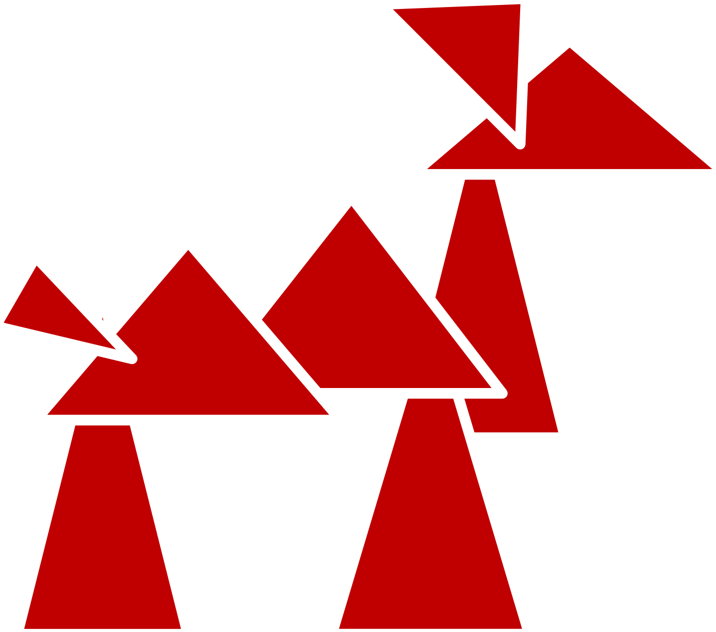
The good, bad & ugly side of

Quantum Satisfiability



D. Gosset, M. Bardoscia, A. Scardicchio,
S. Jordan, H. Nishimura, H. Kobayashi

Daniel Nagaj

universität wien

1 **good**

no frustration
perfect verifiers

bad

ugly

Quantum Satisfiability

1 **good**

no frustration
perfect verifiers

2 **bad**

complete
for QMA_1

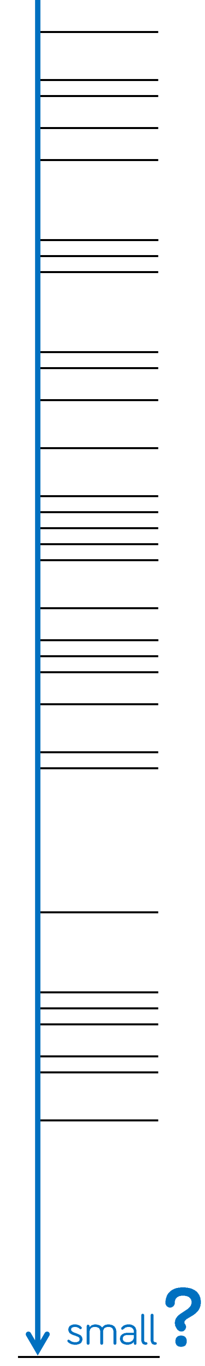
3 **ugly**

SAT/UNSAT
in random qsat

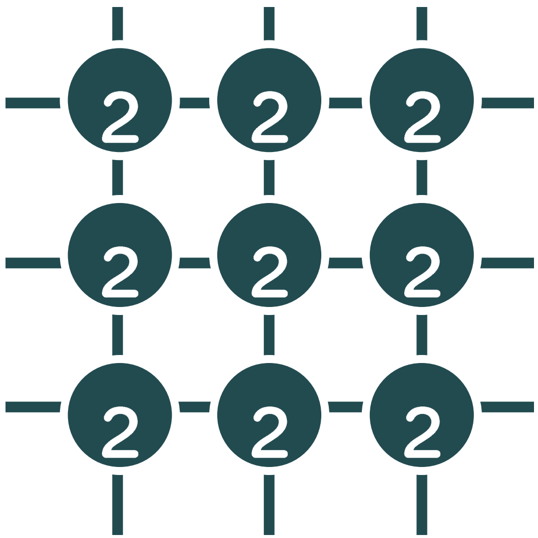
1 The Local Hamiltonian problem

Is
the
ground
state
energy
of a

LH



1 QMA-complete problems: LH

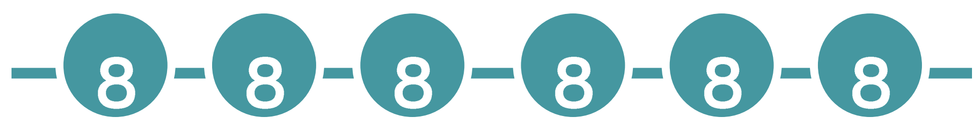


[Oliveira, Terhal '04]

a global minimum

LH

$$\sum H_{jk}$$

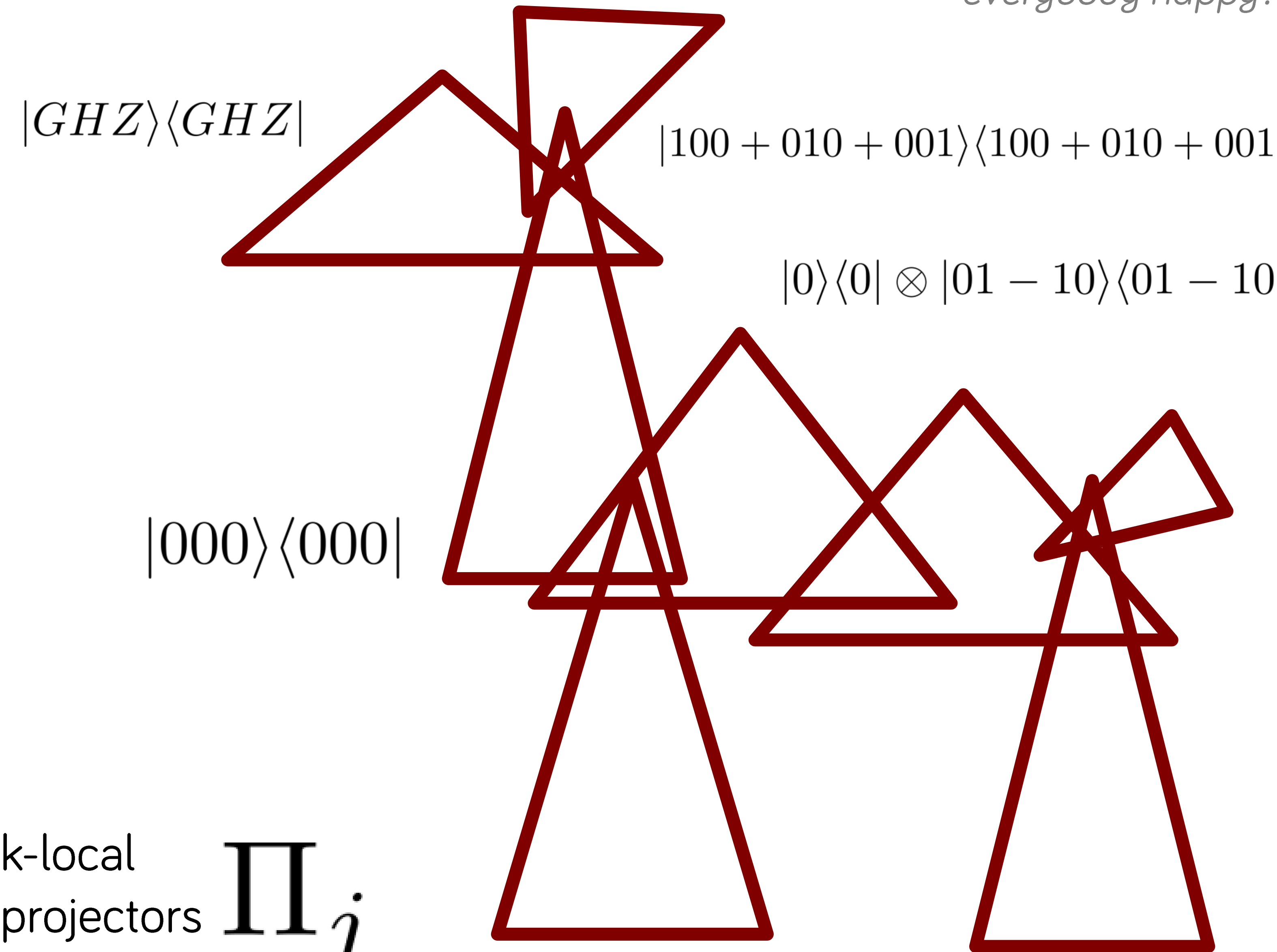


[Hallgren, N., Narayanaswami '13]


frustrated
FRUST
RATED

1 Quantum *k*-SAT

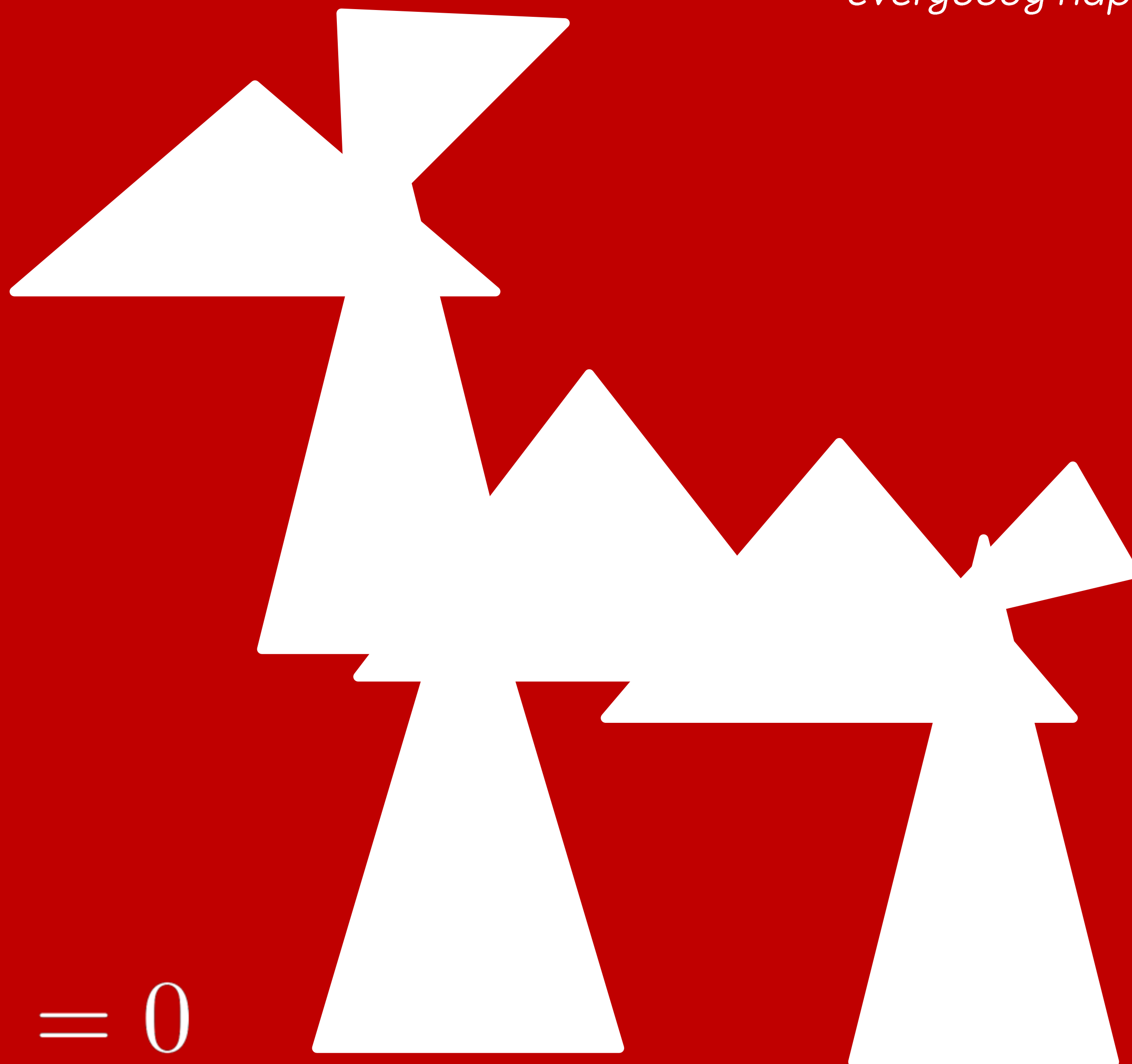
Can we make everybody happy?



- k-local projectors Π_j

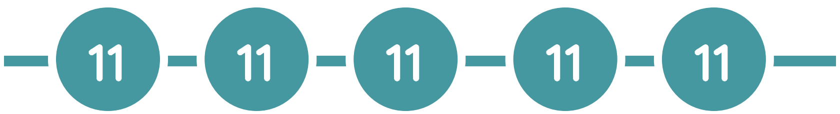
1 Quantum *k*-SAT

Can we make everybody happy?

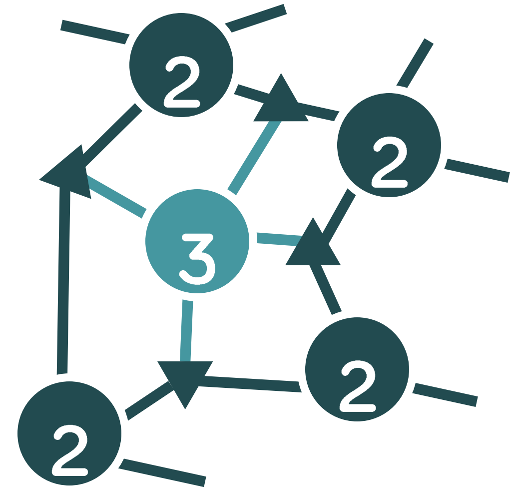


$$\Pi^j |\psi\rangle = 0$$

1 QMA$_1$-complete problems



[N. '08]



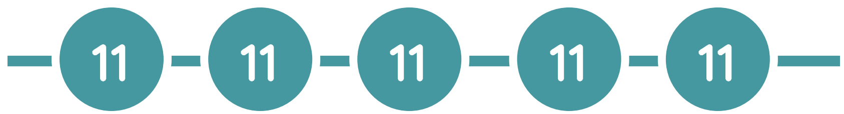
[Moses, N. '07]

Π^j

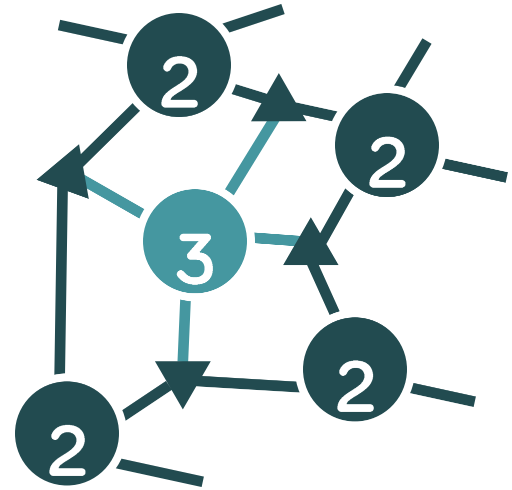
unfrustrated

qSAT

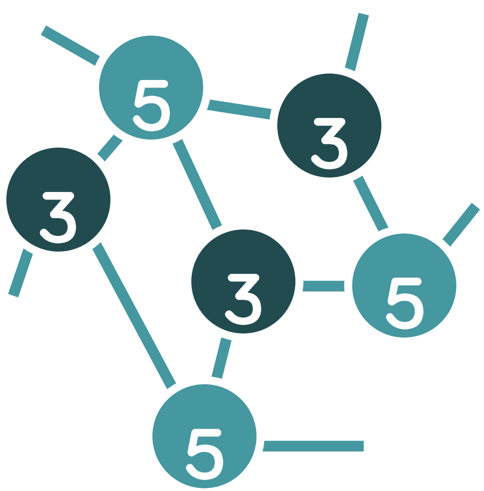
1 QMA$_1$-complete problems



[N. '08]



[Moses, N. '07]

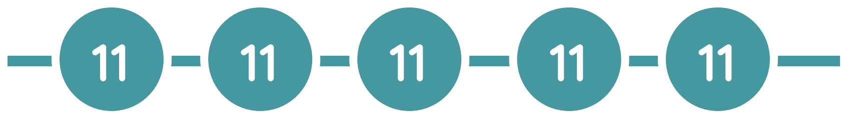


[Eldar, Regev '08]

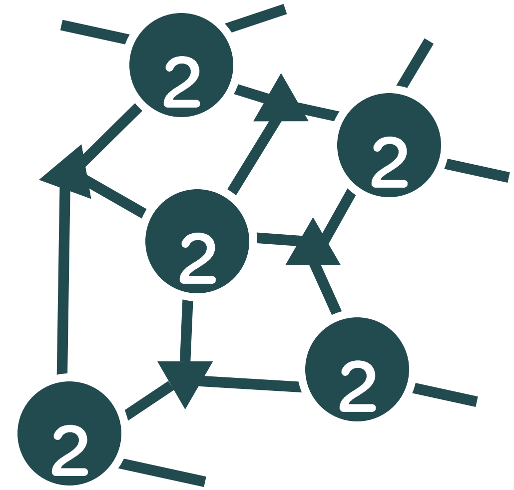
unfrustrated

qSAT

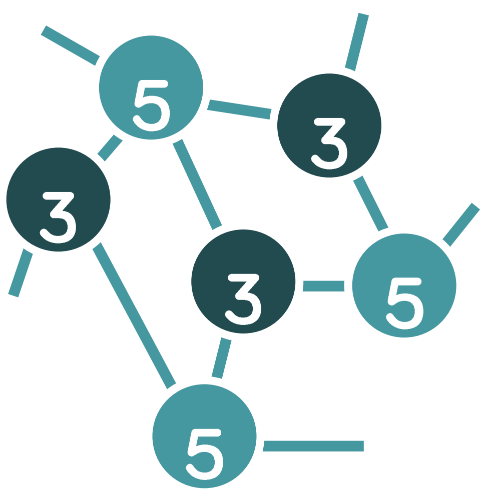
1 QMA_1-complete problems



[N. ’08]



[Gosset, N. ’13]



[Eldar, Regev ’08]

unfrustrated

qSAT

1 Verifying proofs

1 Soundness

NO?
Accept
a fake?

1 Completeness

Did dinosaurs exist?

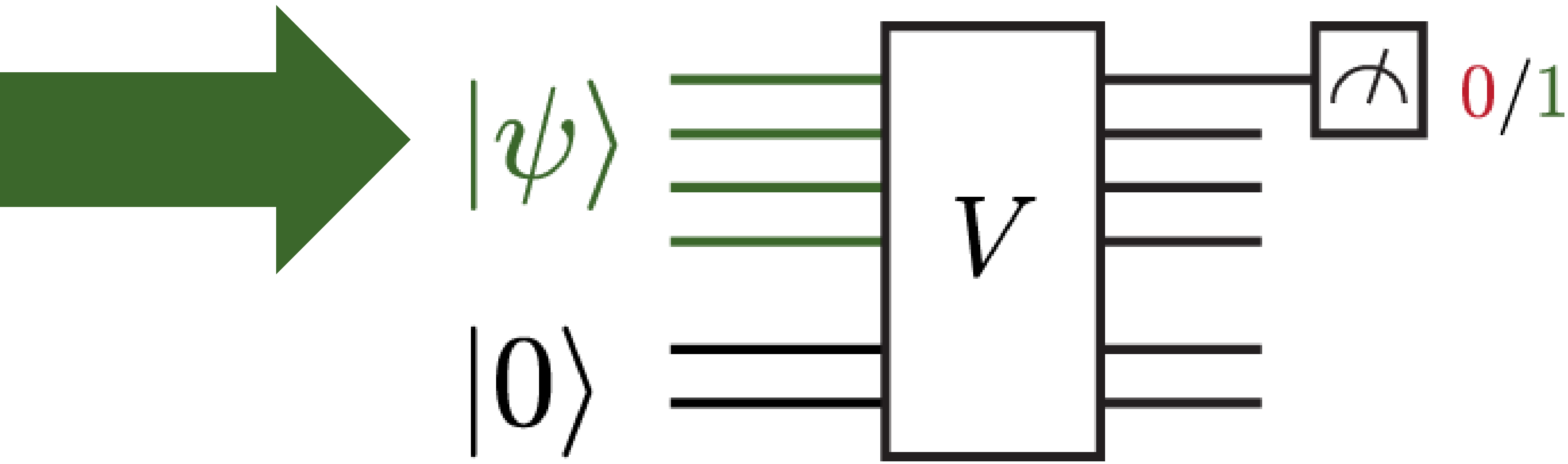
YES?
Accept
a genuine proof
without a doubt.

NO?
Still don't
get fooled
easily.

YES?
Accept
a genuine proof
without a doubt.

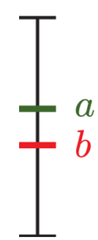
perfect
completeness

4 The QMA protocol

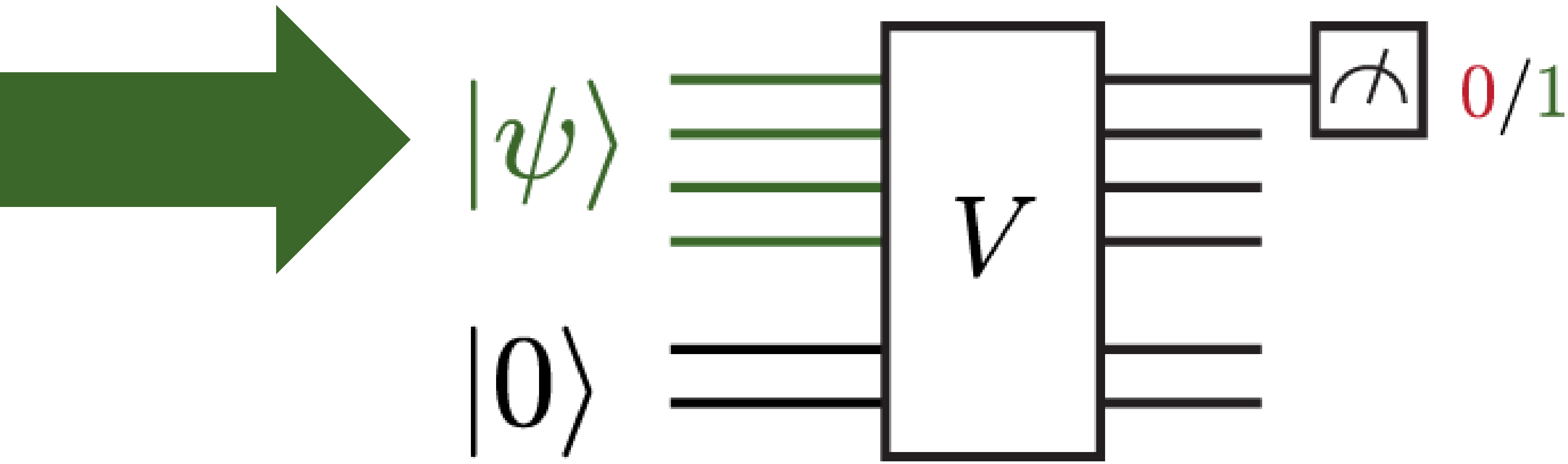


YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.



4 The QMA protocol: amplification



YES? Accept a good proof with $p > a$.

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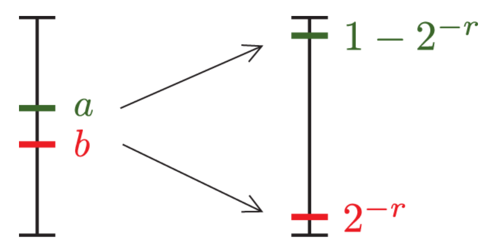


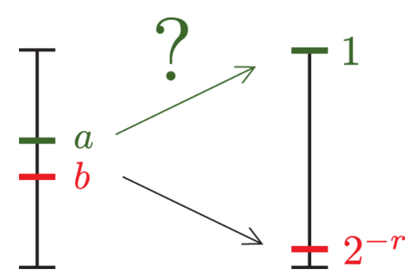
1 MA with one-sided error

perfect classical amplification

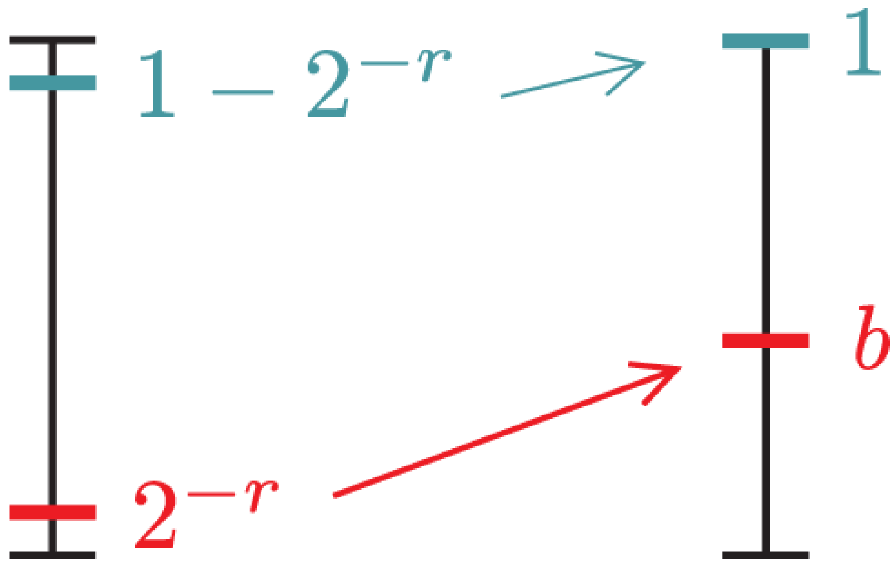
$$\mathrm{MA} = \mathrm{MA}_1$$ [Zachos & Fürer]

YES? Accept a good proof.

NO? Get fooled with small p.



1 QMA? Almost there ...



Probabilistically apply an unknown correction with M's help.

$$\mathrm{QMA} \subseteq \mathrm{QMA}_1^{\text{const. EPR}}$$

a proof by misunderstanding that doesn't work

QIP 2013
rump session

$$\mathrm{QMA} \overset{?}{\subseteq} \mathrm{QMA}_1$$

LH

?

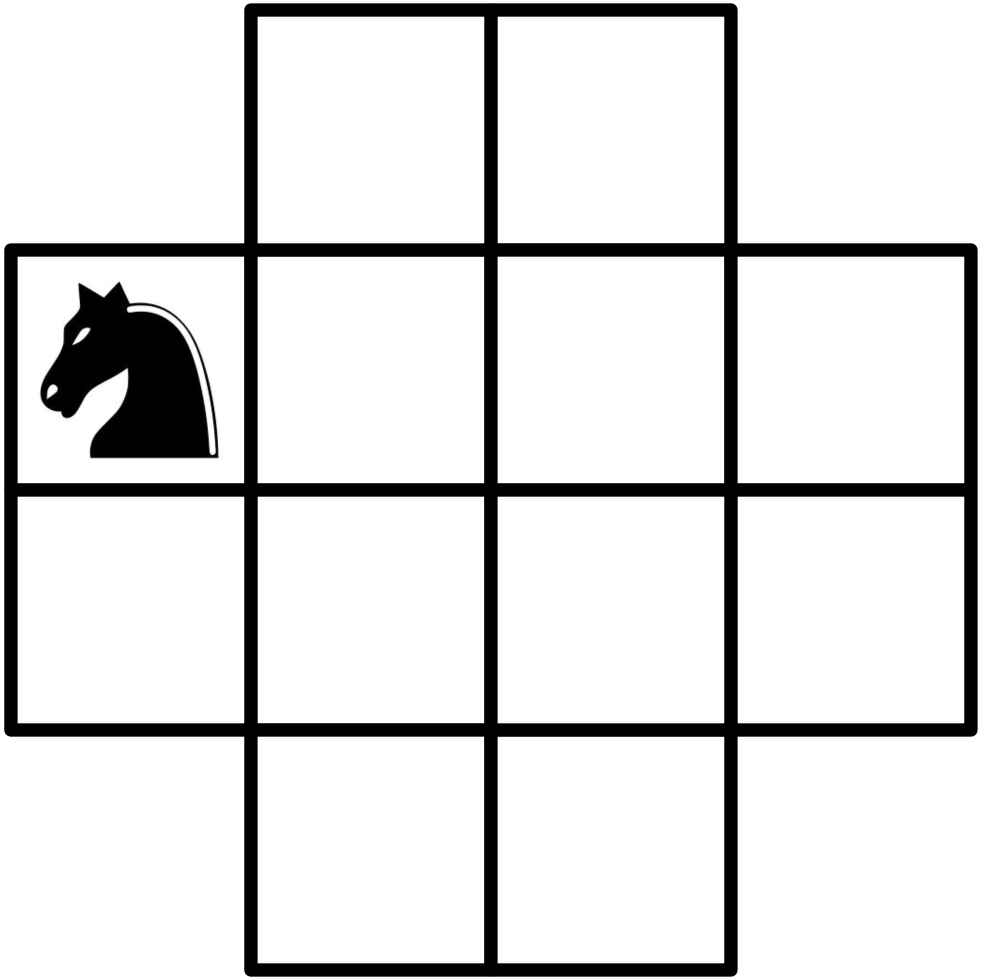
quantum SAT

frustrated

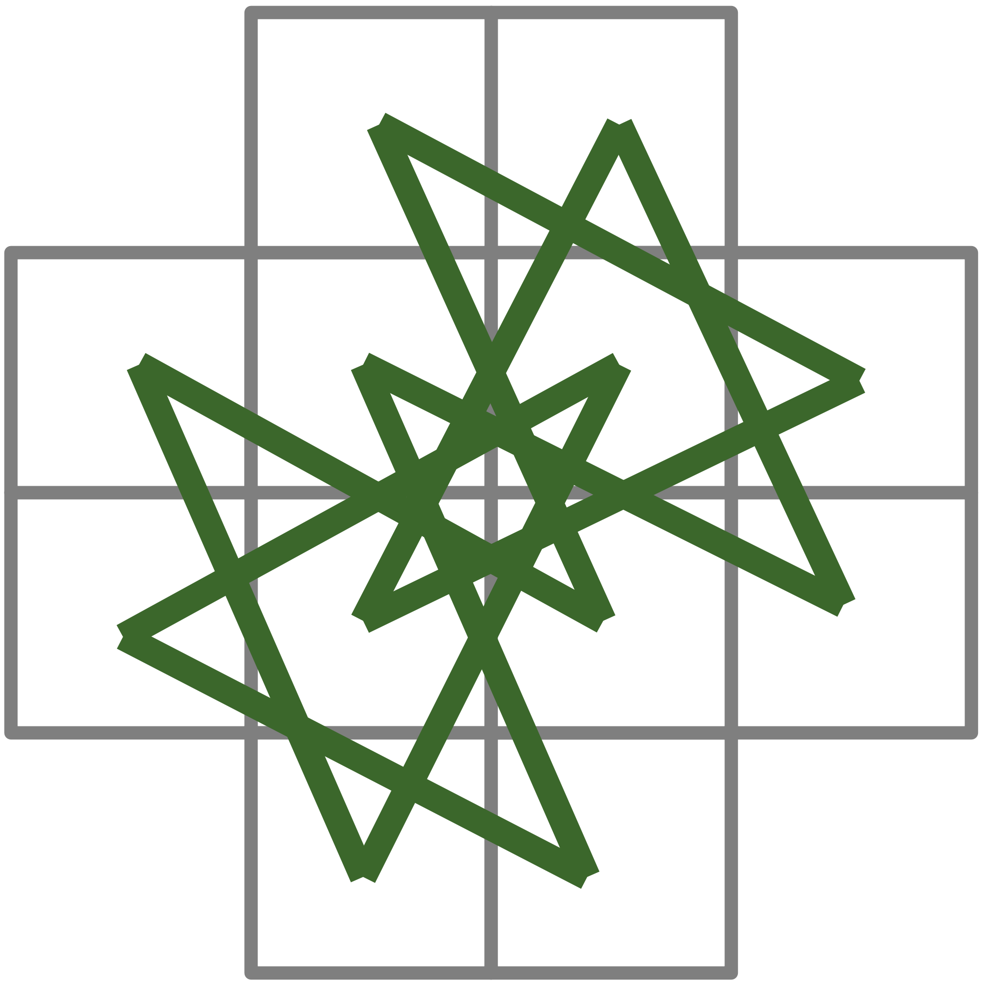
frustration free

proofs and puzzles

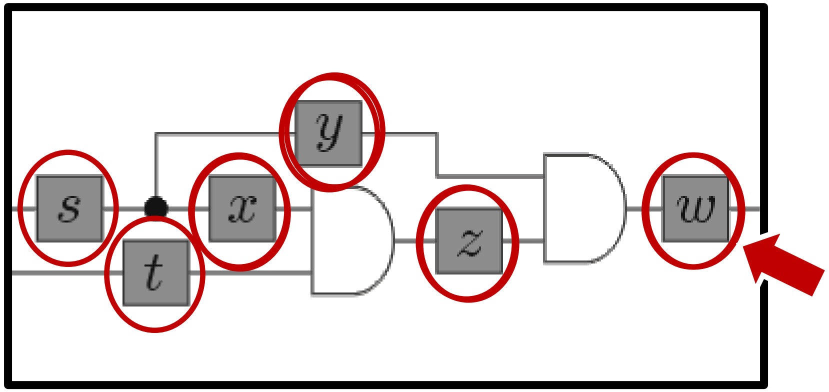
1 A Hamiltonian cycle puzzle



1 A Hamiltonian cycle puzzle



What is hard for NP?



Could this circuit ever output 1?

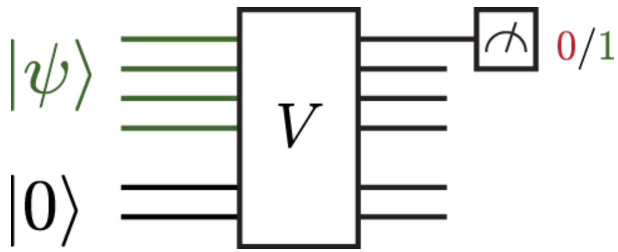
3-local conditions

$$(\cdots \vee \cdots \vee \cdots) \wedge (\cdots \vee \cdots \vee \cdots) \wedge \cdots$$

- 3-SAT is NP-complete. [Cook, Levin]

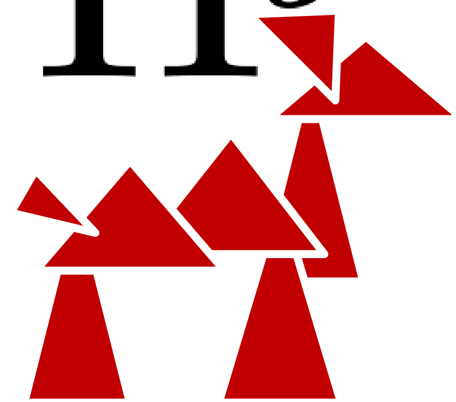
2 A QMA_1-hard problem?

Does this circuit accept?



A common ground state?

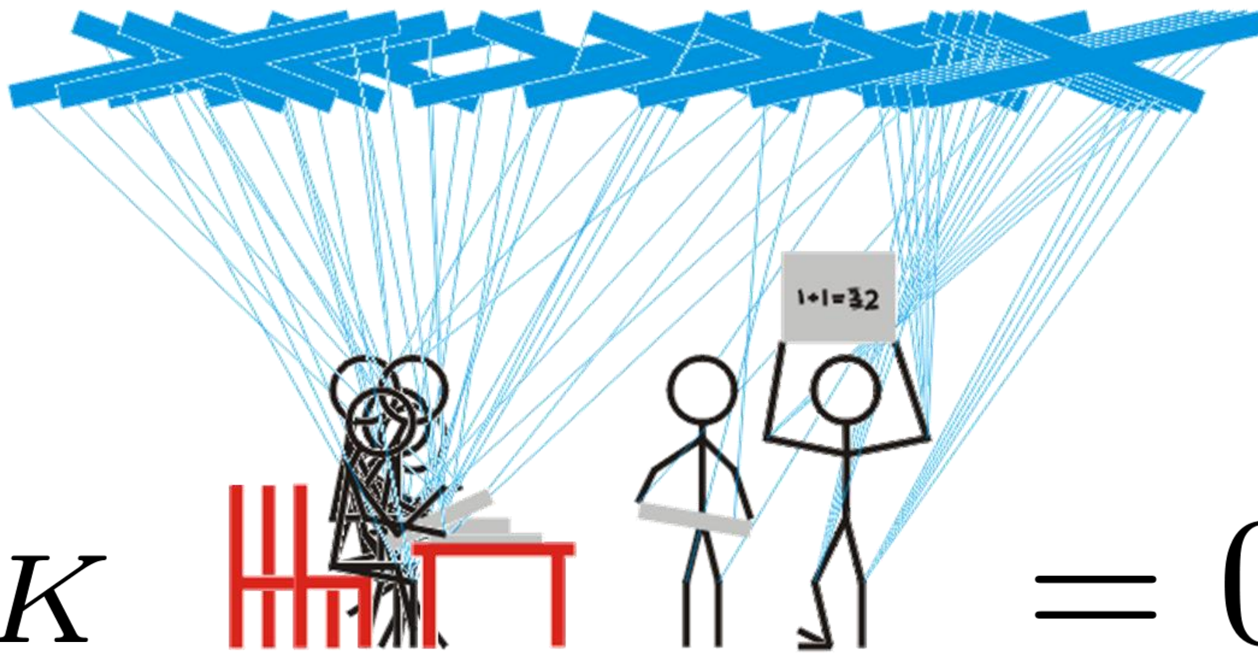
Π^j



quantum 3-SAT

quantum sat
& ground states
& computation

2 The history state: a ground state

H_K  $= 0$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} \underbrace{|\varphi_t\rangle}_{U_t \cdots U_1|\varphi_0\rangle} \otimes |t\rangle$$

2 The history state is a ground state

k-local

c-o-n-d-i-t-i-o-n-s

clock encoding

state progression

initialization

outpu

$$|\cdots 000 \cdots 0\rangle \otimes |0\rangle$$

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

$$|\cdots 1\rangle \otimes |T\rangle$$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

Checking proper computation

Antisymmetry checks.

- uniform superpositions: zero-energy eigenstates

$$H_t = \tfrac{1}{2}\left(|t+1\rangle\langle t+1| + |t\rangle\langle t|\right) - \tfrac{1}{2}\left(U_{t+1}\otimes|t+1\rangle\langle t| + U_{t+1}^{\dagger}\otimes|t\rangle\langle t+1|\right) = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Feynman's Hamiltonian

a projector

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}}\sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

Antisymmetry checks.

- uniform superpositions: zero-energy eigenstates

$$H_t = \tfrac{1}{2}\Big(|\varphi_{t+1}\rangle|t+1\rangle - |\varphi_t\rangle|t\rangle\Big)\Big(\langle\varphi_{t+1}|\langle t+1| - \langle\varphi_t|\langle t|\Big)$$

- an energy penalty for antisymmetric combinations

$$|\varphi_t\rangle \otimes |t\rangle$$
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}}\sum_{t=0}^{T}|\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

$$\sum_{t=1}^{L} H_t = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

positive semidefinite

$$|\varphi_t\rangle \otimes |t\rangle$$
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

$$\sum_{t=1}^{L} H_t = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

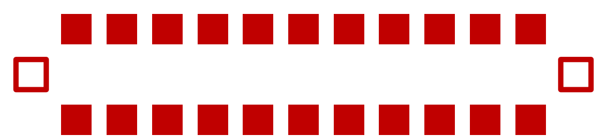
positive semidefinite

local?

$$\sum_t e^{-ipt}|\varphi_t\rangle \otimes |t\rangle$$

eigenvectors: combinations of plane waves

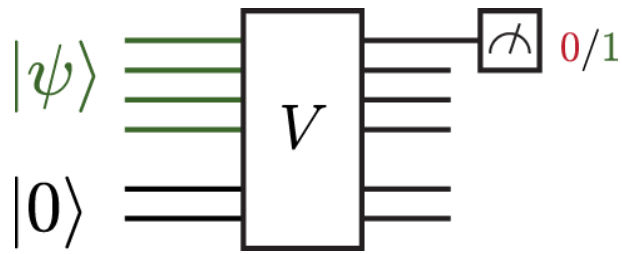
L

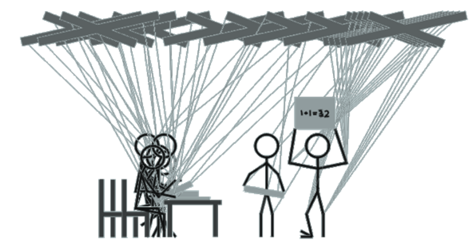


2(L+1)

2 LH and computation

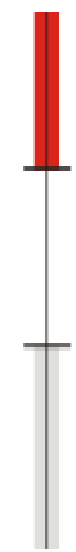
$$H_{clock} + H_{init} + H_{prop} + H_{out}$$





NO no witness accepted by V more likely than ε

any state has a high energy



YES there is a proof accepted by V with probability 1-ε

the history state of V acting on the proof has a low energy a clock workshop

3 Constructing local clocks

- the domain wall 

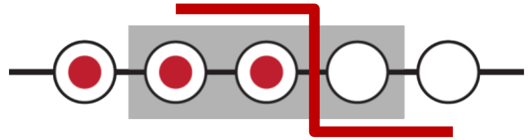
$$|t\rangle = |1\rangle$$
$$= |10000\rangle$$

- 2-local terms "compatible" with **11...1100...00**

?

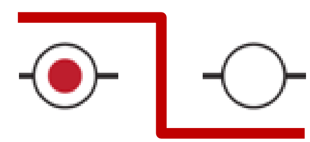
$$|01\rangle\langle 01|$$

3 Constructing local clocks

- the domain wall  transitions: 3-local

$$|t\rangle = |3\rangle$$
$$= |11100\rangle$$

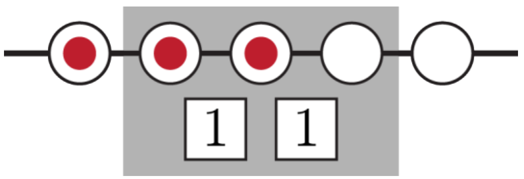
- joining states by transitions? $|100 - 110\rangle\langle 100 - 110|$

- enforce a domain wall: fix the ends 

- the ground state $\cdots + |2\rangle + |3\rangle + \ldots$

3 Constructing local clocks

- the domain wall



transitions: 3-local
2-qubit gates: 5-local

- interacting with data

$$H_t = \tfrac{1}{2}\left(|t+1\rangle\langle t+1| + |t\rangle\langle t|\right) - \tfrac{1}{2}\Big(U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^{\dagger} \otimes |t\rangle\langle t+1|\Big)$$

5-local

3 Local Hamiltonian: putting it all together

- punish bad ancilla initialization

$$\sum_{a=1}^{n_a} |1\rangle\langle 1|_a \otimes |10\rangle\langle 10|_{c_1,c_2} \qquad |\varphi_0\rangle \otimes |0\rangle_c$$

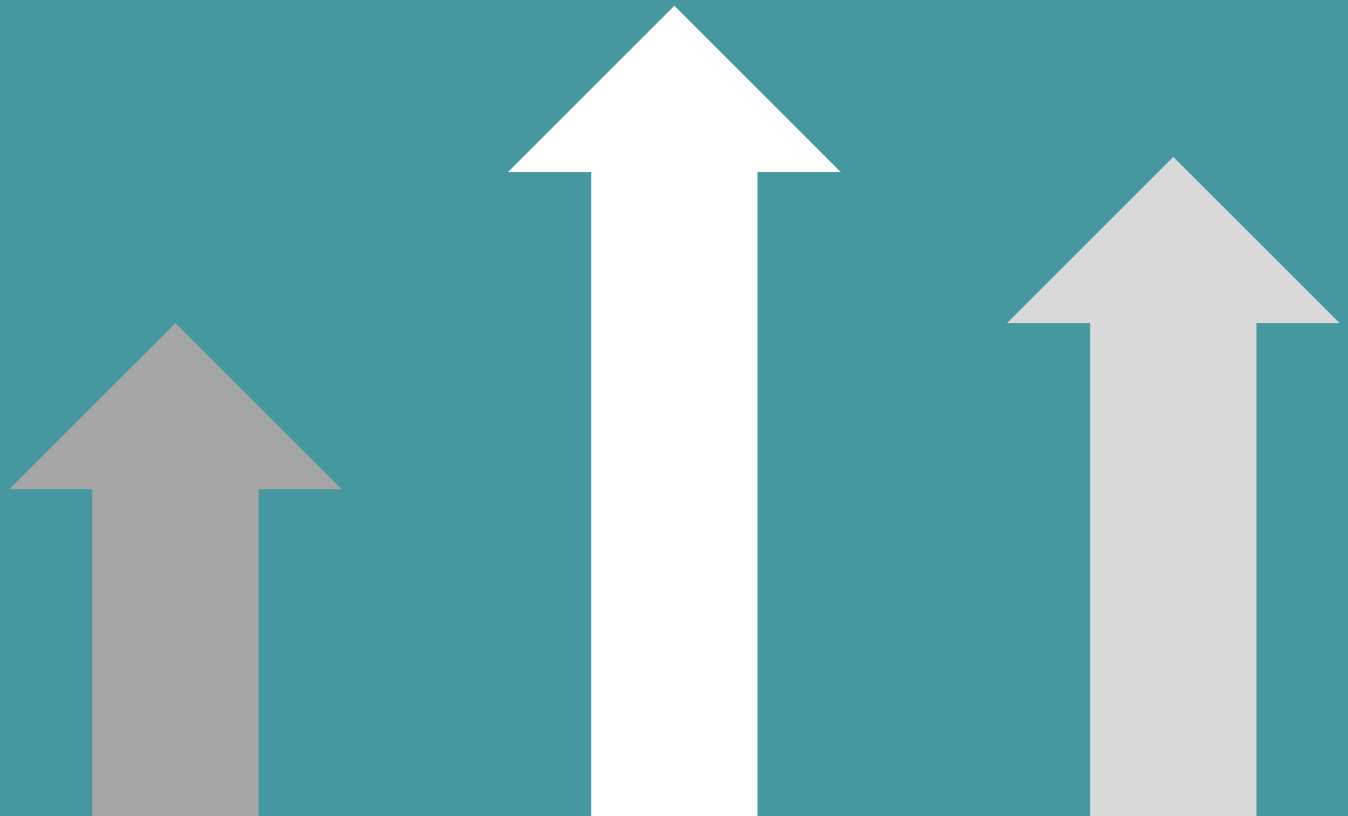
- check the computation

$$\begin{aligned} H_t = \tfrac{1}{2}\left(|t+1\rangle\langle t+1| + |t\rangle\langle t|\right) \\ -\tfrac{1}{2}\left(U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^{\dagger} \otimes |t\rangle\langle t+1|\right) \end{aligned}$$

- punish non-accepting computations

$$|0\rangle\langle 0|_{out} \otimes |1\rangle\langle 1|_{c_L} \qquad |\varphi_L\rangle \otimes |L\rangle_c$$

lower bounding the ground state **energy**



good
clock
states

… 01 …
bad
clock
states

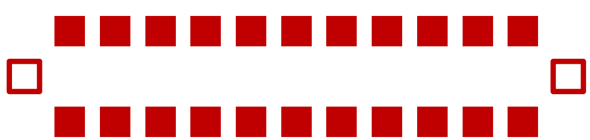
history states

non-uniform
superpositions

history states

a polynomially small gap

$\Delta = O\left(L^{-2}\right)$



history states

well

badly

initialized history states

well

initialized histories

accepted states

well

initialized histories

accepted states

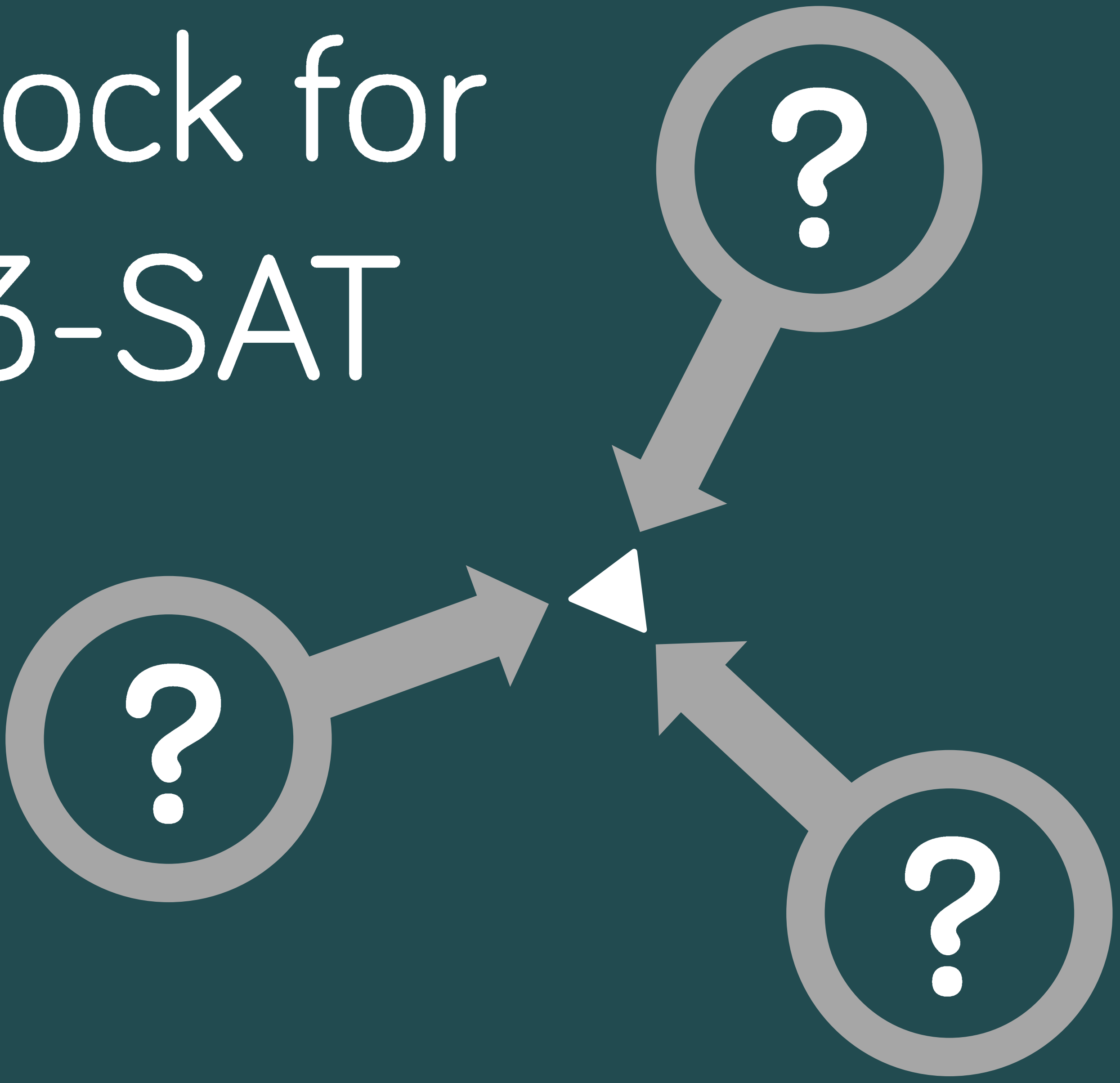
$H_A + H_B$

$$\lambda_0 \geq \sin^2 \frac{\vartheta}{2} \times \min(\Delta_A, \Delta_B)$$

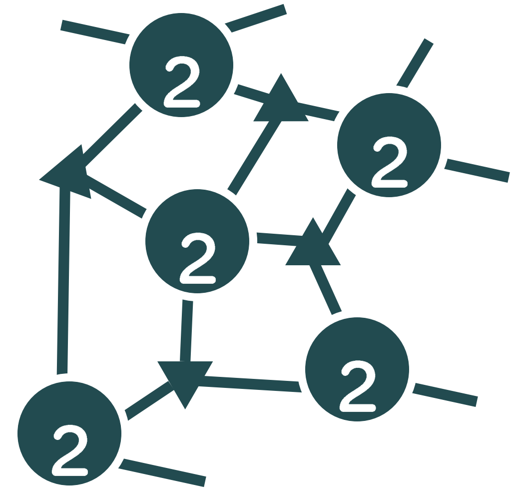
L^{-2}

L^{-1}

a clock for Q-3-SAT



3 Circuits & ground states



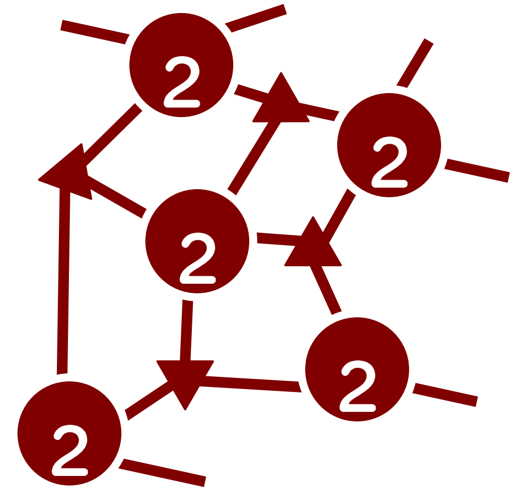
an unfrustrated instance of quantum 3-SAT

$$\frac{1}{4}\left(a + i\,b + \sqrt{2}\,c + i\sqrt{2}\,d\right)$$

CNOT, H & T

a verifier circuit with a perfectly accepted witness

3 Circuits & ground states



a frustrated instance
of quantum 3-SAT

CNOT, H & T

a verifier circuit which doesn't like to accept anything

3 Run the clock, apply 2-qubit gates … 3-locally?

$$|\varphi_{t-2}\rangle \otimes |t-2\rangle$$

$$|\varphi_{t-1}\rangle \otimes |t-1\rangle$$

$$|\varphi_{t}\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

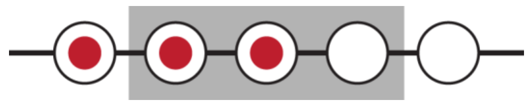
$$|\varphi_{t+2}\rangle \otimes |t+2\rangle$$

$$|\varphi_{t+3}\rangle \otimes |t+3\rangle$$

? ? ?

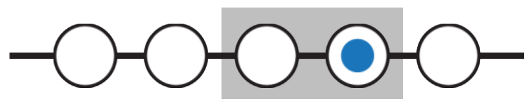
3 Constructing clocks

- the domain wall



transitions: 3-local

- the pulse



transitions: 2-local

$|11\rangle\langle 11|$ clock-checking

needs initialization

- a *good* clock, ticking *2-locally?*

3 Making a composite clock with 2-local progress

1111000000000
1111100000000
1111110000000
1111111000000
1111111100000
1111111110000
1111111111100
1111111111110

3 Making a composite clock with 2-local progress

111	100	000	000
111	110	000	000
111	111	000	000
111	111	100	000
111	111	110	000
111	111	111	000
111	111	111	100
111	111	111	110

3 Making a composite clock with 2-local progress

111 100 000 000
111 110 000 000

111 111 100 000
111 111 110 000

111 111 111 100
111 111 111 110

3 Look at triplets, transform some of them

$\substack{100\\+011}$ 100 000 000

$\substack{100\\+011}$ 110 000 000

$\substack{100\\+011}$ $\substack{100\\+011}$ 100 000

$\substack{100\\+011}$ $\substack{100\\+011}$ 110 000

$\substack{100\\+011}$ $\substack{100\\+011}$ $\substack{100\\+011}$ 100

$\substack{100\\+011}$ $\substack{100\\+011}$ $\substack{100\\+011}$ 110

3 Look at triplets, transform some of them

$+\genfrac{}{}{0pt}{}{100}{011}$ 110 000 000

$+\genfrac{}{}{0pt}{}{100}{011}$ 101 000 000

$+\genfrac{}{}{0pt}{}{100}{011}$ $+\genfrac{}{}{0pt}{}{100}{011}$ 110 000

$+\genfrac{}{}{0pt}{}{100}{011}$ $+\genfrac{}{}{0pt}{}{100}{011}$ 101 000

$+\genfrac{}{}{0pt}{}{100}{011}$ $+\genfrac{}{}{0pt}{}{100}{011}$ $+\genfrac{}{}{0pt}{}{100}{011}$ 110

$+\genfrac{}{}{0pt}{}{100}{011}$ $+\genfrac{}{}{0pt}{}{100}{011}$ $+\genfrac{}{}{0pt}{}{100}{011}$ 101

3 3-local conditions, allowed triplets

$+\substack{100\\011}$	110	000	000
$+\substack{100\\011}$	101	000	000

$+\substack{100\\011}$	$+\substack{100\\011}$	110	000
$+\substack{100\\011}$	$+\substack{100\\011}$	101	000

$+\substack{100\\011}$	$+\substack{100\\011}$	$+\substack{100\\011}$	110
$+\substack{100\\011}$	$+\substack{100\\011}$	$+\substack{100\\011}$	101

3

Enforce sequences of triplets

$+\begin{smallmatrix}100\\011\end{smallmatrix}$ 110 000 000

$+\begin{smallmatrix}100\\011\end{smallmatrix}$ 101 000 000

$+\begin{smallmatrix}100\\011\end{smallmatrix}$ $+\begin{smallmatrix}100\\011\end{smallmatrix}$ 110 000

$+\begin{smallmatrix}100\\011\end{smallmatrix}$ $+\begin{smallmatrix}100\\011\end{smallmatrix}$ 101 000

$+\begin{smallmatrix}100\\011\end{smallmatrix}$ $+\begin{smallmatrix}100\\011\end{smallmatrix}$ $+\begin{smallmatrix}100\\011\end{smallmatrix}$ 110

$+\begin{smallmatrix}100\\011\end{smallmatrix}$ $+\begin{smallmatrix}100\\011\end{smallmatrix}$ $+\begin{smallmatrix}100\\011\end{smallmatrix}$ 101

3 Look at triplets

$+{}^{100}_{011}$ 110 000 000

$+{}^{100}_{011}$ 101 000 000

$+{}^{100}_{011}$ $+{}^{100}_{011}$ 110 000

$+{}^{100}_{011}$ $+{}^{100}_{011}$ 101 000

$+{}^{100}_{011}$ $+{}^{100}_{011}$ $+{}^{100}_{011}$ 110

$+{}^{100}_{011}$ $+{}^{100}_{011}$ $+{}^{100}_{011}$ 101

a projector on

$\left|\, {}_{100} - 011 \right\rangle$

3 Look at triplets, combine some lines

$$\overset{+}{\substack{100\\011}}\quad 110\quad 000\quad 000$$

$$\overset{+}{\substack{100\\011}}\quad 101\quad 000\quad 000$$

$$+\quad \overset{+}{\substack{100\\011}}\quad \overset{+}{\substack{100\\011}}\quad 110\quad 000$$

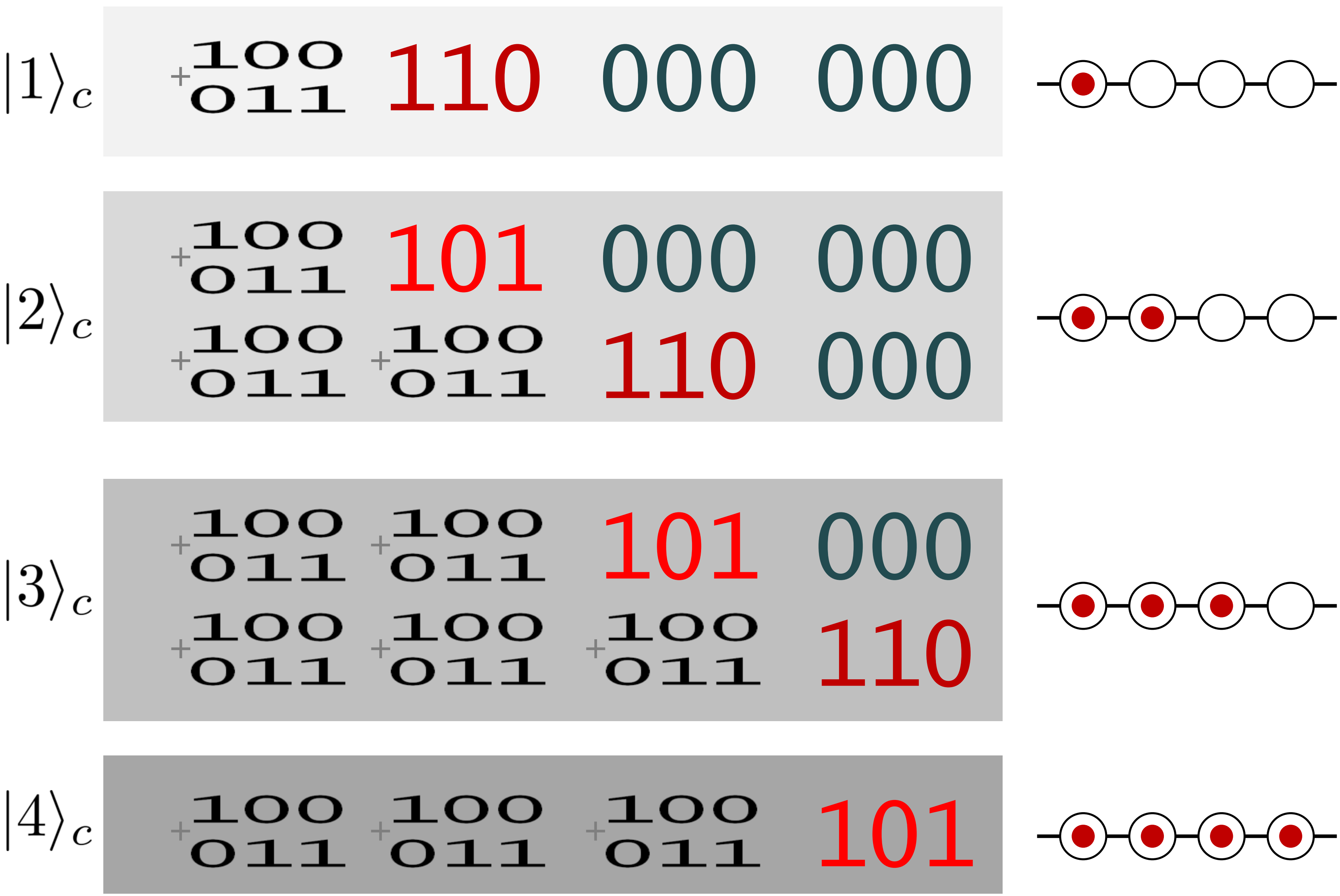
a projector on $\left|\,{}_{100} - 011\right\rangle$

$$\overset{+}{\substack{100\\011}}\quad \overset{+}{\substack{100\\011}}\quad 101\quad 000$$

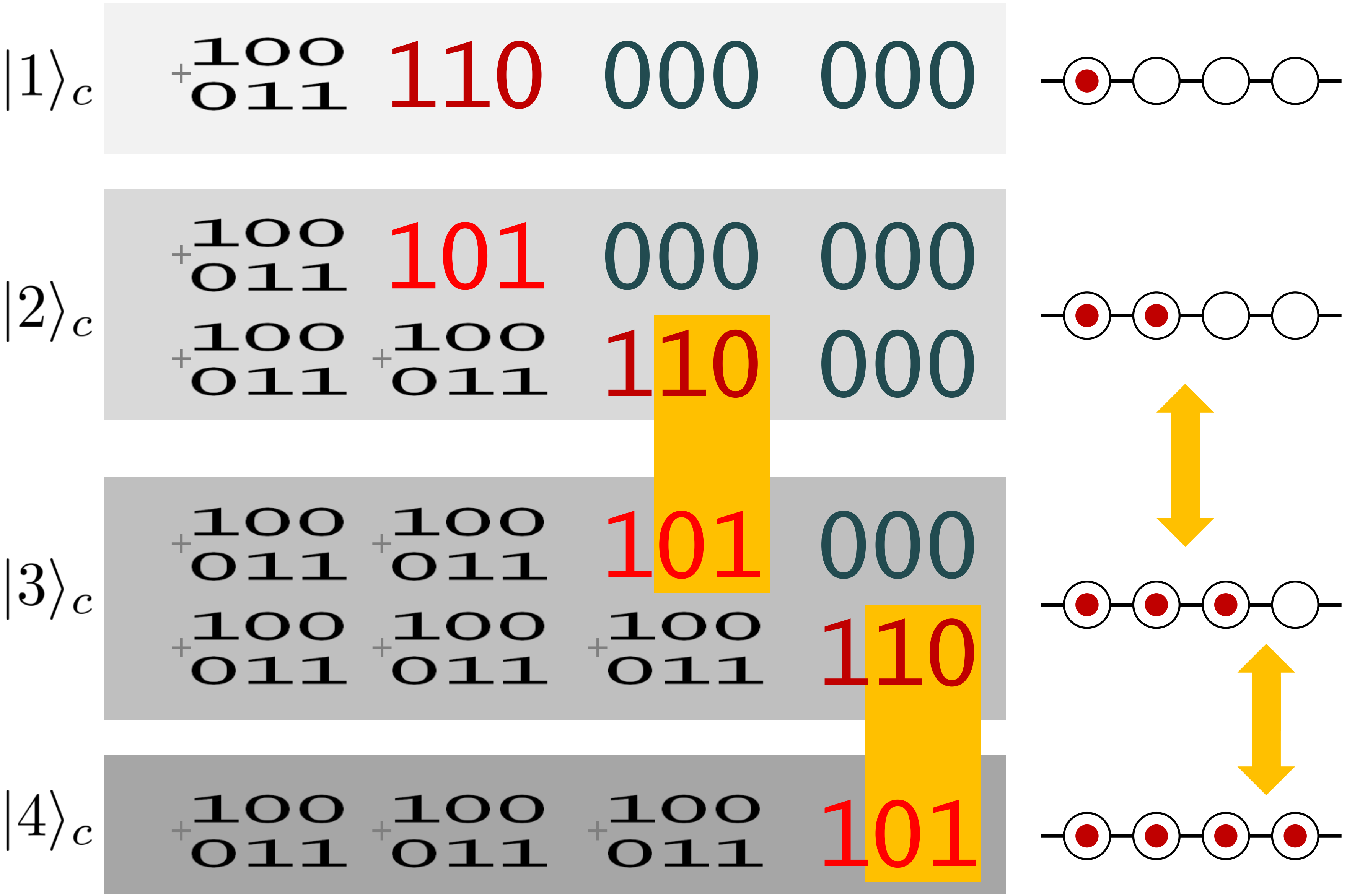
$$+\quad \overset{+}{\substack{100\\011}}\quad \overset{+}{\substack{100\\011}}\quad \overset{+}{\substack{100\\011}}\quad 110$$

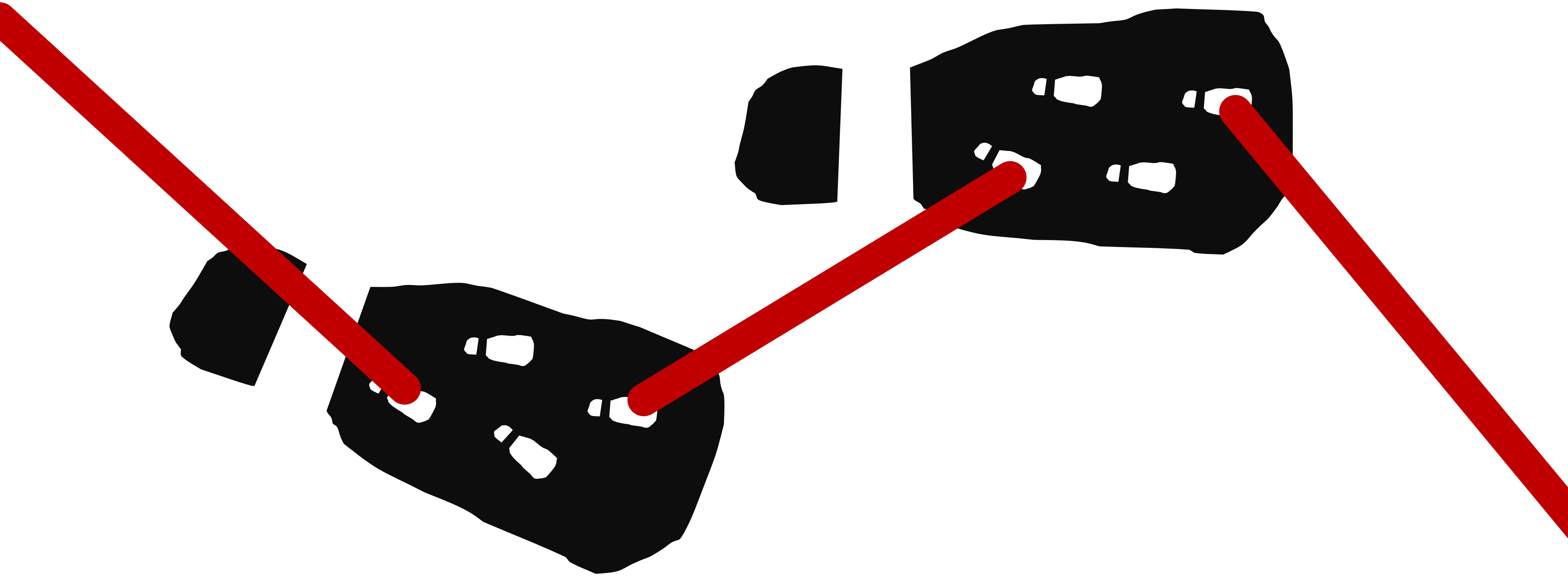
$$\overset{+}{\substack{100\\011}}\quad \overset{+}{\substack{100\\011}}\quad \overset{+}{\substack{100\\011}}\quad 101$$

3 A clock made from superpositions



3 A clock with **2-local progress**

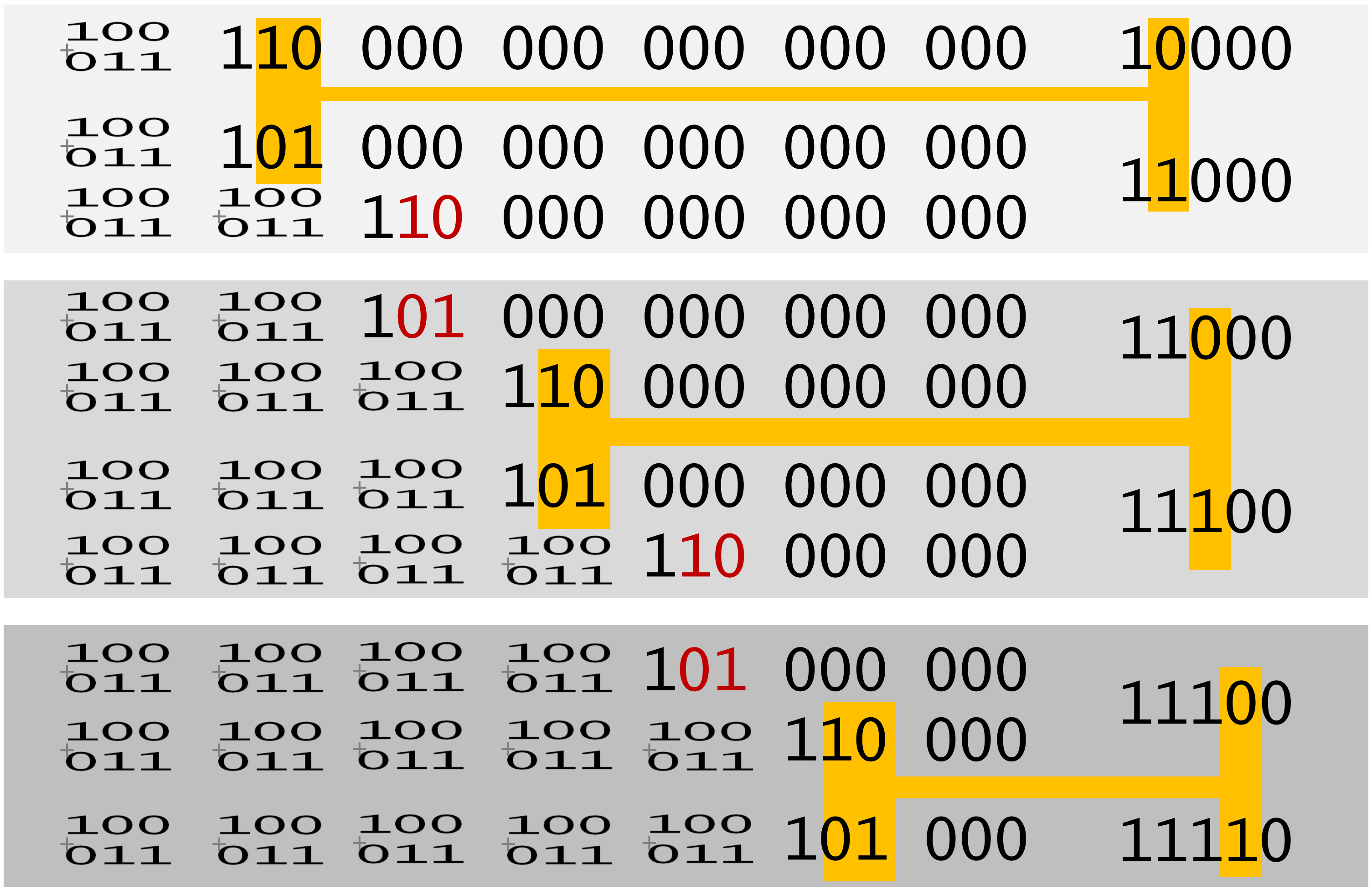




3 Even larger superpositions

$\frac{100}{+011}$	110	000	000	000	000	000	10000
$\frac{100}{+011}$	101	000	000	000	000	000	11000
$\frac{100}{+011}$	$\frac{100}{+011}$	110	000	000	000	000	
$\frac{100}{+011}$	$\frac{100}{+011}$	101	000	000	000	000	11000
$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	110	000	000	000	
$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	101	000	000	000	11100
$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	110	000	000	
$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	101	000	000	11100
$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	110	000	
$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	$\frac{100}{+011}$	101	000	11110

3 Even larger superpositions



3 Even larger superpositions

$\vert 1\rangle_c$	$\substack{100\\+011}$	110	000	000	000	000	000	10000
	$\substack{100\\+011}$	101	000	000	000	000	000	11000
	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	000	000	000	
$\vert 2\rangle_c$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	000	000	000	11000
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	000	000	
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	000	000	11100
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	000	
$\vert 3\rangle_c$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	000	11100
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	11110

- a single active spot, a domain wall structure

3 Even larger superpositions

$\|1\rangle_c$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	110	000	000	000	000	000	10000
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	101	000	000	000	000	000	11000
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	110	000	000	000	000	
$\|2\rangle_c$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	101	000	000	000	000	11000
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	110	000	000	000	
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	101	000	000	000	11100
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	110	000	000	
$\|3\rangle_c$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	101	000	000	11100
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	110	000	
	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	$\begin{smallmatrix}100\\+011\end{smallmatrix}$	101	000	11110

- 2-local connections

3 Even larger superpositions – to identify time **1-locally**.

$\|1\rangle_c$	$\substack{100\\+011}$	110	000	000	000	000	000	10000
	$\substack{100\\+011}$	101	000	000	000	000	000	11000
	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	000	000	000	
$\|2\rangle_c$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	000	000	000	11000
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	000	000	
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	000	000	11100
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	000	
$\|3\rangle_c$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	000	11100
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	110	000	
	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	$\substack{100\\+011}$	101	000	11110

$t \geq 2$

- 2-local connections, tell time 1-locally

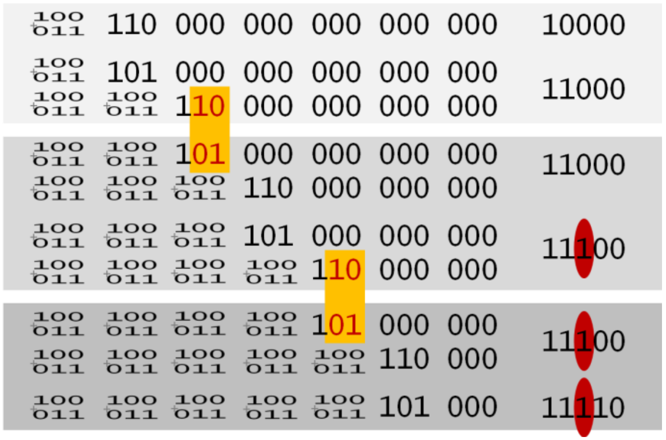
3 Constructing clocks

- find out I'm late

1-locally

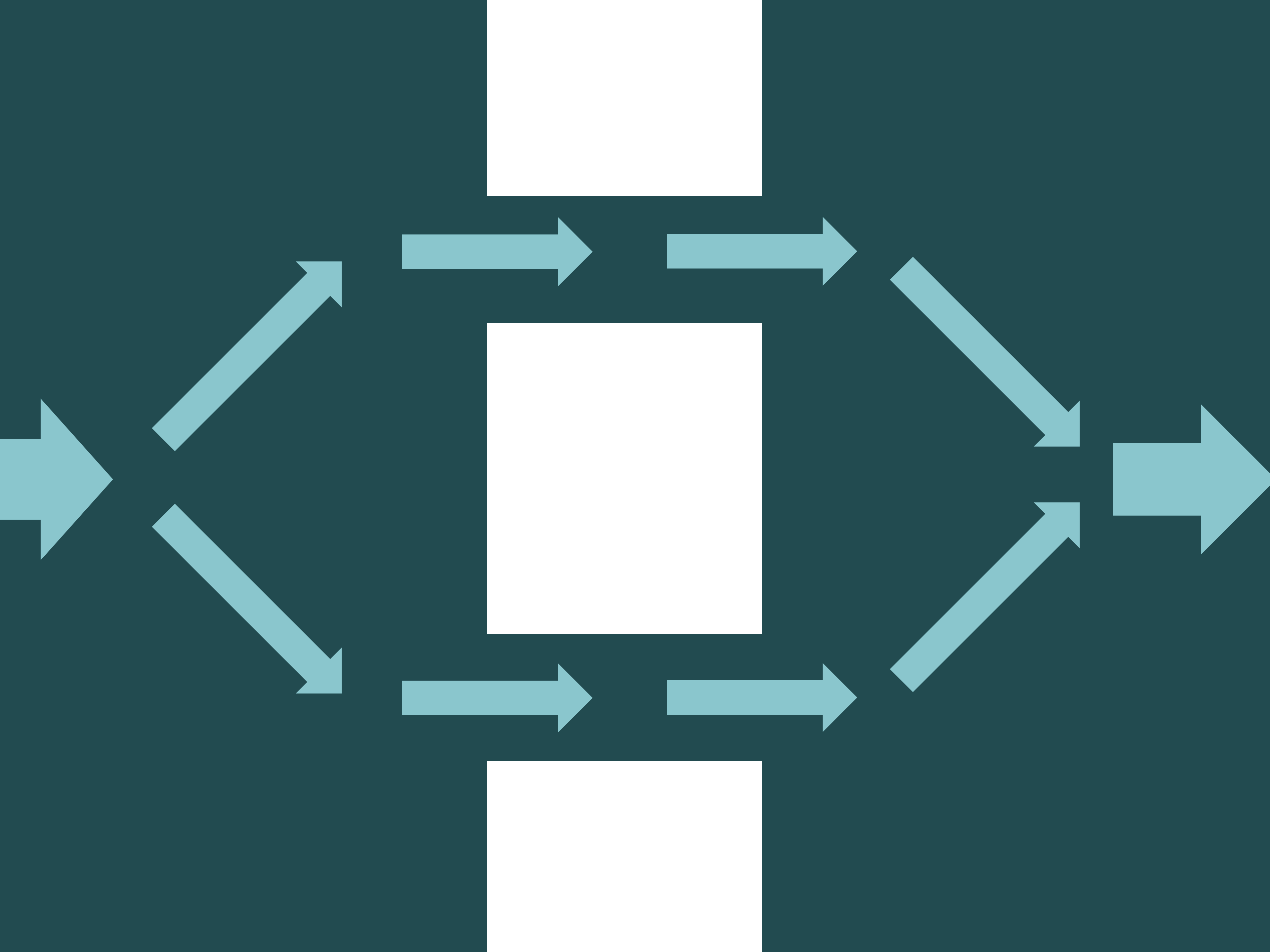
- advance the clock

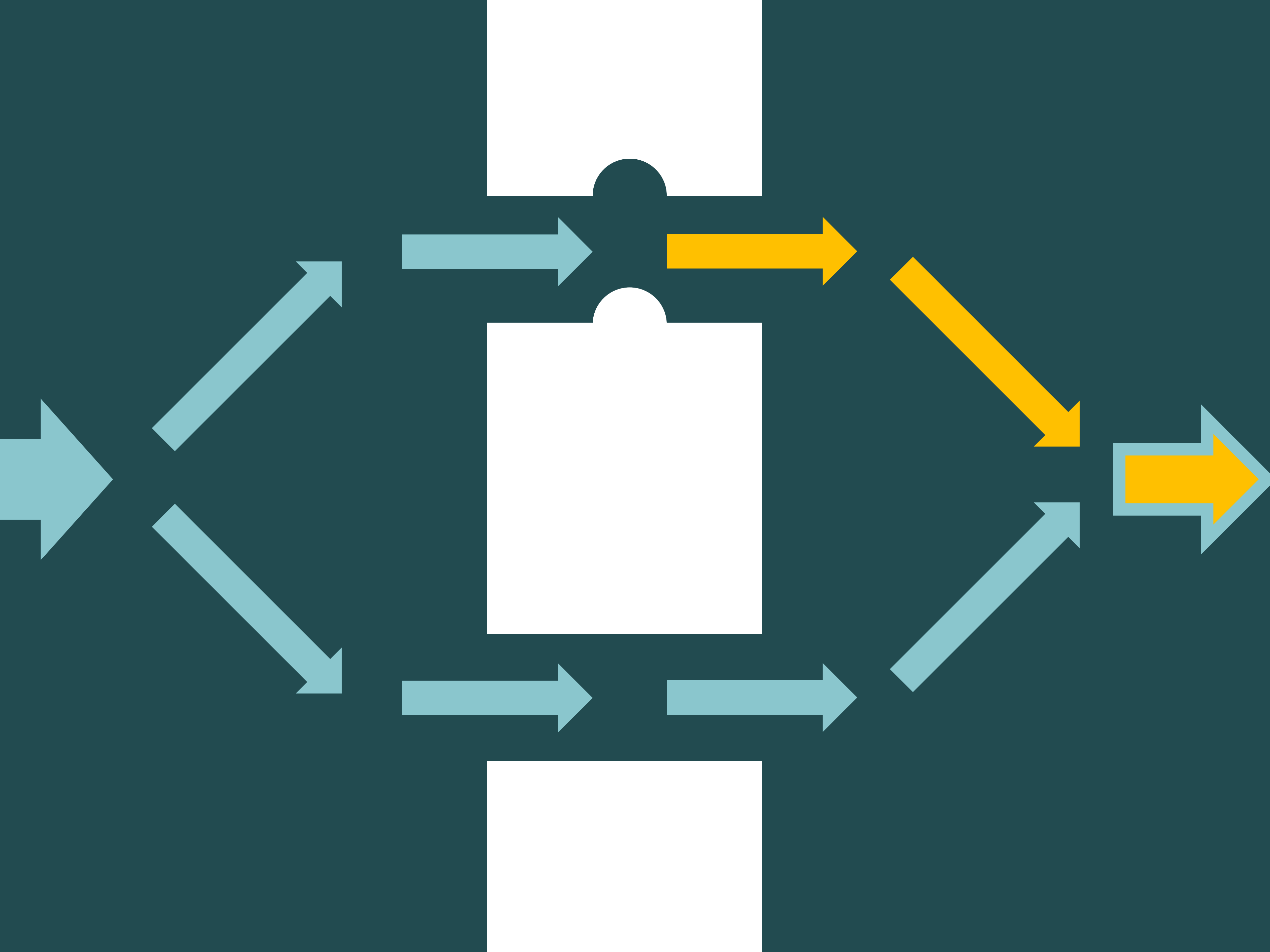
2-locally



- apply a CNOT

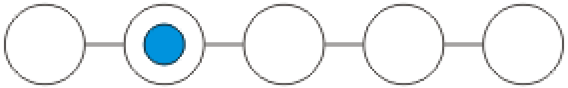
3-locally?





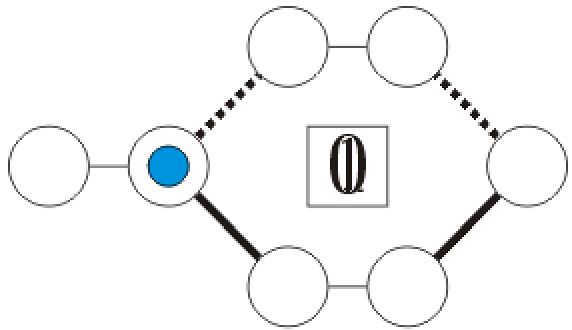
4 Applying 2-qubit gates 3-locally

- the railroad switch



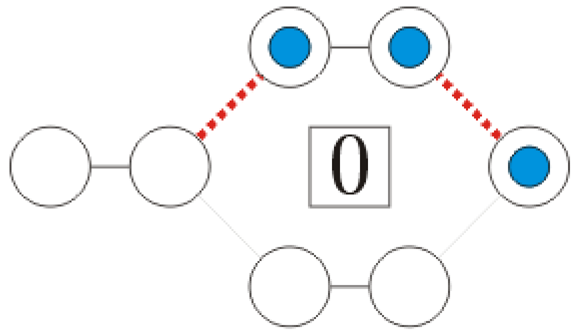
4 Applying 2-qubit gates 3-locally

- the railroad switch



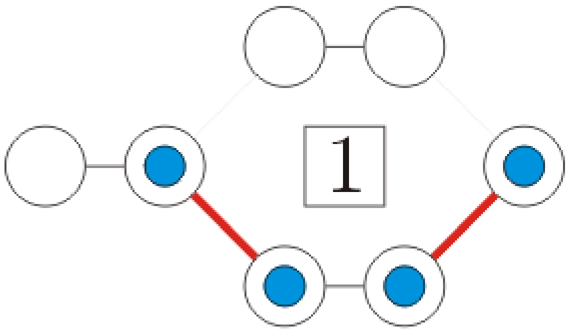
4 Applying 2-qubit gates 3-locally

- the railroad switch



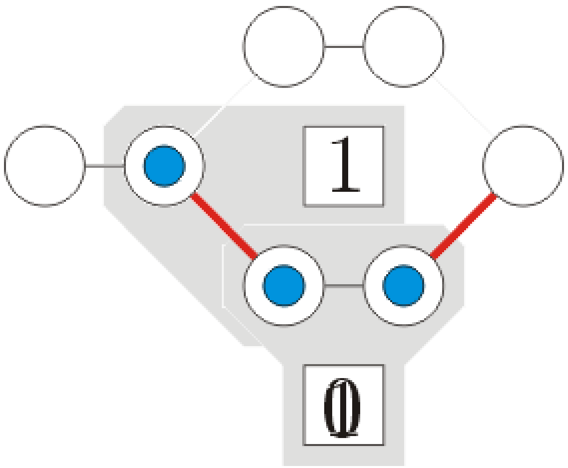
4 Applying 2-qubit gates 3-locally

- the railroad switch



4 Applying 2-qubit gates 3-locally

- the railroad switch



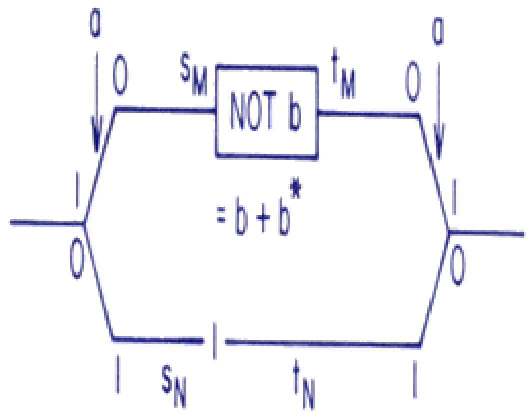
CNOT: 3-local

with a pulse
needs initialization

4 Applying 2-qubit gates 3-locally

[Feynman's paper]

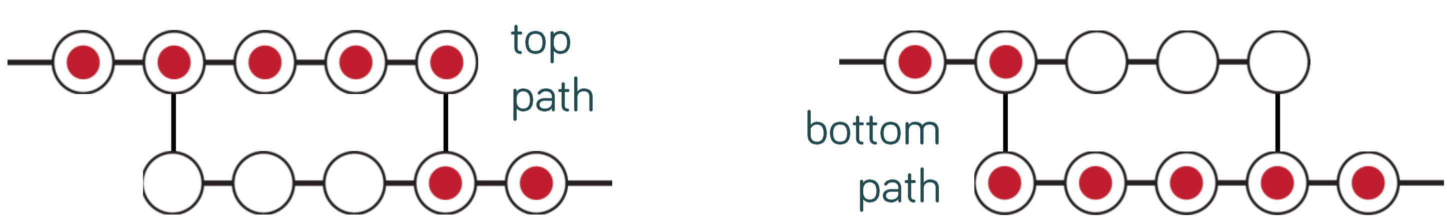
- the railroad switch



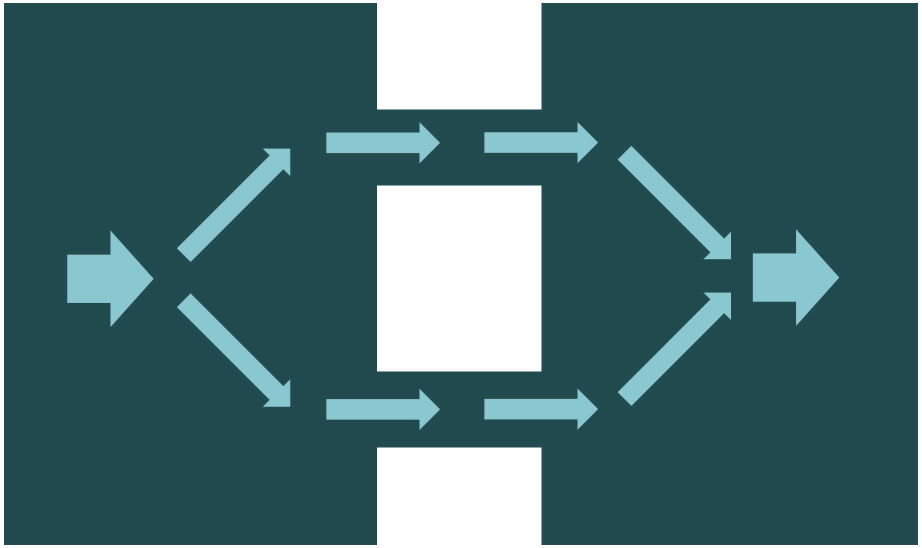
CNOT: 3-local

with a pulse
needs initialization

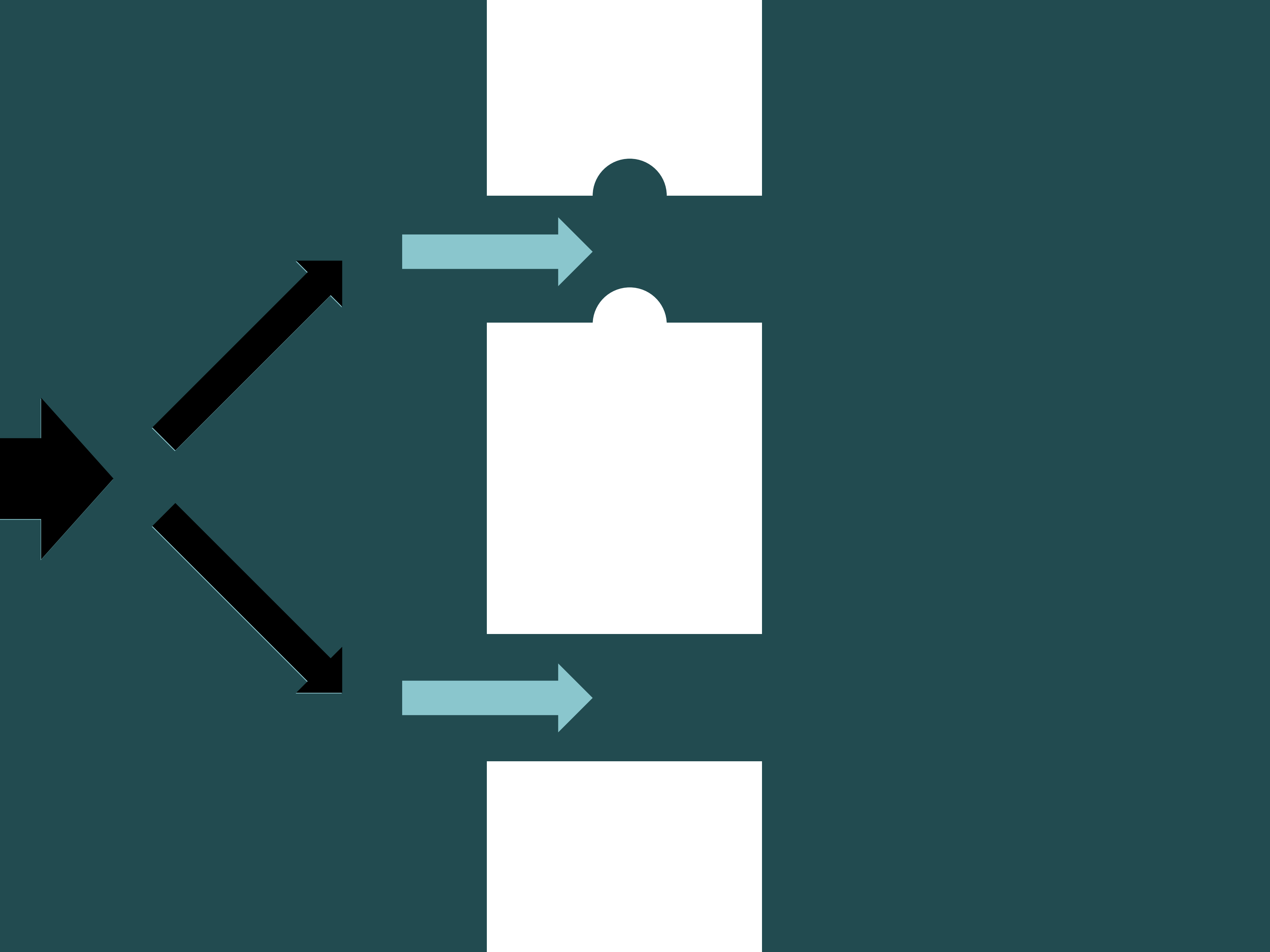
- a domain wall switch?

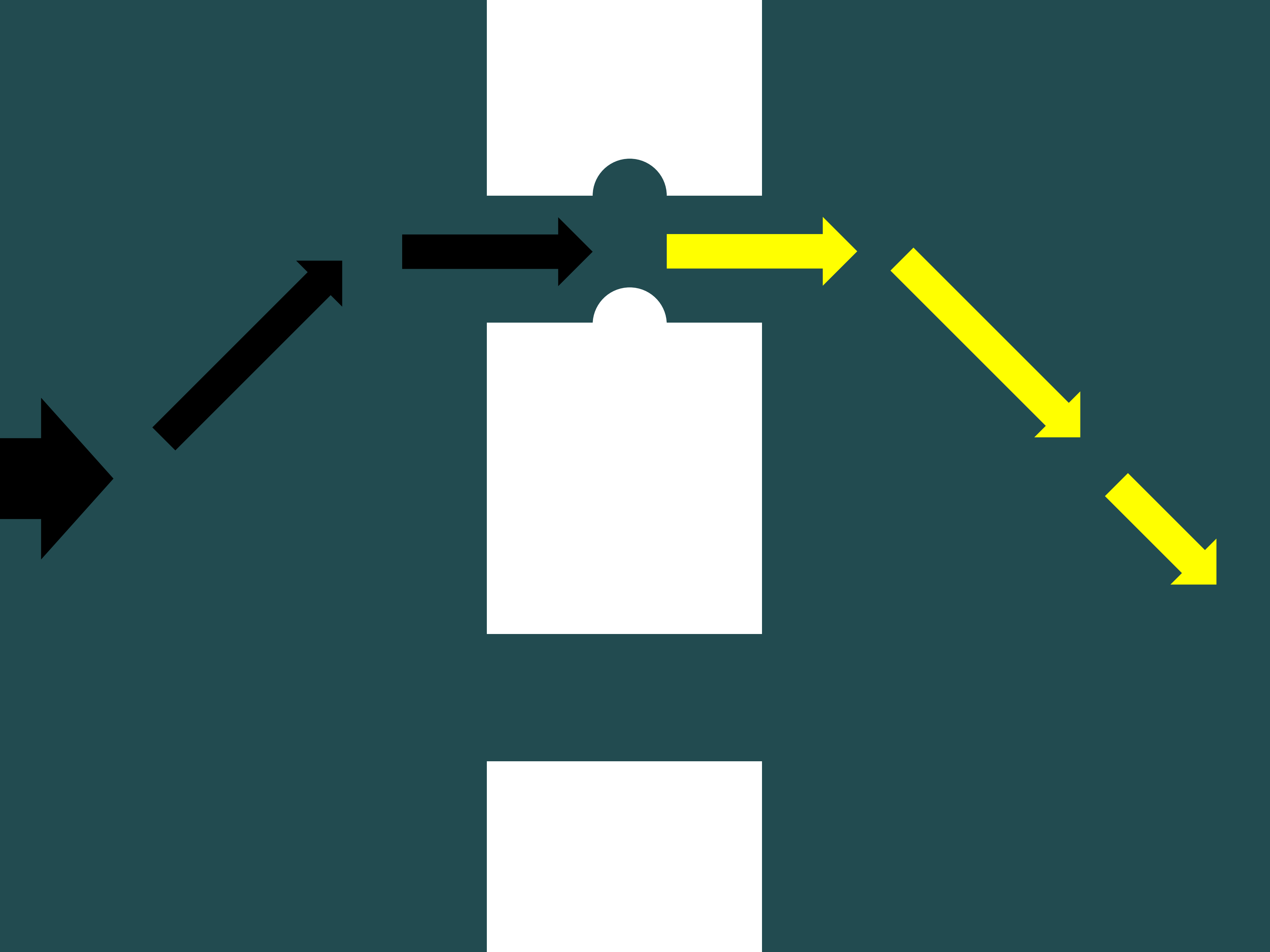


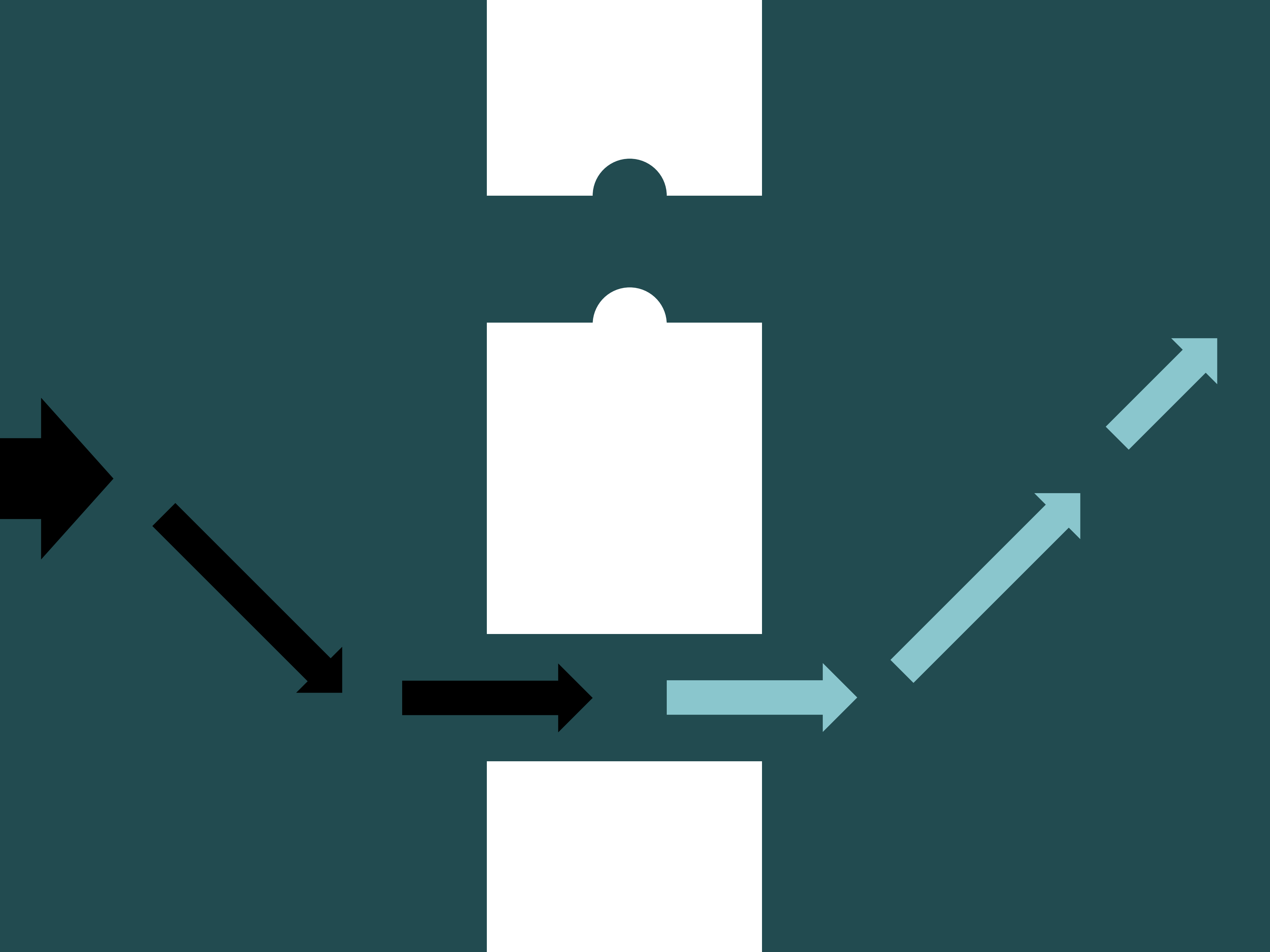
4 A double-slit experiment



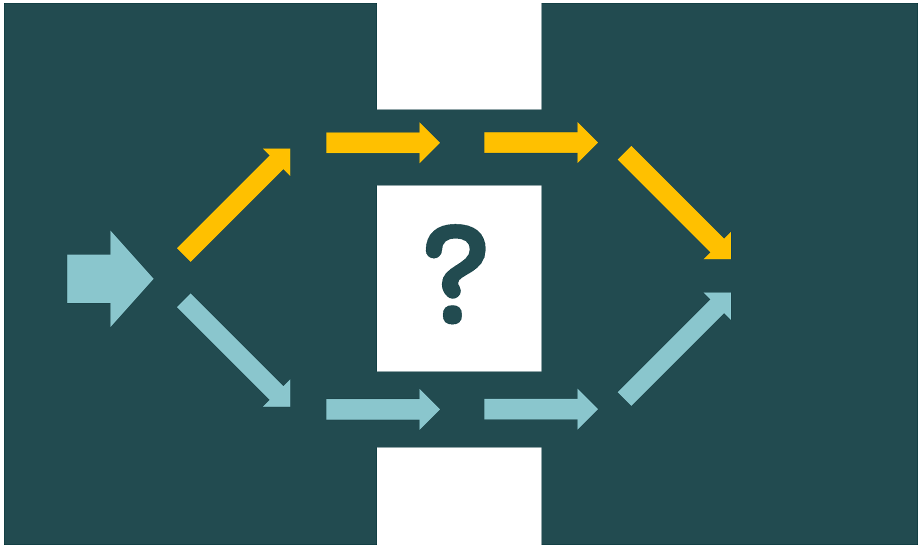
- a domain wall switch?





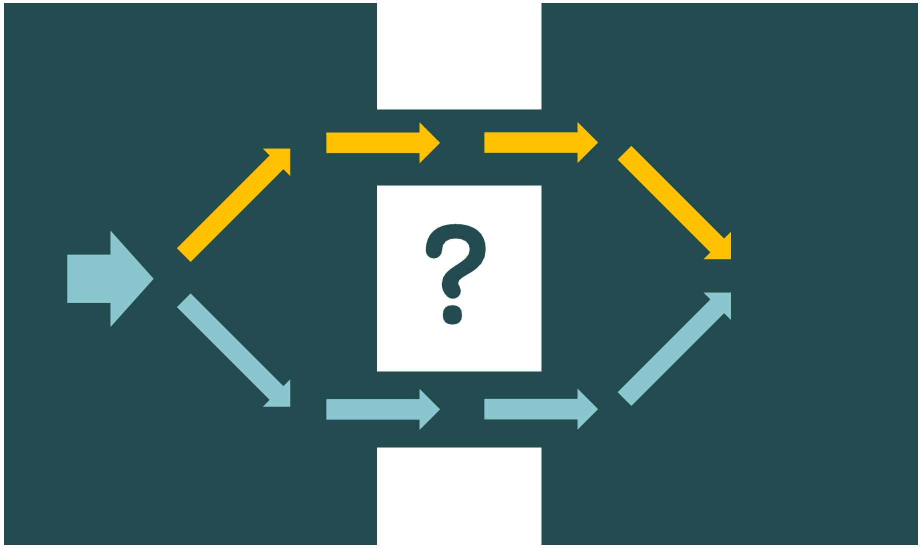


4 Can we make it work with a domain wall?



- a domain wall switch?

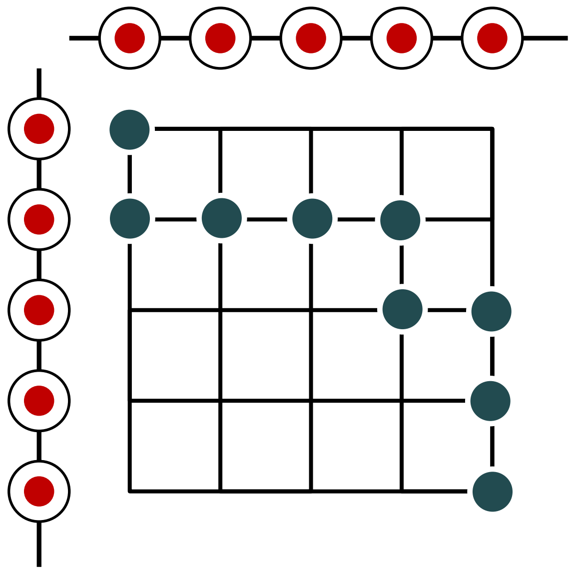
4 Can we make it work with a domain wall?



- a domain wall switch?

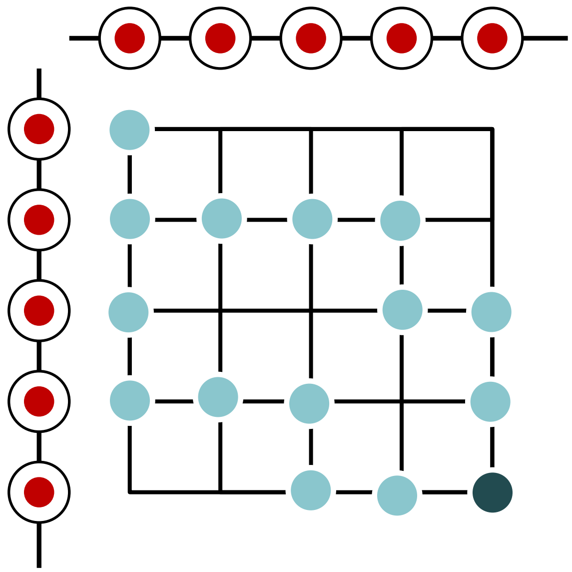
4 2D clocks (with two registers)

- two clocks



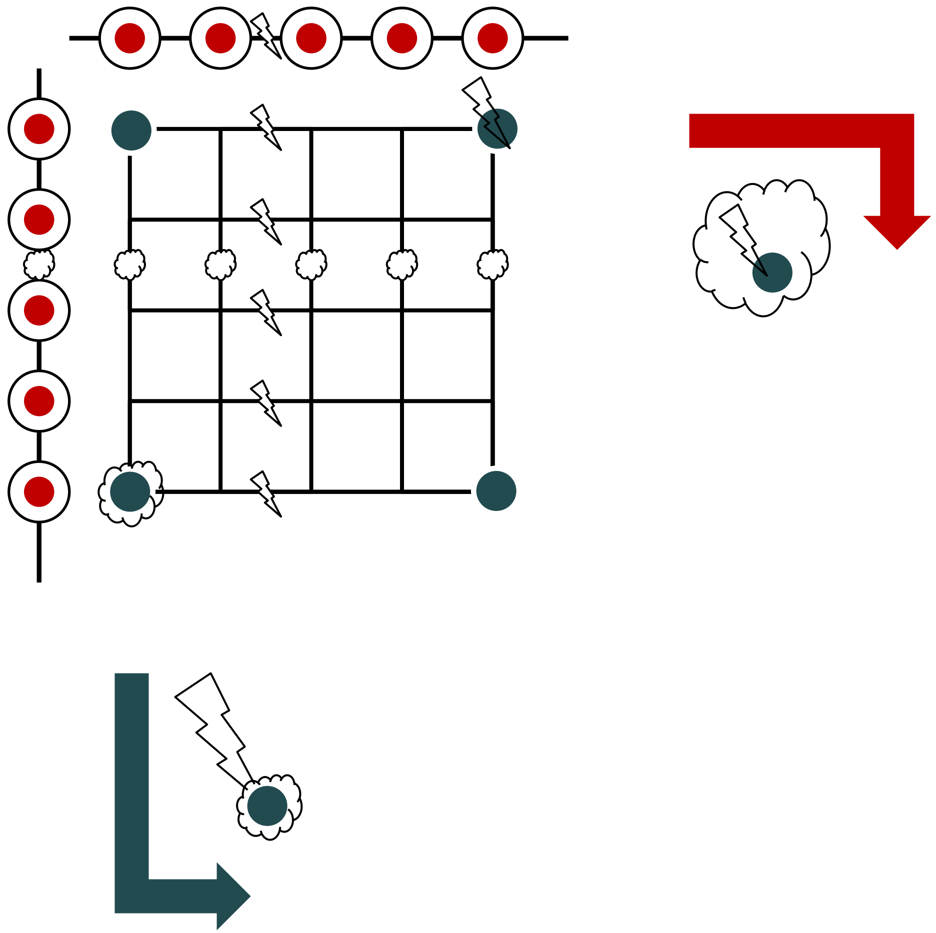
4 2D clocks (with two registers)

- two clocks



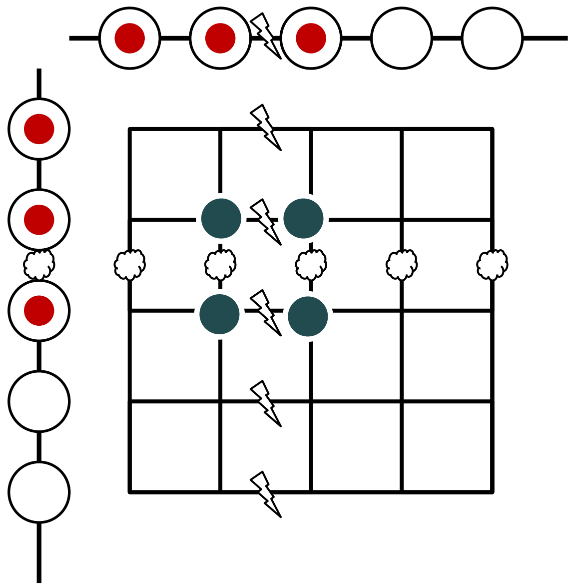
4 2D clocks (with two registers)

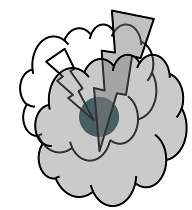
- add non-commuting (data) operations



4 2D clocks (with two registers)

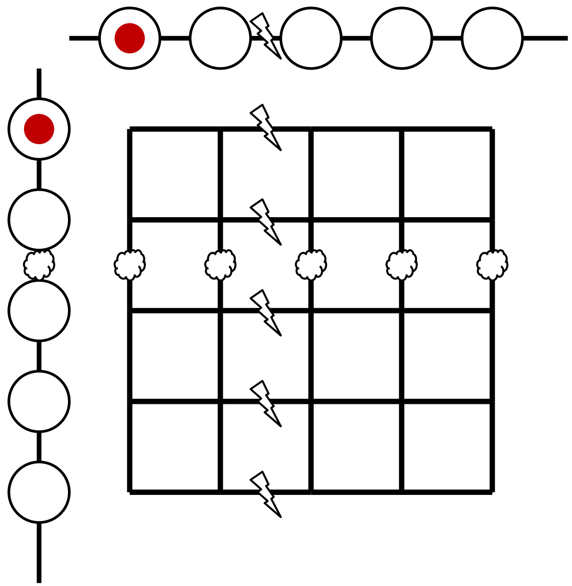
- add non-commuting (data) operations





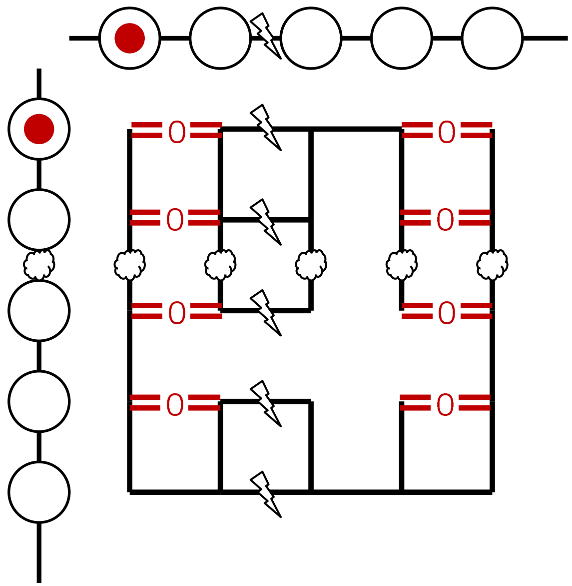
4 2D clocks (with two registers): fixing the mess

- remove some transitions



4 2D clocks (with two registers): fixing the mess

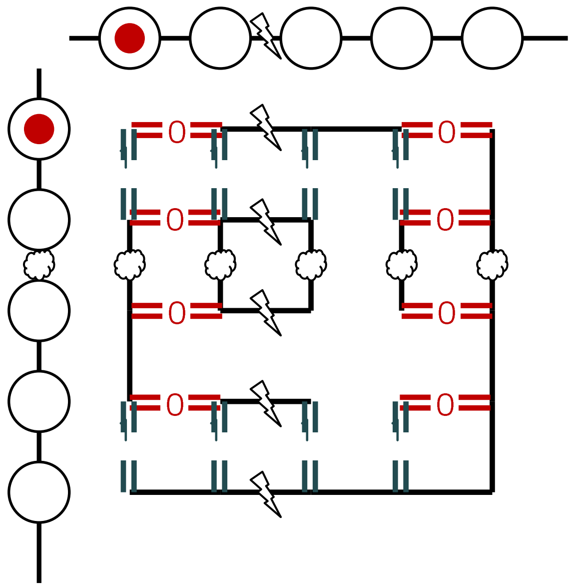
- condition transitions on a data (control) qubit



control: **0**

4 2D clocks (with two registers): fixing the mess

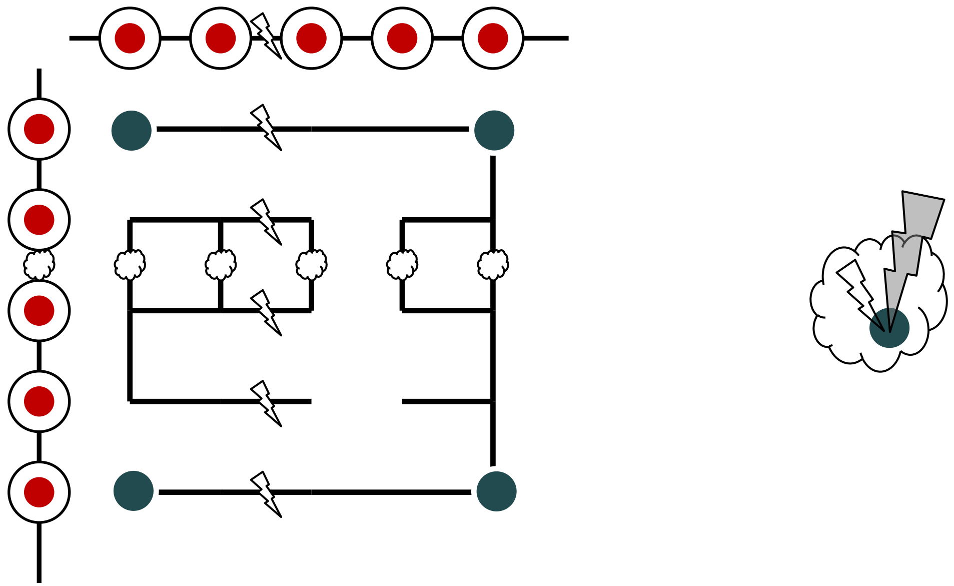
- condition transitions on a data (control) qubit



control: 0, 1

4 2D clocks (with two registers)

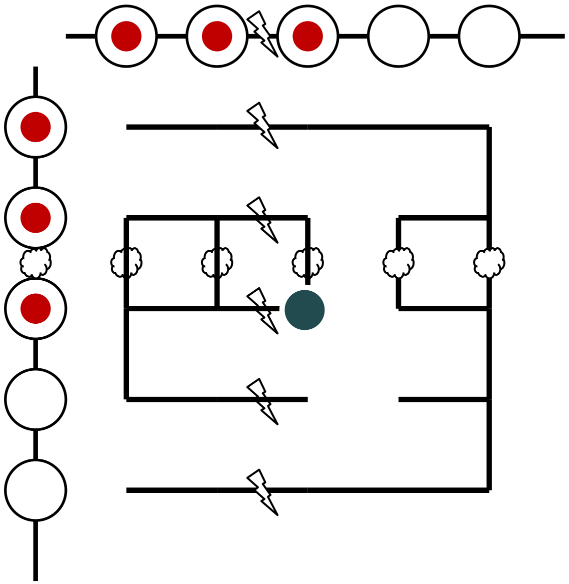
- condition transitions on a data (control) qubit

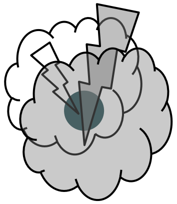


control: 0

4 2D clocks (with two registers)

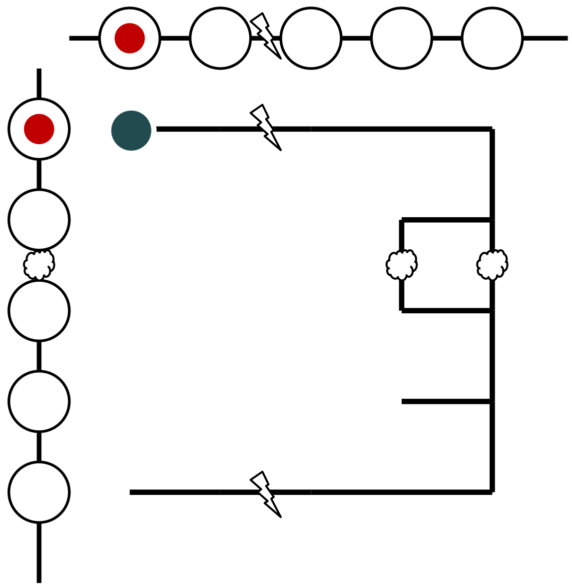
- there is no "bound" ground state (,  don't commute)

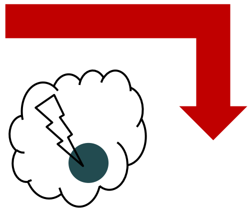




control: 0

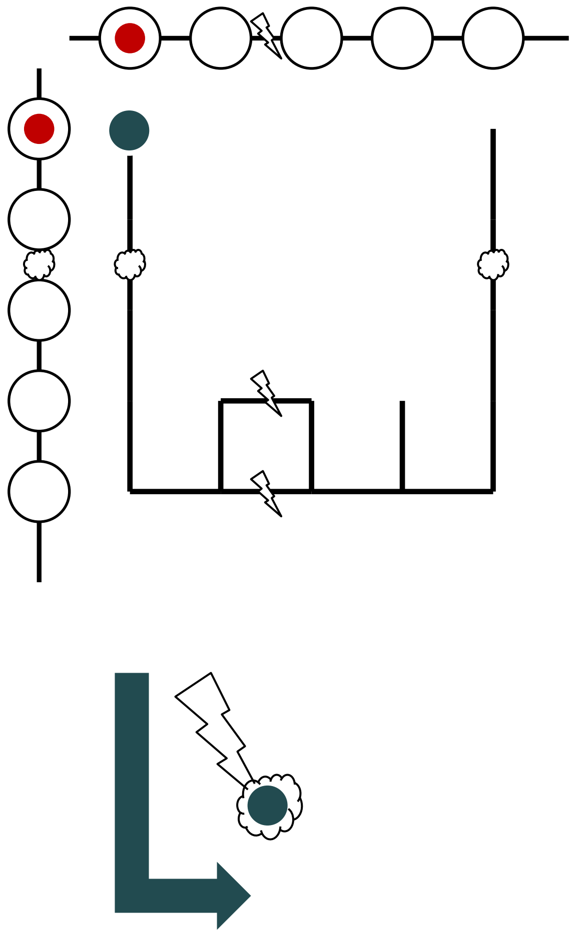
4 2D clocks (with two registers)





control: 0

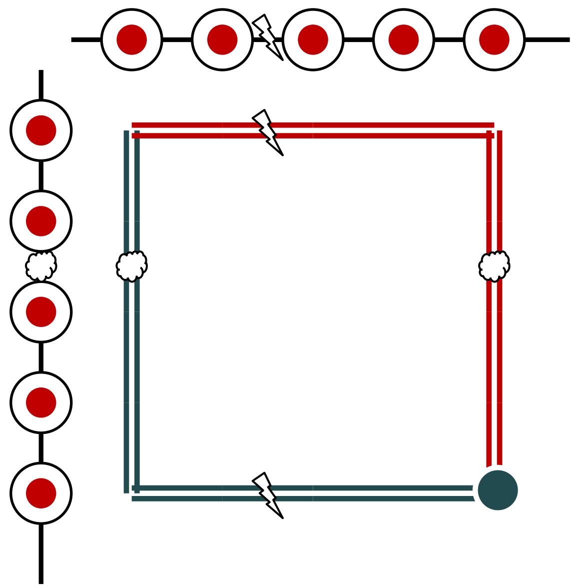
4 2D clocks (with two registers)

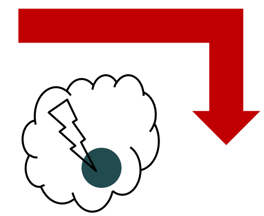


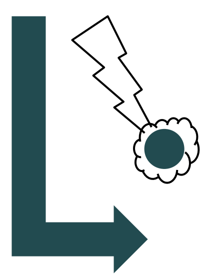
control: 1

4 2D clocks (with two registers)

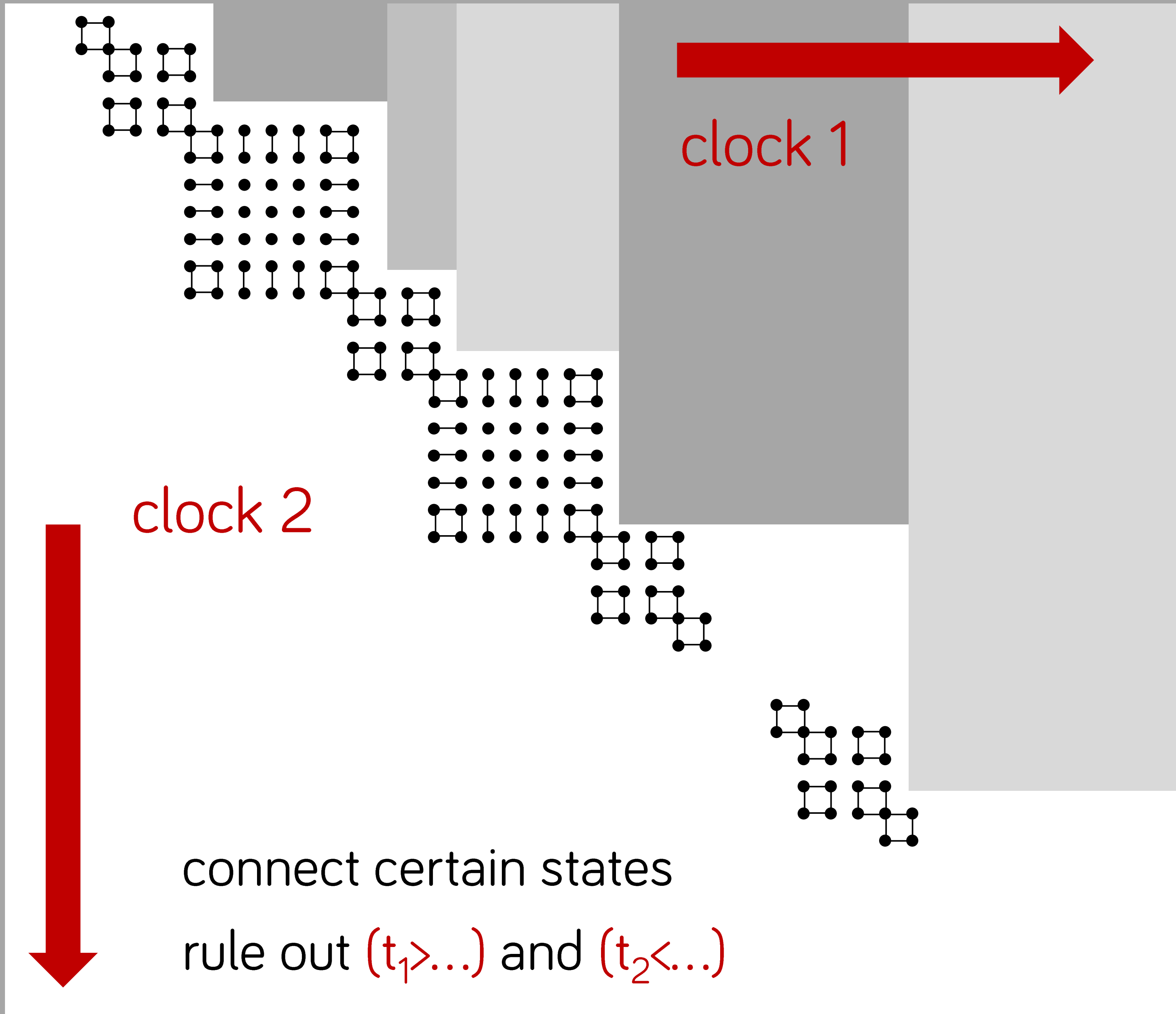
- like a railroad switch... with a guaranteed single active site

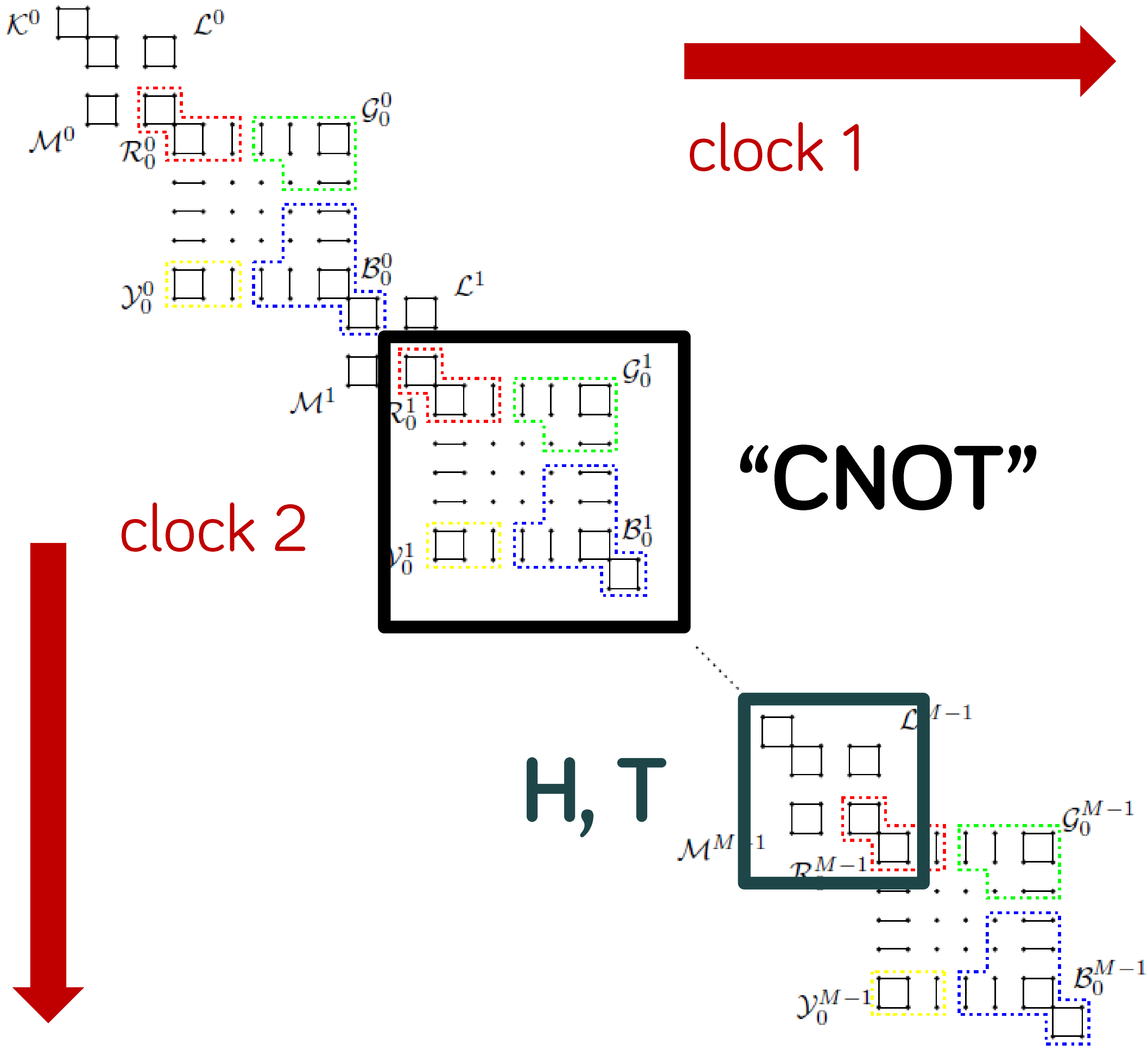






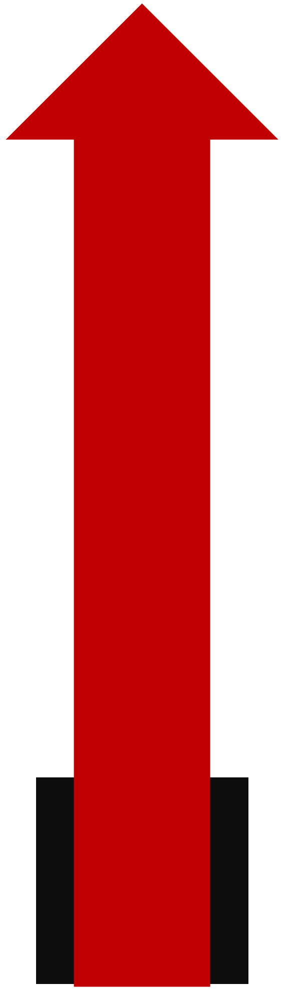
control: 0,1


clock 1
clock 2
connect certain states
rule out (t_1>...) and (t_2<...)


$\mathcal{K}^0$
$\mathcal{L}^0$
$\mathcal{M}^0$
$\mathcal{R}_0^0$
$\mathcal{G}_0^0$
$\mathcal{Y}_0^0$
$\mathcal{B}_0^0$
$\mathcal{L}^1$
$\mathcal{M}^1$
$\mathcal{R}_0^1$
$\mathcal{G}_0^1$
$\mathcal{Y}_0^1$
$\mathcal{B}_0^1$
$\mathcal{L}^{M-1}$
$\mathcal{M}^{M-1}$
$\mathcal{R}_0^{M-1}$
$\mathcal{G}_0^{M-1}$
$\mathcal{Y}_0^{M-1}$
$\mathcal{B}_0^{M-1}$
clock 1
clock 2
“CNOT”
H, T

soundness

projection lemma



projection lemma

no solution?
all states have
a high energy

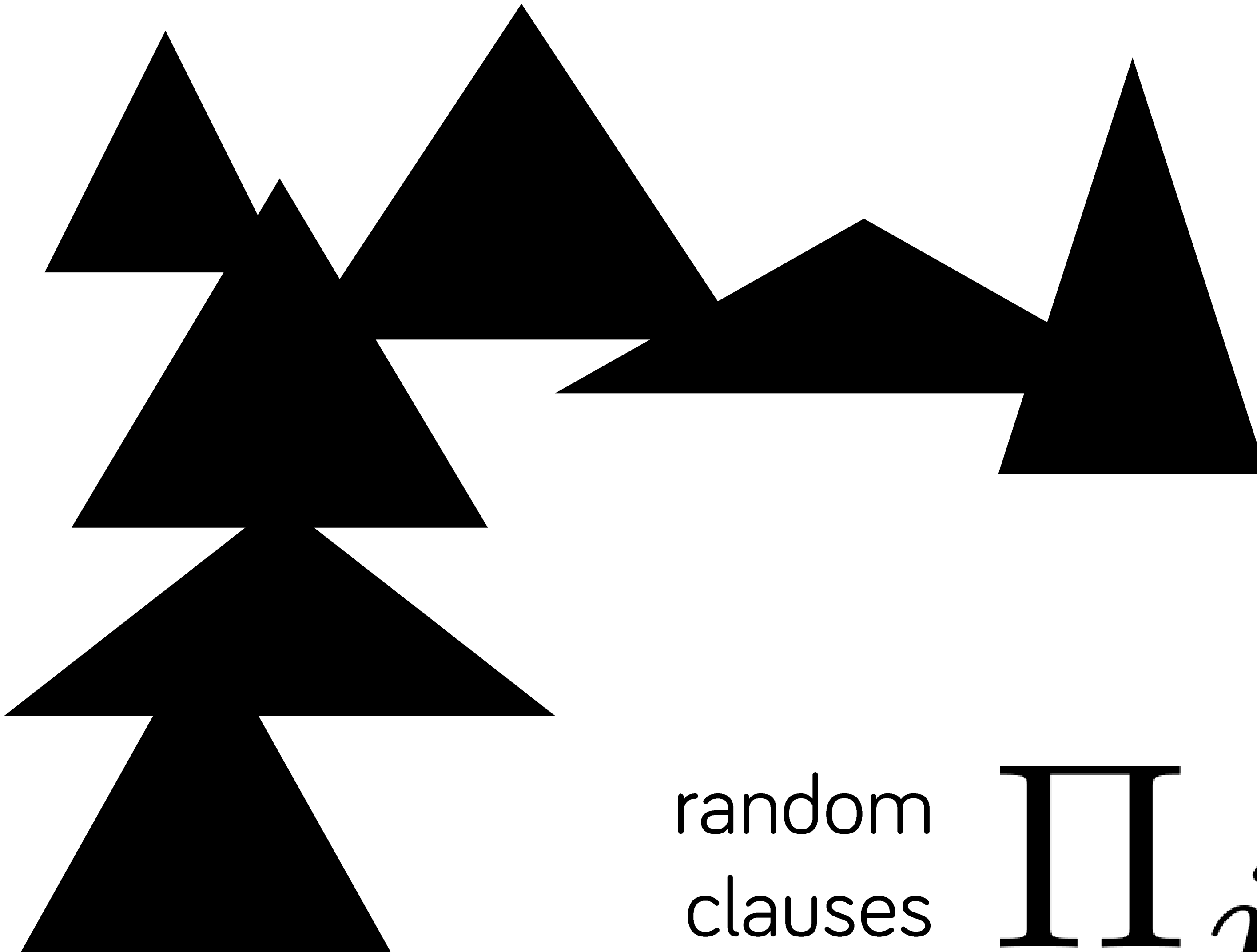
quantum 3-SAT

is QMA_1-complete

[Gosset, N. '13]

random q-SAT

5 Random QSAT



random clauses Π_j

SAT

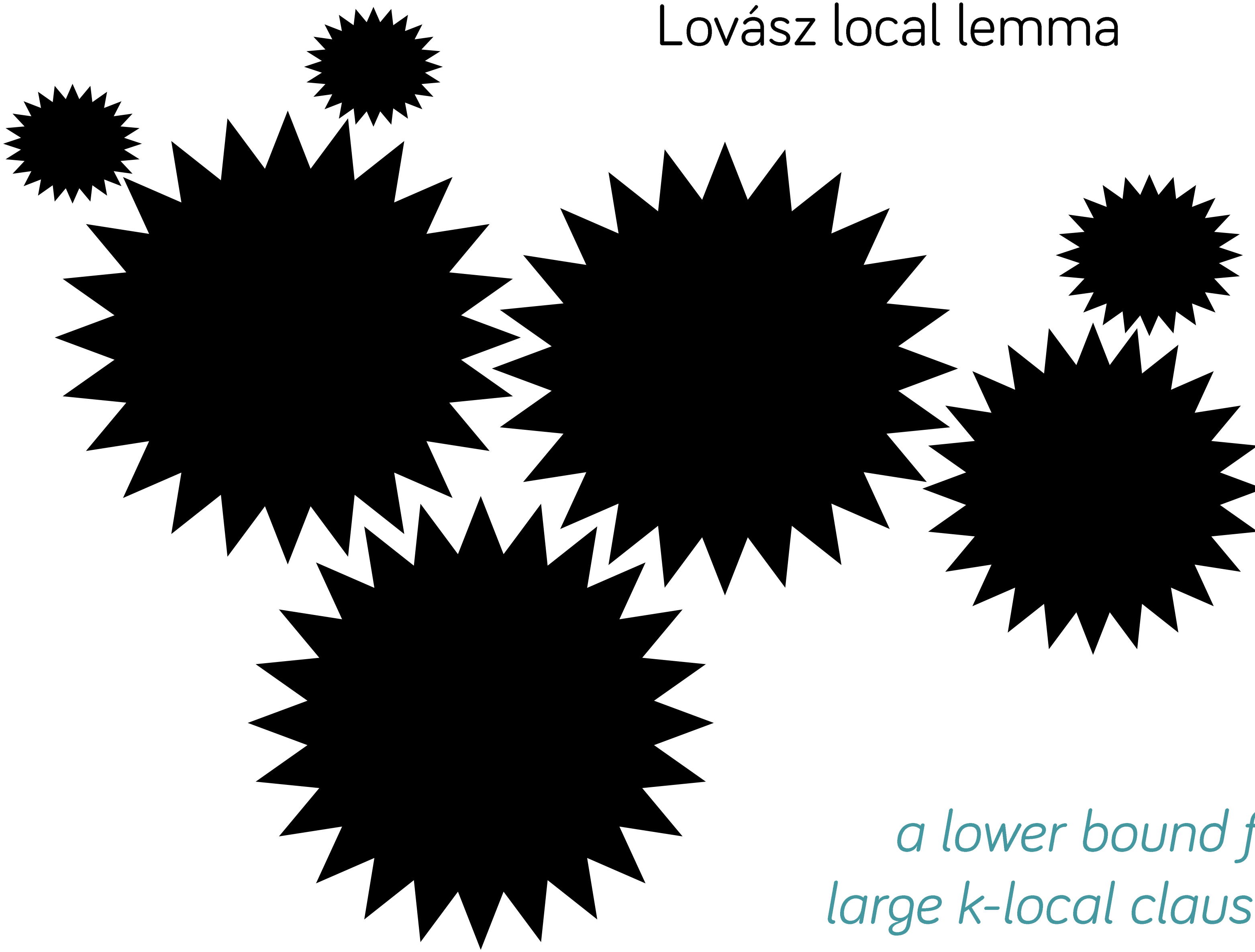

SAT

UNSAT

product solutions

$$\frac{M}{N} < 0.92$$

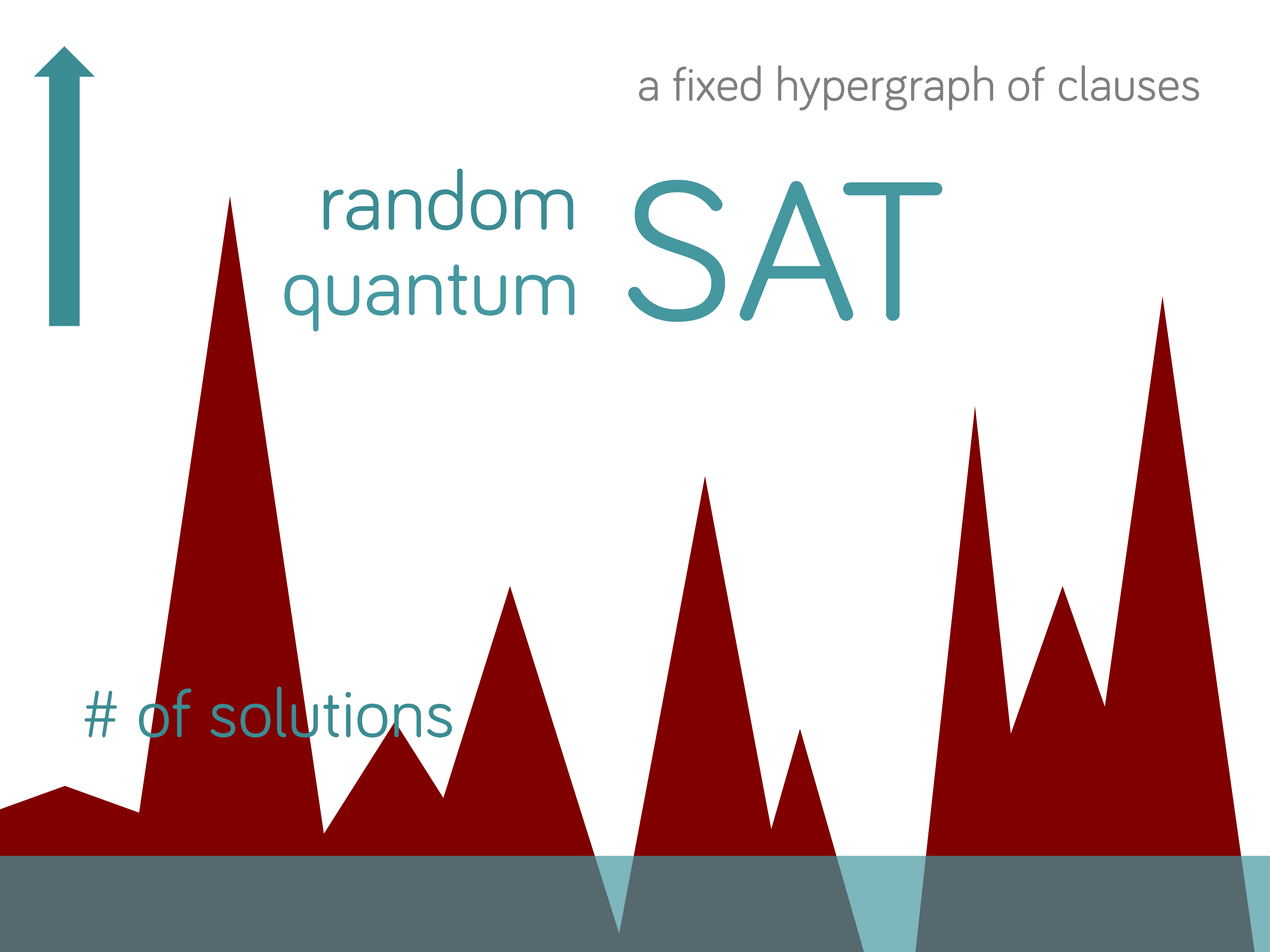
Lovász local lemma



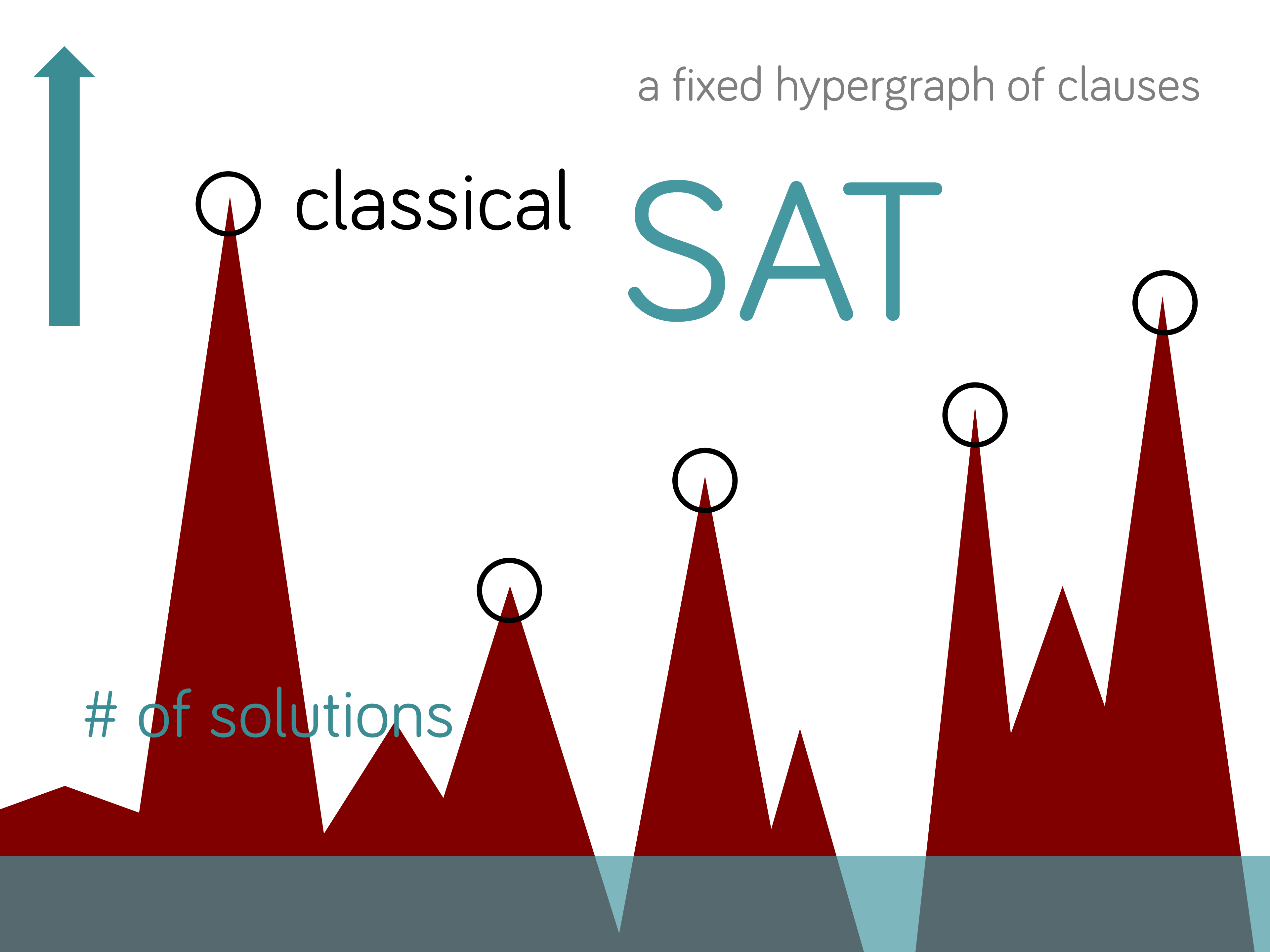
a lower bound for
large k-local clauses

in practice

$M/N \sim 1$

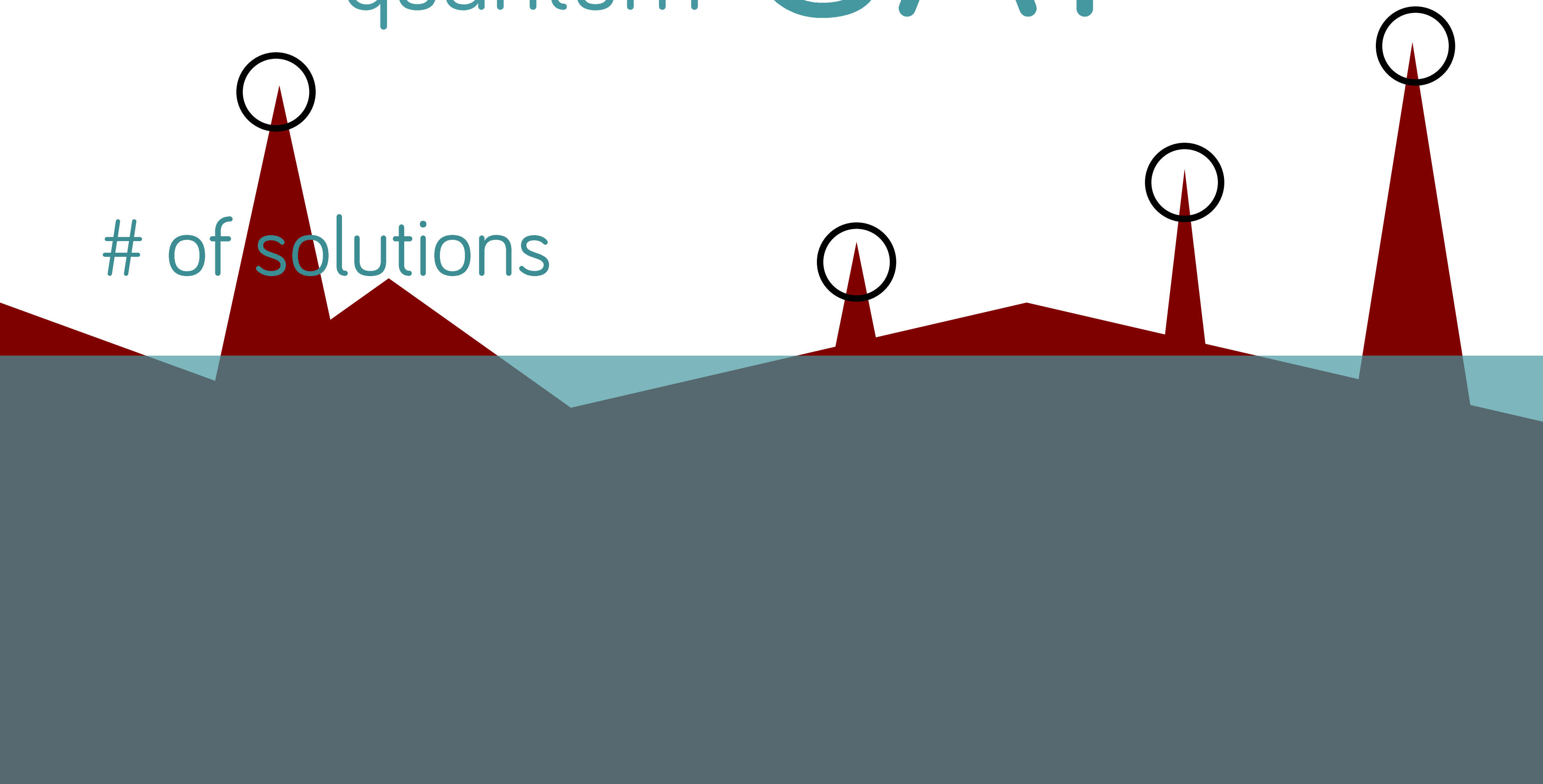


a fixed hypergraph of clauses
random
quantum
SAT
# of solutions


a fixed hypergraph of clauses
classical
SAT
# of solutions

another hypergraph

quantum SAT



classical SAT worst case

# of solutions

quantum SAT random case

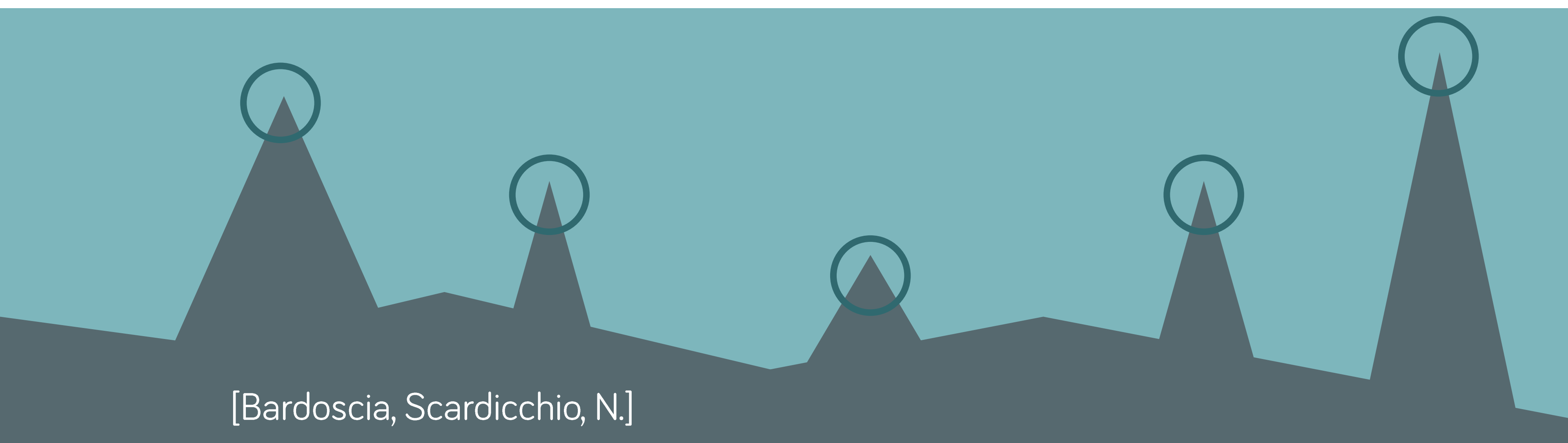
adversary **SAT** worst

classical case

fix a random hypergraph

are there killer clauses?

of solutions



adversary **SAT** worst

classical case

fix a random hypergraph

are there killer clauses?

random Q-SAT: no solutions

# of solutions

How can we show that there are no solutions?

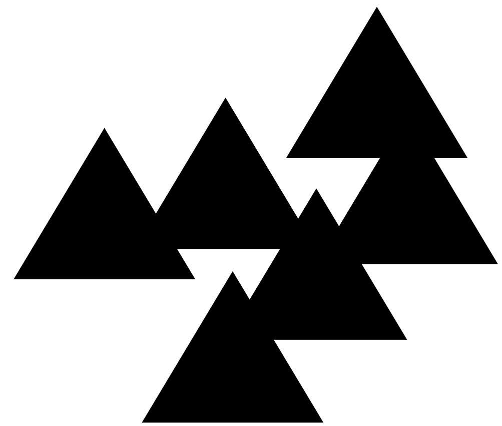
adversary SAT

fix a hypergraph

count solutions

look for killer clauses

greedy



adversary SAT

start balanced

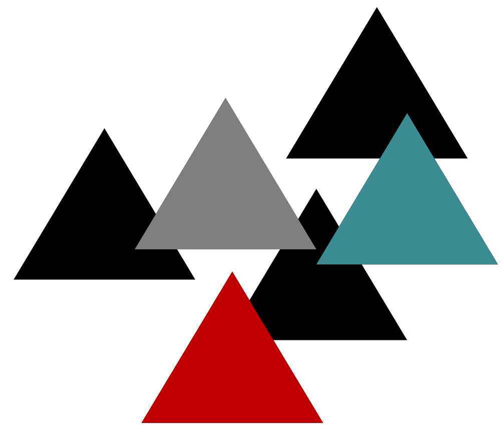
fix a hypergraph

count solutions

look for killer clauses

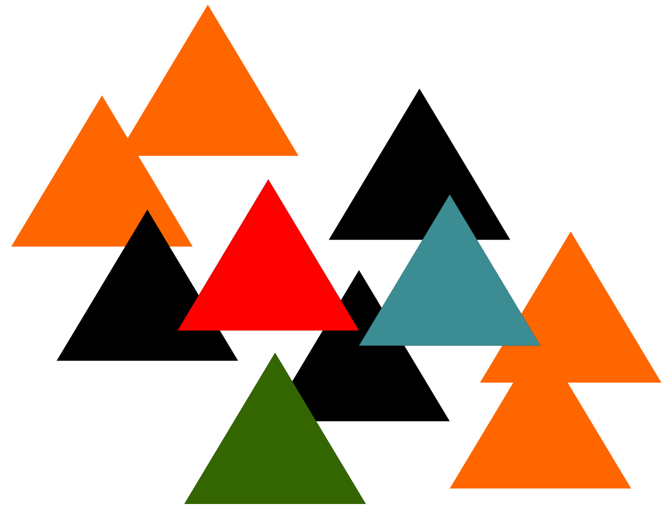
greedy

annealing



adversary SAT

start balanced



fix a hypergraph

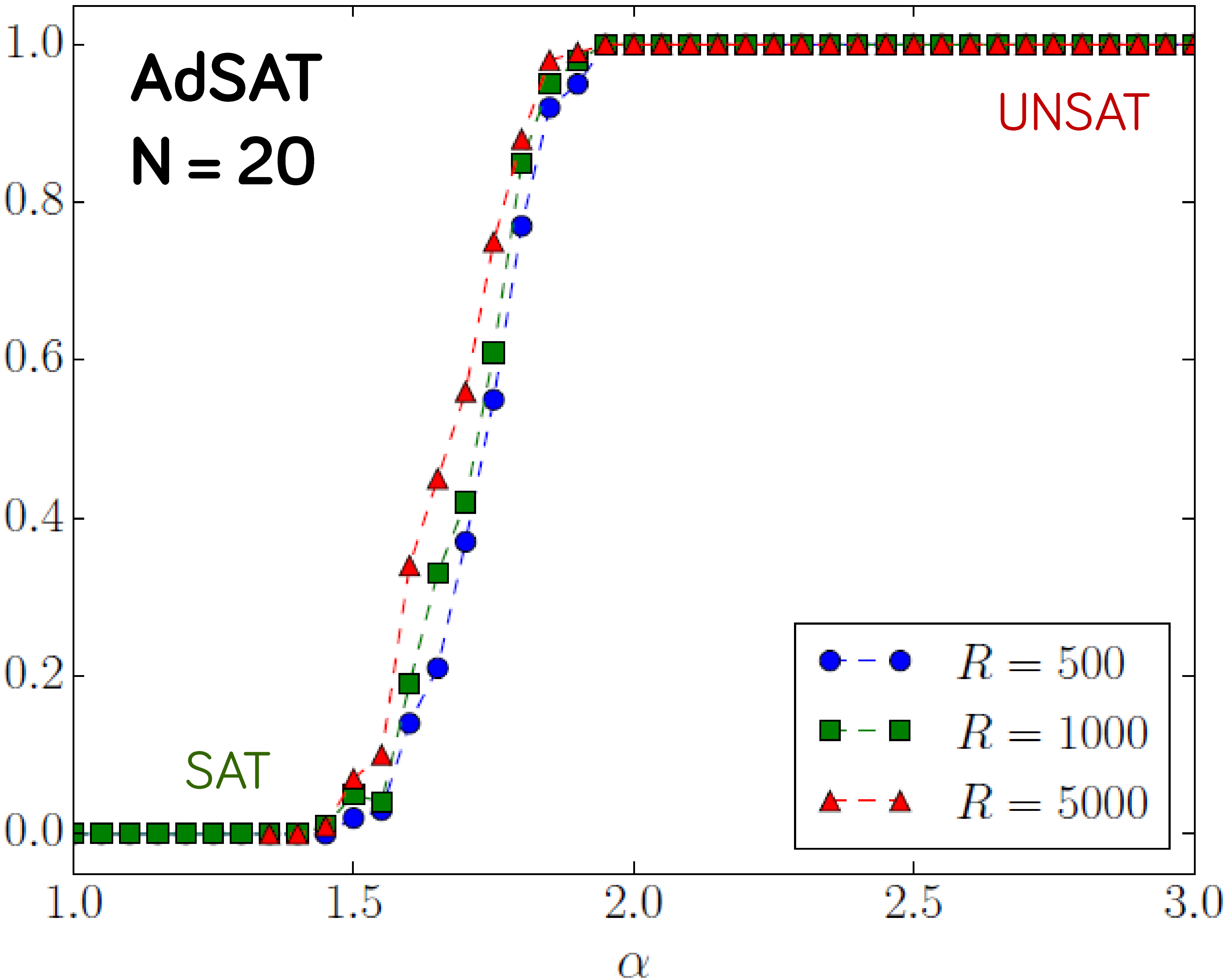
count solutions

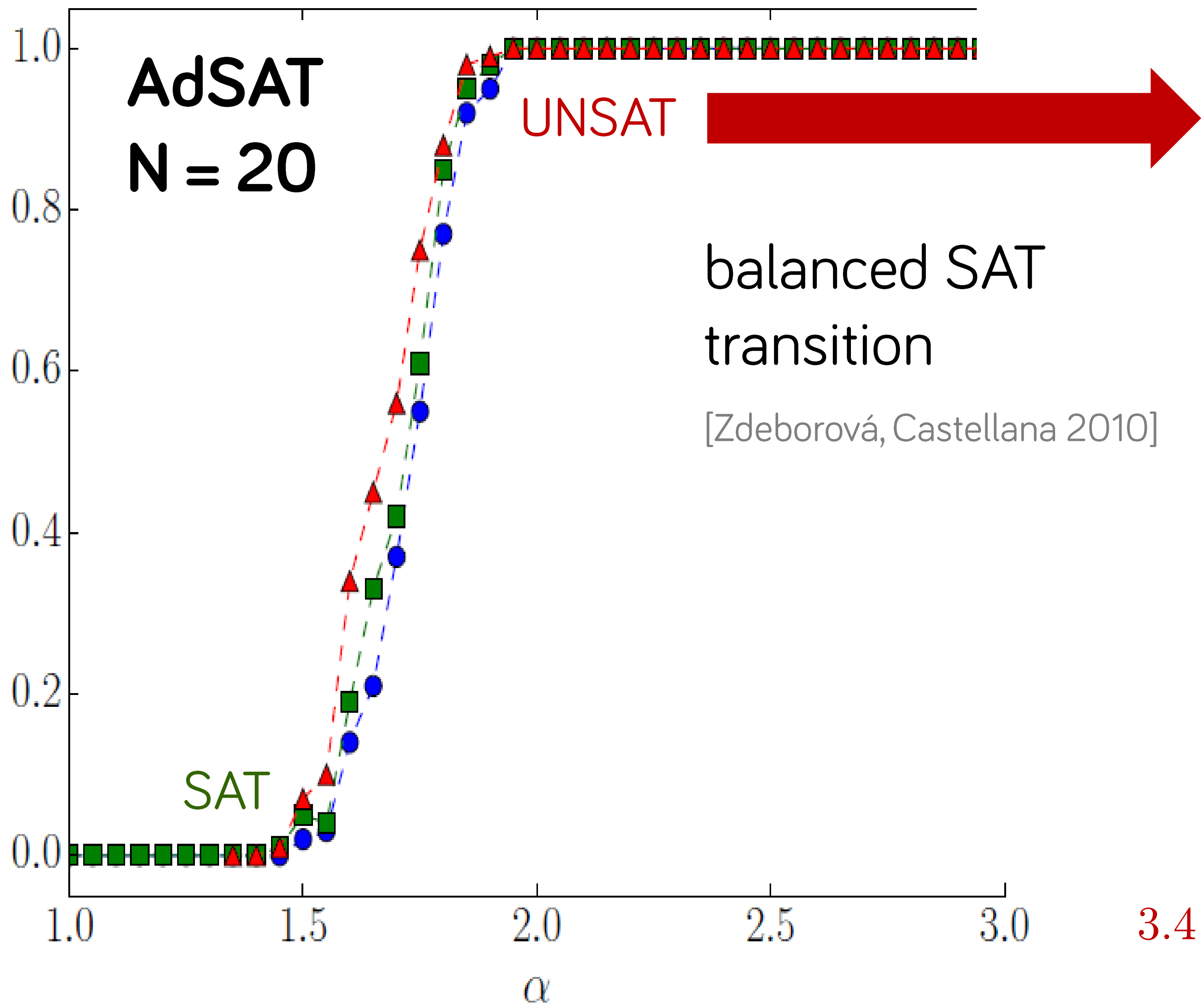
look for killer clauses

- overkill
- annealing
- restarts

UNSAT

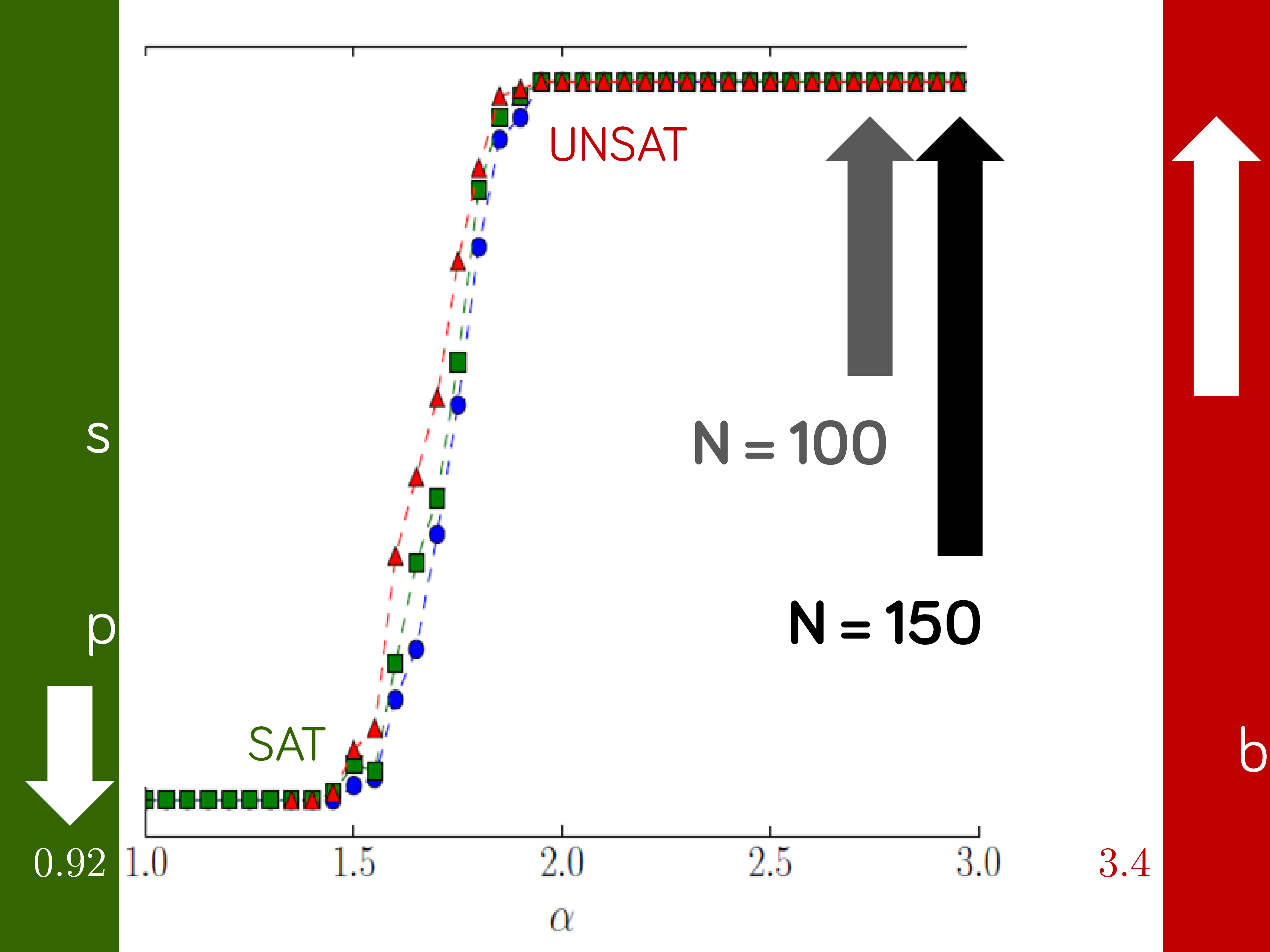
[Bardoscia, Scardicchio, N.: arXiv: 1310.0967]


AdSAT
N = 20
UNSAT
SAT
1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.5
2.0
2.5
3.0
α
$R = 500$
$R = 1000$
$R = 5000$



balanced SAT transition

[Zdeborová, Castellana 2010]

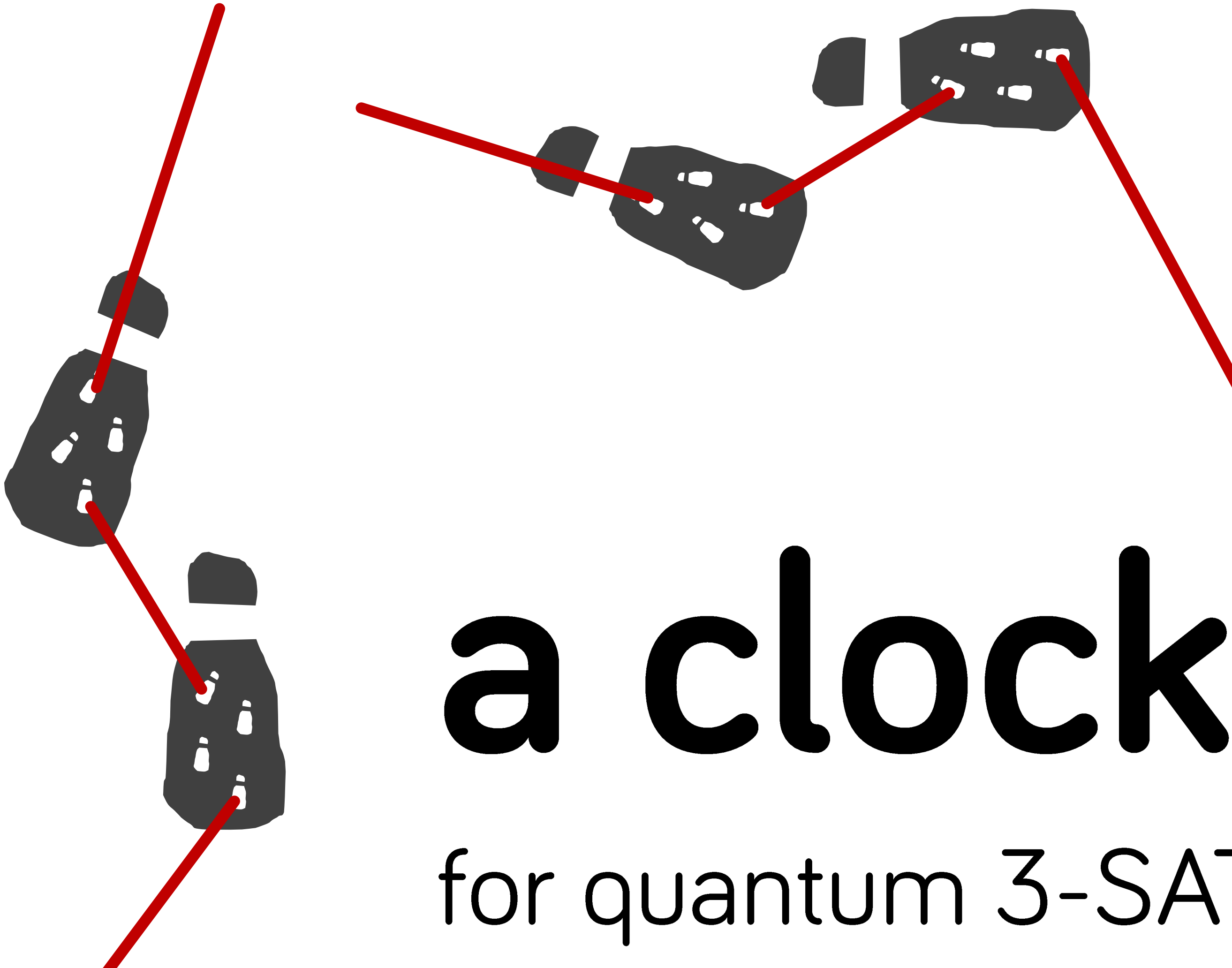


s
p
b
0.92
3.4
UNSAT
SAT
N = 100
N = 150
1.0
1.5
2.0
2.5
3.0
α

perfect
verifiers

&

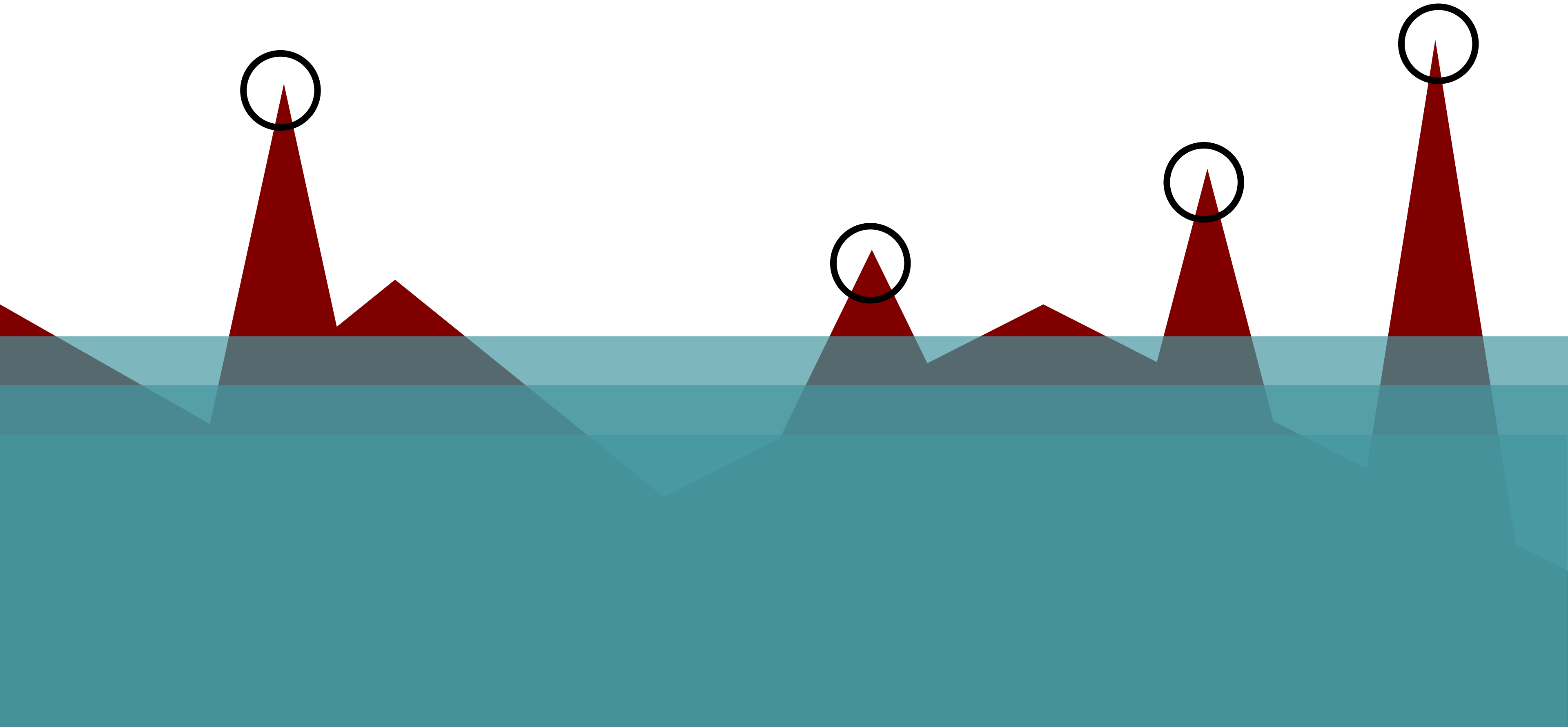
really
convincing
proofs

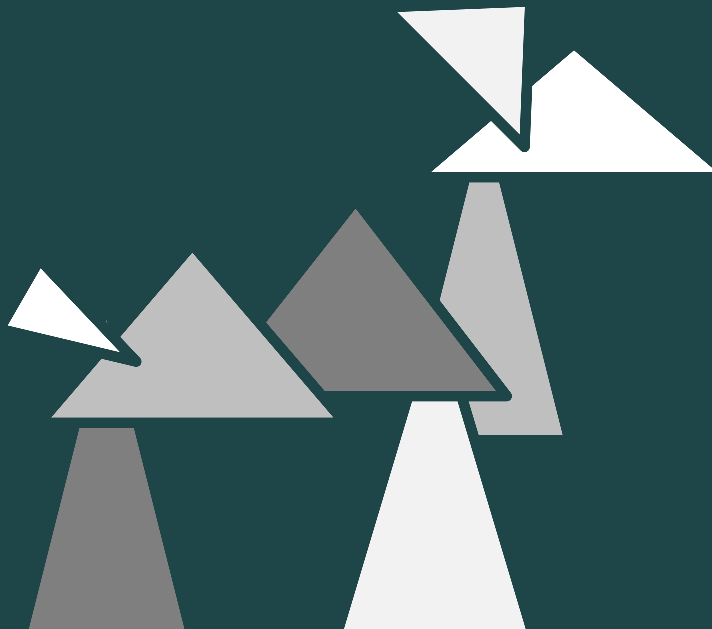


a clock

for quantum 3-SAT

adversary
random **q**- **SAT**





Three looks at

Quantum Satisfiability

D. Gosset, M. Bardoscia, A. Scardicchio,
S. Jordan, H. Nishimura, H. Kobayashi

Daniel Nagaj

universität wien