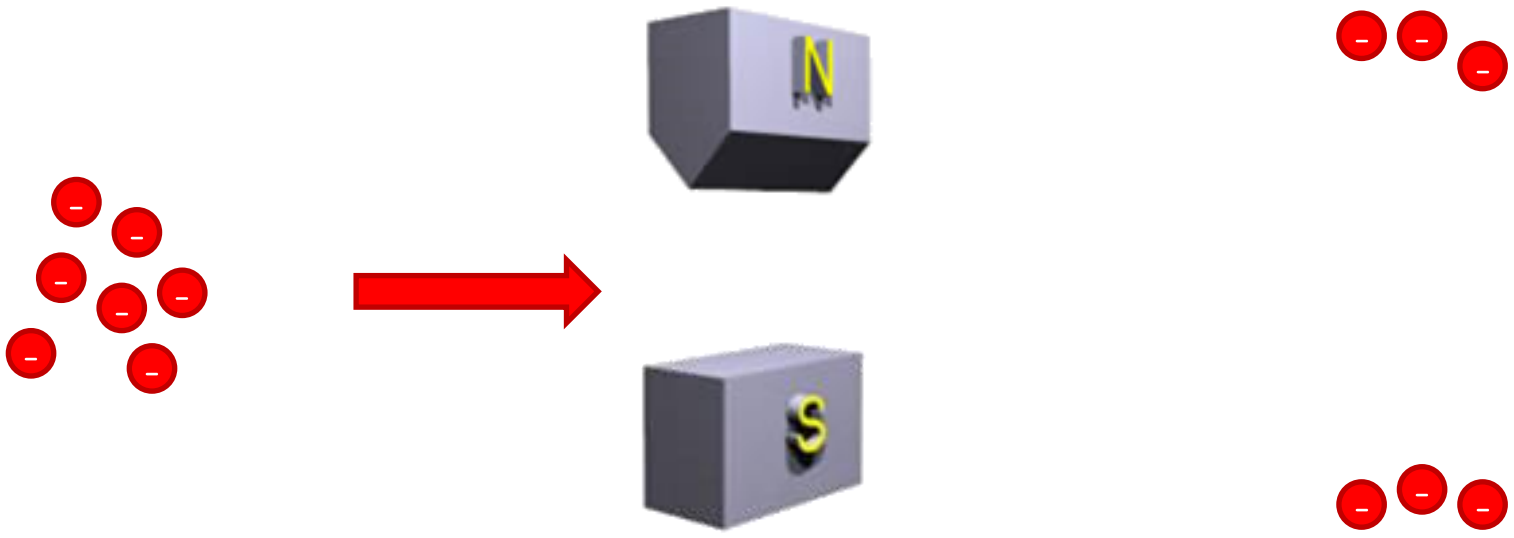
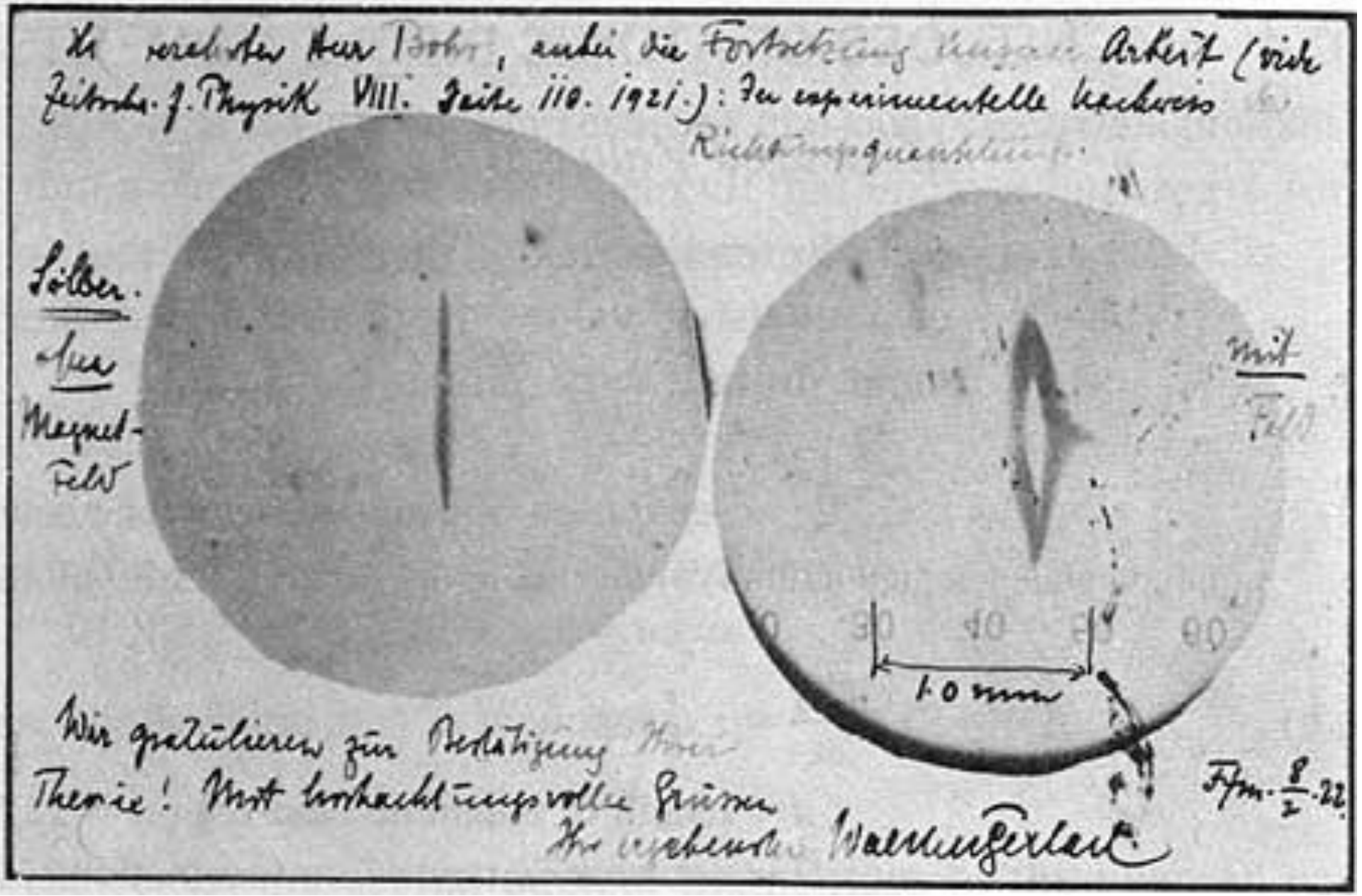


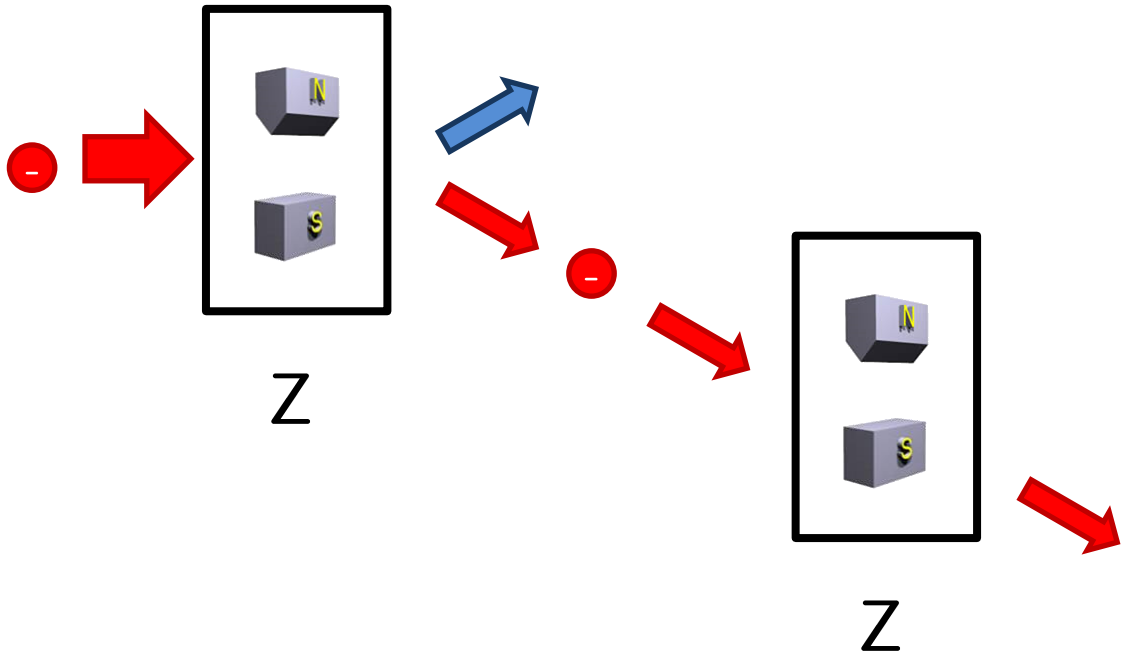


Gerlach's postcard to Bohr

[Niels Bohr Archive, Copenhagen]

1 Electrons and their spin

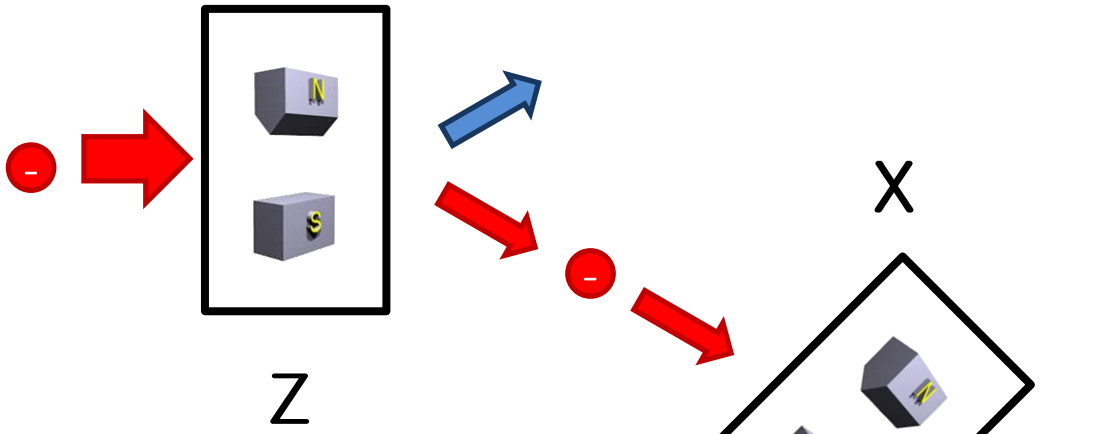
Predeterminted measurement results?



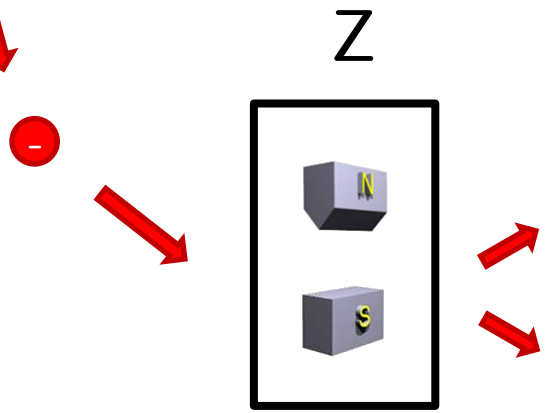
- **eigenstates** repeated measurements of the same type don't change the results anymore

1 Electrons and their spin

Predeterminted measurement results?

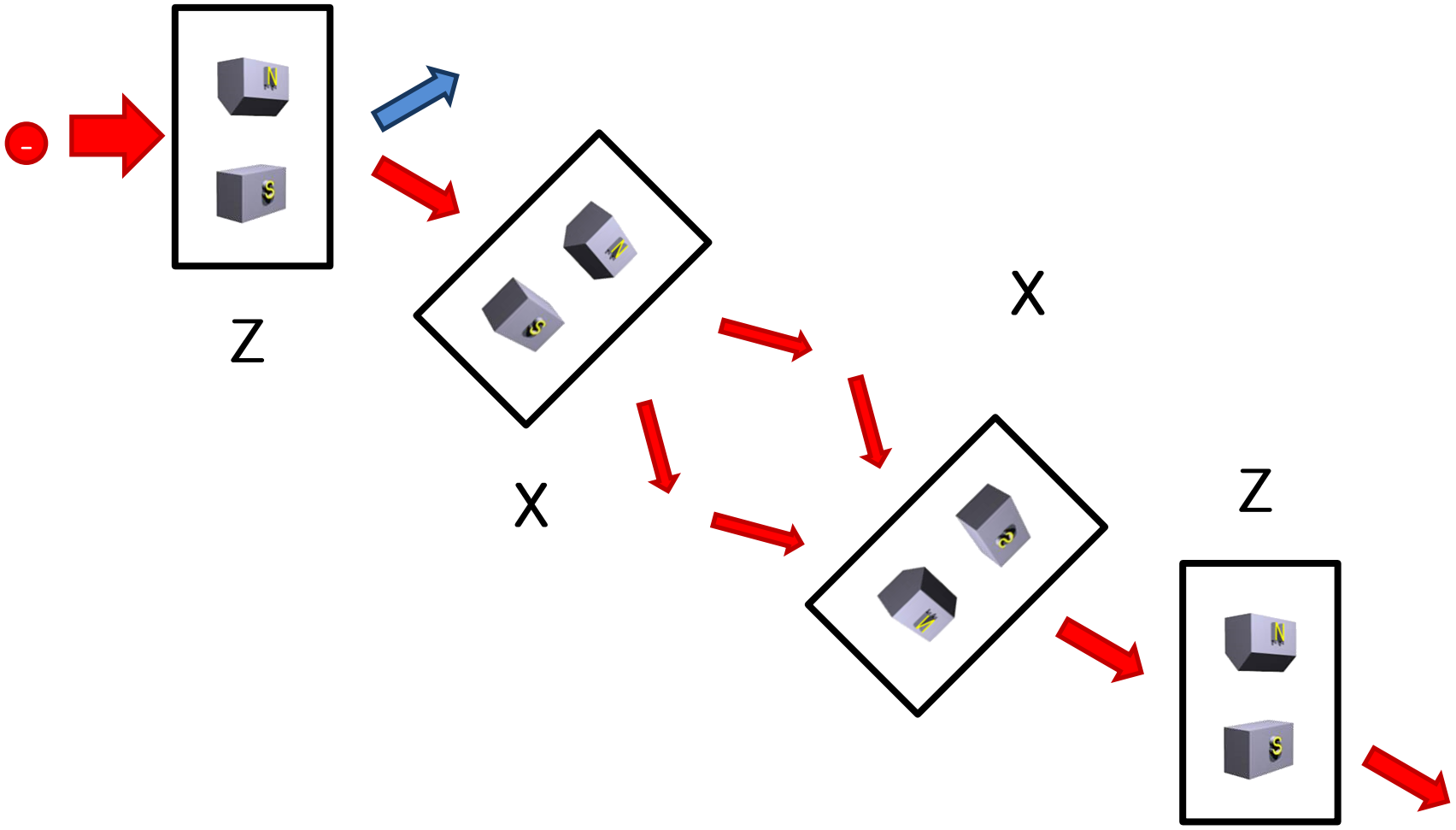


Data in a backpack?



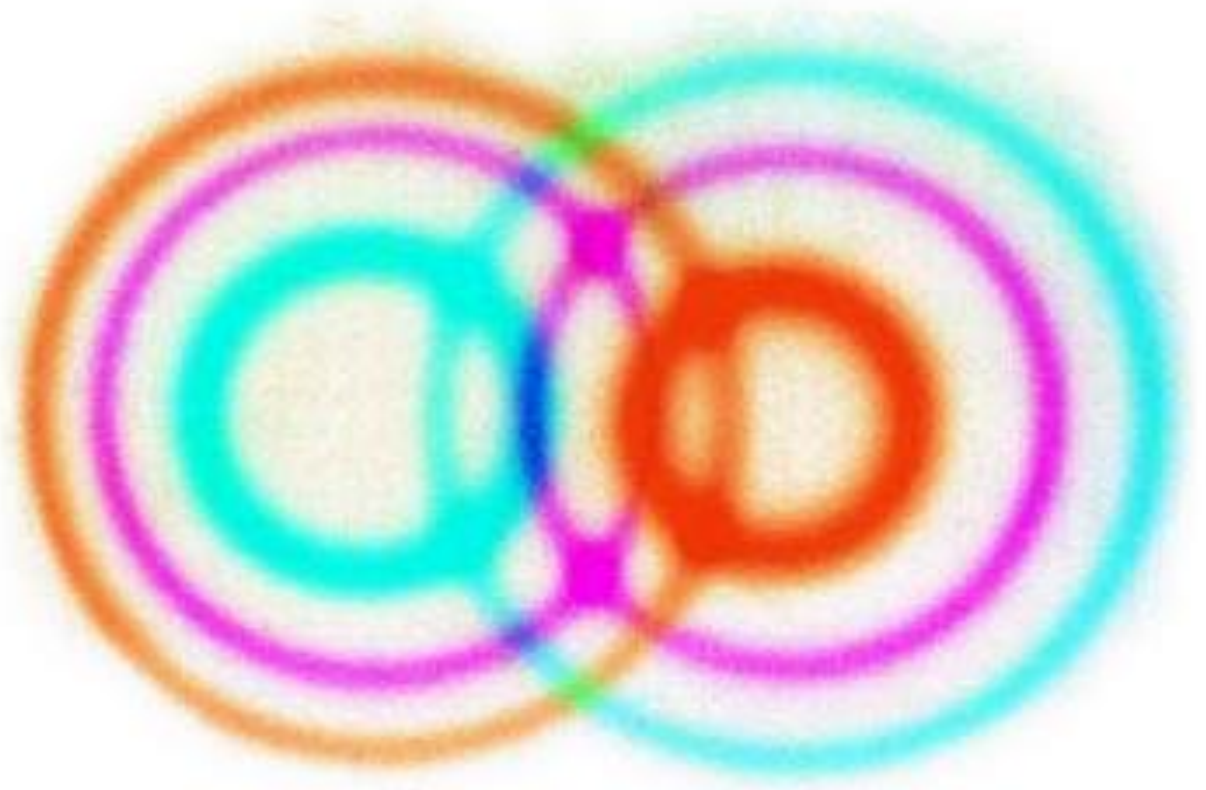
1 Electrons and their spin

Predeterminted measurement results?



1 Superpositions are necessary.

0's and 1's are can't describe photon polarizations and spins.



[qubit-ulm.com]

2 The playground of QM

0 1 bits

$|0\rangle$ $|1\rangle$ qubits

$|+\rangle$ $|-\rangle$ superpositions

polarized photons
(electron) spins
ground/excited atomic states
superconducting circuits
quantum dots

2 The quantum-mechanical playground

The state of a single qubit.

a bit 0 or 1

a qubit a (normalized, complex) vector in a 2D Hilbert space

$$|\psi\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle$$

- How many parameters?

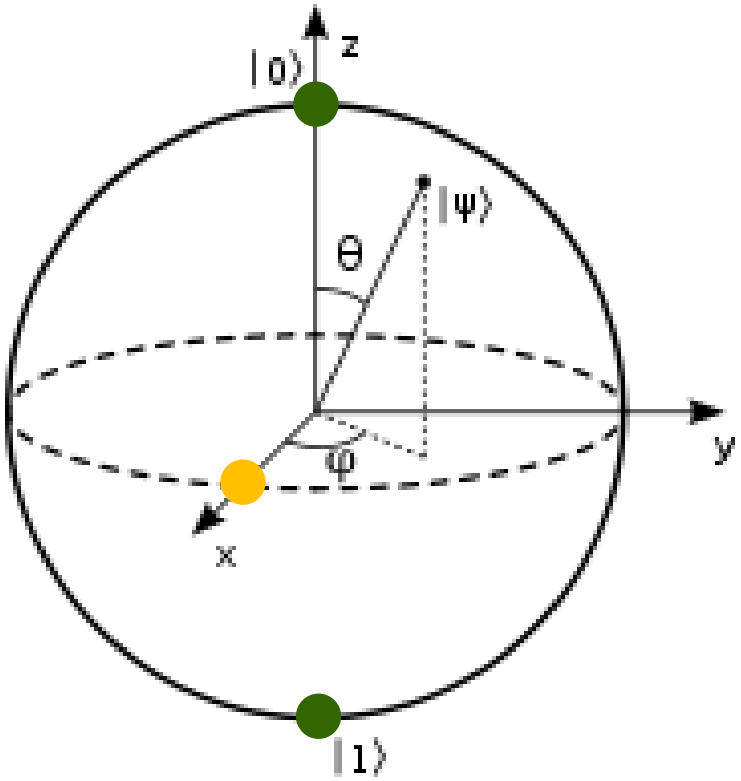
$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$|a|^2 + |b|^2 = 1$$

2 A qubit: the Bloch sphere

A two-angle parametrization.

- $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- $|+\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



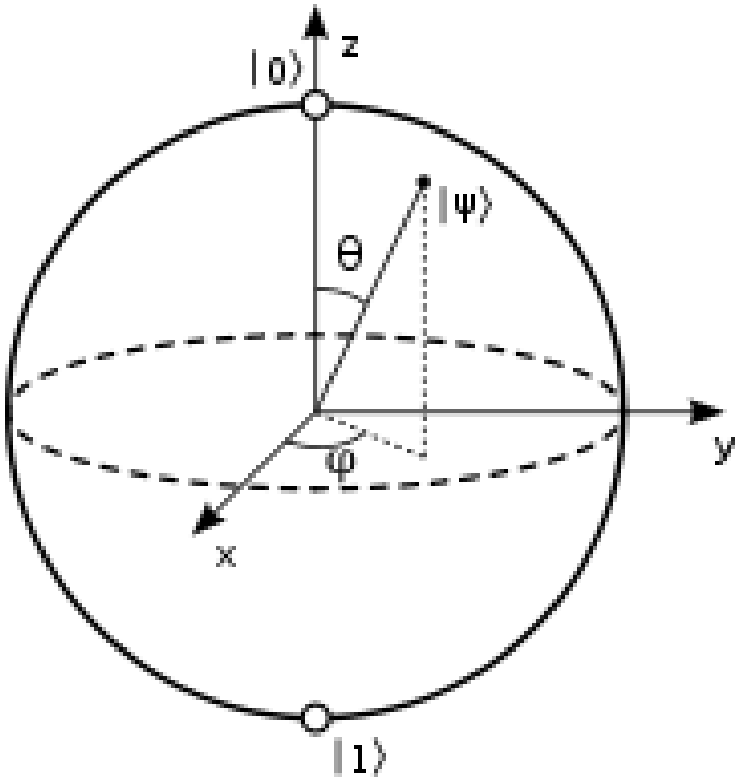
- How come the basis states are opposite from each other in the picture?

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \quad |a|^2 + |b|^2 = 1$$

2 A qubit: the Bloch sphere

A two-angle parametrization.



- How much information can we store in a qubit?
- How can we distinguish the states of a qubit?

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

- **unitary transformations**
  projective measurements

2 The Dirac bra-c-ket notation for states

Pure states of a qubit, overlaps and probabilities.

a "ket" a vector of amplitudes

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

the Z-basis

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

probability of finding "0"

$$p_0 = |a|^2$$

- Which state has a 50% probability to be up or down? $\begin{bmatrix} & \end{bmatrix}$
- How about another state...

2 The Dirac bra-c-ket notation for states

Pure states of a qubit, overlaps and probabilities.

a "ket" a vector of amplitudes

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

the Z-basis

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

probability of finding "ϕ" calculate the overlap of the states

$$p_\phi = |\langle\phi|\psi\rangle|^2$$

a "bra"

$$\langle\psi| = [|\psi\rangle]^\dagger = [\, a^* \;\; b^* \,]$$

eats a ket, spits out a number

2 The players and rules of QM

States, operators, measurements and evolution.

state a vector of amplitudes

operator a Hermitian matrix
what we observe
real eigenvalues

energy
position
momentum
correlation

$$\bar{A} = \langle\psi|A|\psi\rangle$$

Schrödinger equation $i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$

the Hamiltonian H $U(t) = e^{-iHt}$ generates unitary evolution

2 The players and rules of QM

States, operators, measurements and evolution.

state a vector of amplitudes

operator a Hermitian matrix
what we observe
real eigenvalues

energy
position
momentum
correlation

$$\bar{A} = \langle \psi | A | \psi \rangle$$

transformation a unitary matrix
something we can do
a reversible operation

a rotation
a “gate”
a conditional operation

$$|\psi'\rangle = U|\psi\rangle$$

2 What do you know about the Pauli operators?

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Hermiticity? Unitarity? Trace?
Eigenvalues? Eigenvectors?
Multiplication rules?
Commutation rules?
Exponentiation?
How do they act on
the others' eigenvectors?
How can we exchange
between the X and Z bases?

$$e^{-i\varphi Z}$$

$$e^{-i\varphi X}$$

$$\vec{\sigma} = (X, Y, Z)$$

$$e^{-i\varphi(\hat{r}\cdot\vec{\sigma})}$$

3 A spin in a magnetic field.

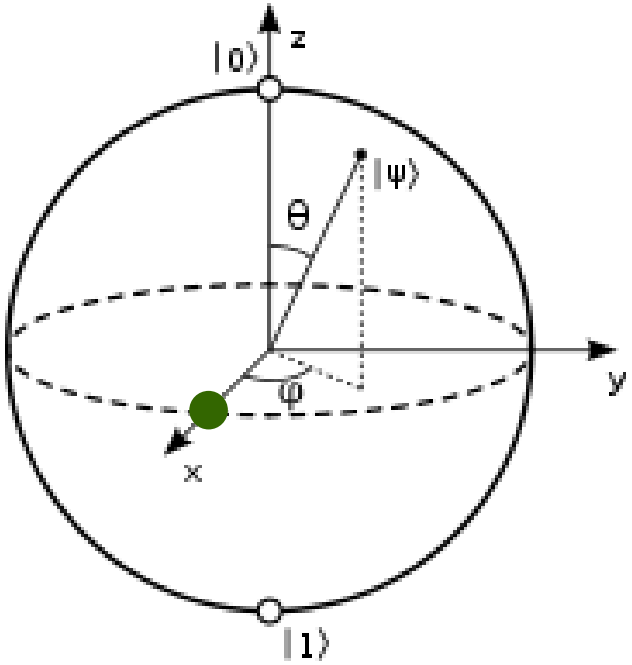
- A magnetic field in the z-direction.

$$H = -\mu\,\vec{\sigma}\cdot\vec{B} = -cZ = -c\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

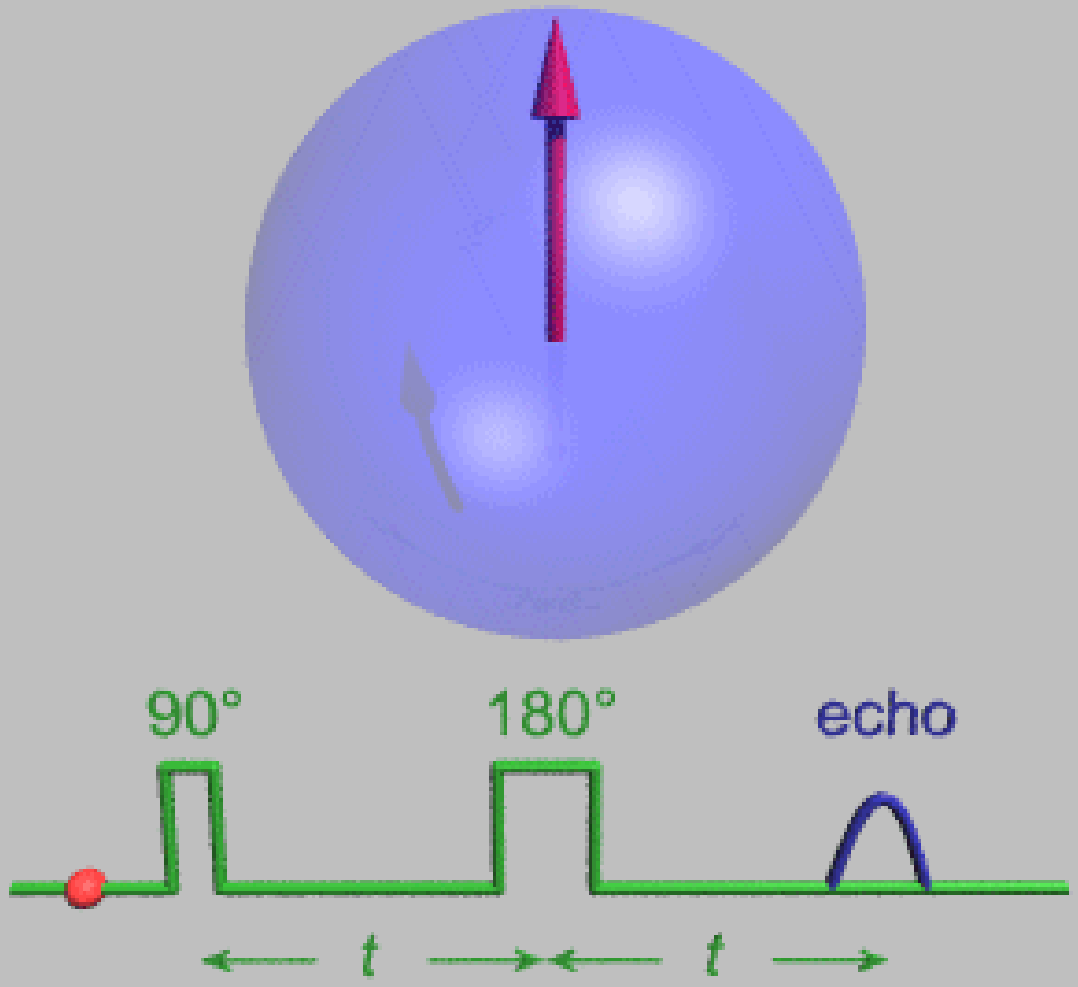
- Our initial state: spin in the x-direction.

$$|x+\rangle = |+\rangle = \tfrac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

- What will happen and why?



3 Spin echo.



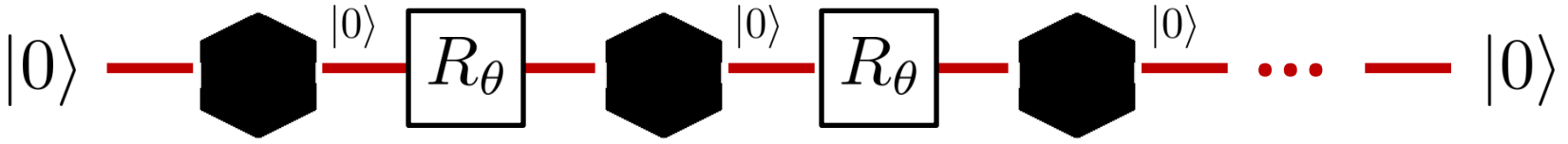
[Gavin W. Morley]

4 Detecting a bomb without detonating it [Elitzur-Vaidman].

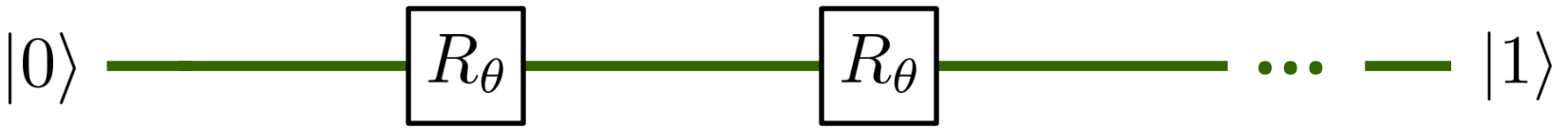
- A bomb explodes we decide to look at it.
- If we could use a qubit as our “control”...

  $|0\rangle$ don’t look $|1\rangle$ let’s test our luck

Rotate the qubit a little and test again...

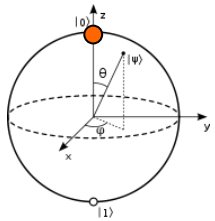


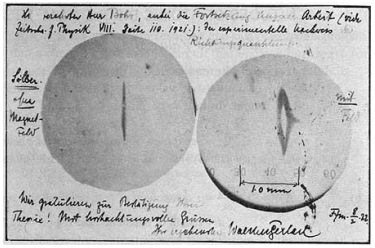
- What if there was no bomb?

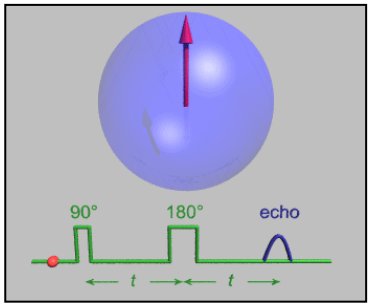


1 we need a qubit

and we can use it

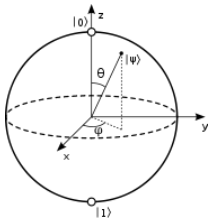






1 **we need a qubit**

and we can use it

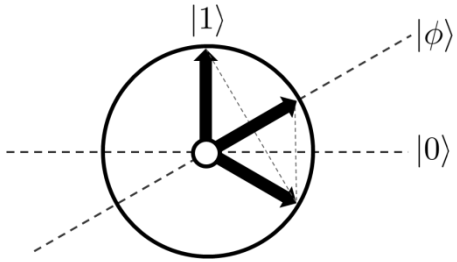


2 **EPR pairs**

give us cool 2-qubit protocols

3 **the algorithms**

that make quantum computing tick



4 **error correction**

can we really scale up this stuff?