Quantum 3-SAT

is QMA_1-complete

FOCS '13

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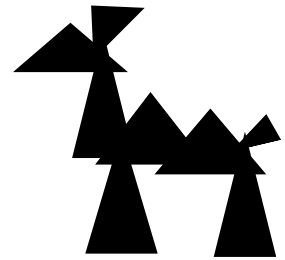
Daniel Nagaj universität wien

classical	*quantum*
2SAT	q2SAT
MAX 2-SAT	2-local Hamiltonian
3SAT	q3SAT

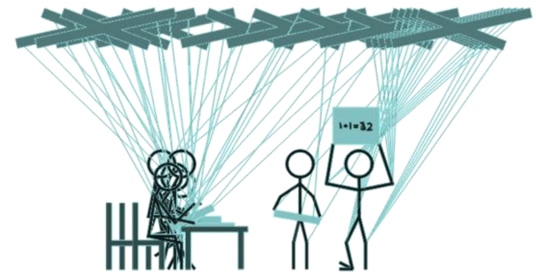
1 quantum sat

without frustration



2 the history

of (a) quantum computation

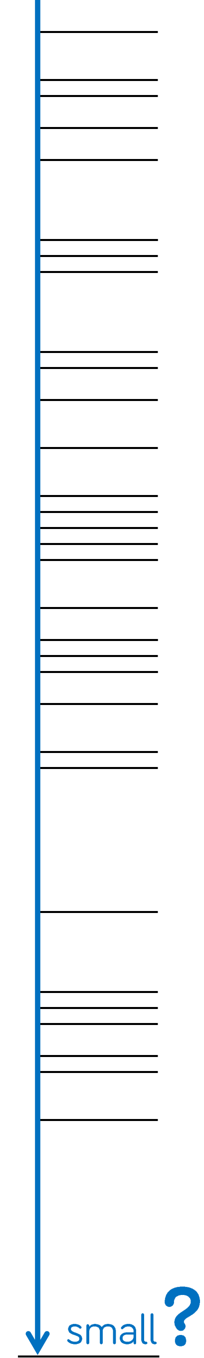


3 a local clock

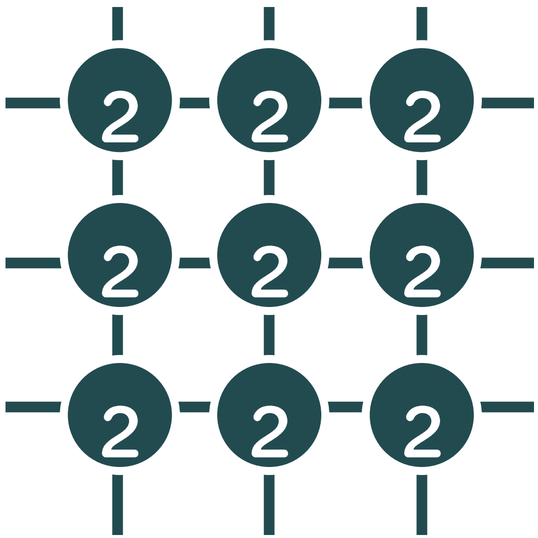
superpositions & interference

1 The Local Hamiltonian problem

Is the ground state energy of a **LH**

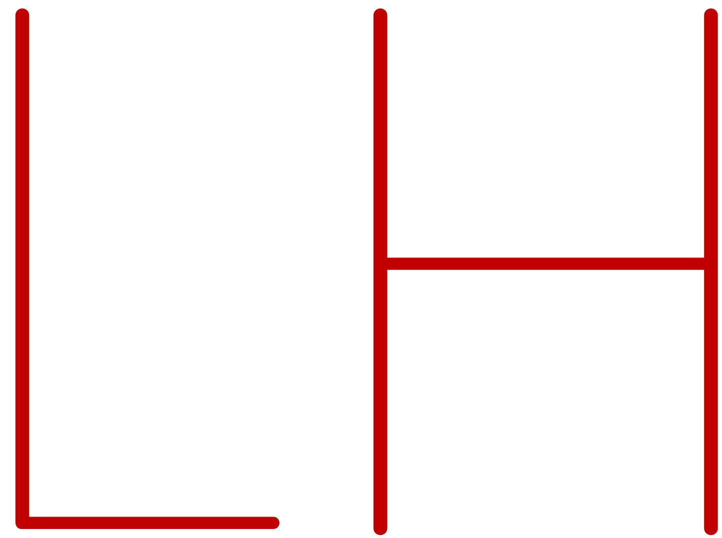


1 2-local Hamiltonian: QMA-complete

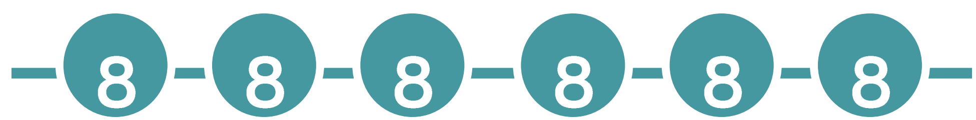


[Oliveira, Terhal ’04]

a global minimum



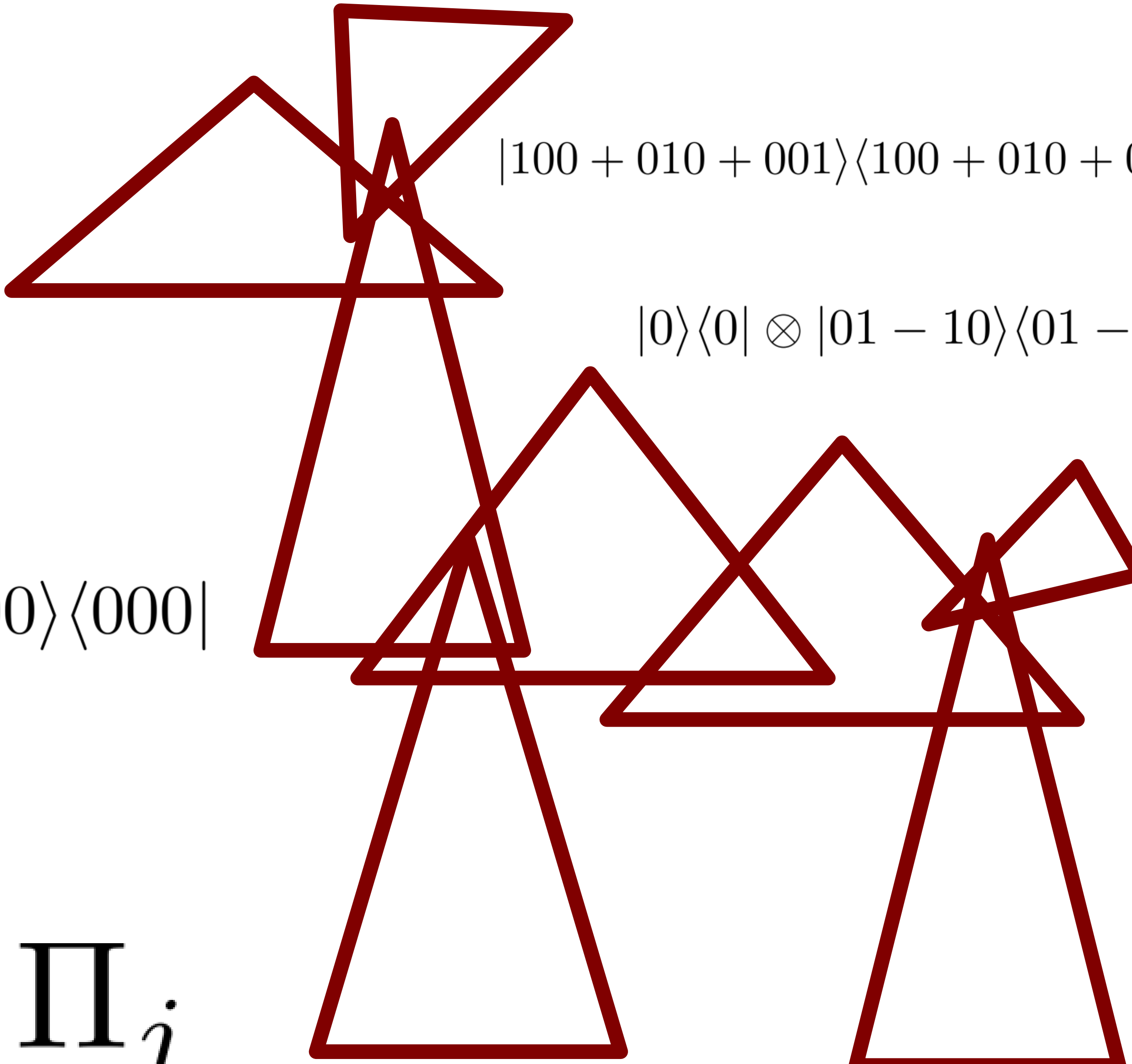
$$\sum H_{jk}$$



[Hallgren, N., Narayanaswami ’13]


frustrated
FRUST
RATED

1 Quantum 3-SAT



$|100+010+001\rangle\langle 100+010+001|$

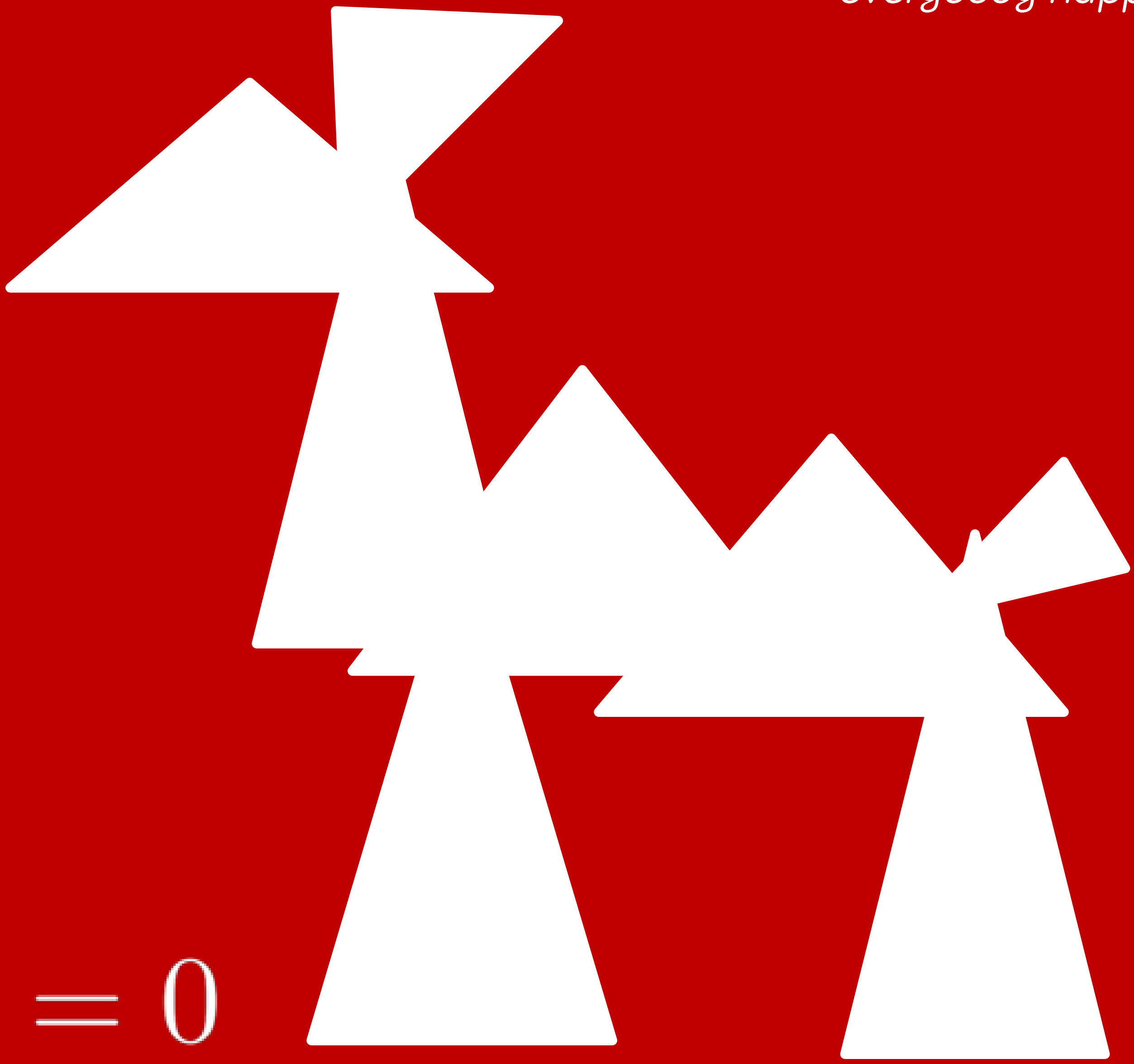
$|0\rangle\langle 0| \otimes |01-10\rangle\langle 01-10|$

$|000\rangle\langle 000|$

- 3-local projectors Π_j

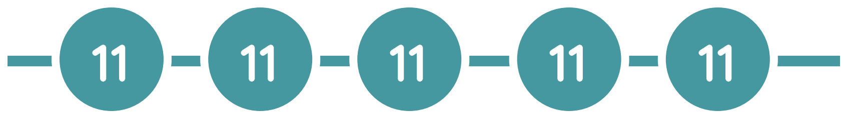
1 Quantum 3-SAT

Can we make everybody happy?



$$\Pi_j|\psi\rangle = 0$$

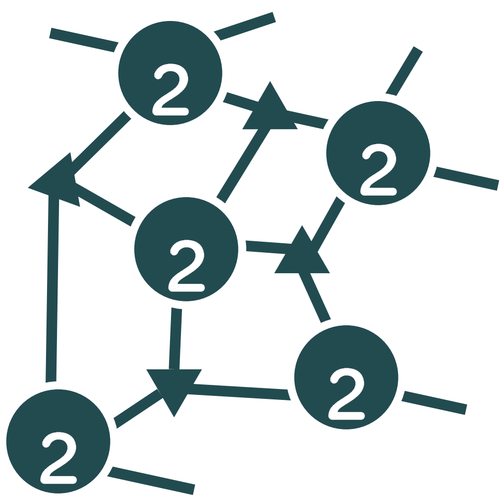
1 qSAT: QMA$_1$-complete



[N. ’08]

$$\Pi_j$$

[Gosset, N. ’13]



unfrustrated

qSAT

puzzles &
proofs

NO?
Don't get fooled easily.

YES?
Accept a genuine proof without a doubt.

NO?
Don't get fooled easily.

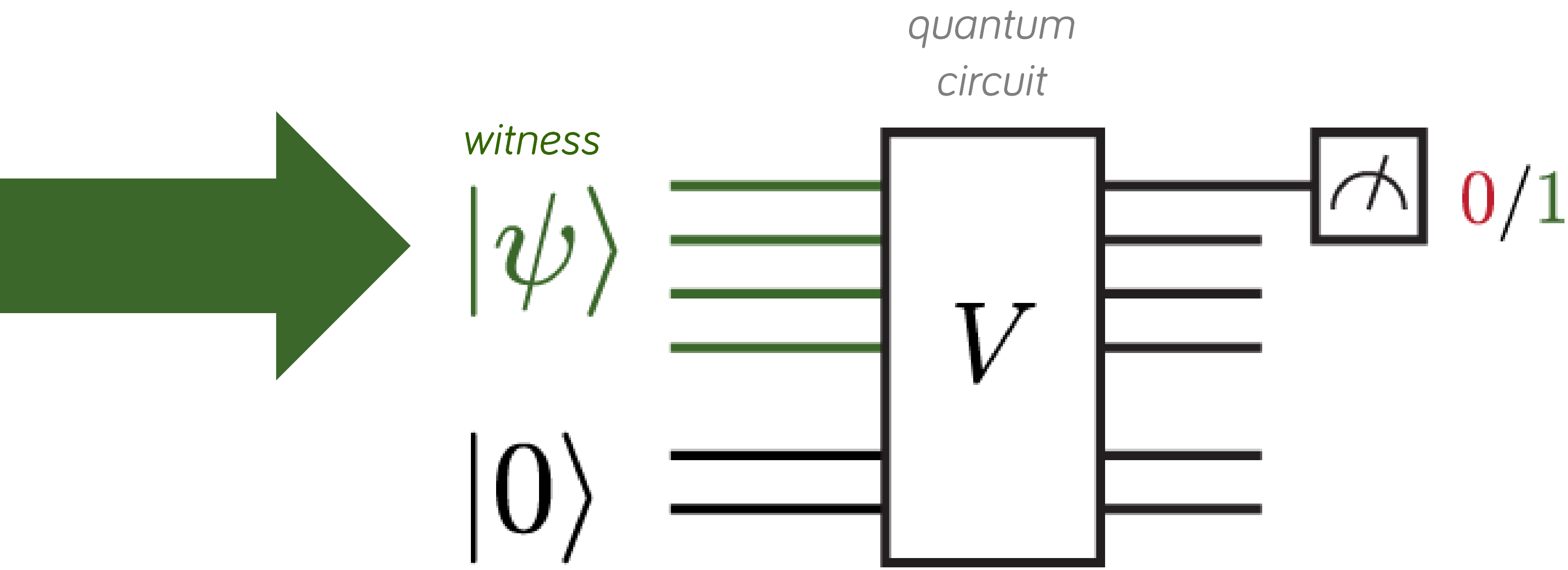
YES?
Accept a genuine proof without a doubt.

MA_1

perfect

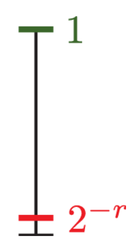
completeness

1 A quantum analogue: QMA_1

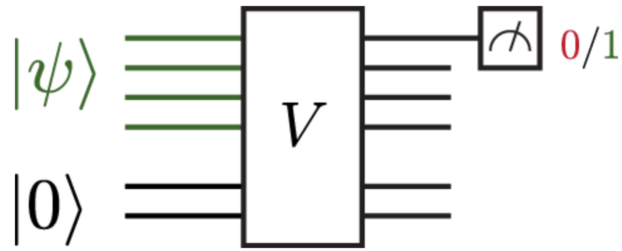


YES? Accept a good proof.

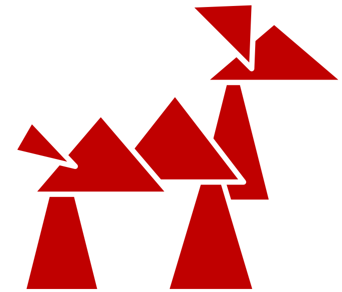
NO? Get fooled with small *p*.



1 A candidate for a QMA_1-hard problem



Does this circuit accept?



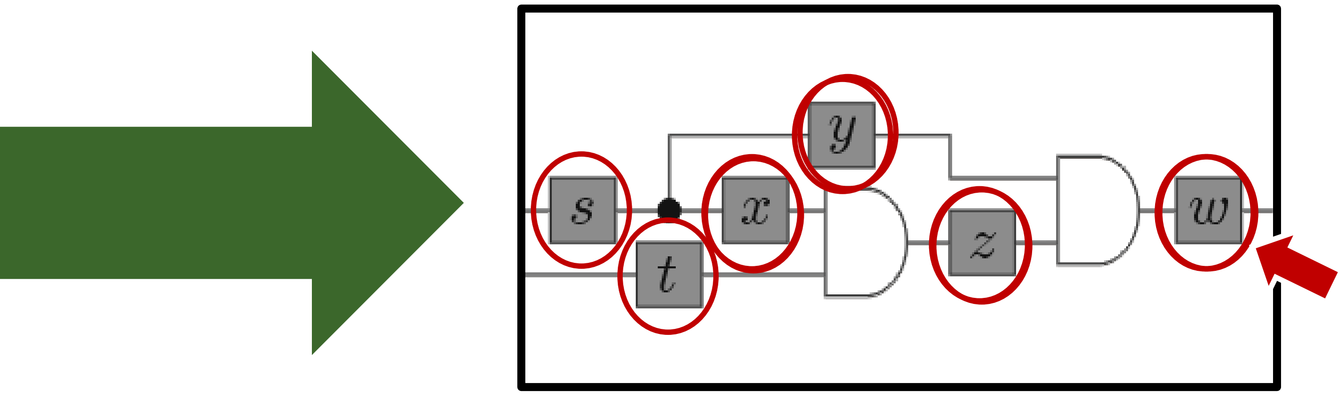
$$\Pi_j$$

Is there a common ground state?

computation
ground states
& quantum sat

2 What is hard for NP?

- 3-SAT is NP-complete. [Cook, Levin]



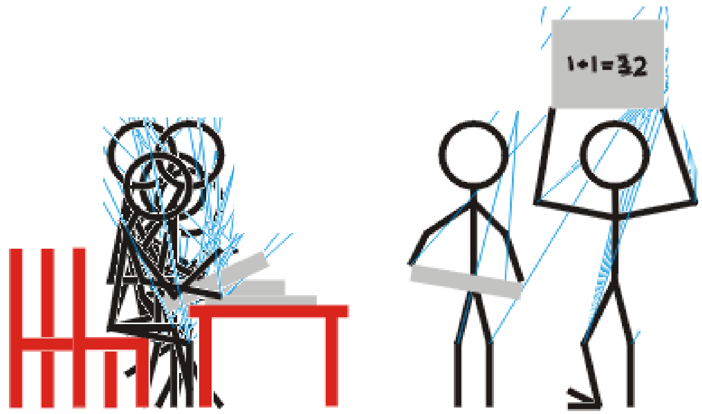
Could this circuit ever output 1?

3-local conditions

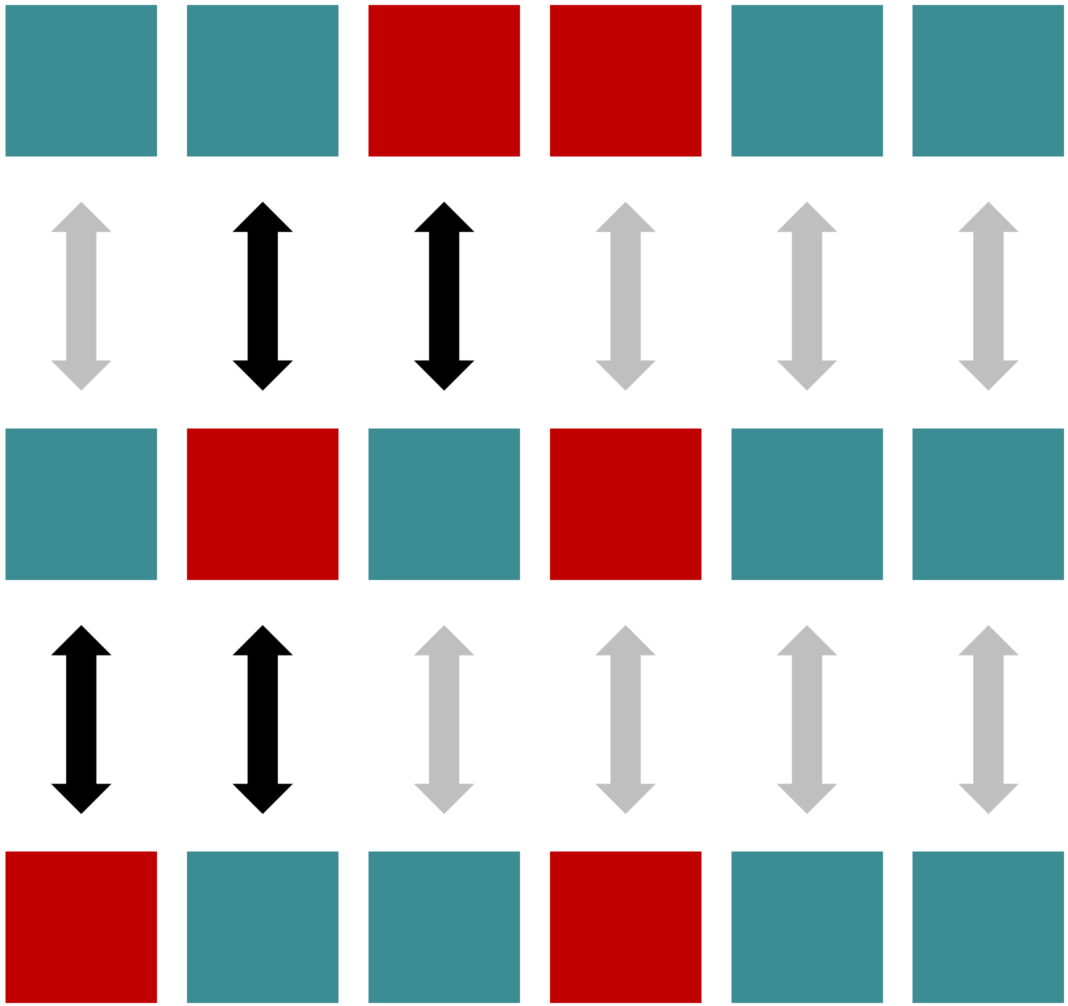
$$(\cdots \vee \cdots \vee \cdots) \wedge (\cdots \vee \cdots \vee \cdots) \wedge \cdots$$

- Can we do the same for a quantum computation?

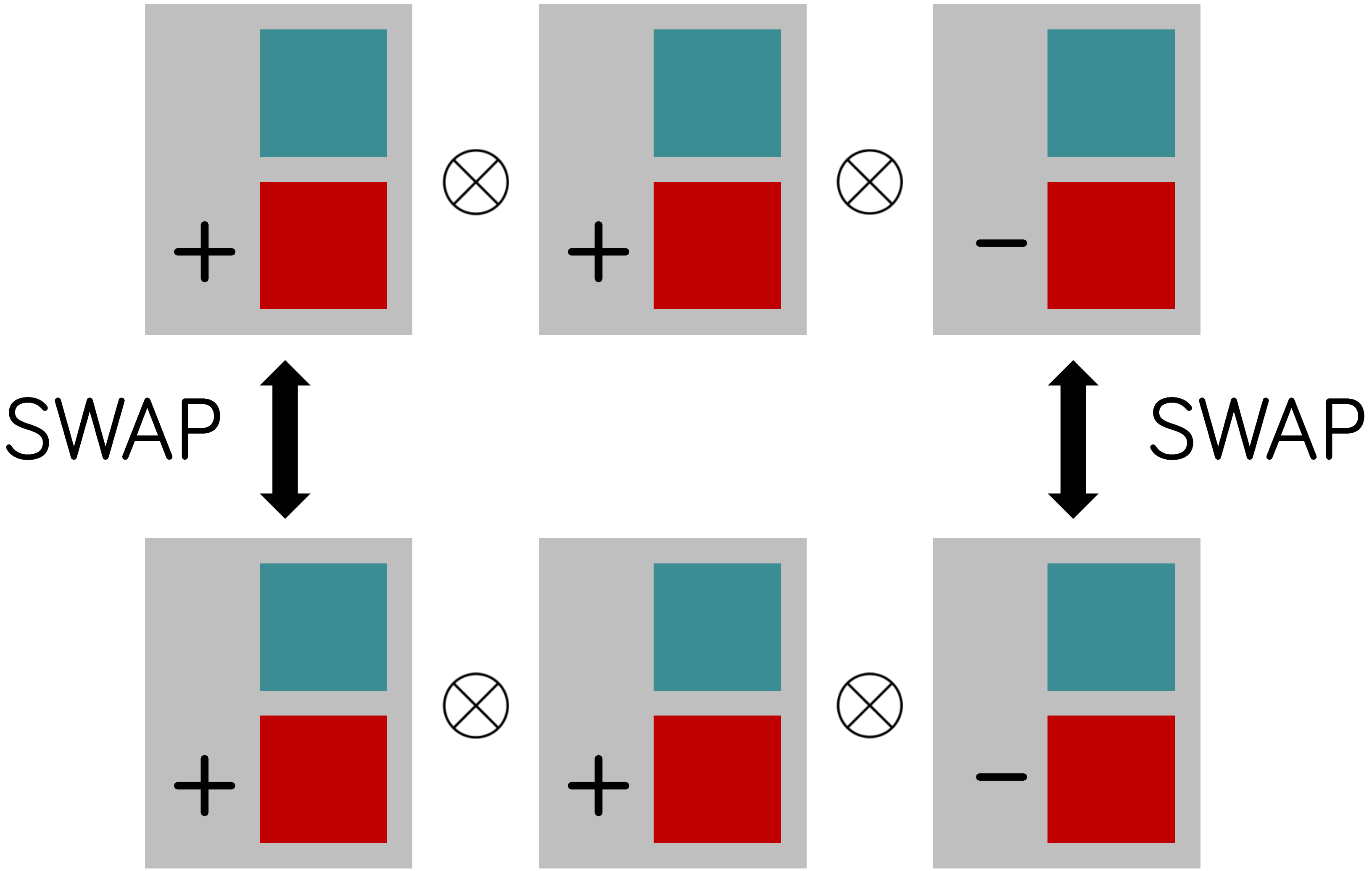
2 Snapshots of a computation



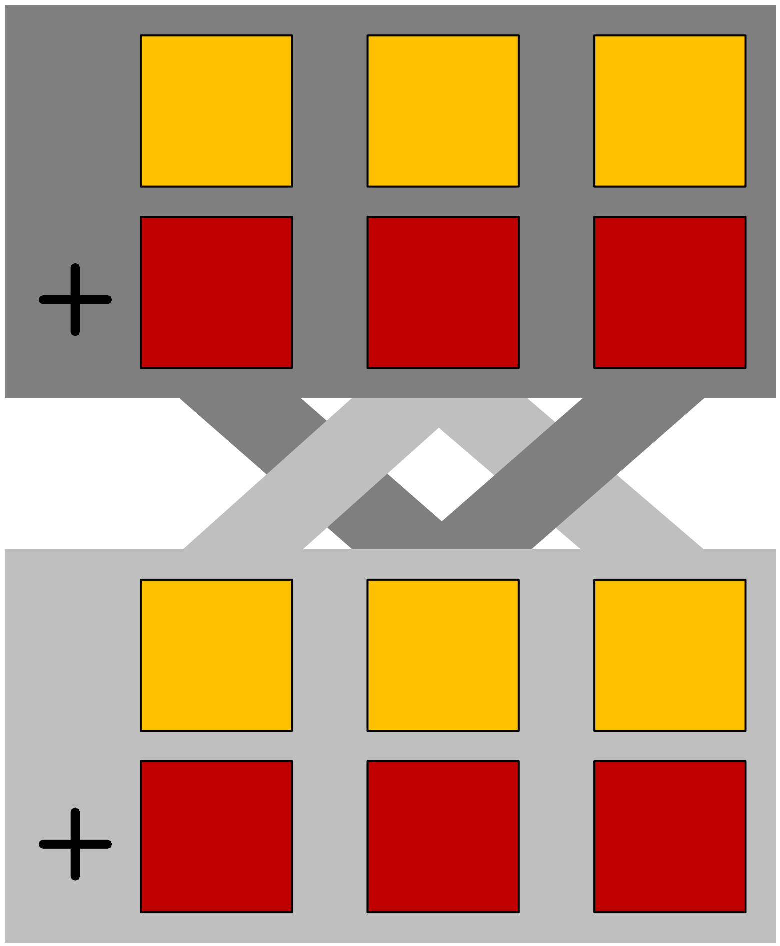
Locally comparing strings.



Locally comparing product states: SWAP.

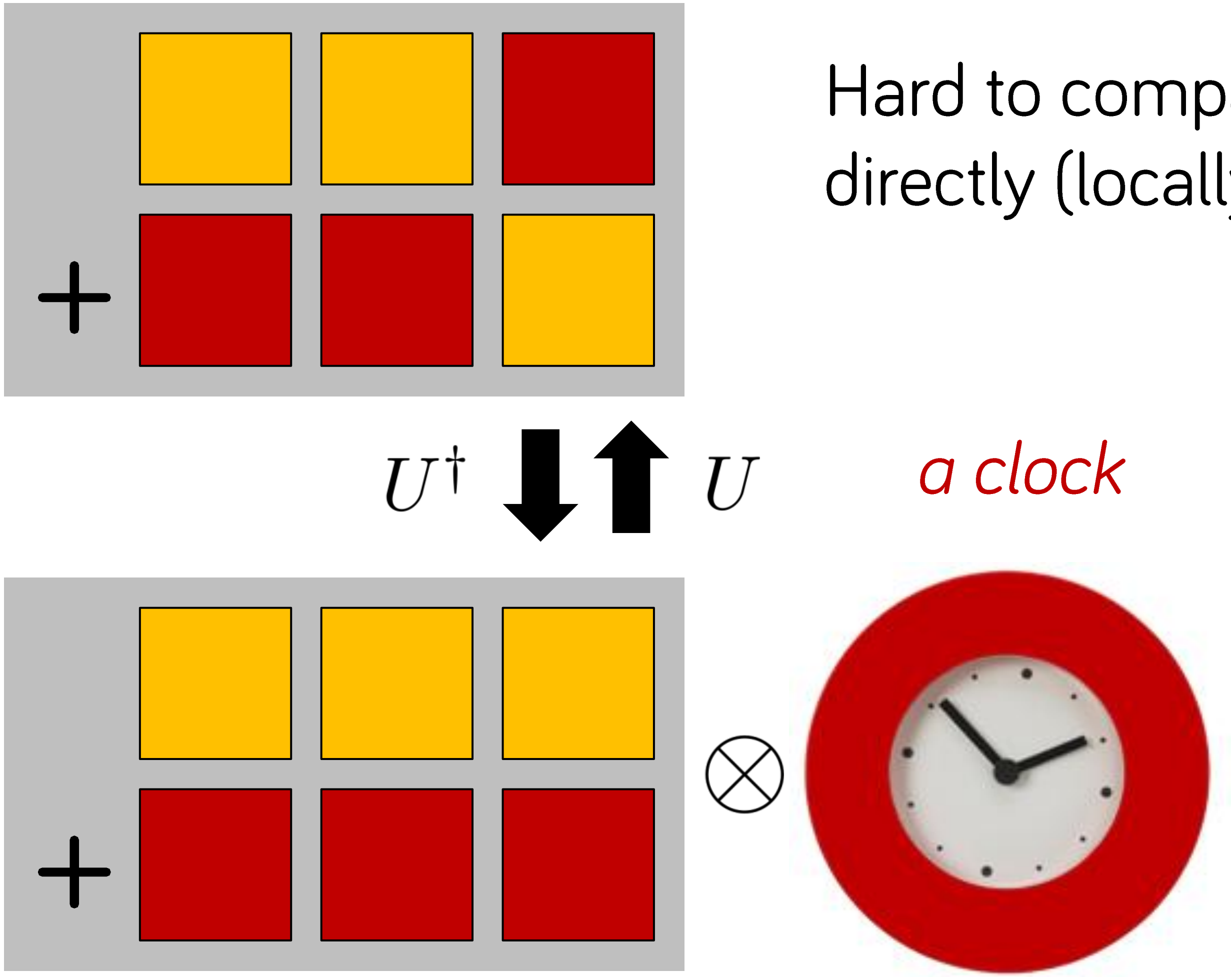


Locally comparing entangled states?



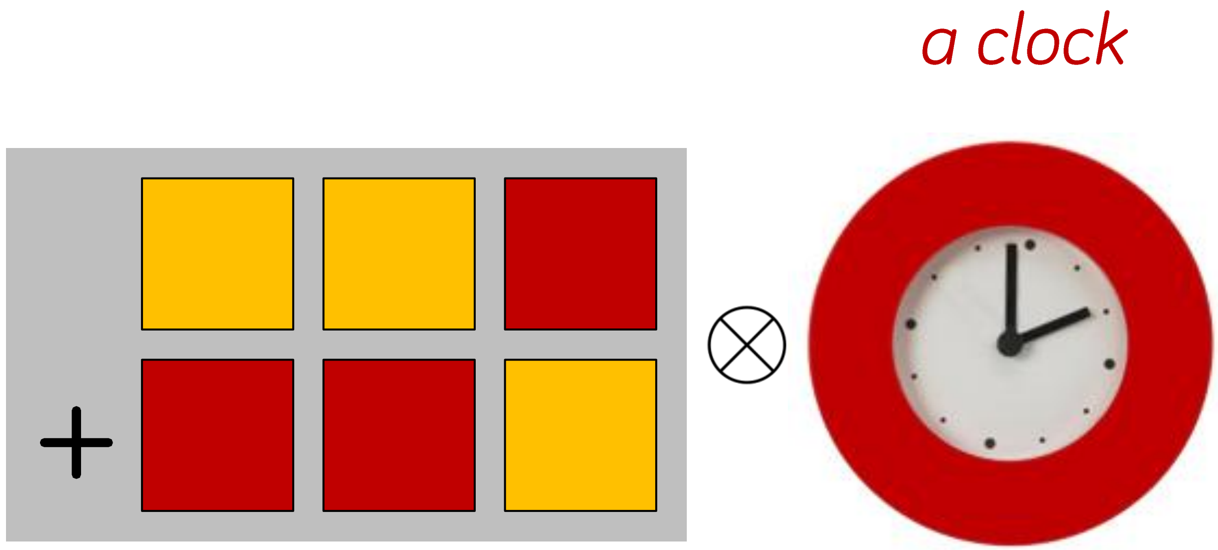
UGH!

2 The data & the clock

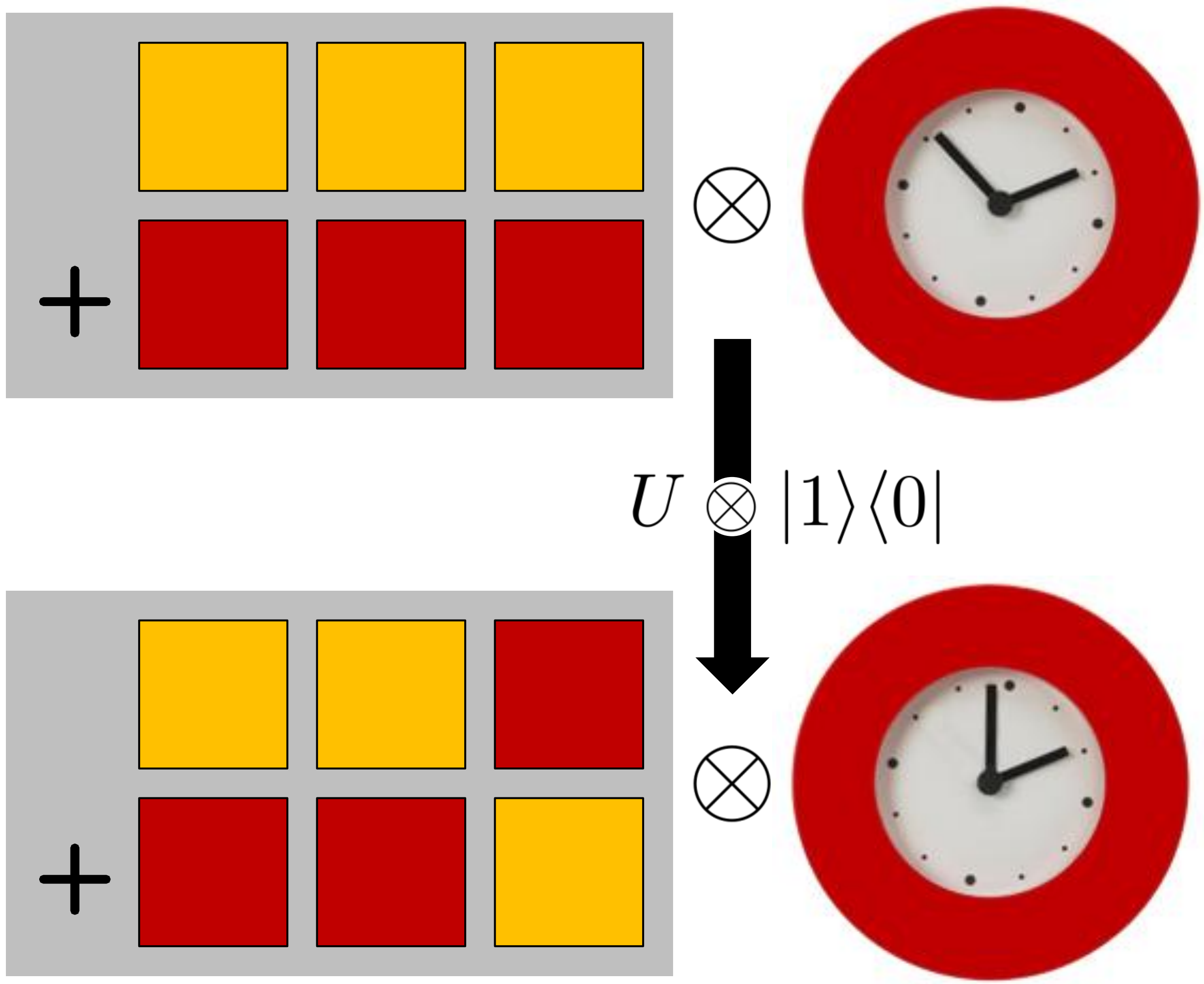


Hard to compare directly (locally).

2 The data & the clock



2 The data & the clock



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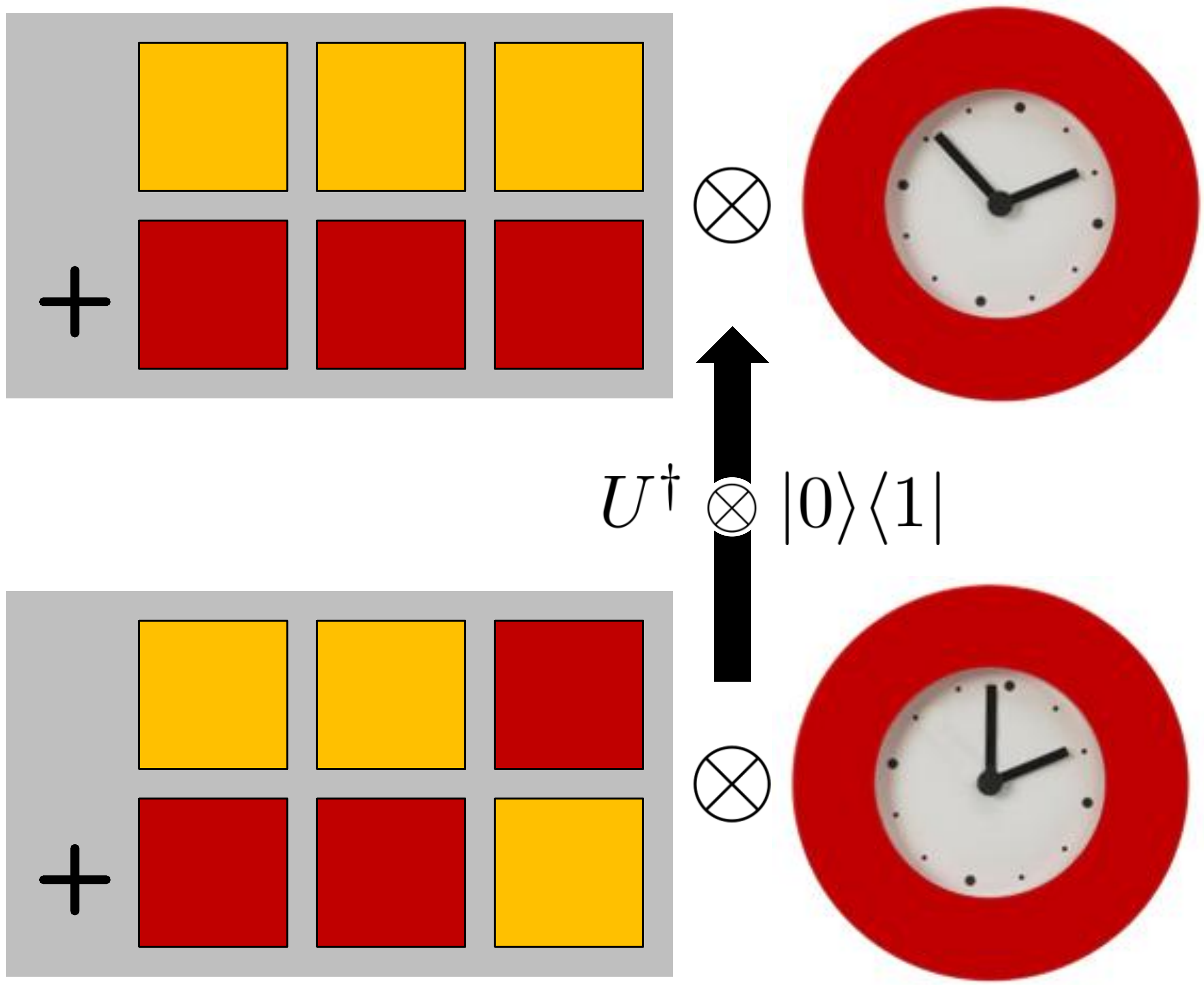
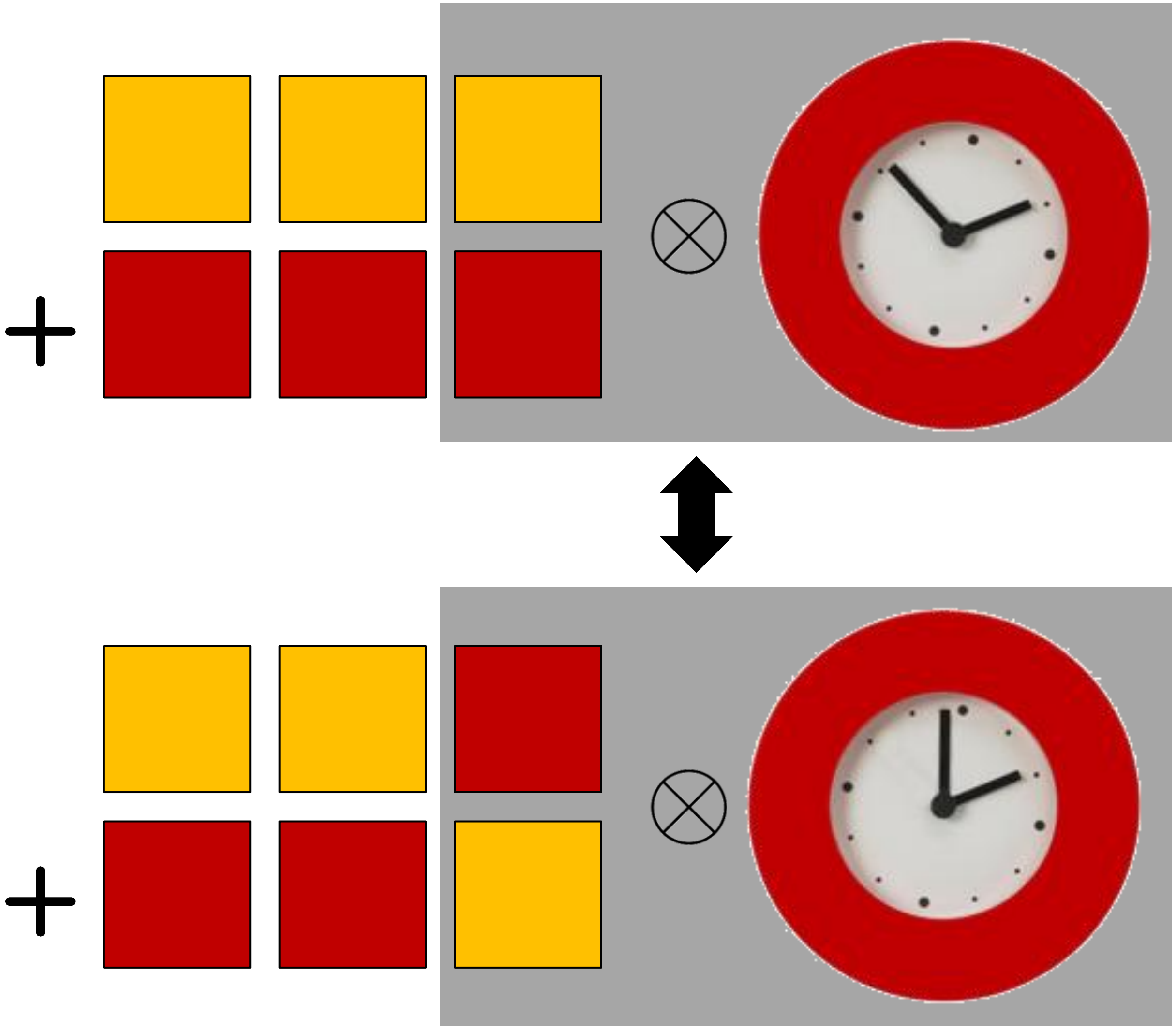
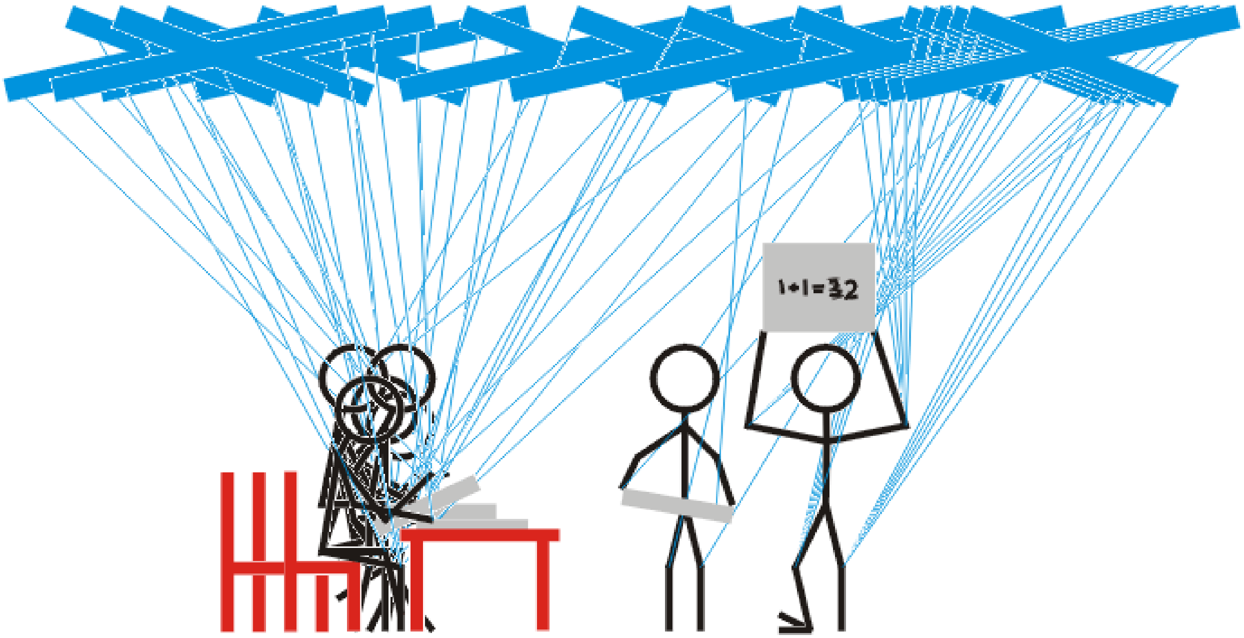


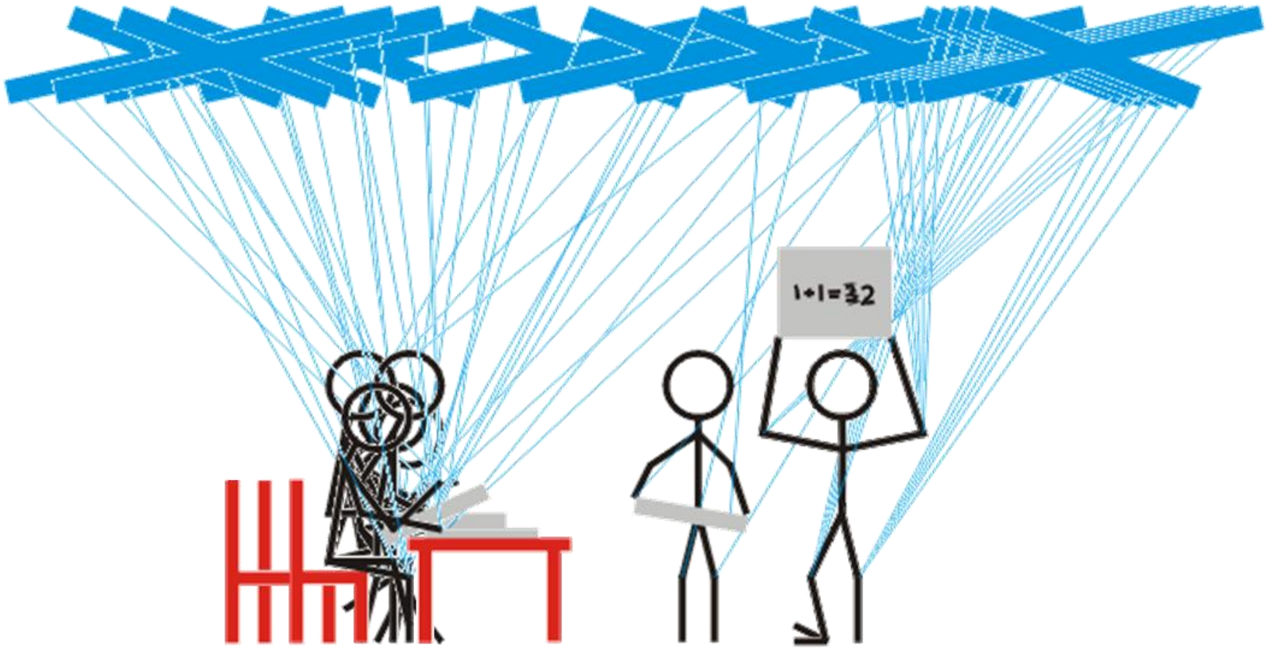
2 The data & the clock



2 The history state



2 The history state



$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} \underbrace{|\varphi_t\rangle}_{U_t \cdots U_1 |\varphi_0\rangle} \otimes |t\rangle$$

2 The history state: a ground state

Π_j  $= 0$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} \underbrace{|\varphi_t\rangle}_{U_t \cdots U_1 |\varphi_0\rangle} \otimes |t\rangle$$

2 Do we have a history state?

k-local

c-o-n-d-i-t-i-o-n-s

clock encoding

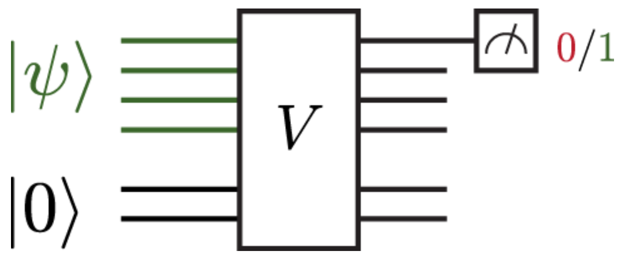
state progression

$$|\varphi_t\rangle \otimes |t\rangle$$

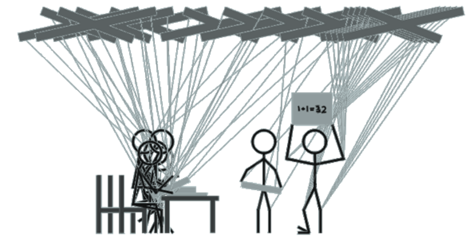
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

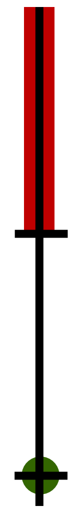
2 Computation and Quantum SAT



$$H = \sum \Pi_j$$



NO nothing is likely to be accepted



any state has a high energy

YES there is a perfectly accepted proof

its history state has energy 0

- YES

ground state

- NO

lower bound on the ground state energy

good
clock
states

bad
clock
states

history states

non-uniform
superpositions

well	badly
initialized history states	

well

initialized histories

accepted states

well

initialized histories

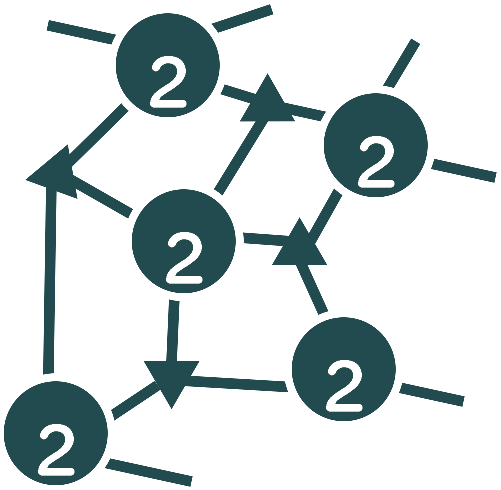
$H_A + H_B$

$$\lambda_0 \geq \sin^2 \frac{\vartheta}{2} \times \min(\Delta_A, \Delta_B)$$

accepted states

L^{-2} L^{-1}

L^{-3} promise gap (soudness)

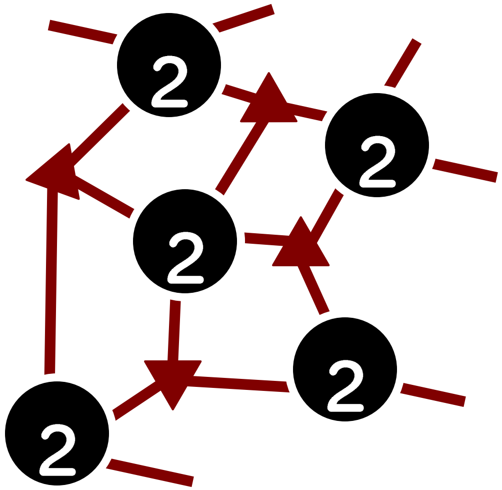


unfrustrated

quantum 3-SAT

a verifier circuit with

a genuine witness



frustrated

quantum 3-SAT

a verifier circuit with

no likely witness

a clock workshop

3 Run the clock, apply 2-qubit gates ...

$$|\varphi_{t-2}\rangle \otimes |t-2\rangle$$

$$|\varphi_{t-1}\rangle \otimes |t-1\rangle$$

$$|\varphi_{t}\rangle \otimes |t\rangle$$

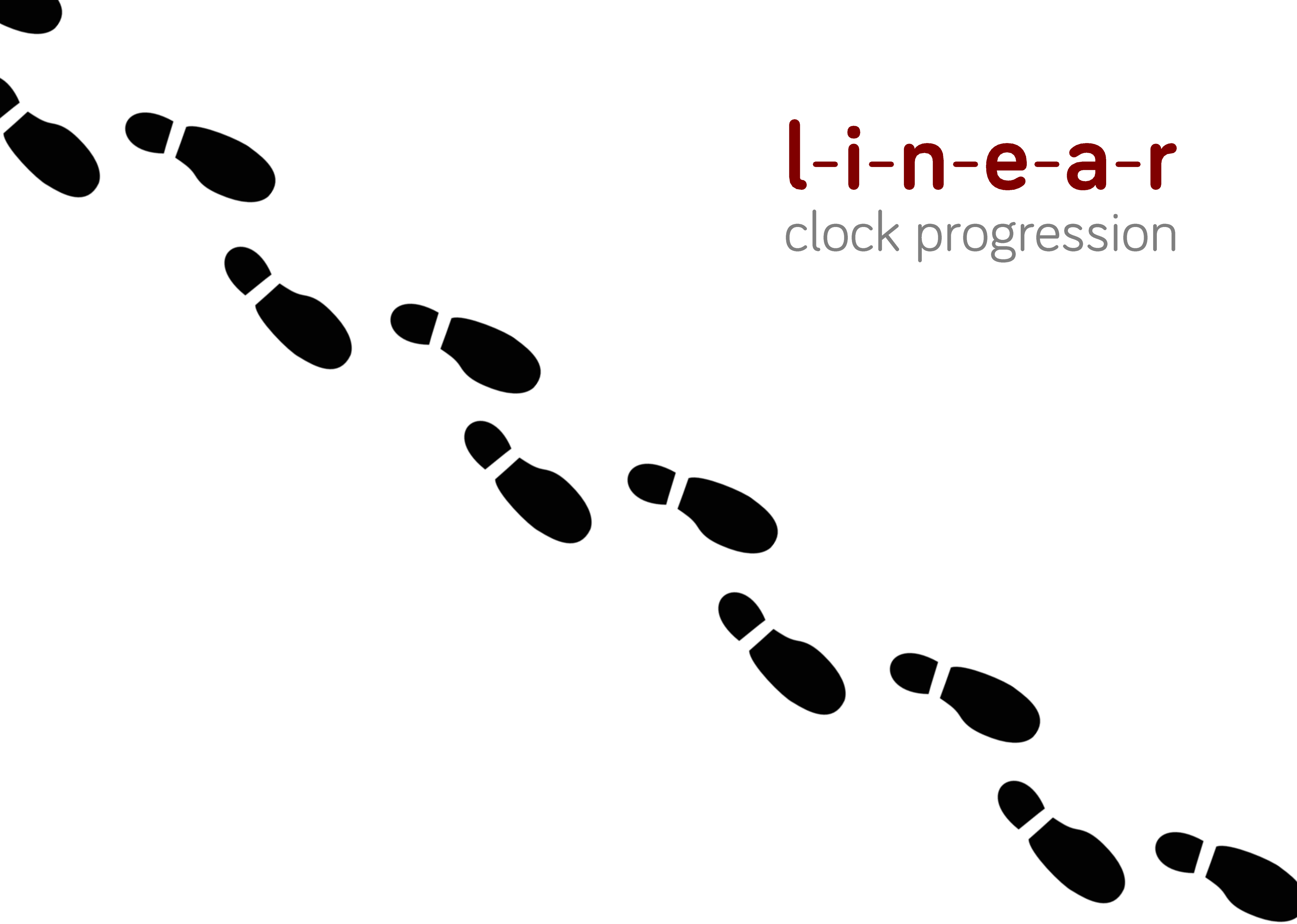
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

$$|\varphi_{t+2}\rangle \otimes |t+2\rangle$$

$$|\varphi_{t+3}\rangle \otimes |t+3\rangle$$

l-i-n-e-a-r

clock progression

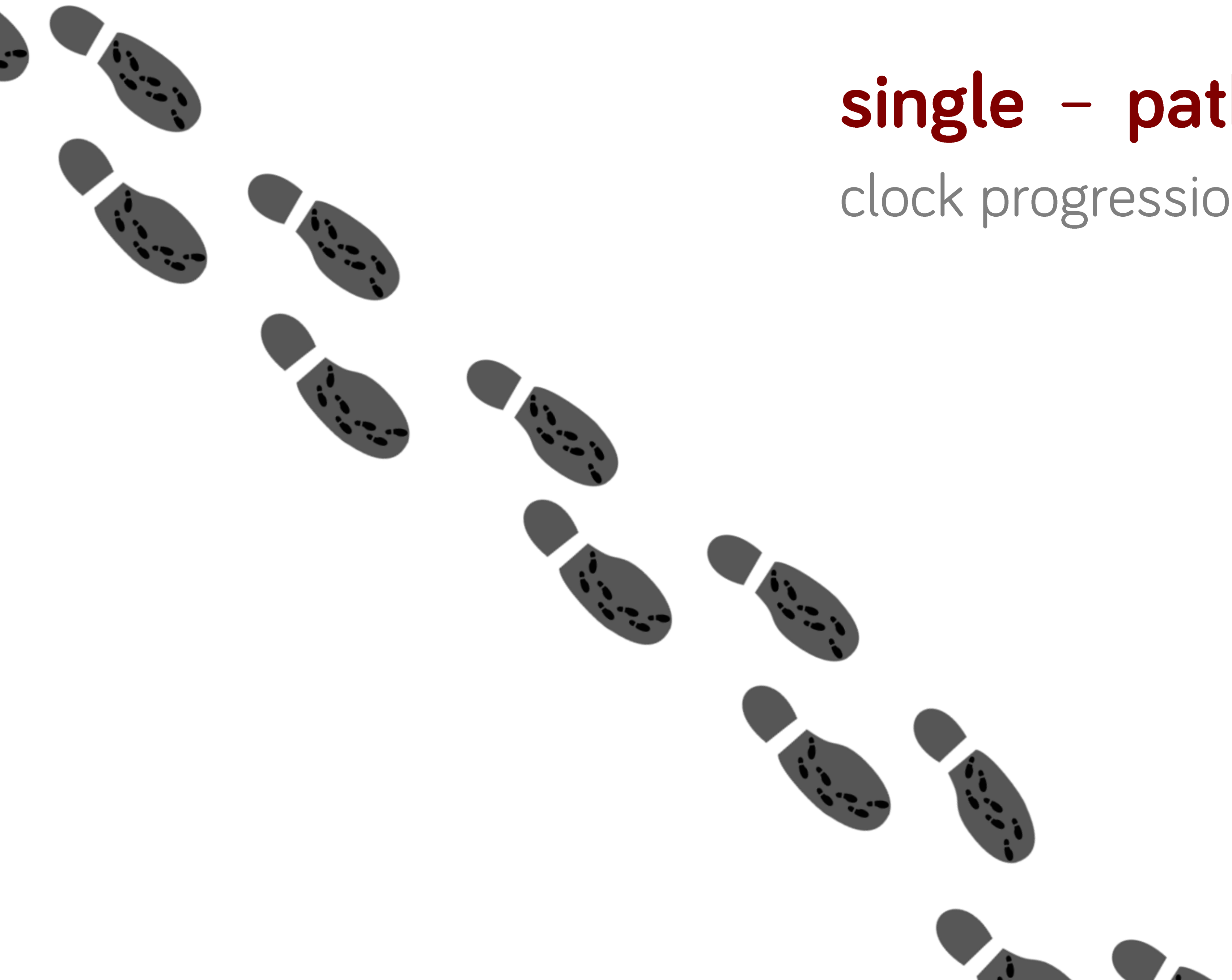


com-pos-ite

clock progression

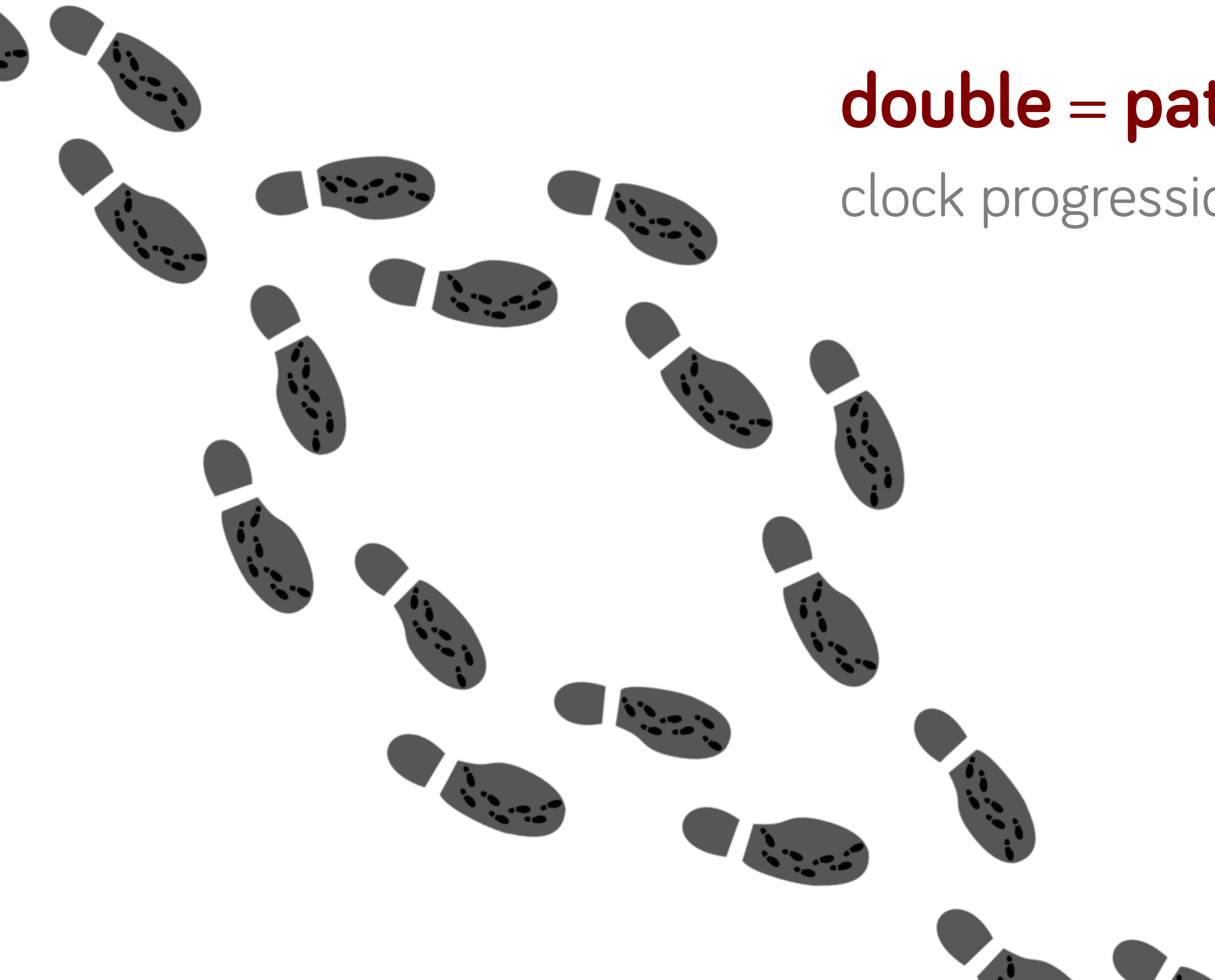
single – path

clock progression



double = path

clock progression

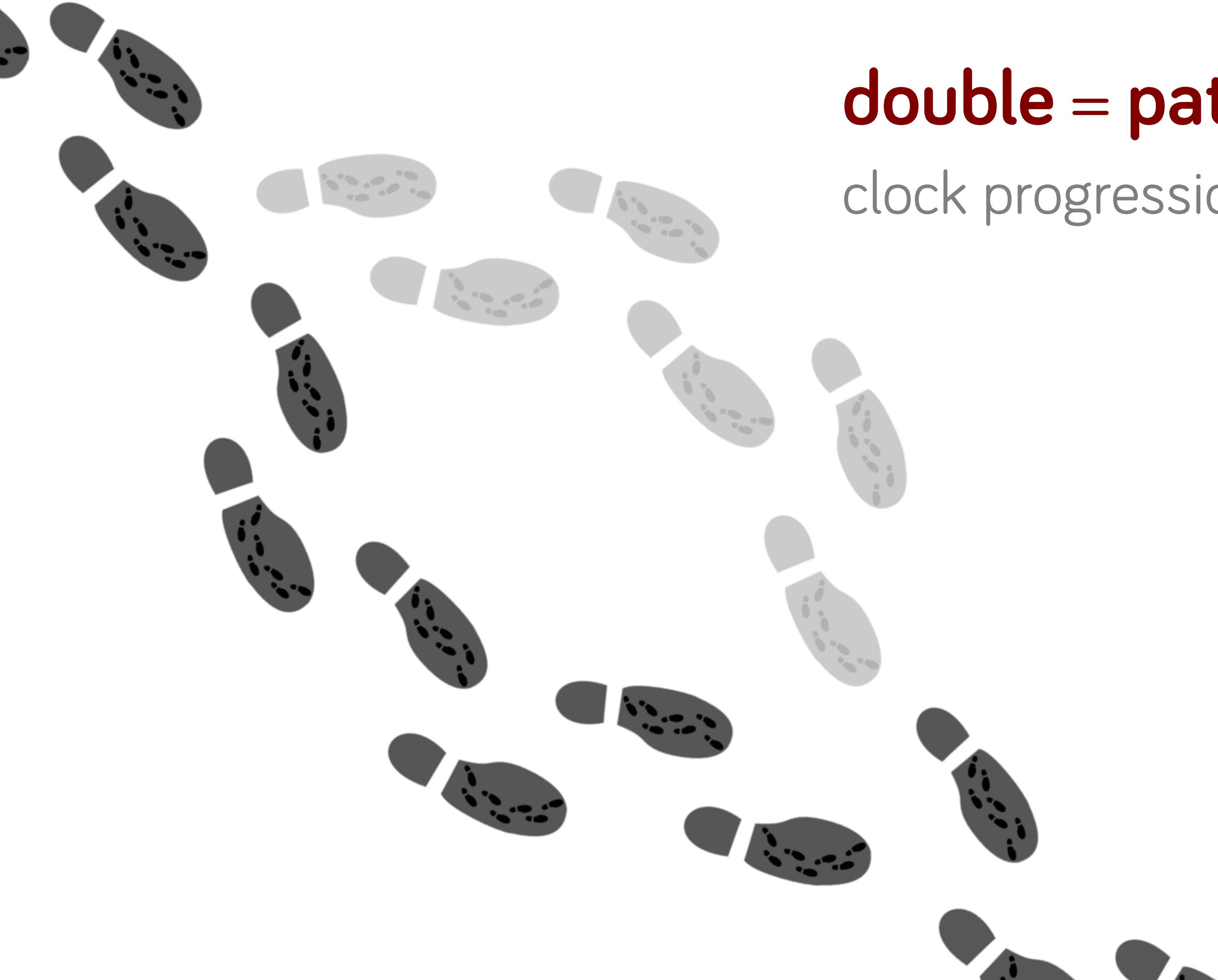


double = path

clock progression

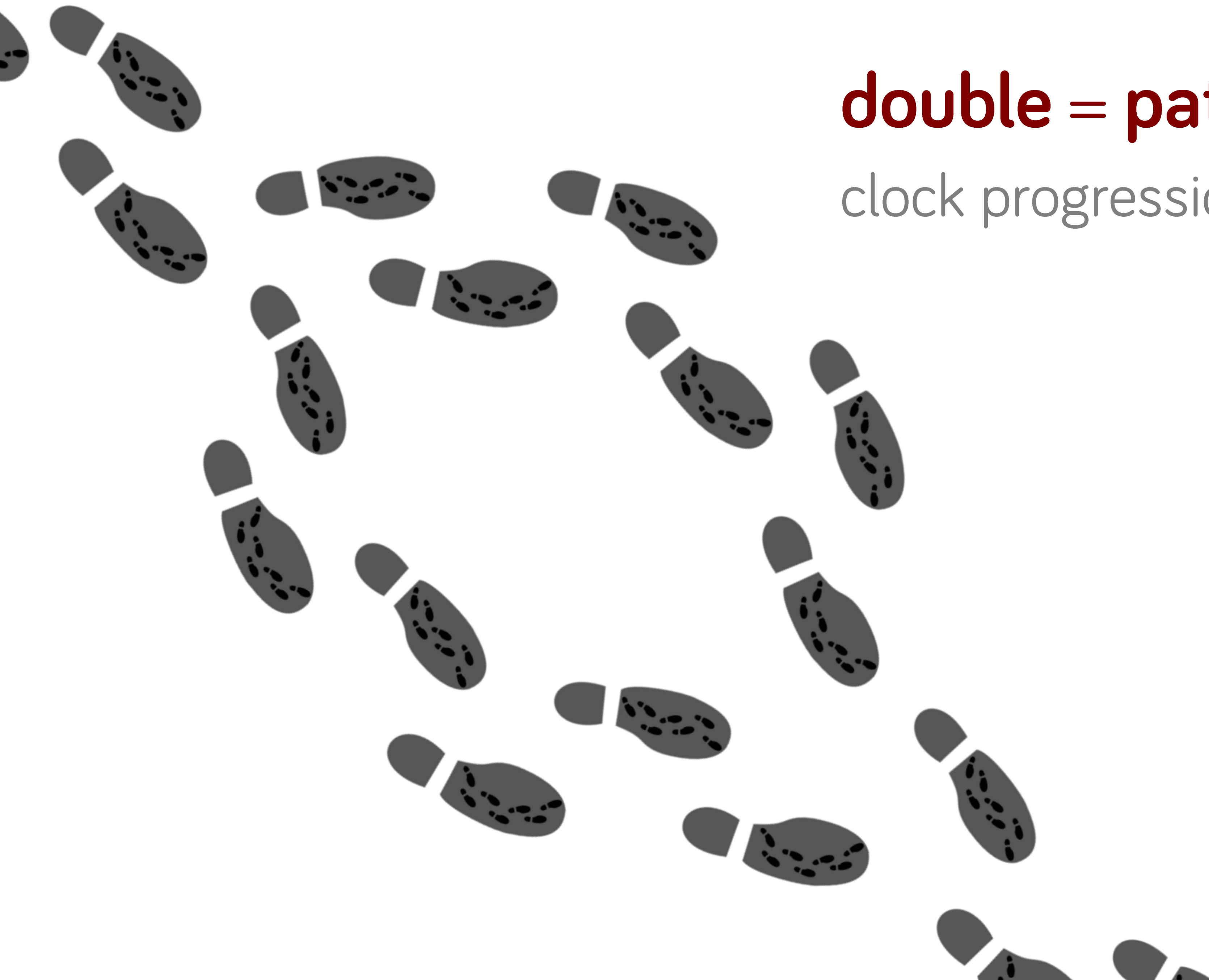
double = path

clock progression



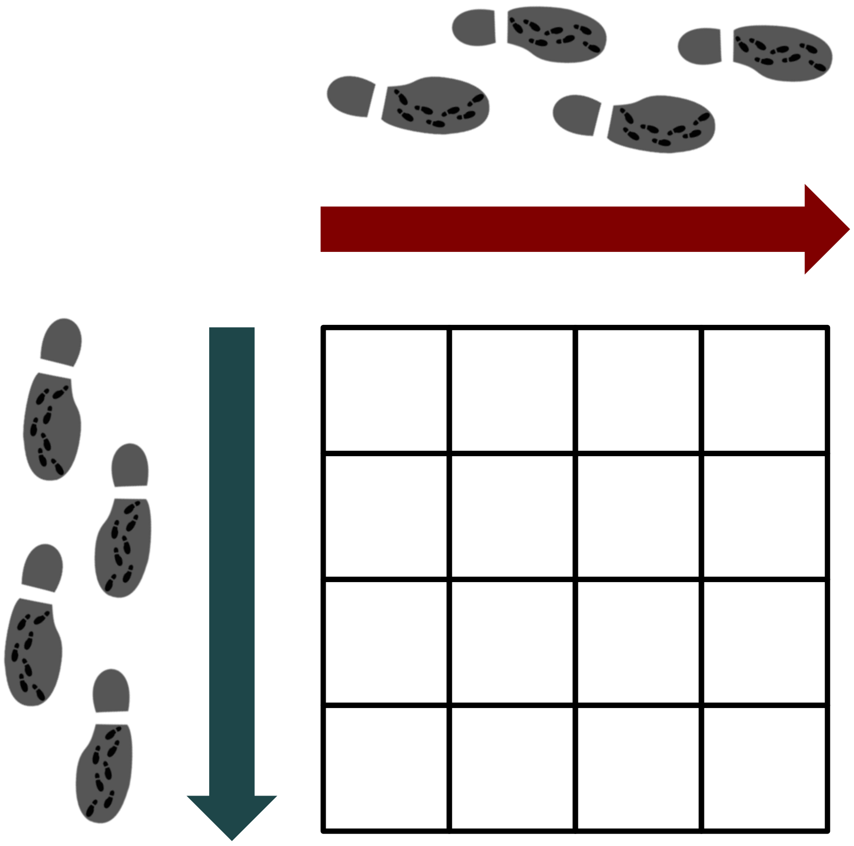
double = path

clock progression



2 clocks: 2D

clock progression

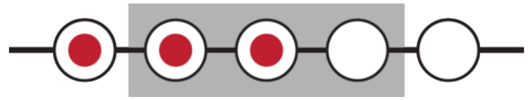


3 Constructing a clock: unary

- the domain wall 

$$|t\rangle = |4\rangle$$
$$= |11110\rangle$$

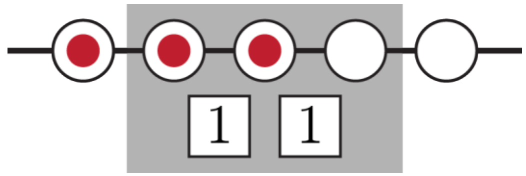
3 Constructing a clock: tick, tock

- the domain wall 

$$
\begin{aligned}
|t\rangle &= |3\rangle \\
&= |11100\rangle
\end{aligned}
$$

- unique transitions: 3-local

3 Constructing a clock: data-clock interaction

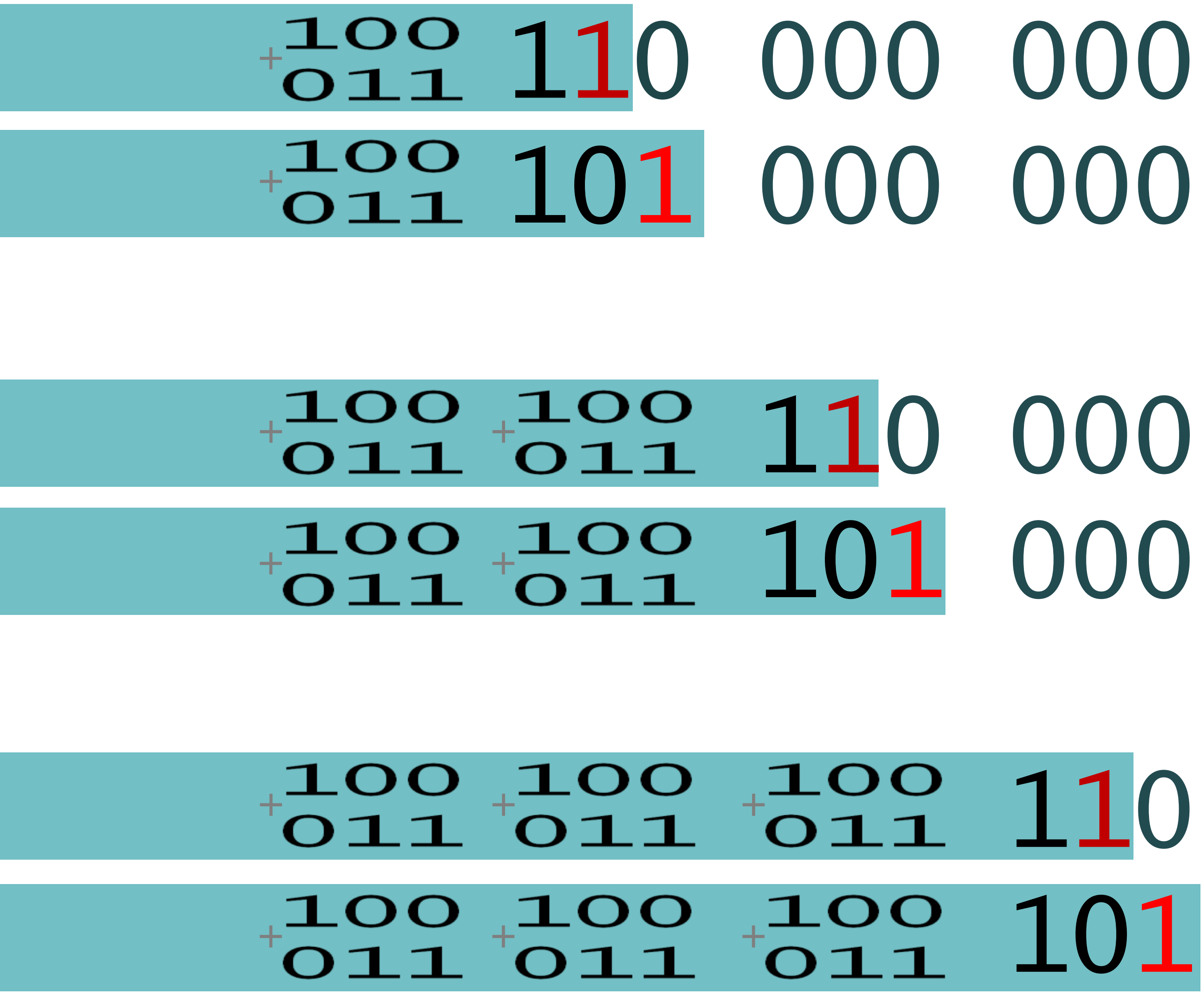
- the domain wall  5-local

- 2-qubit gates: 5-local
- unique transitions: 3-local

quantum 5-SAT [A. Kitaev]

quantum 4-SAT [S. Bravyi]

3 A “composite” domain wall clock



3 Made of “legal” triplets

100 + 011	110	000	000
100 + 011	101	000	000

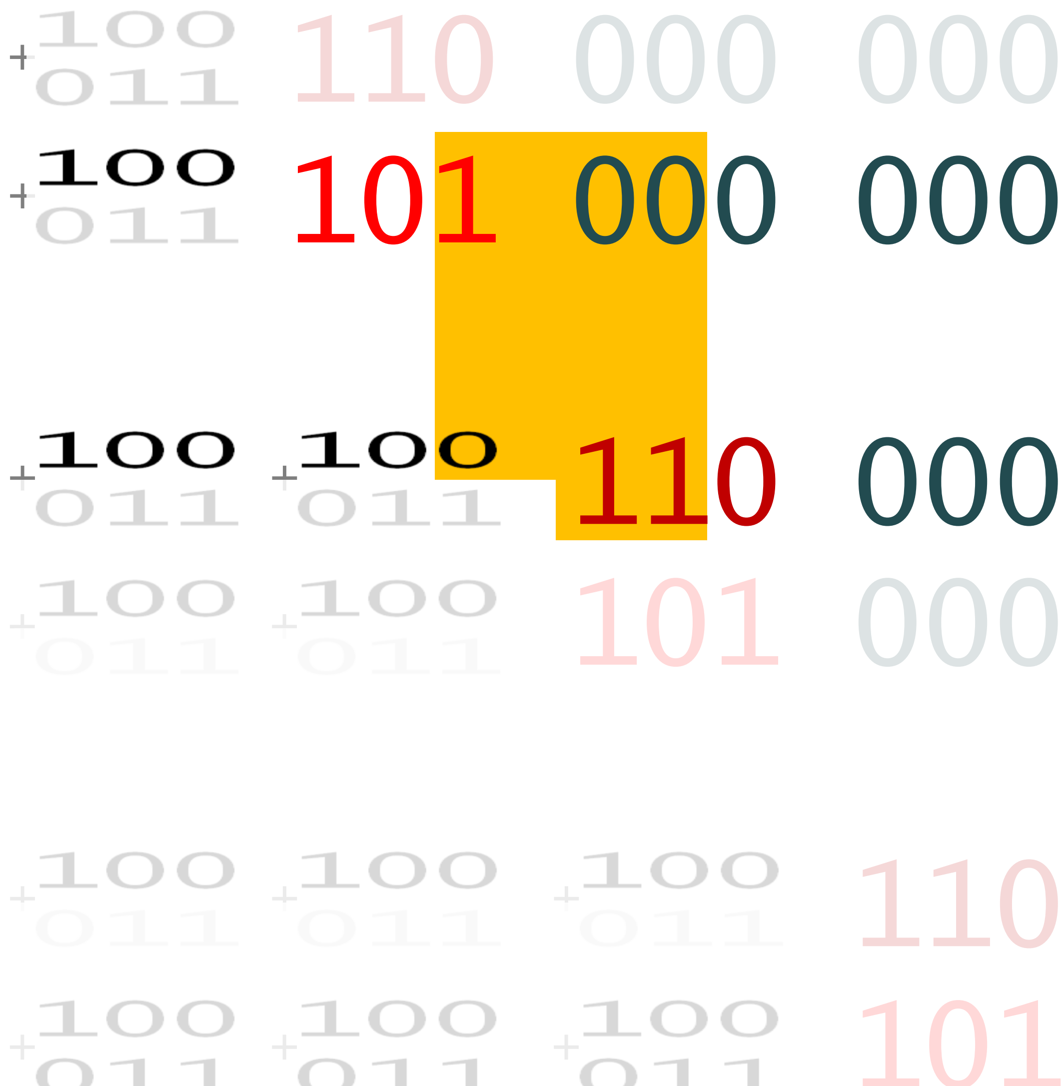
100 + 011	100 + 011	110	000
100 + 011	100 + 011	101	000

100 + 011	100 + 011	100 + 011	110
100 + 011	100 + 011	100 + 011	101

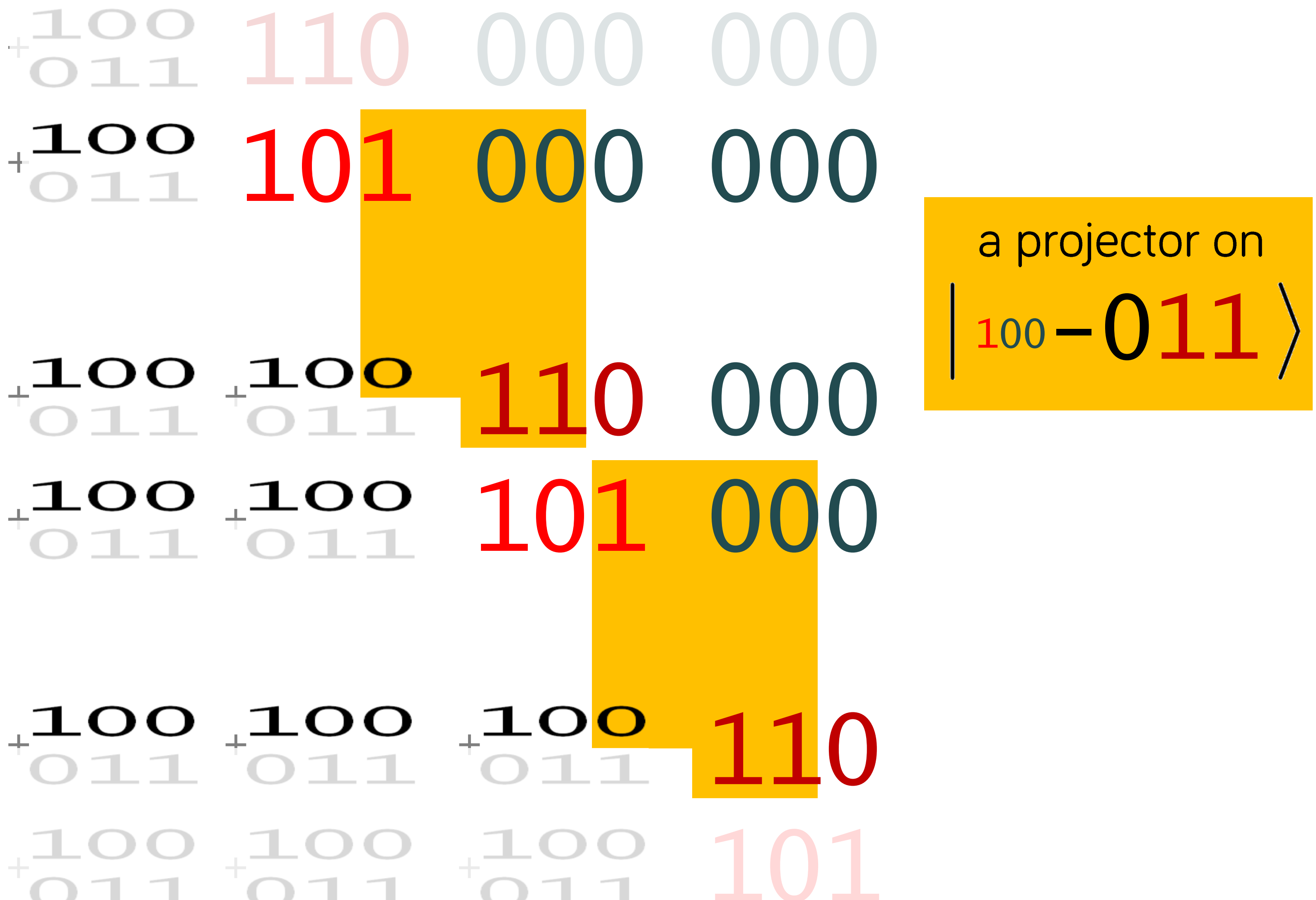
3 Enforce sequences of triplets

100 + 011	110	000	000
100 + 011	101	000	000
100 + 011	100 + 011	110	000
100 + 011	100 + 011	101	000
100 + 011	100 + 011	100 + 011	110
100 + 011	100 + 011	100 + 011	101

3 Look at the differences between lines



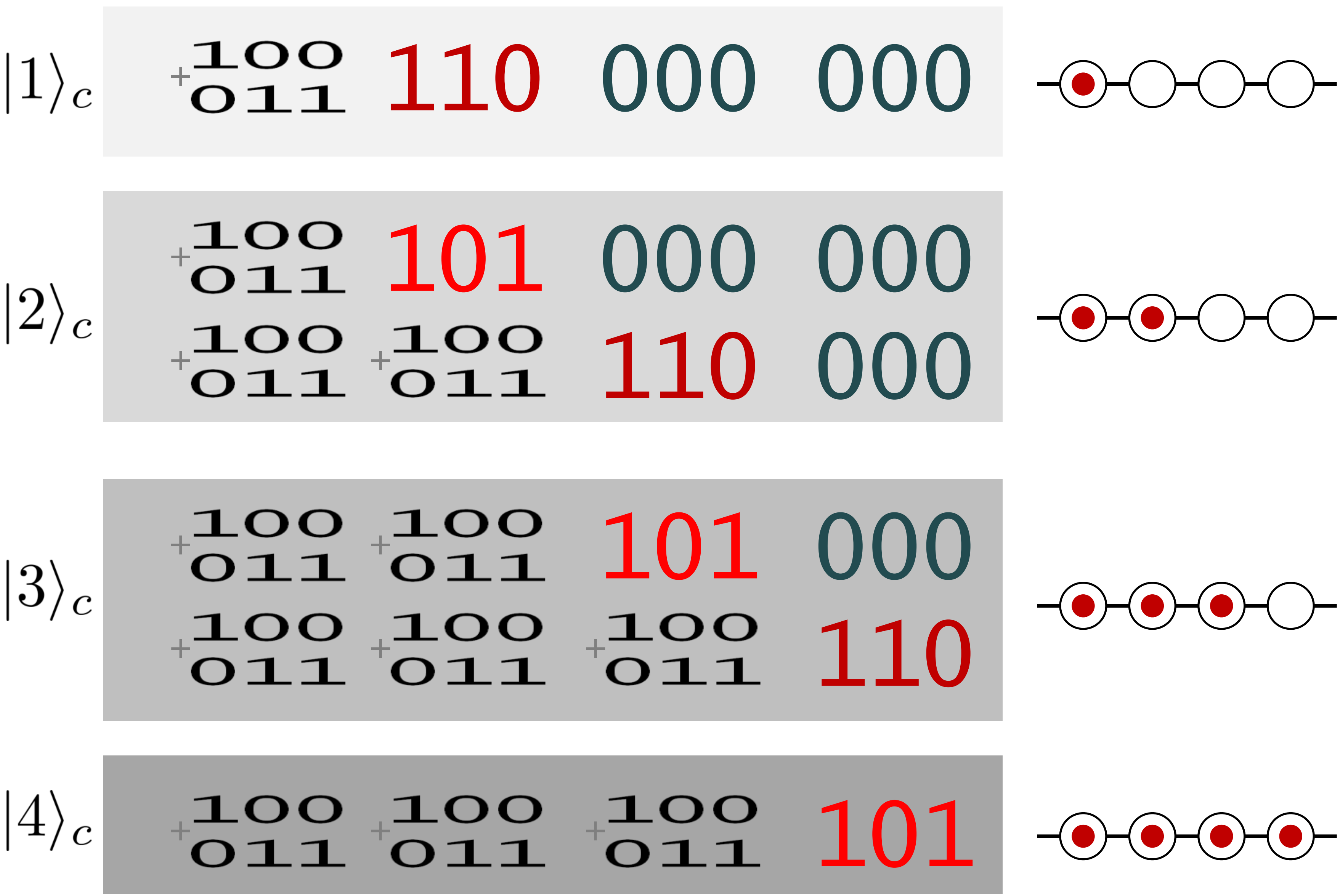
3 Look at the differences between lines



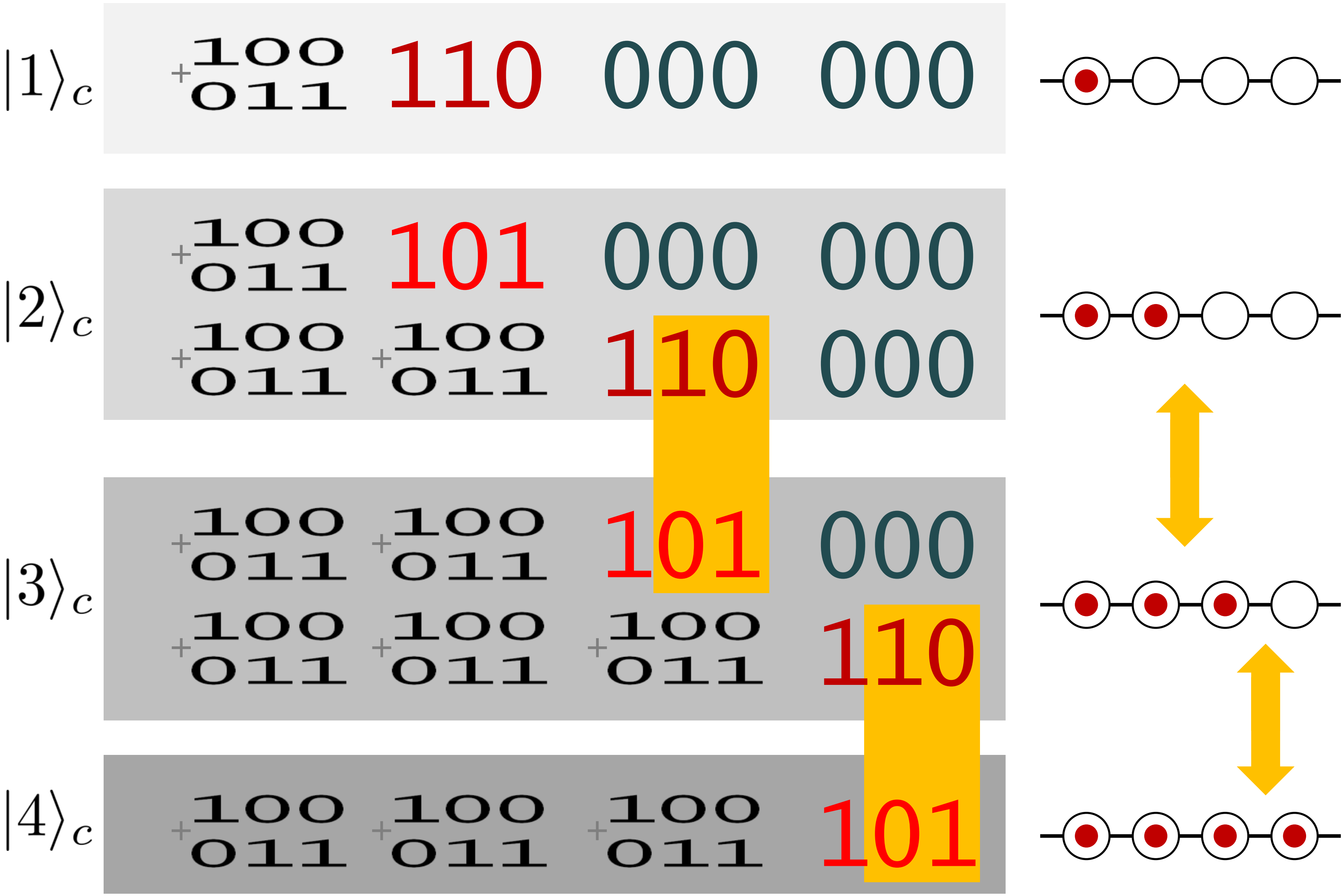
3 Look at the differences, combine lines

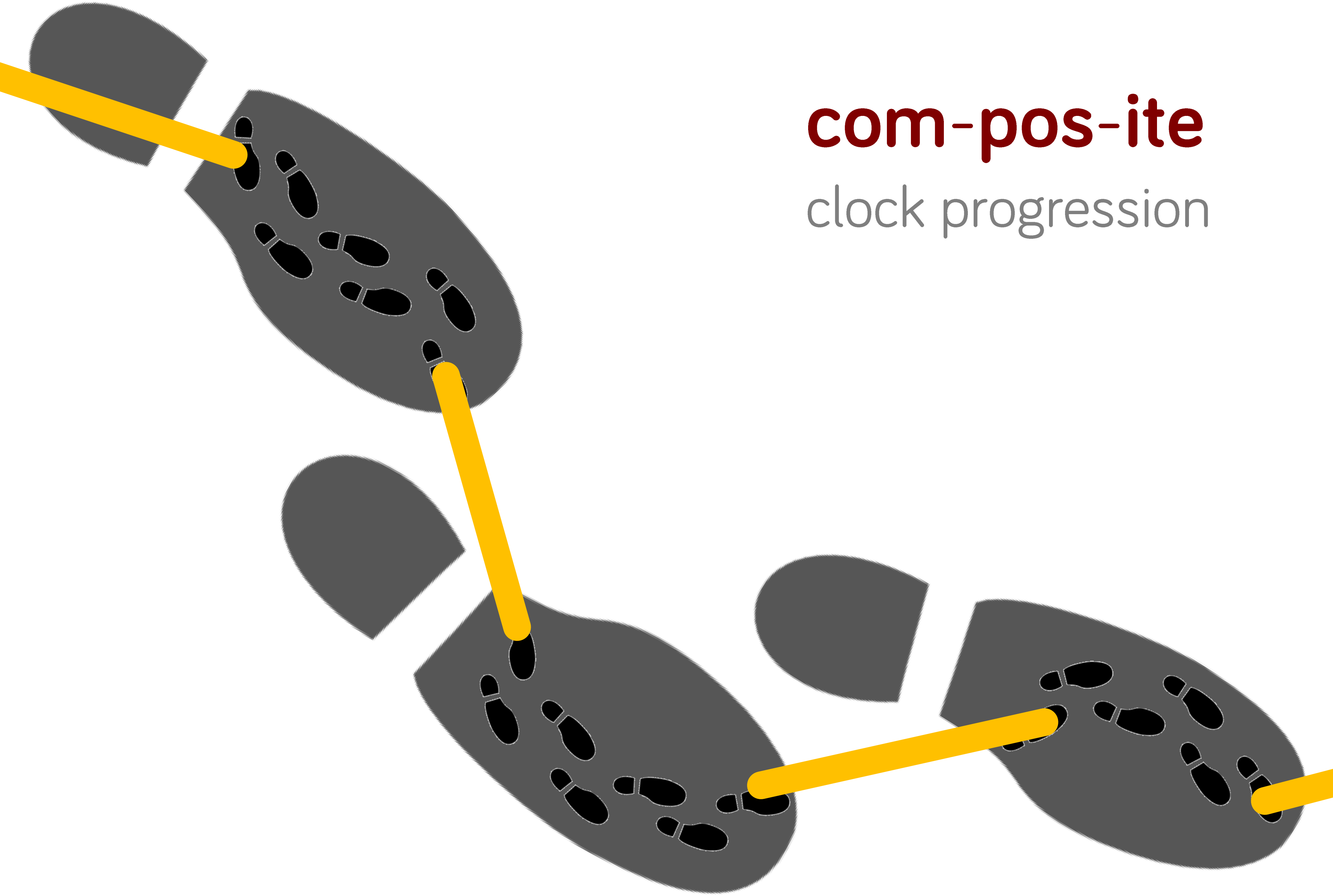
a projector on $|100-011\rangle$ ensures they appear together

3 A clock made from superpositions



3 A clock with **2-local progress**



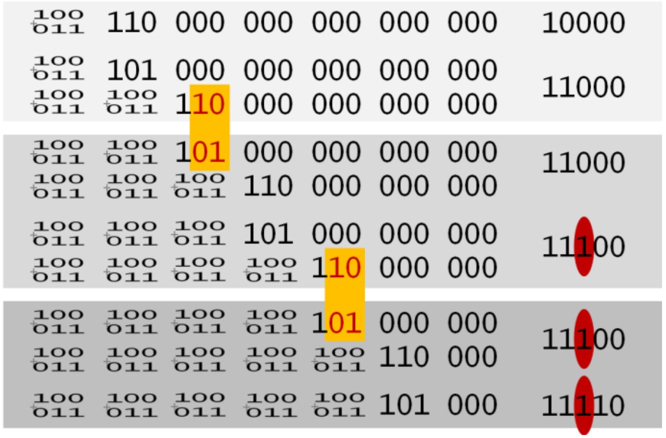


com-pos-ite

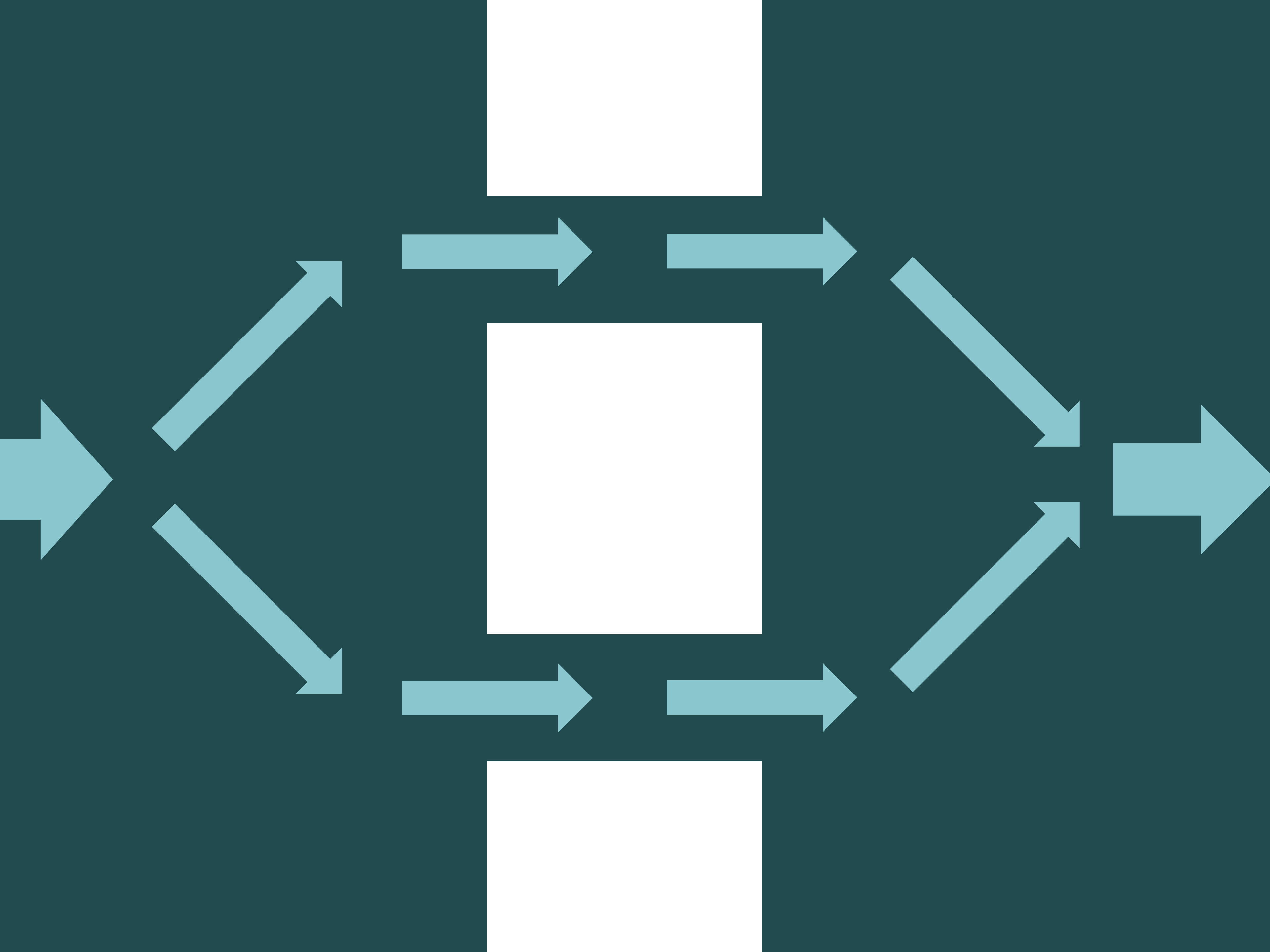
clock progression

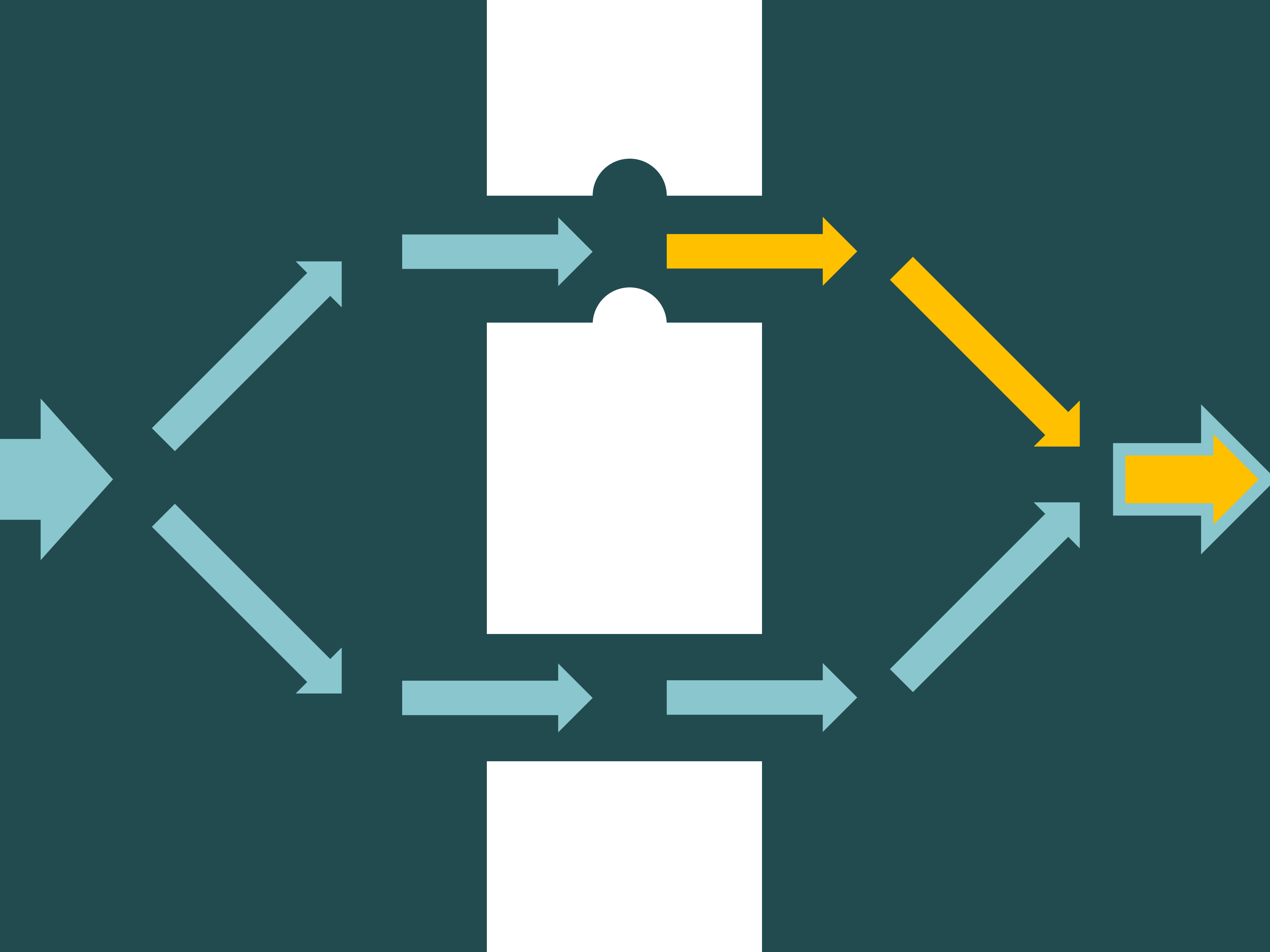
3 Constructing clocks

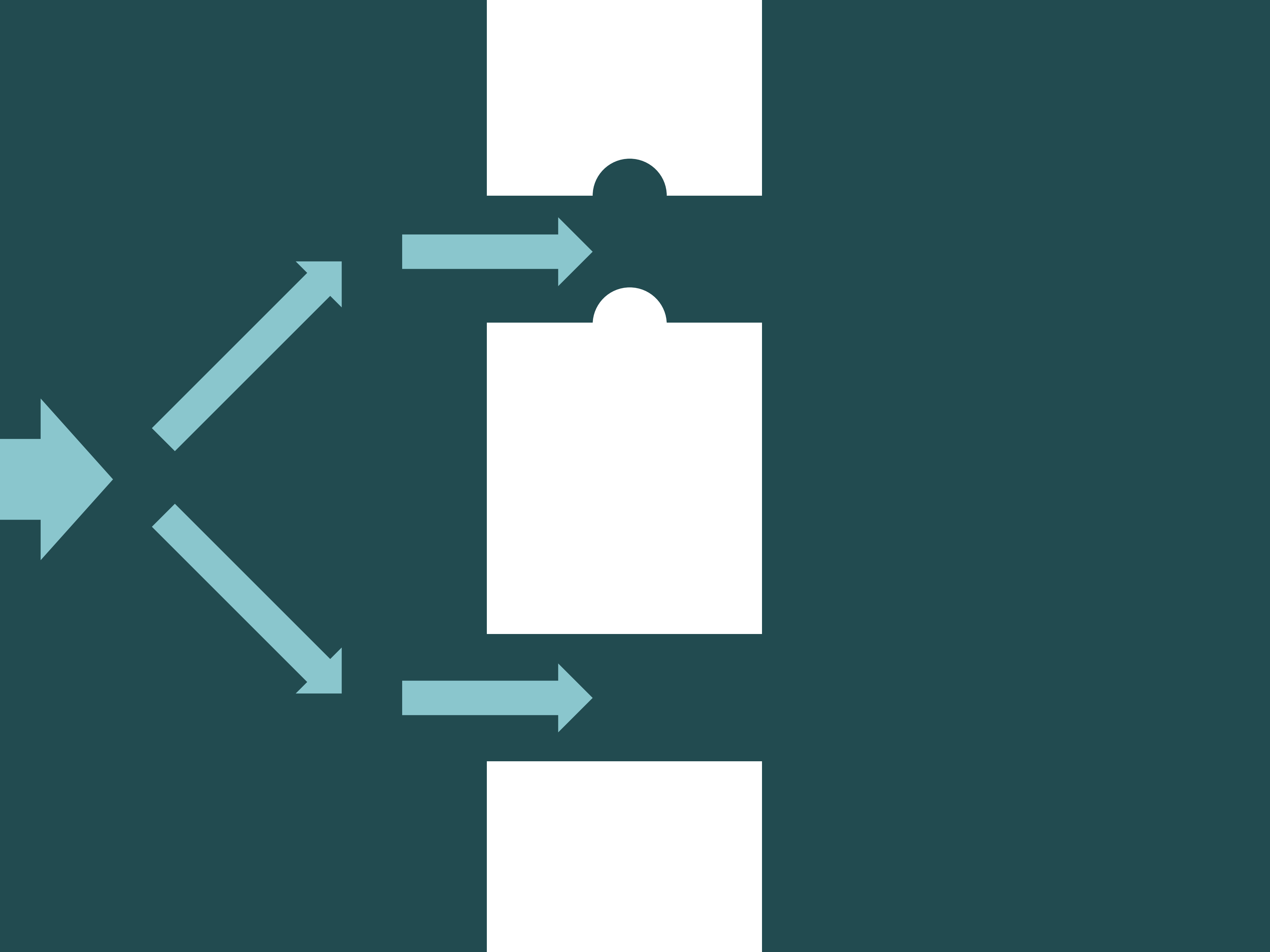
- advance the clock — *2-locally*
- find out I'm late — *1-locally*



- apply a CNOT — *3-locally?*

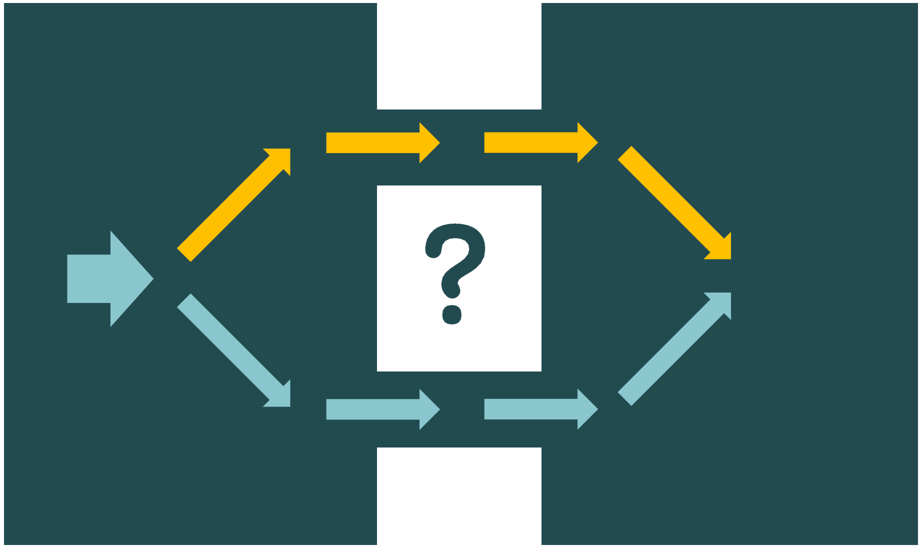




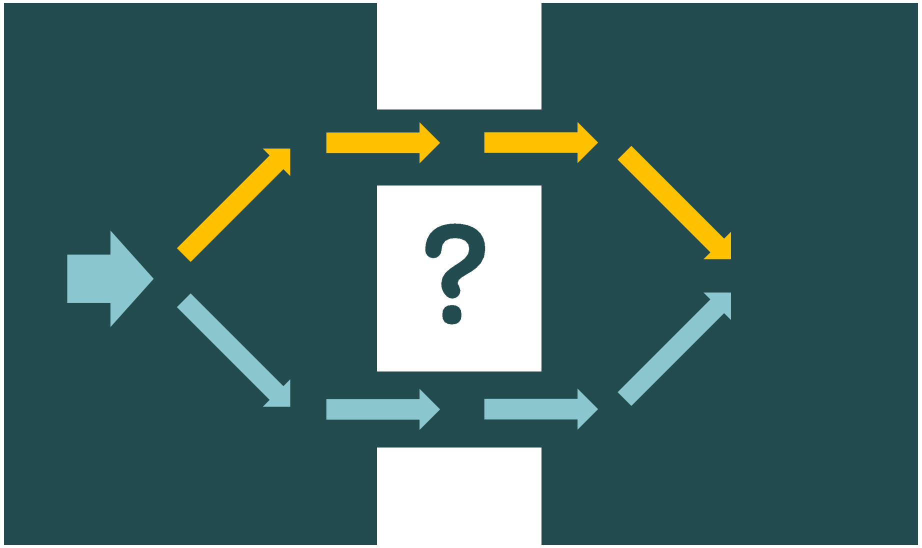


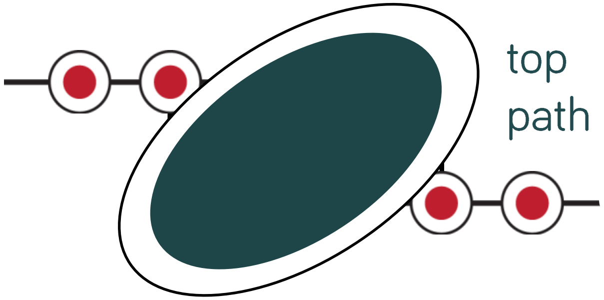
4 A botched double-slit experiment

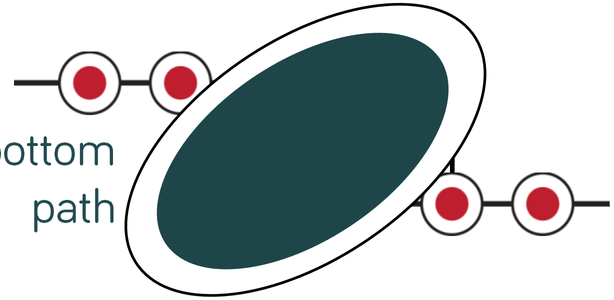
4 Can we make it work?



4 Can we make it work?

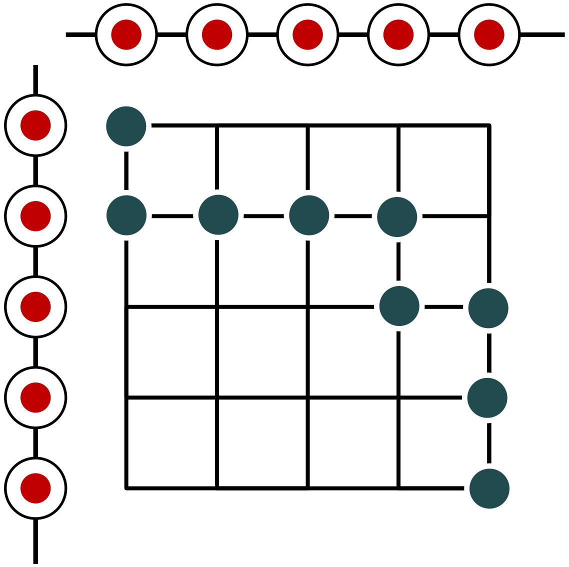






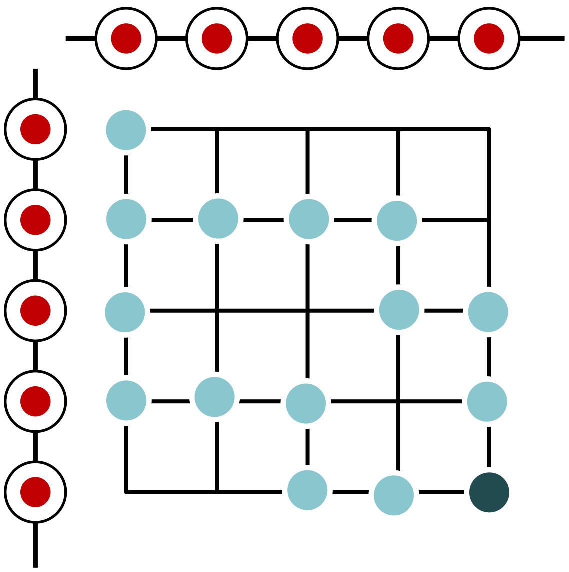
4 2D time

- two clocks



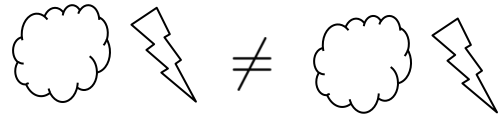
4 2D time

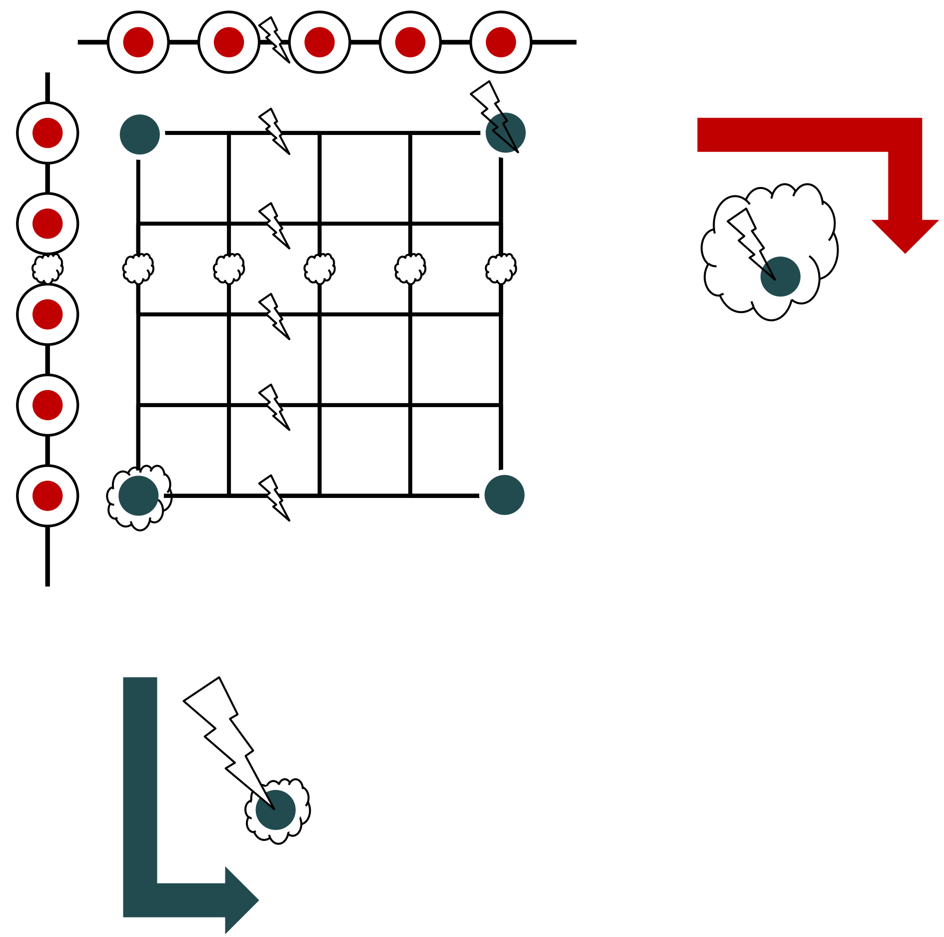
- two clocks



4 2D time

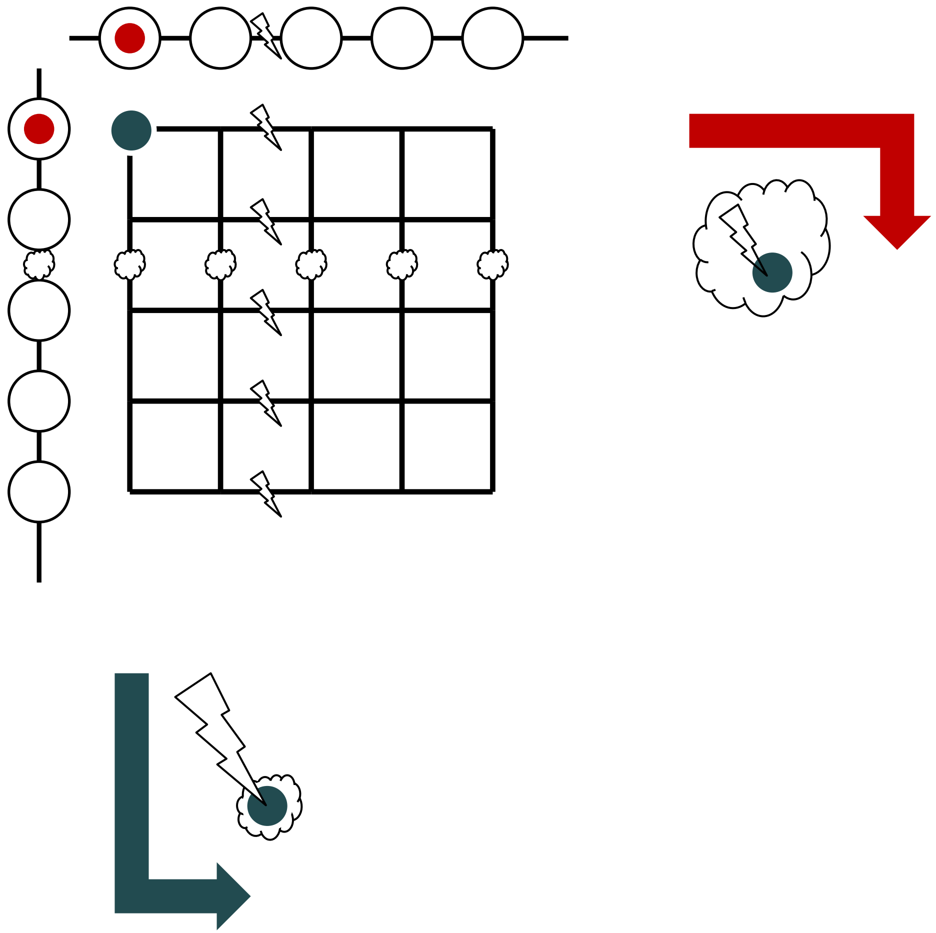
- non-commuting (data) operations





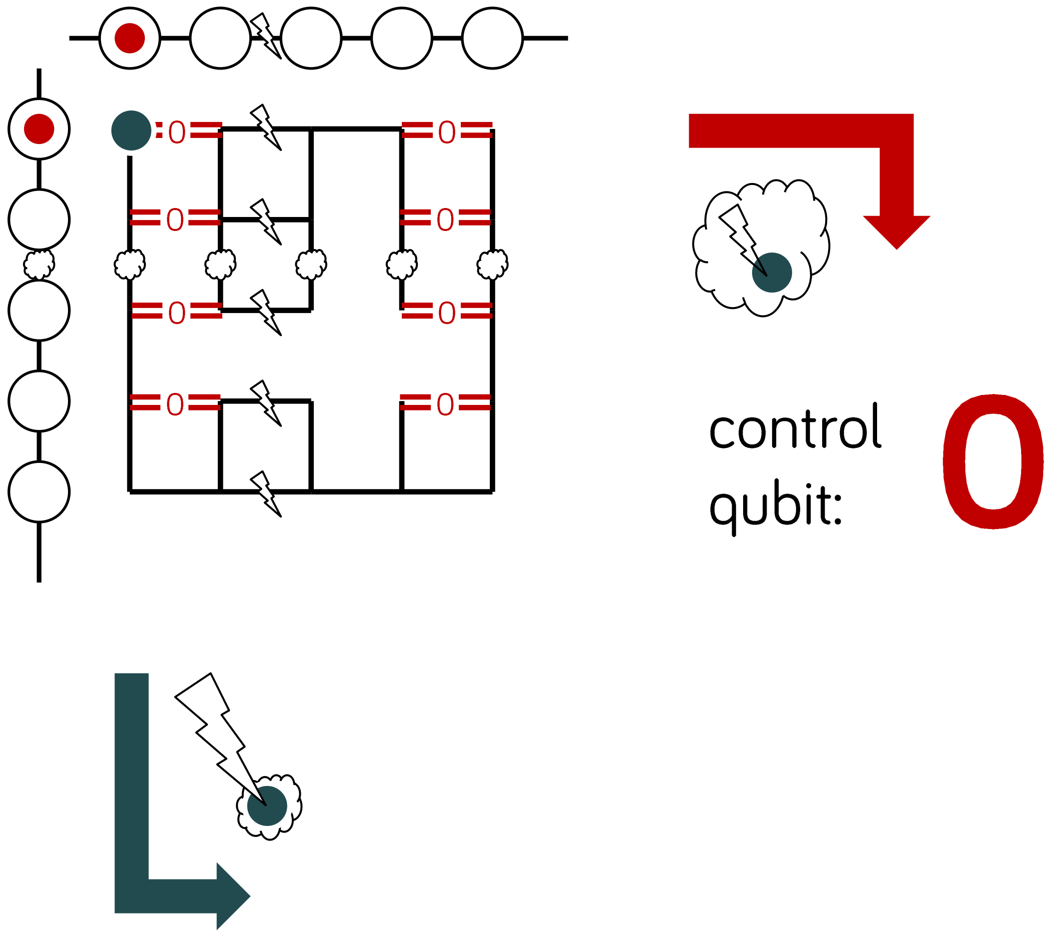
4 2D time + data

- remove/control transitions



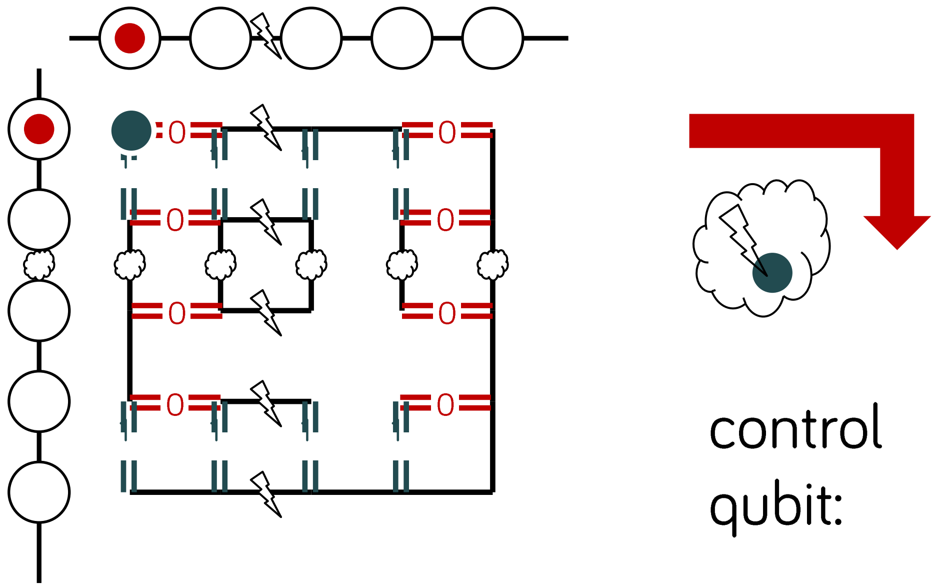
4 2D time + data

- remove/control transitions



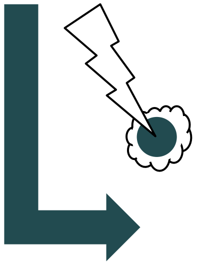
4 2D time + data

- remove/control transitions



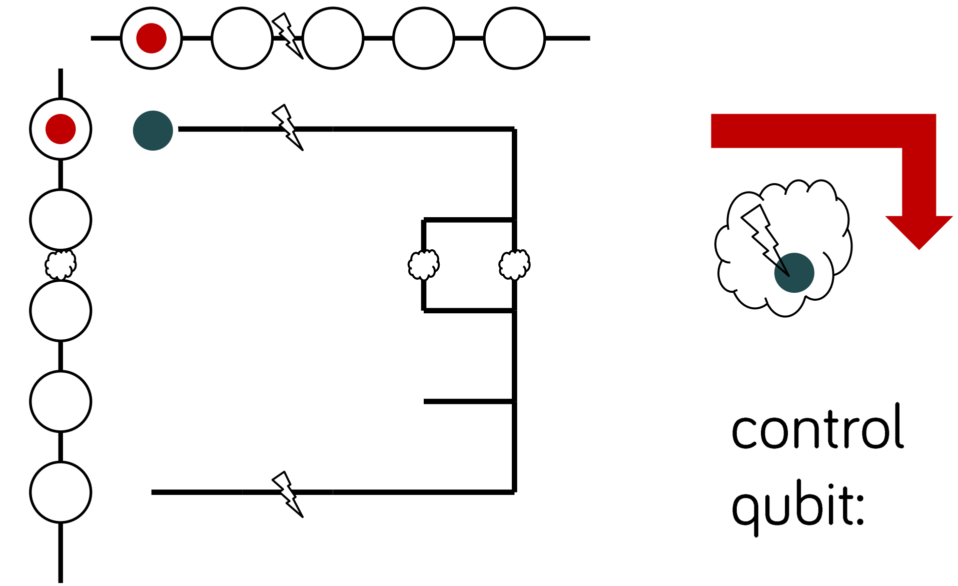
control qubit: 0

control qubit: 1



4 2D time + data

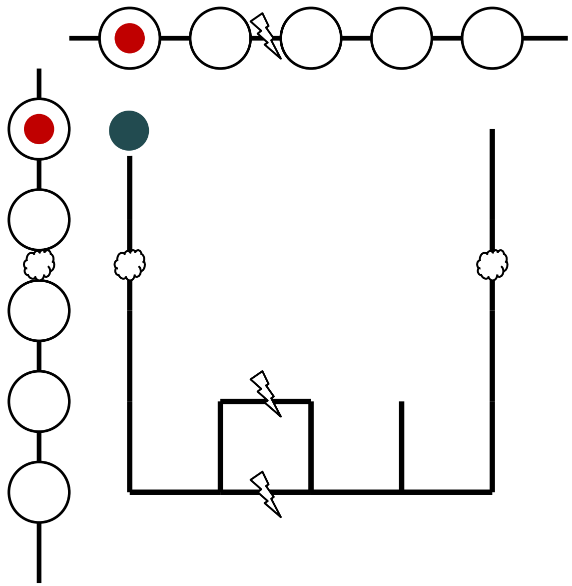
- remove/control transitions



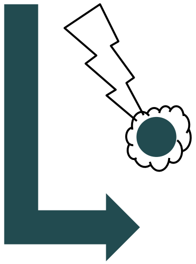
control qubit: **0**

4 2D time + data

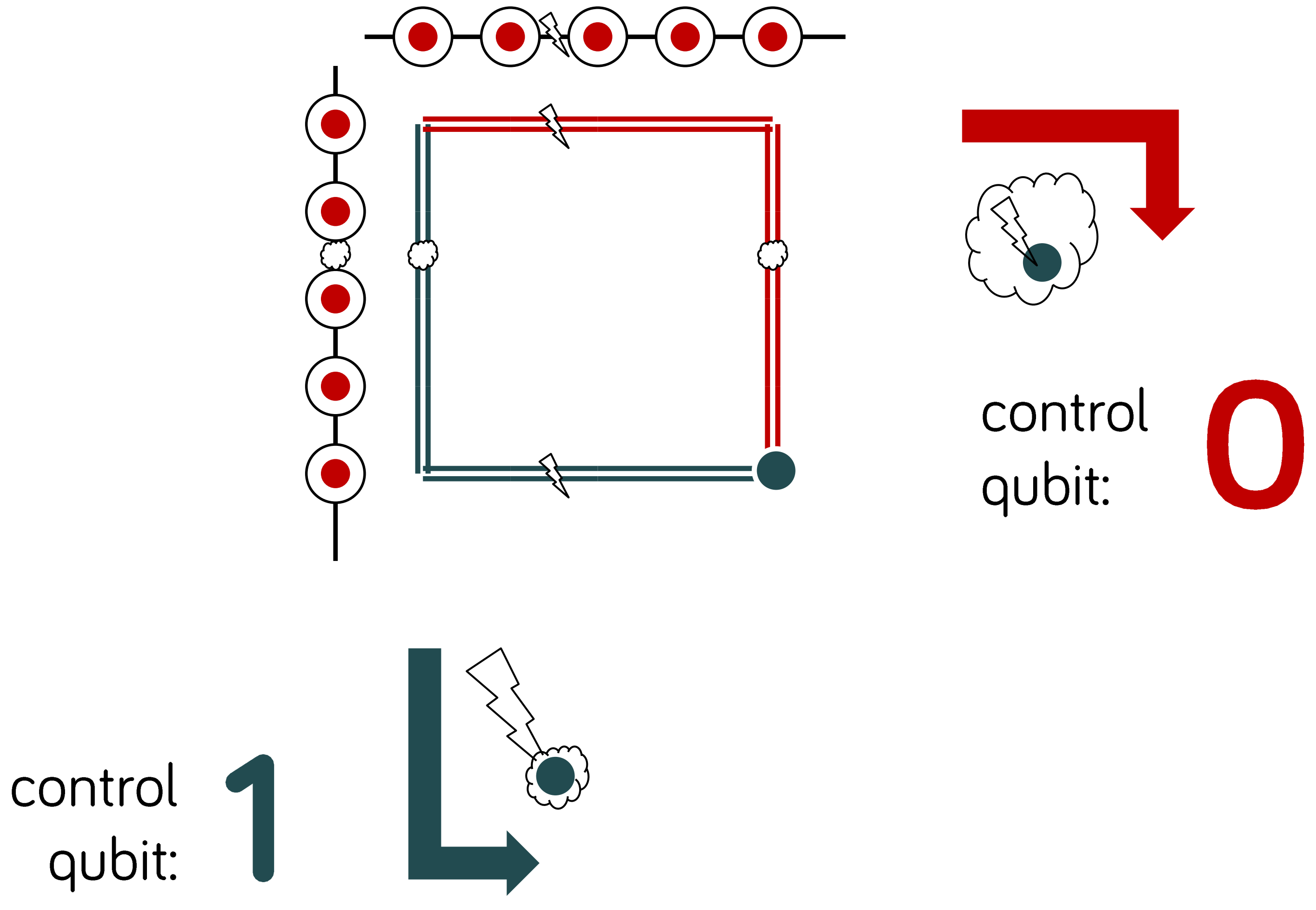
- remove/control transitions

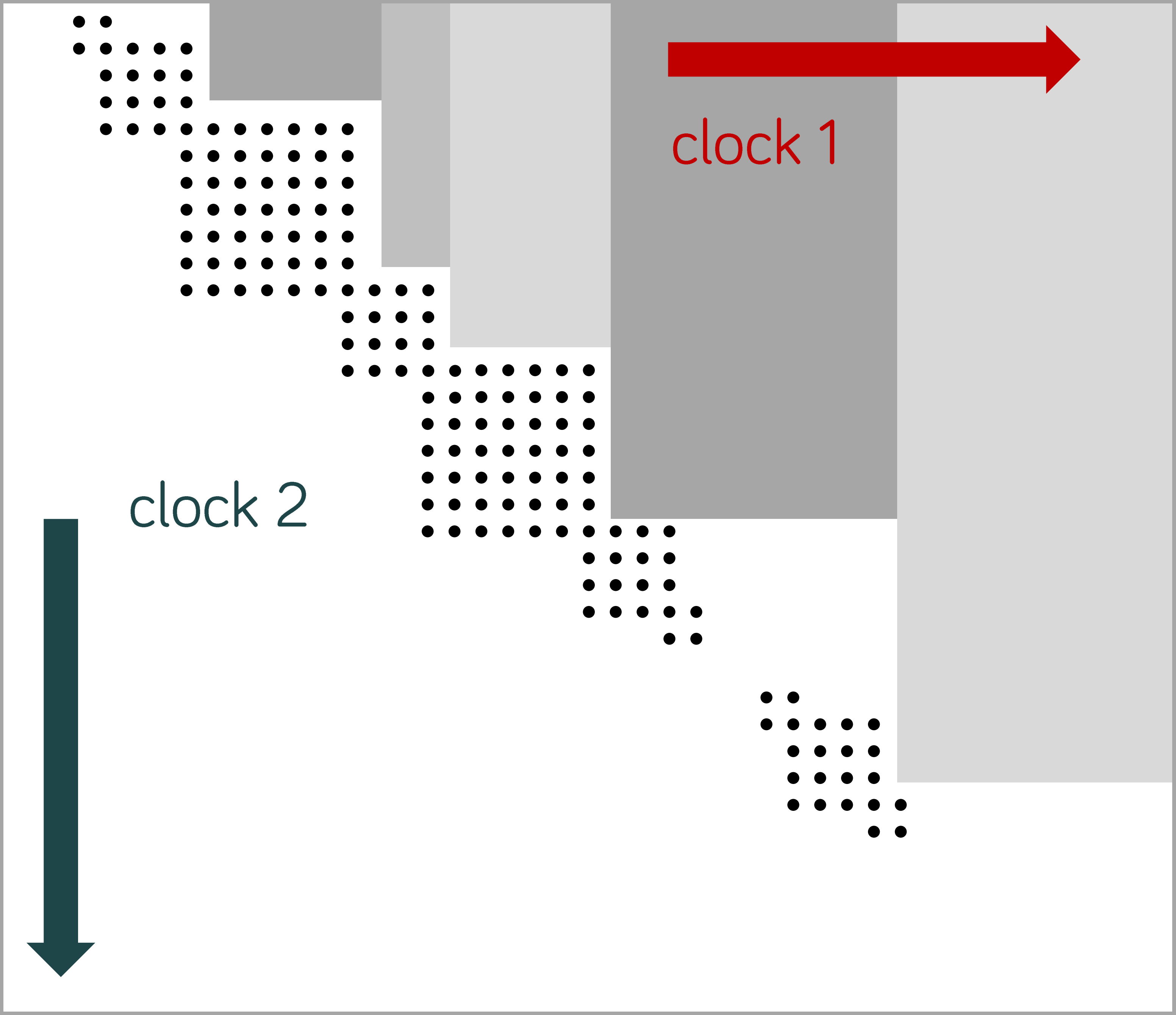


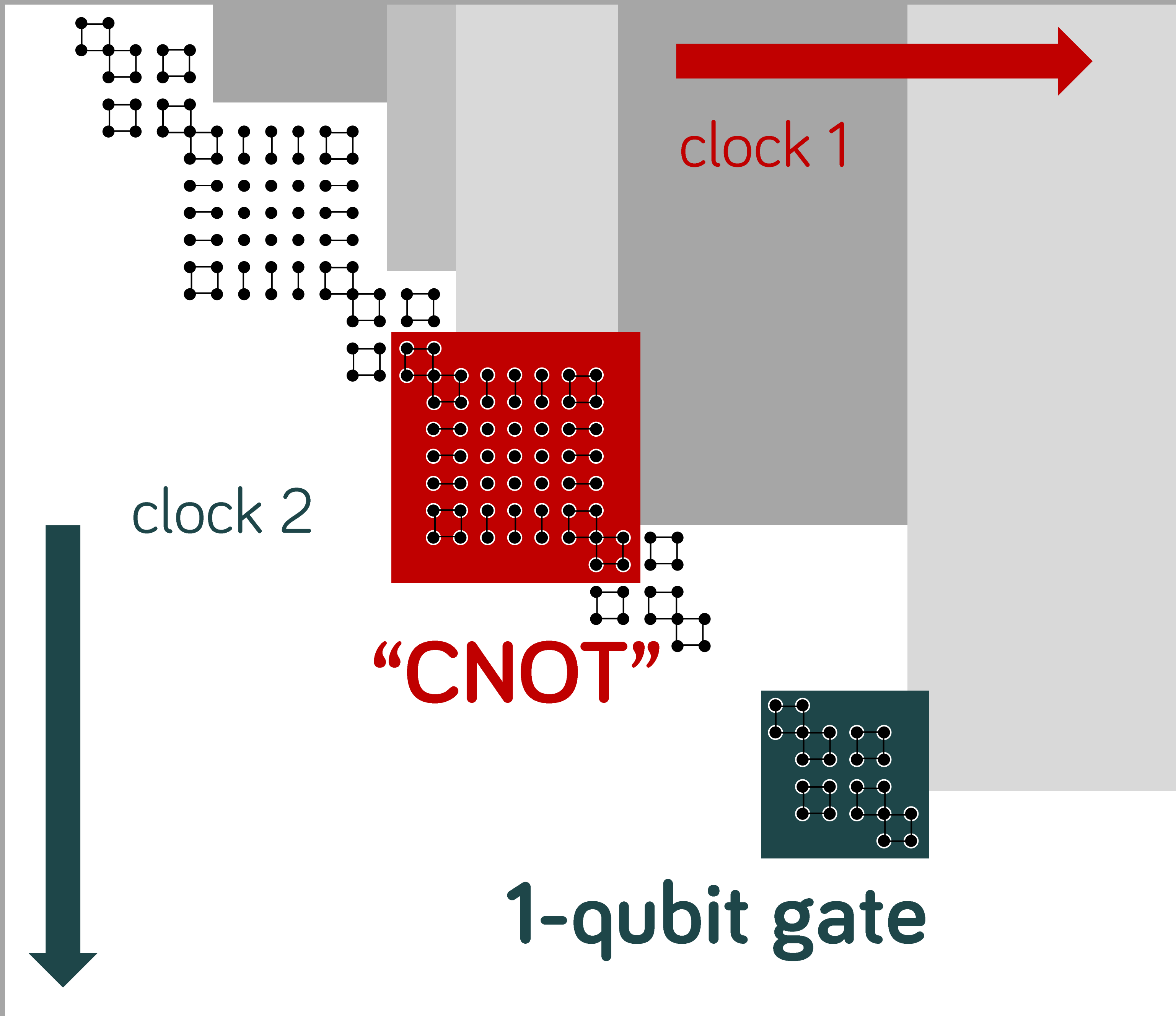
control qubit: 1



4 Check a controlled gate



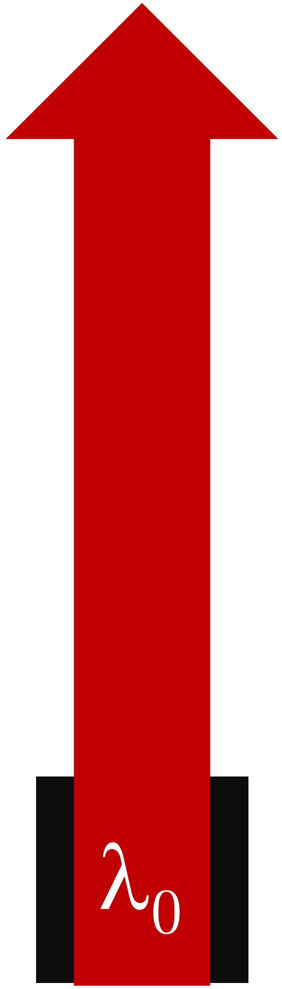

clock 1
clock 2


clock 1
clock 2
“CNOT”
1-qubit gate

soundness

Is the smallest eigenvalue large?

$$H_A + H_B + H_C + H_D + H_E$$



$$H_A + H_B + H_C + H_D + H_E + H_F$$

no solution?
all states have
a high energy

quantum 3-SAT

is QMA_1-complete

[Gosset, N. '13]

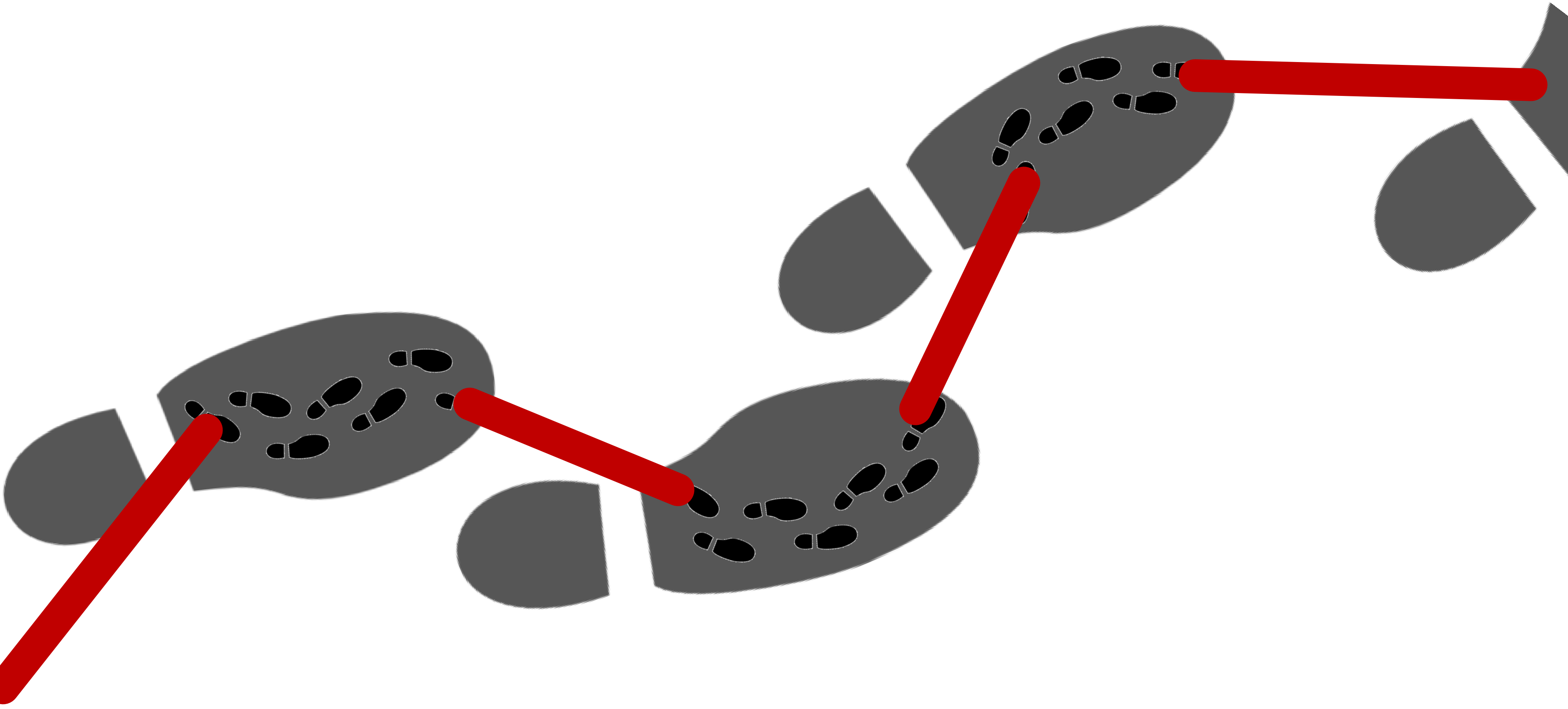
quantum

3-sat

&

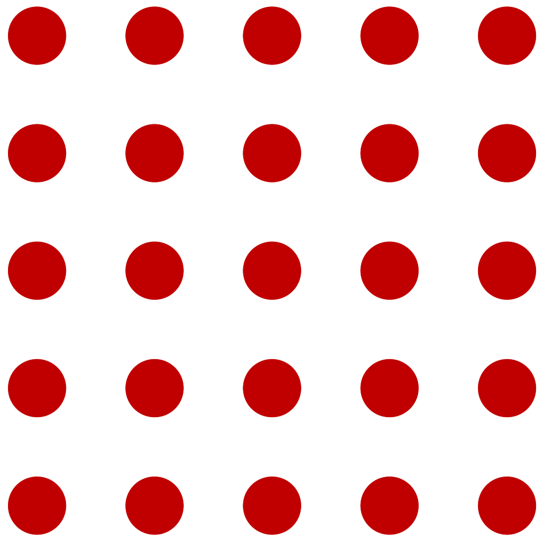
projectors
no frustration
computation

a clock

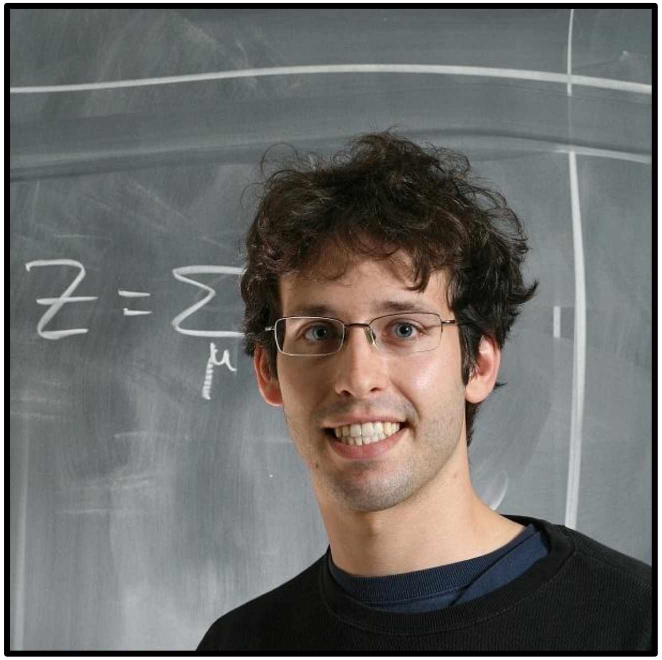


a clock

for quantum computation



3-local composite runs in 2D



Quantum 3-SAT

is QMA_1-complete

David Gosset UNIVERSITY OF WATERLOO

Daniel Nagaj universität wien

FOCS '13