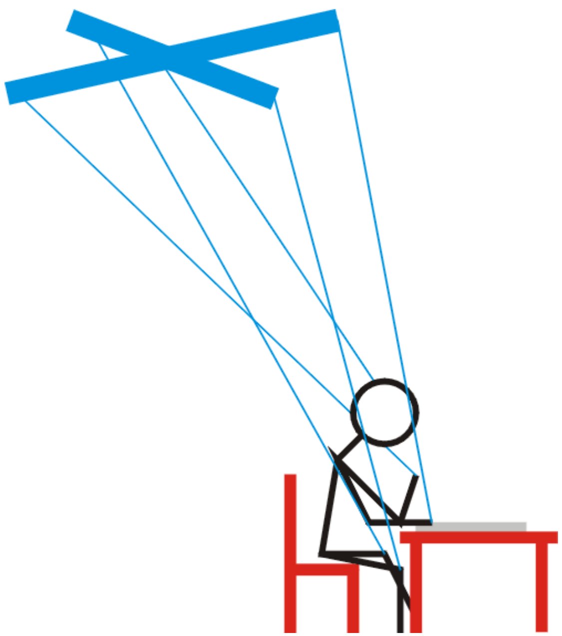


An introduction to

Local Hamiltonians & Quantum Complexity

D. Gosset, S. Hallgren, S. Narayanaswami,
S. Mozes, P. Wocjan, O. Regev, P. Love, P. Shor, S. Jordan,
S. Lloyd, A. Landahl, S. Irani, D. Gottesman, S. Bravyi,
E. Farhi, J. Goldstone, D. Aharonov, T. Morimae,
R. Movassagh, F. Brandao, J. Eisert, J. Whitfield, ...

Daniel Nagaj

universität wien


Pico Posets | 3371m | 7/2013

Local

Hamiltonian

[wisegeek.com]

TODAY QMA

- 0 why Hamiltonians
- 1 a litte complexity
- 2 QMA, ground states and local Hamiltonians
- 3 what can we do in 1D

TODAY QMA

TOMORROW QMA_1

0 why Hamiltonians

1 a little complexity

2 QMA, ground states and local Hamiltonians

3 what can we do in 1D

4 no frustration

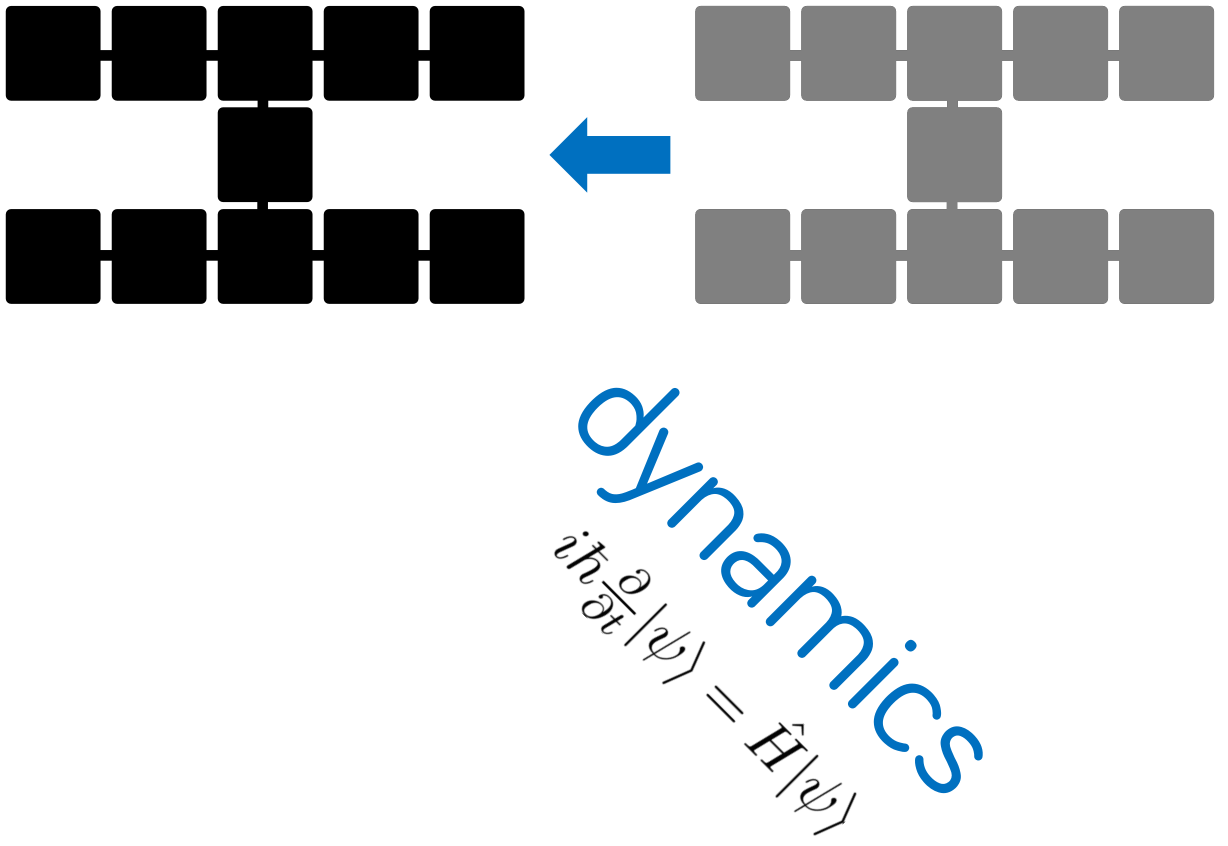
5 perfect verifiers

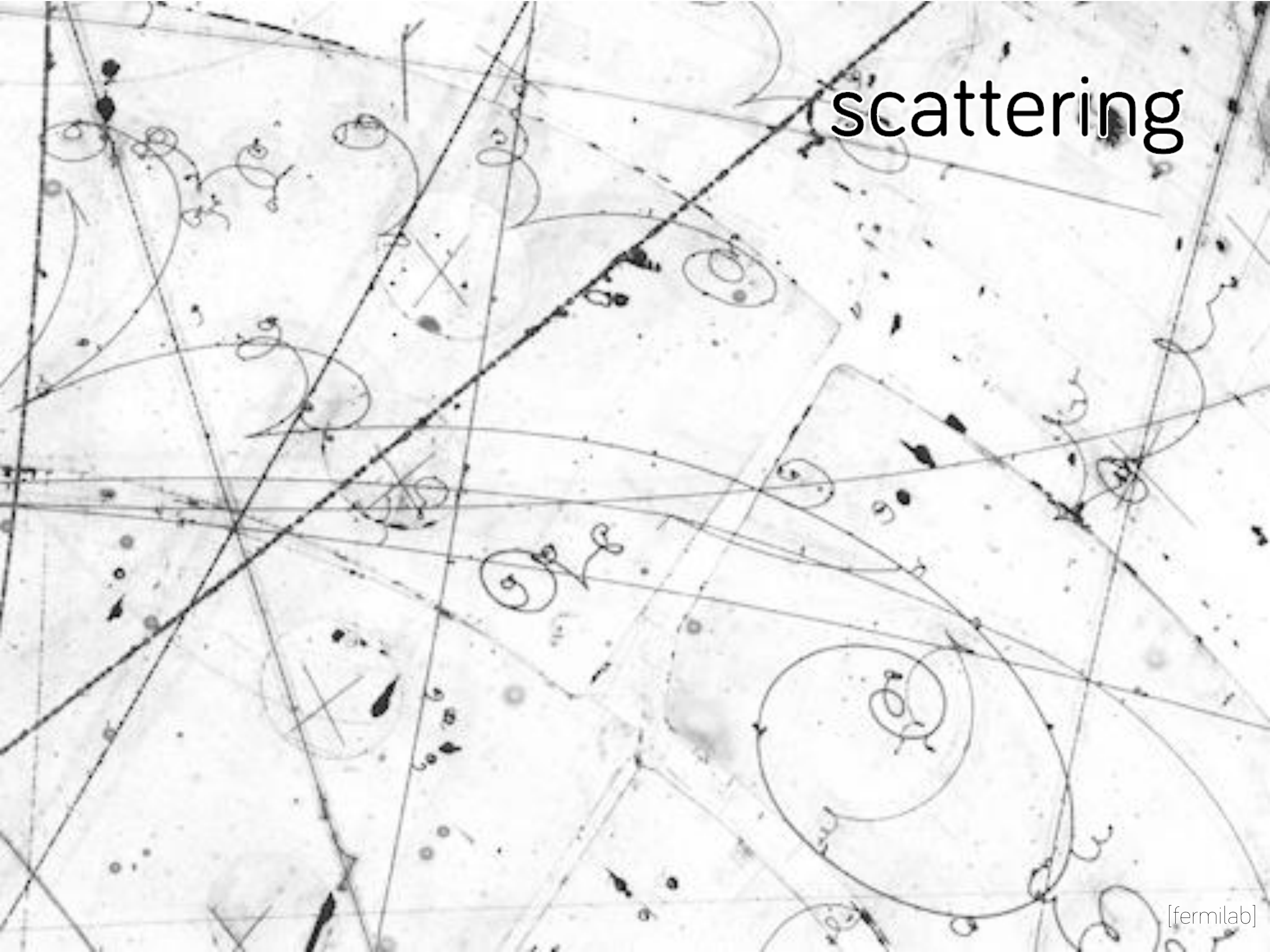
6 quantum 3-SAT

7 random quantum SAT

8 quantum PCP

Hamiltonians


dynamics
$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$

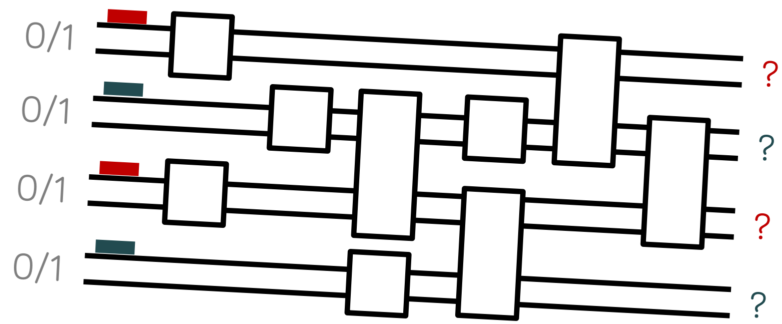

scattering
[fermilab]

scattering

Universal computation by multi-particle quantum walk

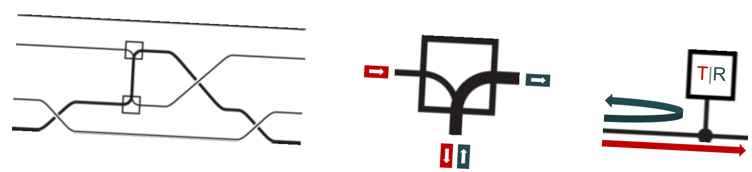
- dual-rail encoding
  N wavepackets

$$a_j^\dagger a_k + a_k^\dagger a_j$$

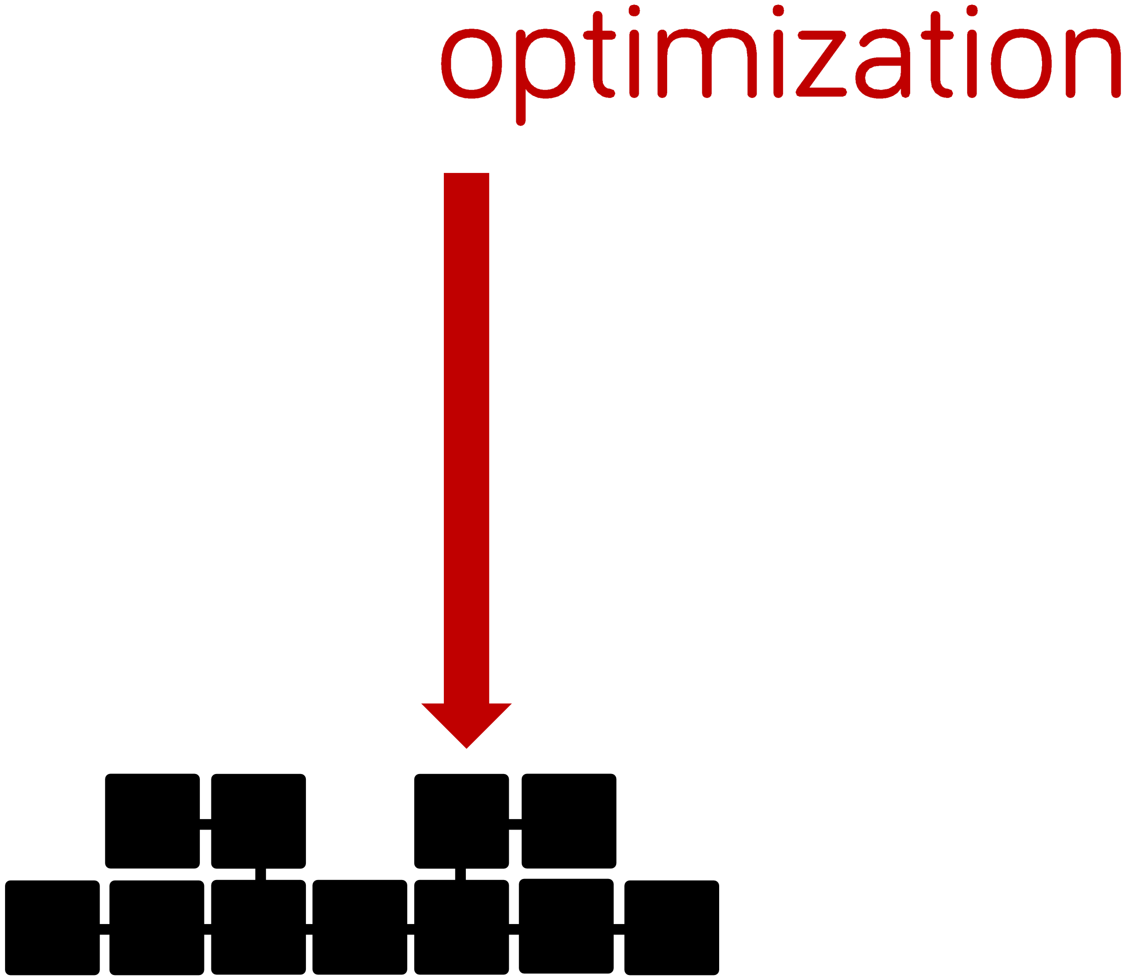


- CPHASE: interaction

$$a_j^\dagger a_k^\dagger a_j a_k$$

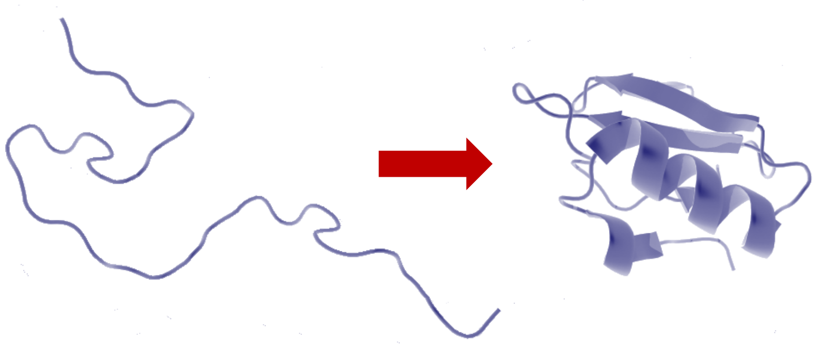


[Childs, Gosset, Webb, Science 339, 791 (2013)]

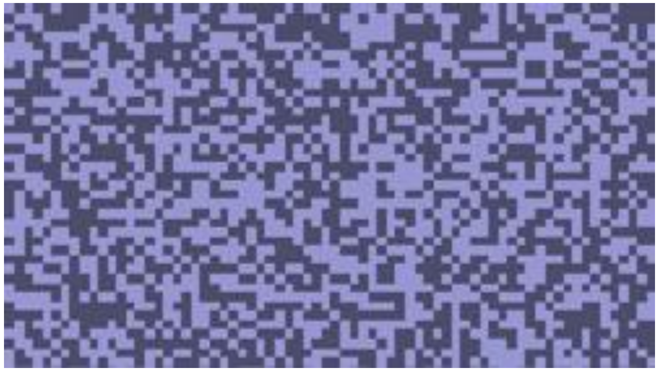

optimization

protein folding

spin glasses



[wikipedia]



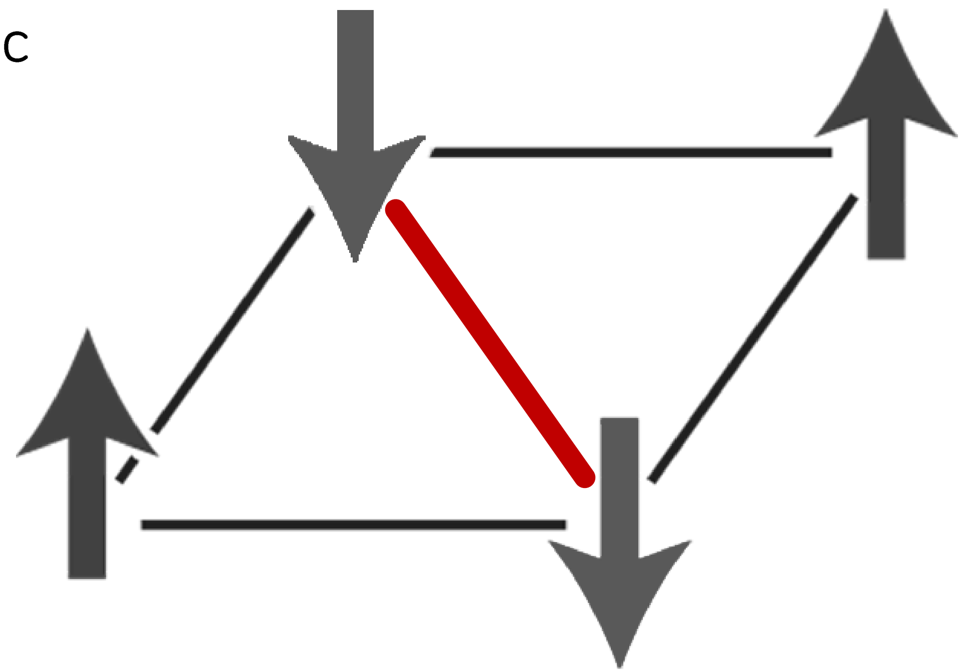
[uni-koeln.de]

local Hamiltonians

0 Frustrated systems

You can't make everybody happy.

antiferromagnetic
spin glass



a global
ground state

HARD?

find & describe it?
is it entangled?


frustrated
FRUST
RATED

0 Local Hamiltonians

Many local questions.

- What are they like?

ground state (energy)

QMA-complete problems

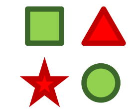
$$H(t) = \sum_j H_j(t)$$

- What are they good for?

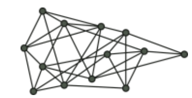
quantum computing, chemistry, control, transport

BQP universality

- local particle dimension

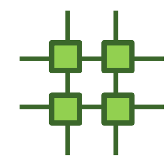


- interaction geometry

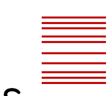


- time independence

- translational invariance



- promise and eigenvalue gaps

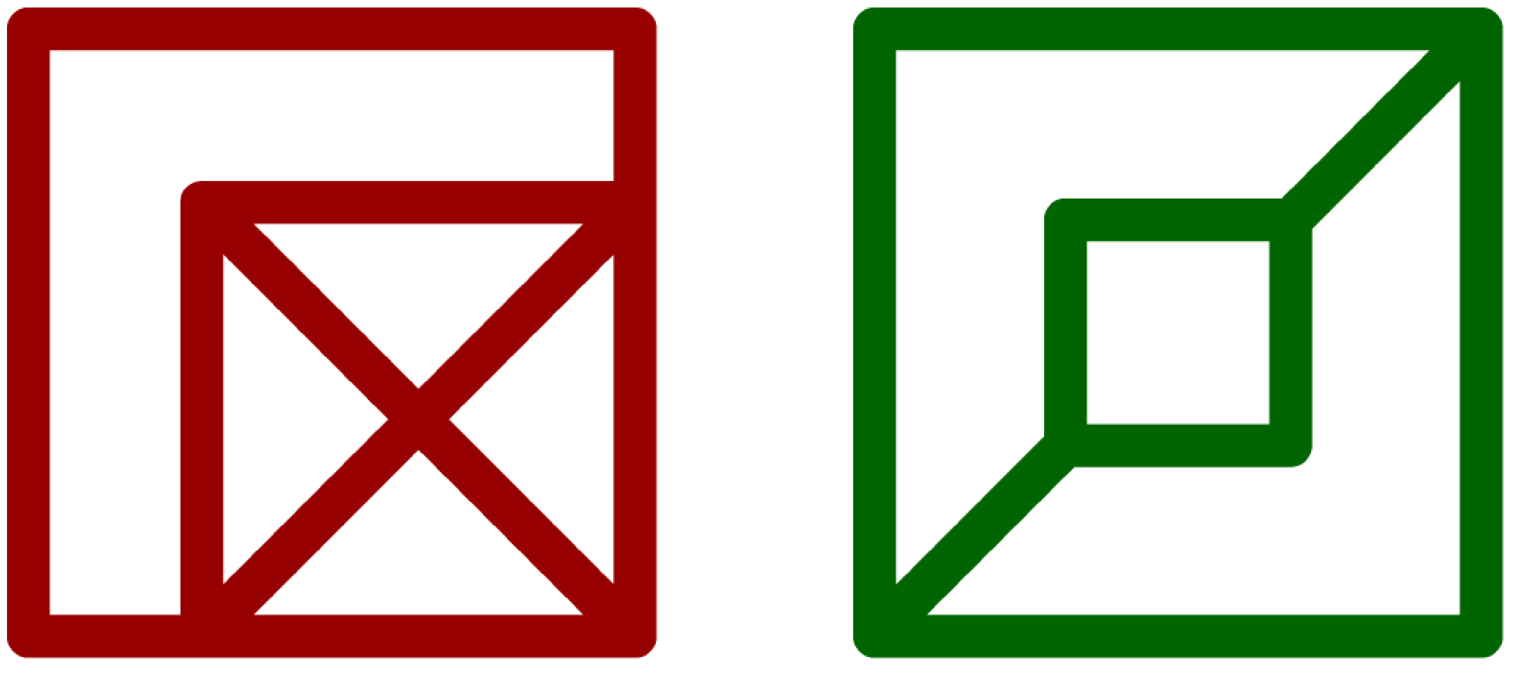


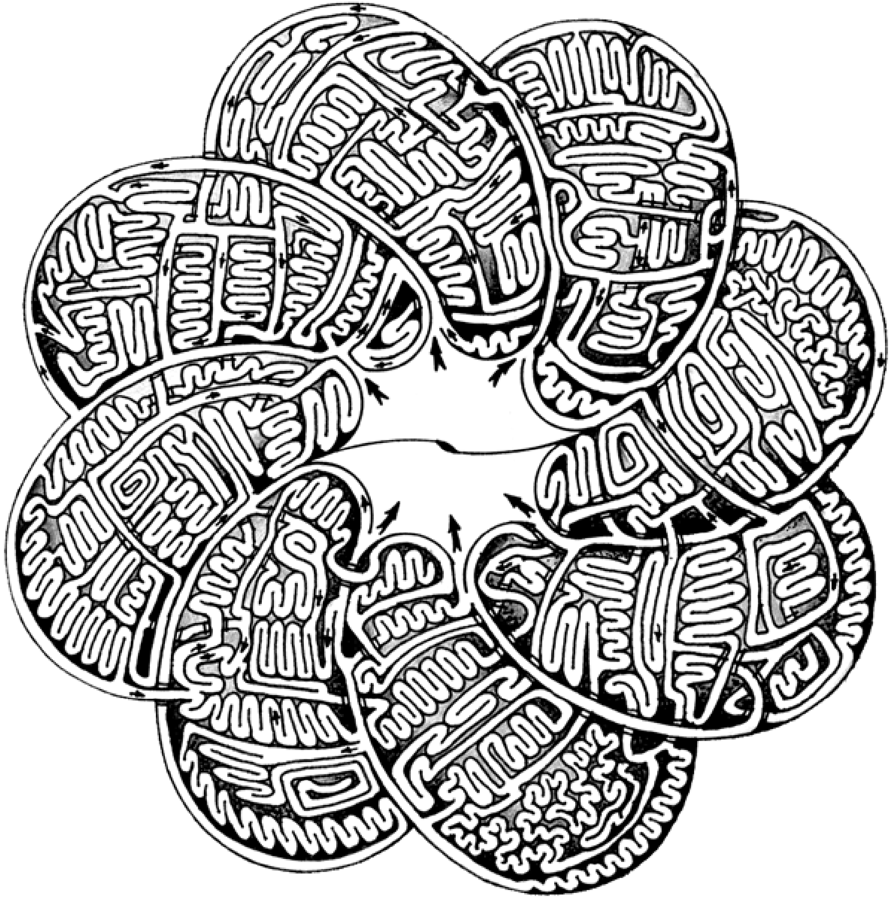
- energy × time cost

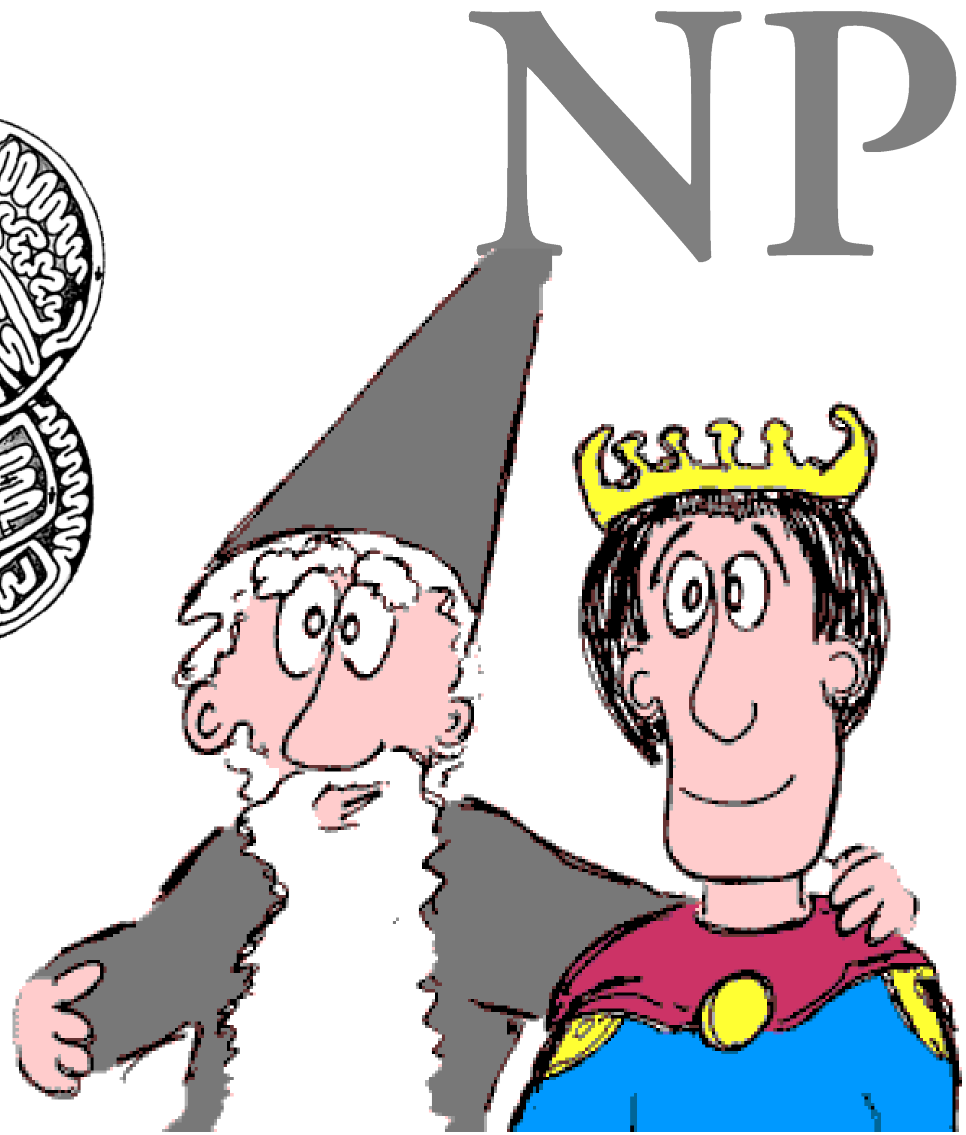
how hard
is this
question

[J. Howard Miller]

P

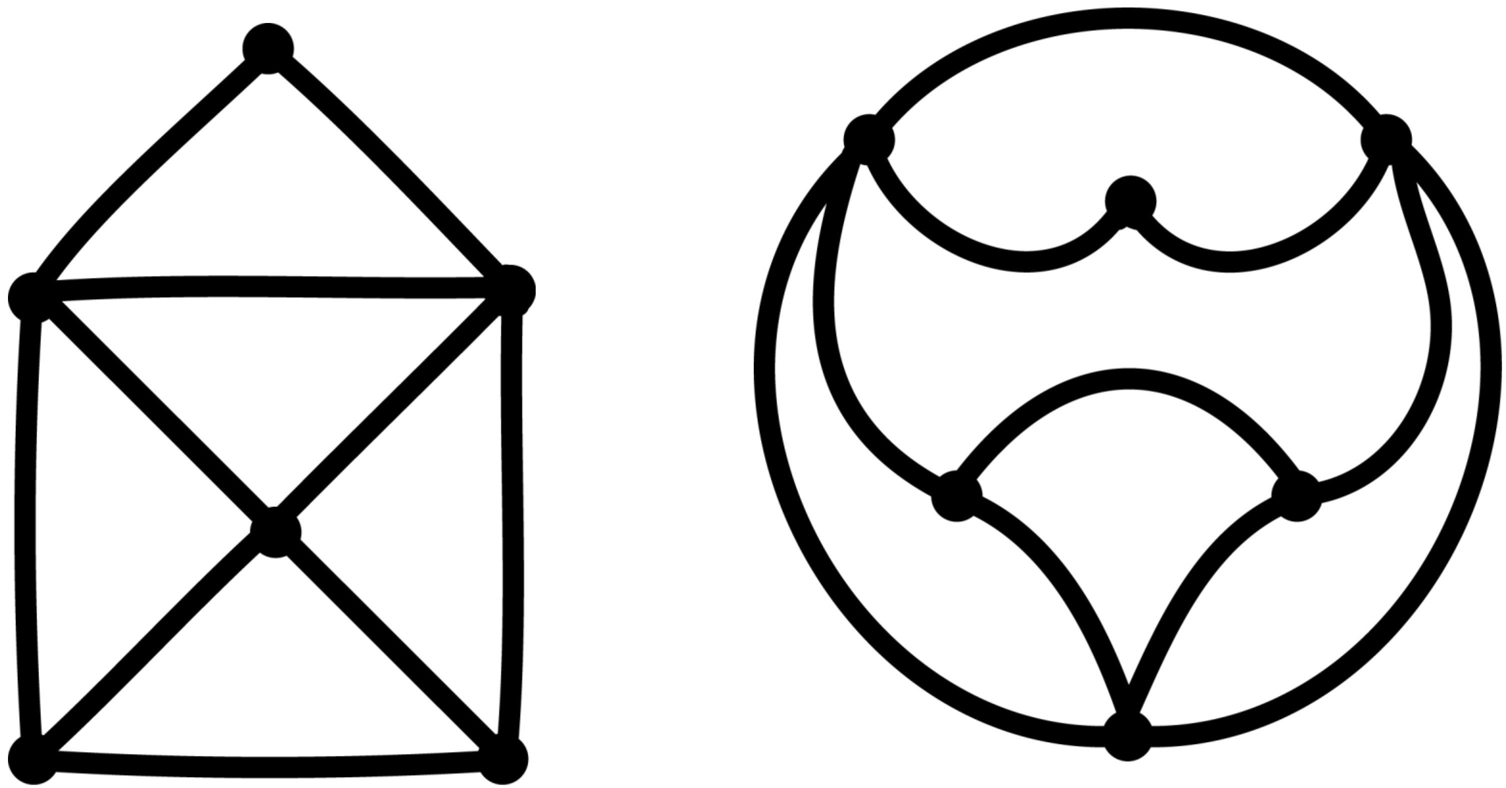




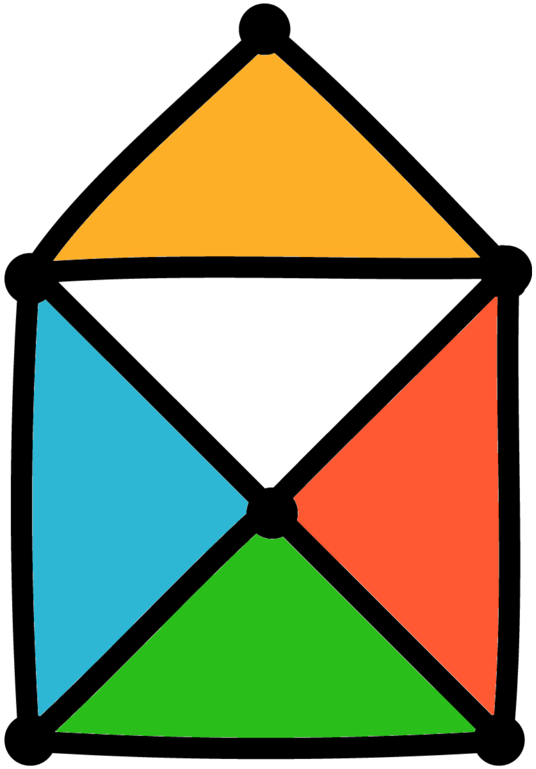


[maze: Andrew Bernhardt]
[A+M: primaryresources.co.uk]

1 A graph isomorphism puzzle



1 A graph isomorphism puzzle



1 A cryptarithmetic puzzle

```
   DID
+DINOS
------
 CROAK
```

?

[www.iread.it/cryptarithms.php]

1 A cryptarithmetic puzzle

$$\begin{array}{r} 595 \\ +\ 59842 \\ \hline 60437 \end{array}$$

!

1 The NP protocol

Did dinosaurs exist?

?

a proof

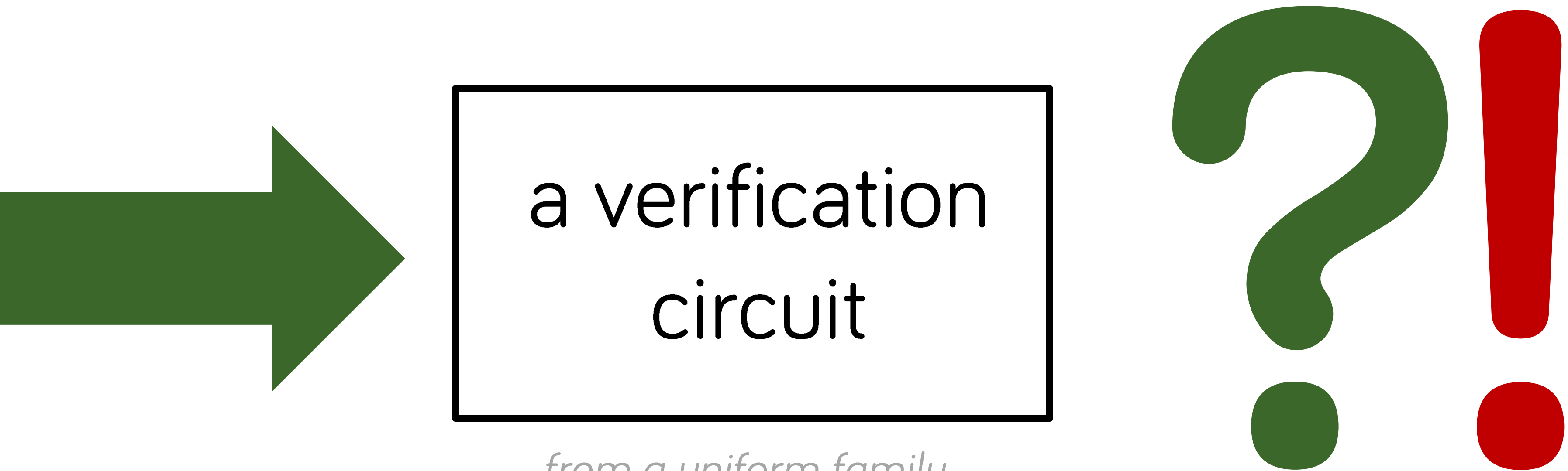
1 The NP protocol

Did dinosaurs exist?

a witness

1 The class NP

Yes/no questions, easy to verify solutions.



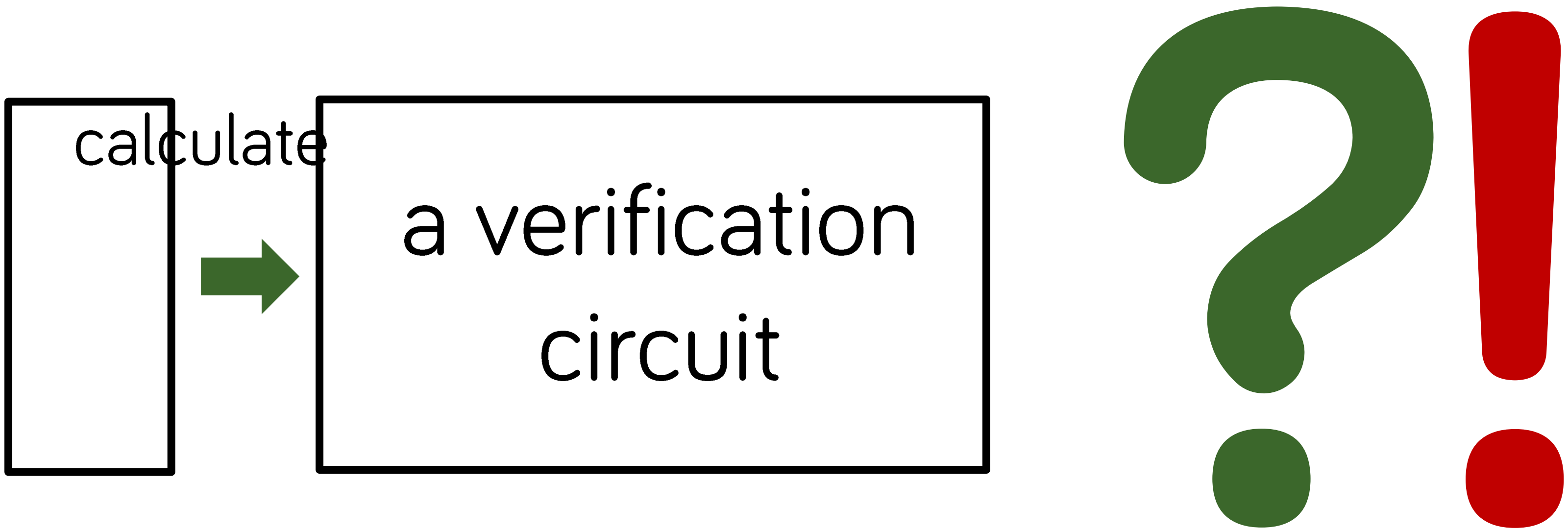
from a uniform family

YES? Accept a good proof.

NO? Reject any witness.

1 The class P

Yes/no questions that we can answer.



YES? Figure it out by yourself.

NO? Figure it out by yourself.

1 The class NP

Yes/no questions, easy to verify solutions.

a verification circuit

from a uniform family

?!

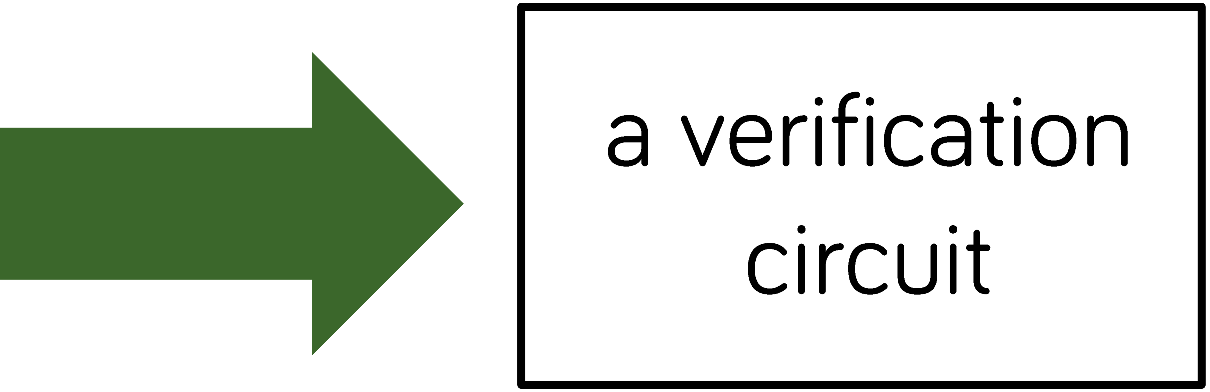
YES? Accept a good proof.

NO? Reject any witness.

1 An NP-hard problem

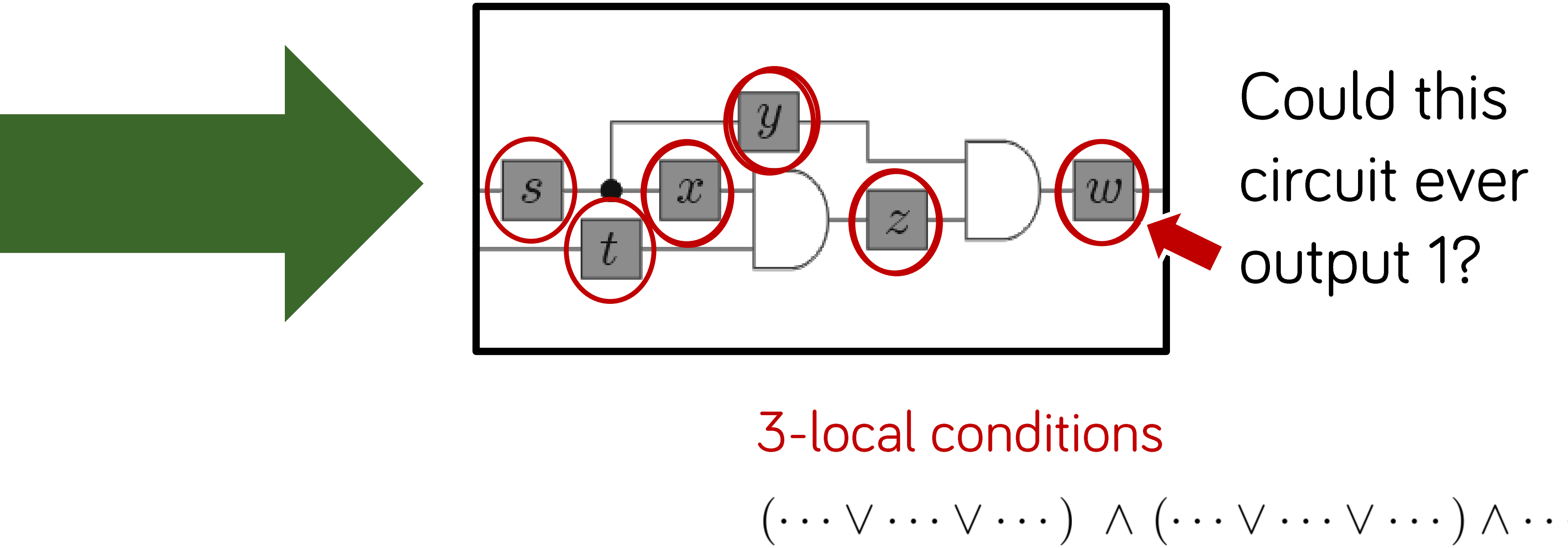
The mother of them all.

- An NP-hard problem solver solves anything in NP.



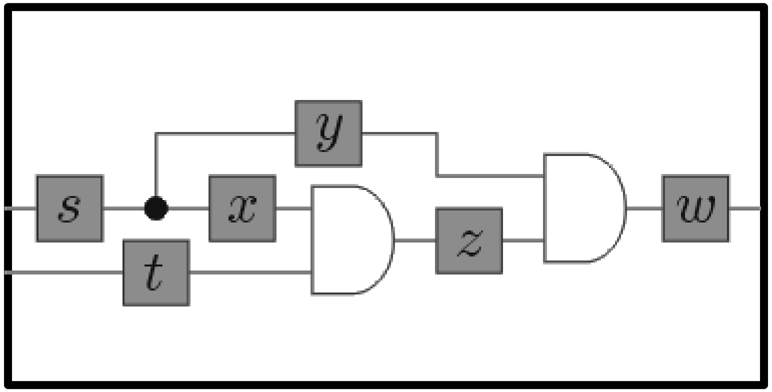
1 An NP-hard problem

- An NP-hard problem solver solves anything in NP.



- 3-SAT is NP-hard.
- 3-SAT is in NP.

- An NP-hard problem solver solves anything in NP.

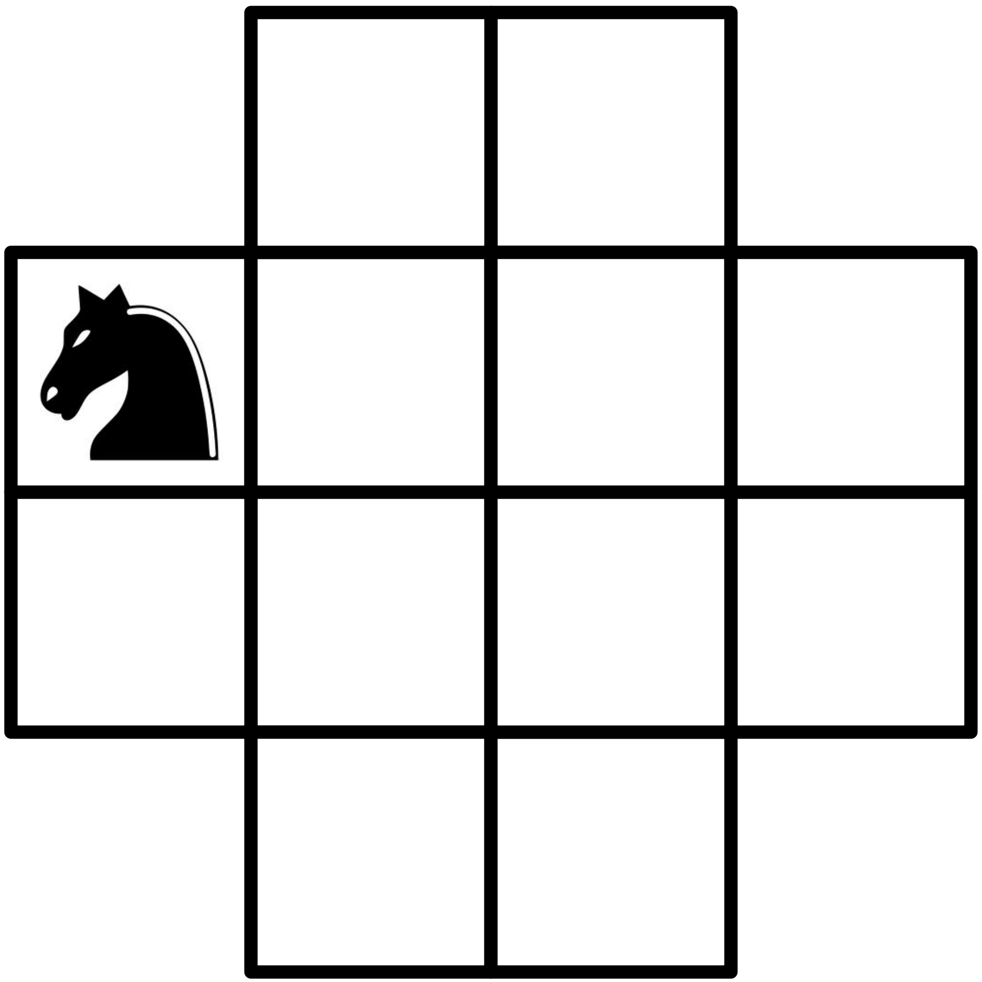


Could this circuit ever output 1?

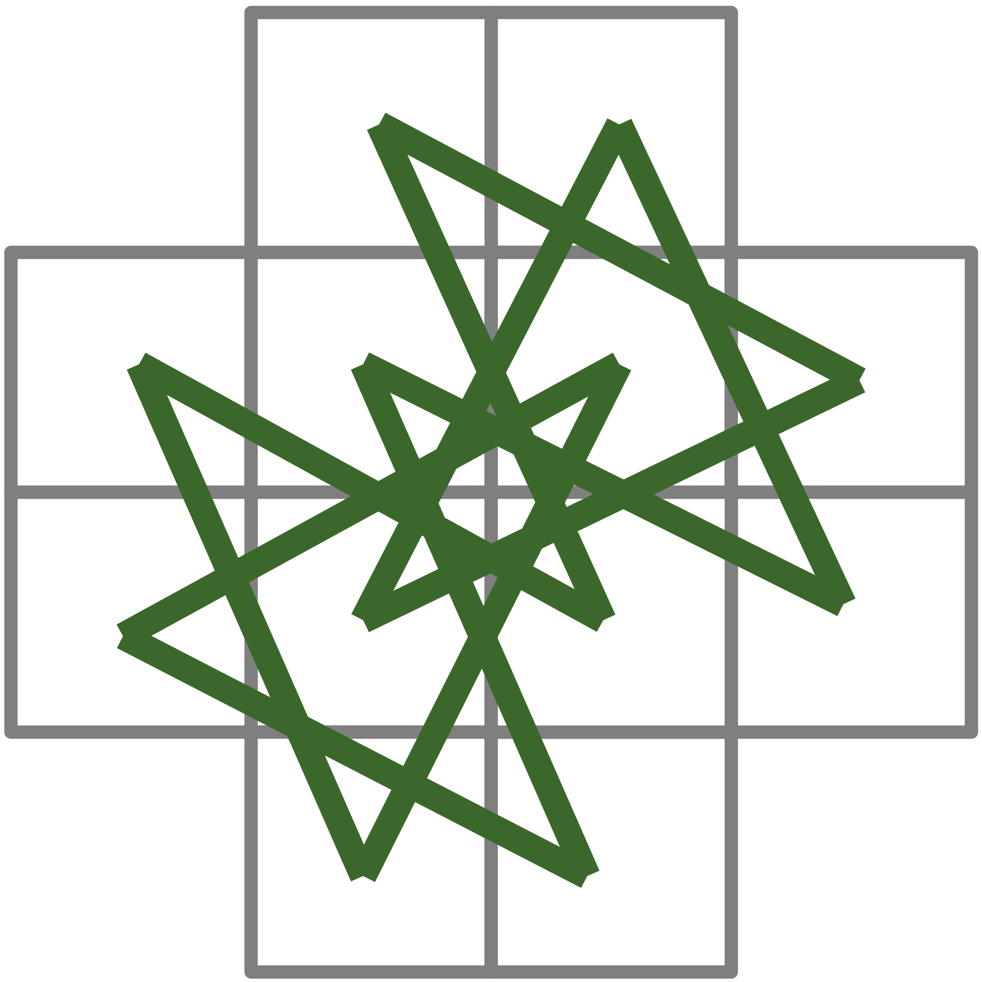
3-local conditions

- 3-SAT is NP-complete. [Cook, Levin]

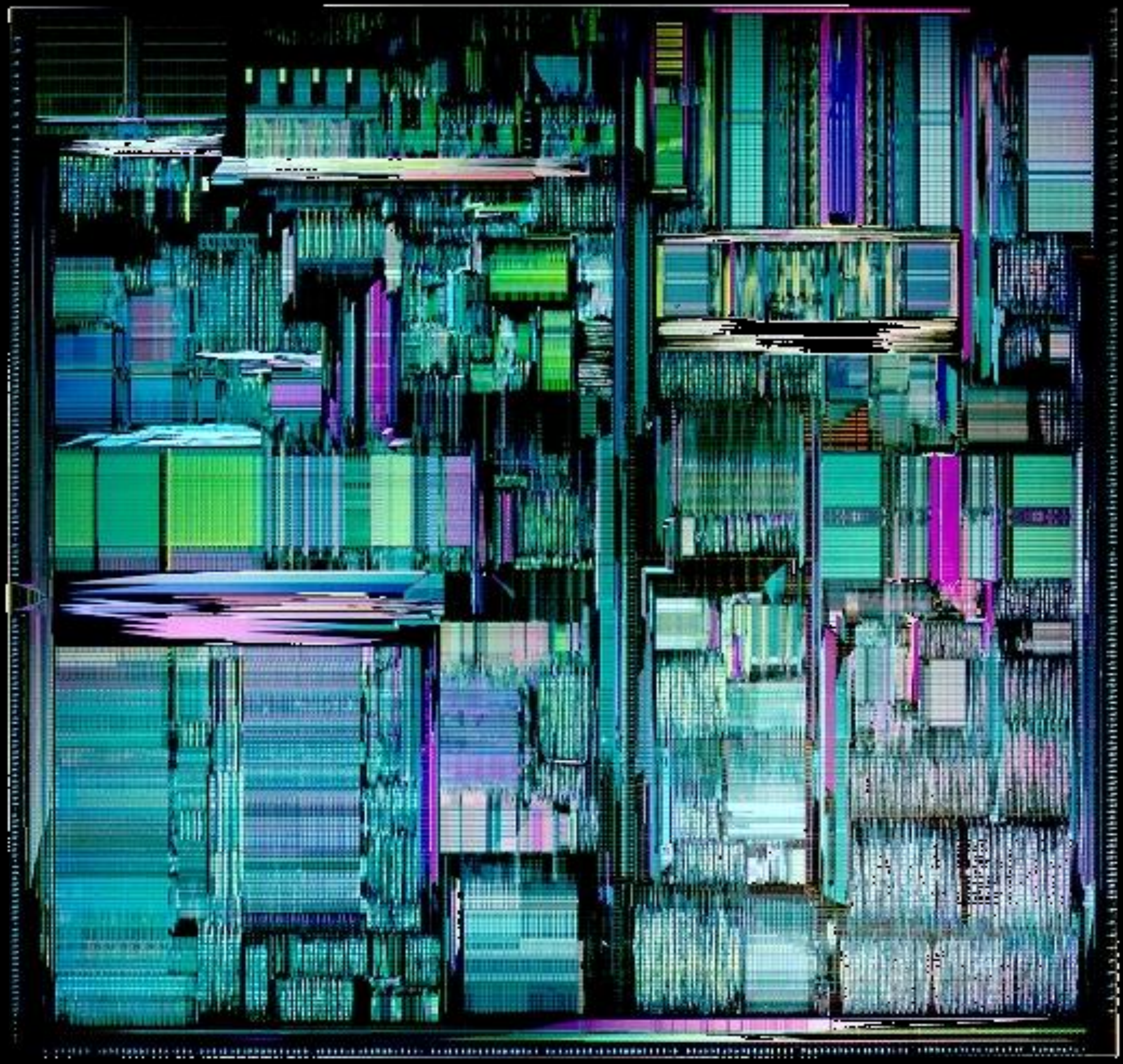
1 NP-complete problems: Hamiltonian cycle



1 NP-complete problems: Hamiltonian cycle

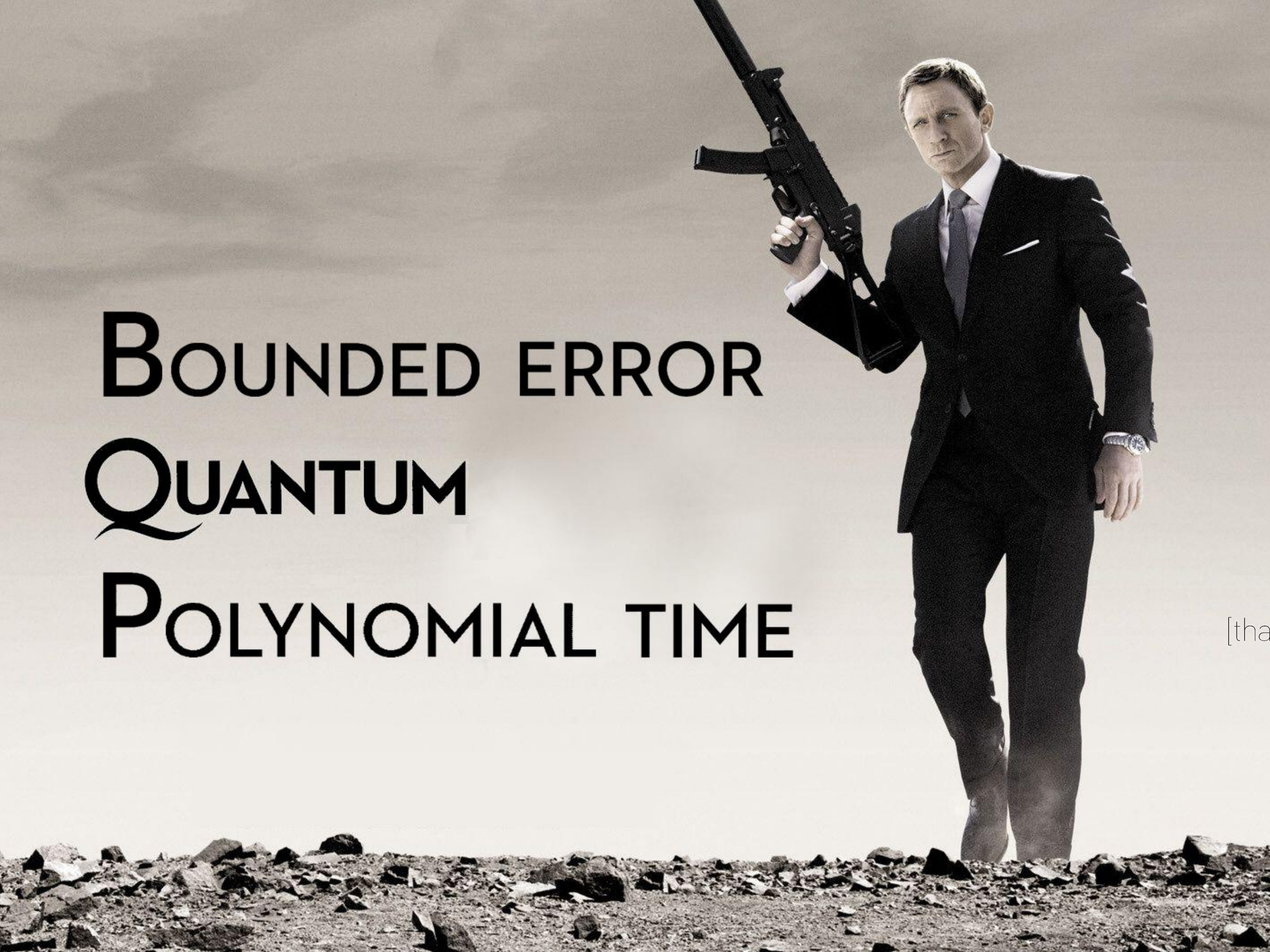


the puzzles of QMA



[1995 Pentium Pro

[wire


Bounded error
Quantum
Polynomial time

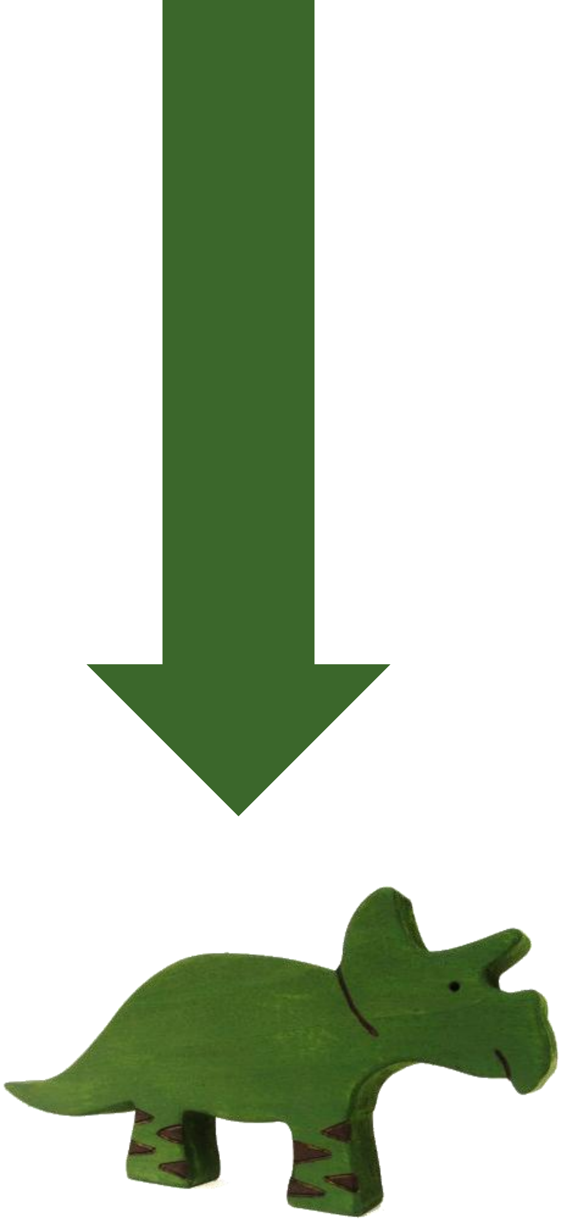
NP

Did dinosaurs exist?

?

2 The MA protocol

Did dinosaurs exist?



[wooden animals: Imagination Kids Toys]

2 The MA protocol

Did dinosaurs exist?

YES?
Eager to be convinced.

[magnifying glass: hillllall]

2 The MA protocol

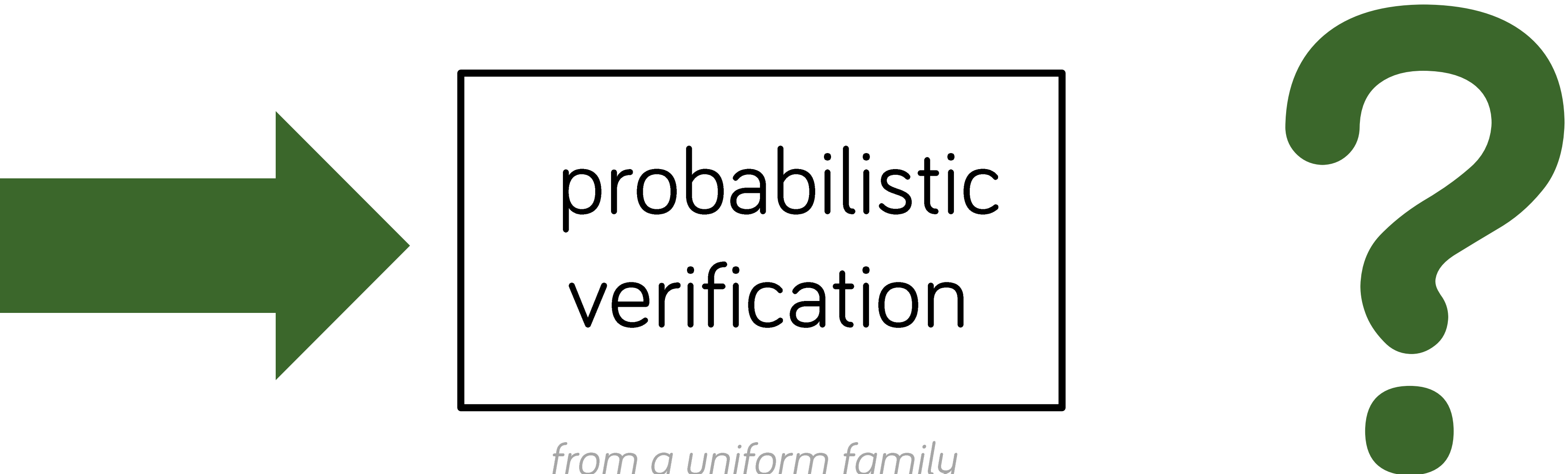
NO?
Don't be
fooled
easily.

2 Probabilistic checks

Sometimes reject a genuine proof?

Accept a fake?

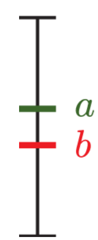
Probabilistic checks.



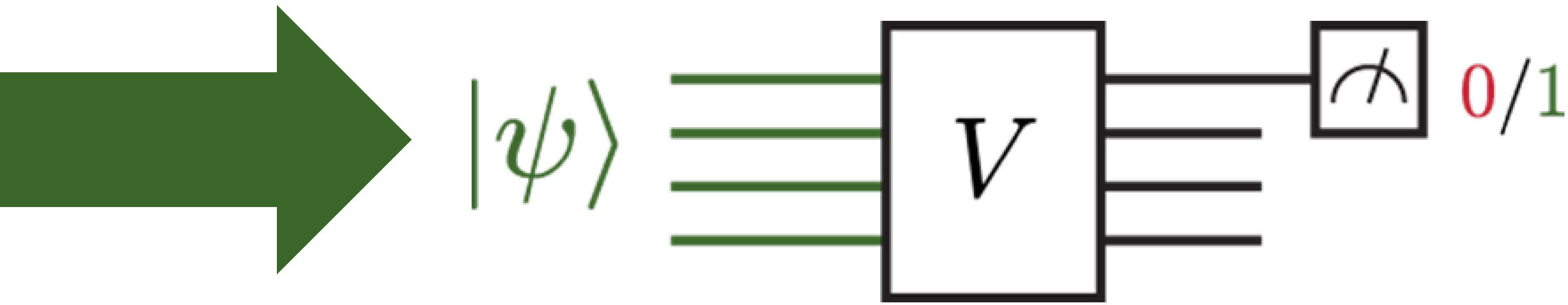
from a uniform family

YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.

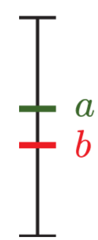


a quantum verifier



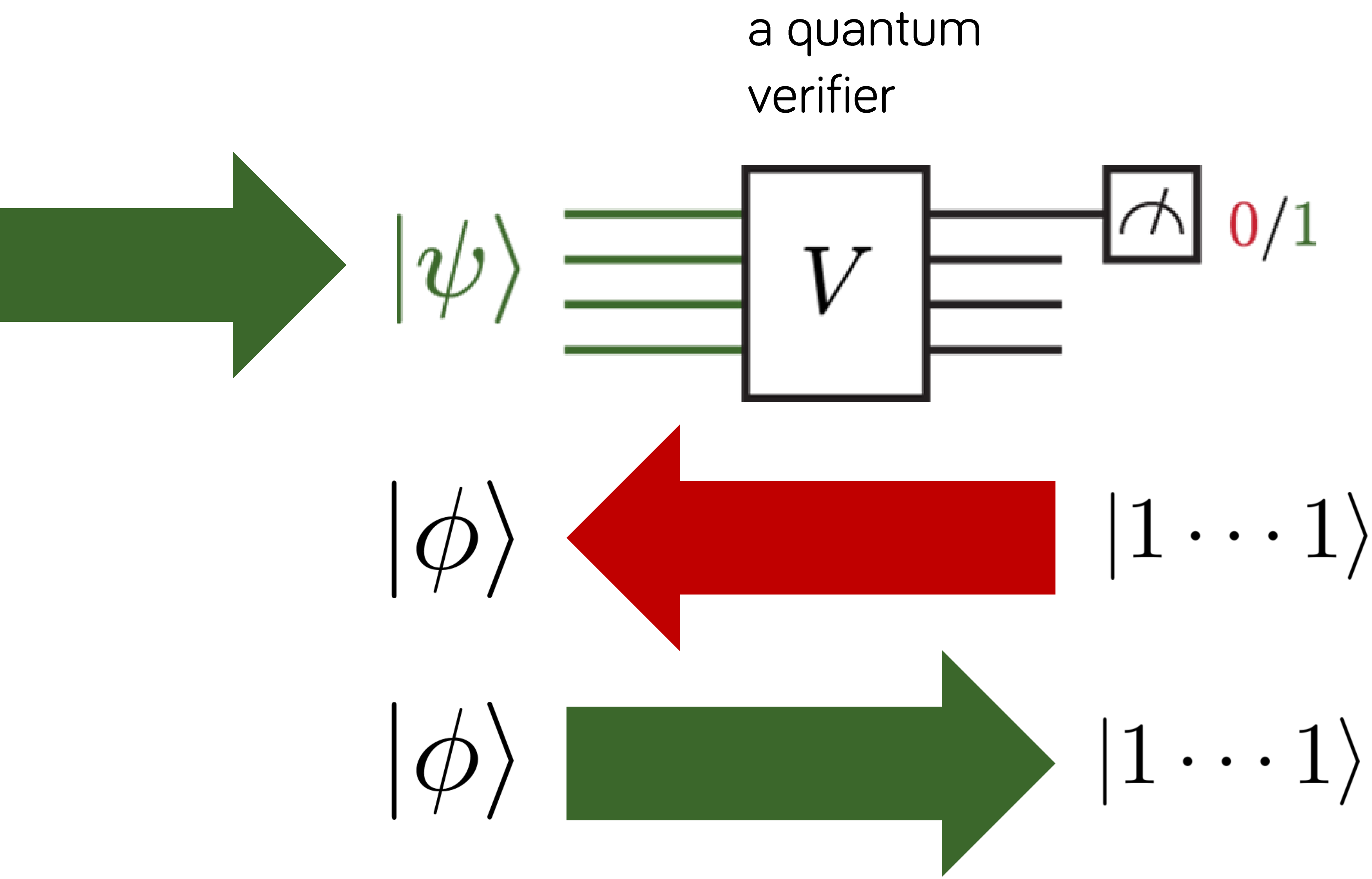
YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.



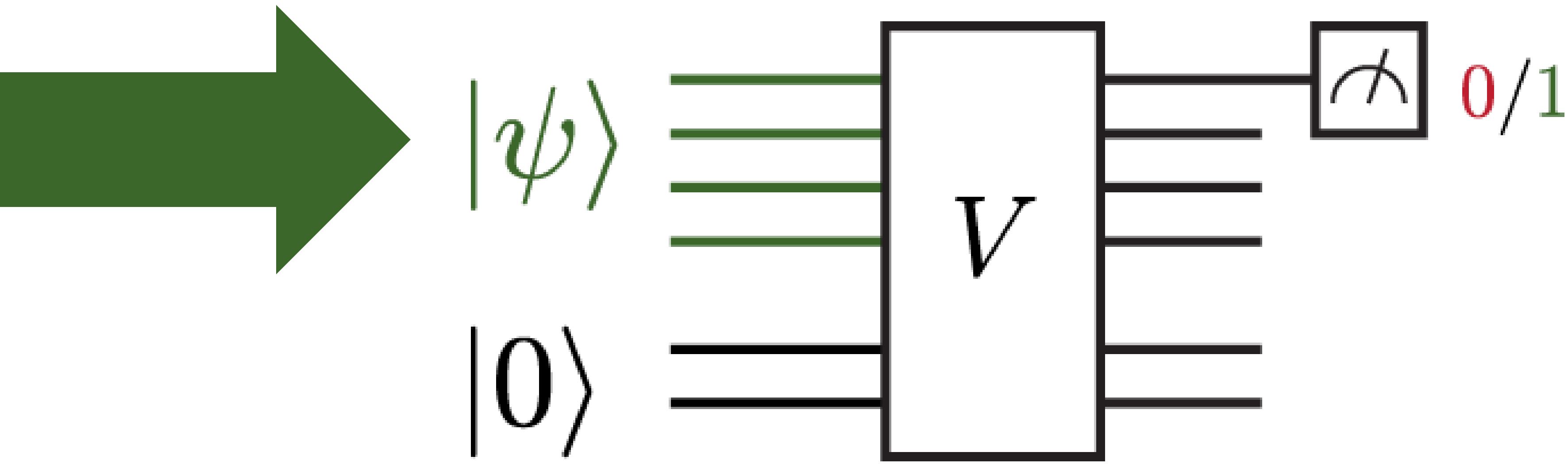
The QMA protocol

This is too simple.



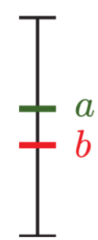
2 The QMA protocol

Ancillas are necessary.

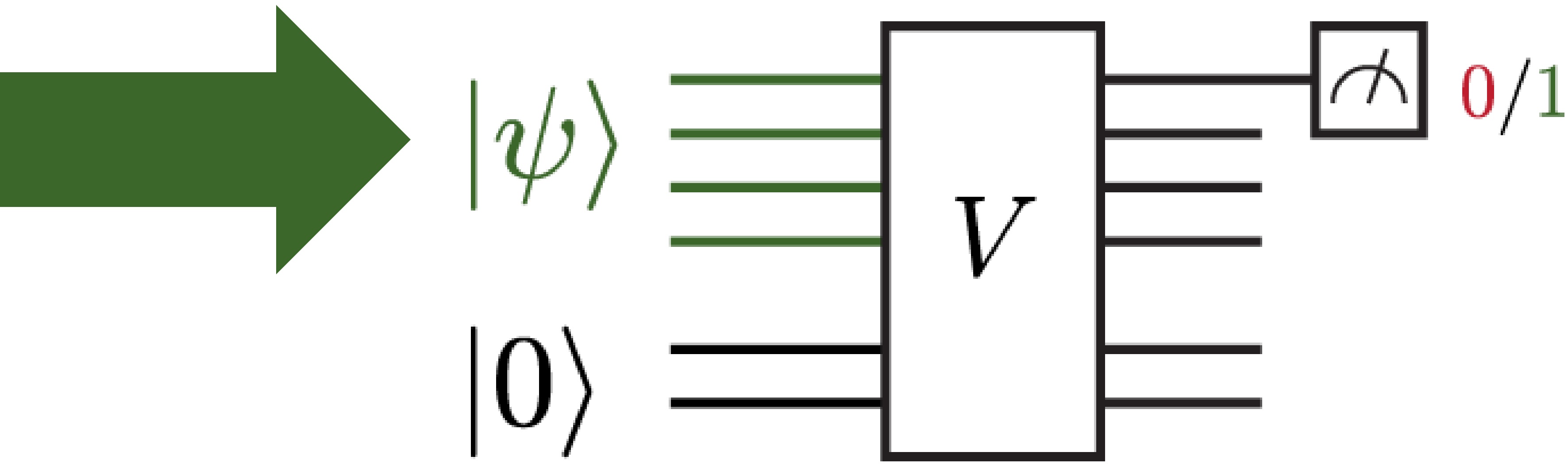


YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.



2 A QMA-hard question



Could we feed this quantum verifier something that likely outputs 1?

2 Implementing reversible quantum circuits

- The Schrödinger equation

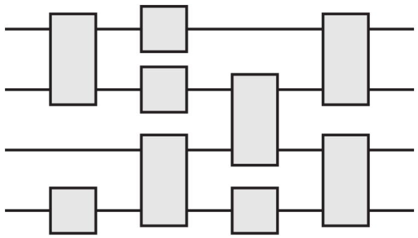
  a Hamiltonian *H*
  $$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

  made of local terms
  $$H(t) = \sum_j H_j(t)$$

  unitary time evolution
  $$|\psi(t)\rangle = U_{t,0}|\psi(0)\rangle$$

- Implementing a circuit U

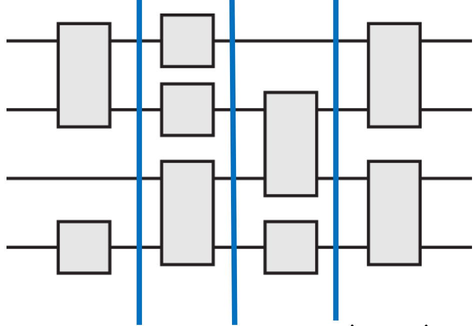
  how to make a system compute?



2 Making a quantum system compute

- Evolving a quantum system

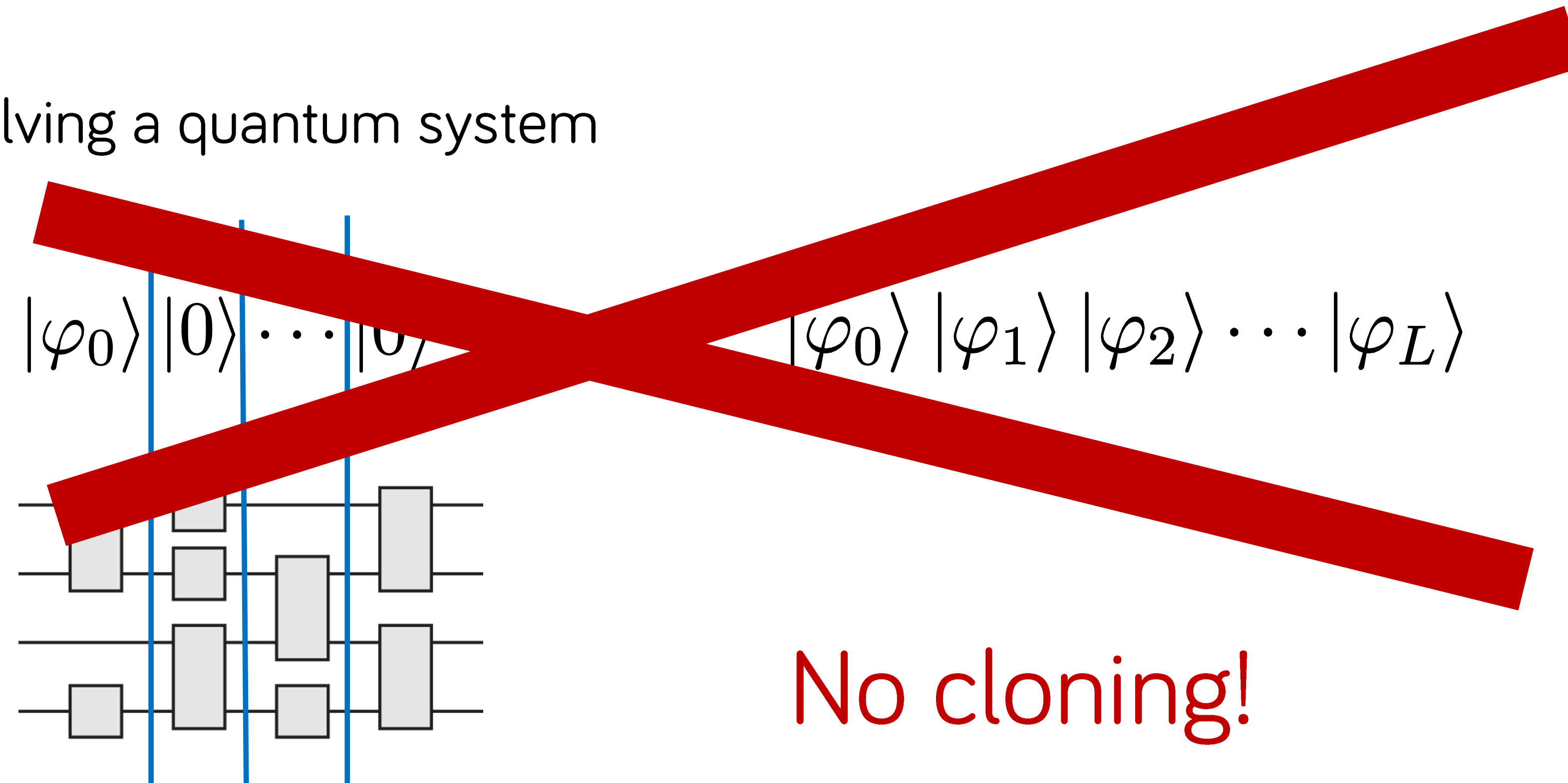
$$|\varphi_0\rangle \, |0\rangle \cdots |0\rangle \;\Longrightarrow\; |\varphi_0\rangle \, |\varphi_1\rangle \, |\varphi_2\rangle \cdots |\varphi_L\rangle$$



$$|\varphi_t\rangle = U_t U_{t-1} \dots U_2 U_1 \, |\varphi_0\rangle$$

2 Making a quantum system compute

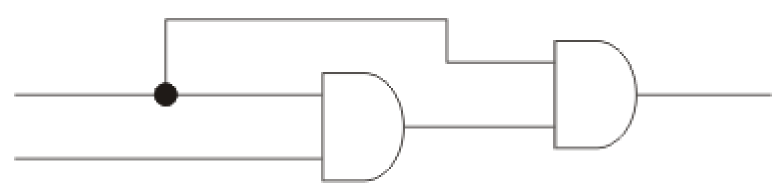
- Evolving a quantum system



$$|\varphi_t\rangle = |\varphi_0\rangle$$

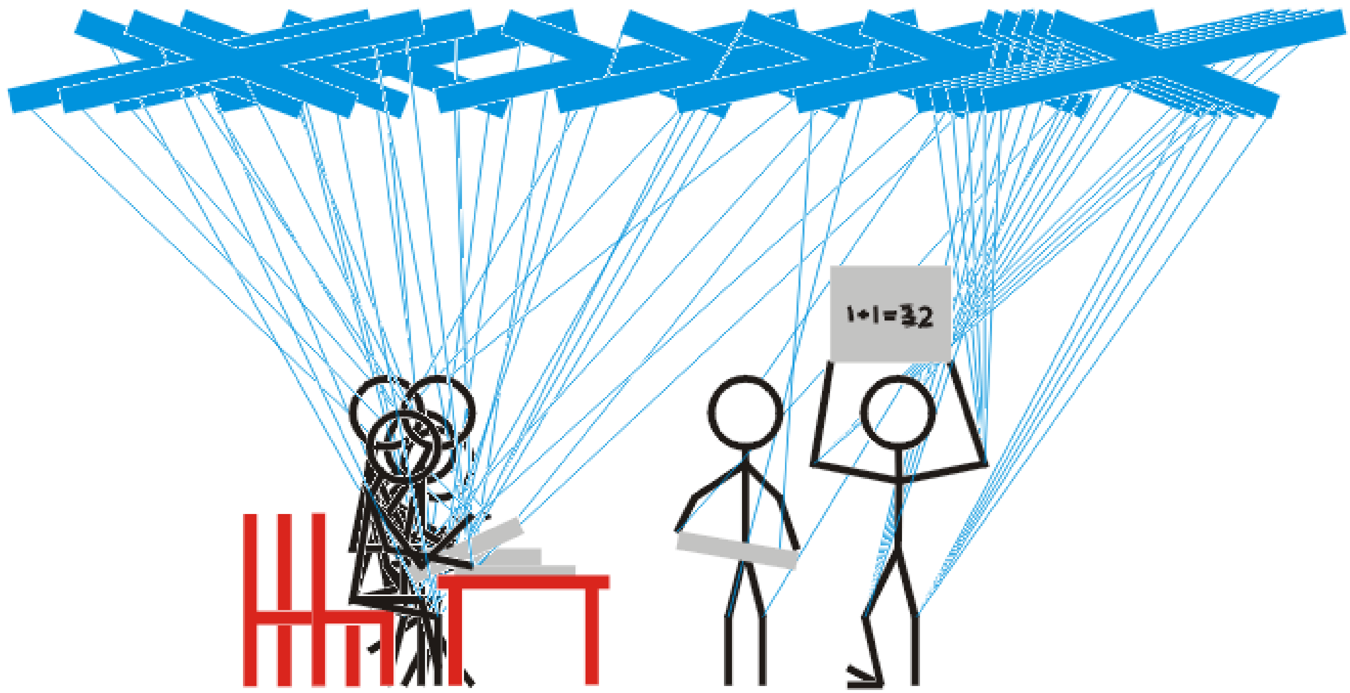
No cloning!

- Copying is OK classically.

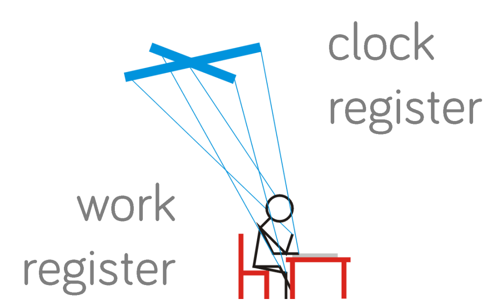


2 Feynman's computer

The data & a pointer.



2 Feynman's (Hamiltonian) computer

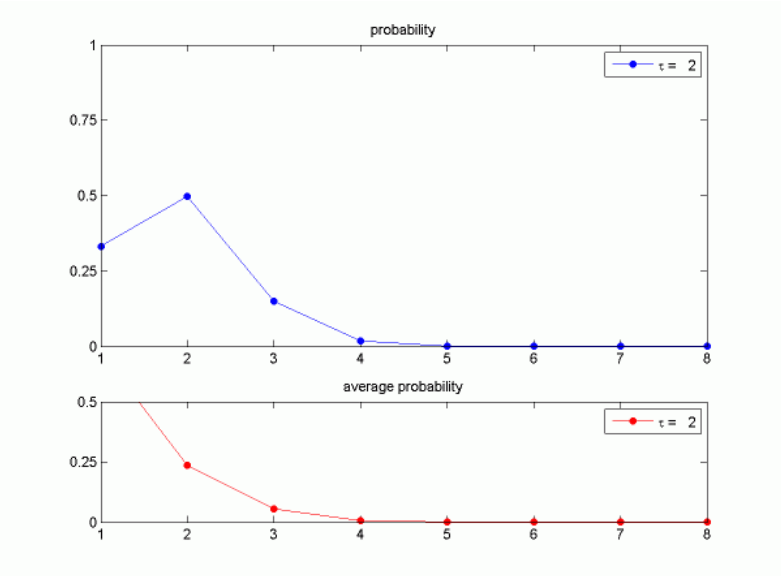


- The Hamiltonian

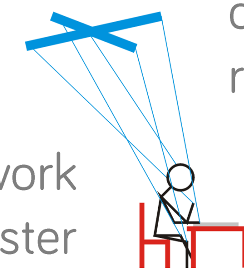
$$H_F = -\sum_{t=1}^{L} \left(U_t \otimes |t\rangle \langle t-1| + U_t^{\dagger} \otimes |t-1\rangle \langle t| \right)$$

- A quantum walk on a "line"

$$|\varphi_0\rangle \otimes |0\rangle_c$$
$$U_1 |\varphi_0\rangle \otimes |1\rangle_c$$
$$U_2 U_1 |\varphi_0\rangle \otimes |2\rangle_c$$
$$U_3 U_2 U_1 |\varphi_0\rangle \otimes |3\rangle_c$$



2 Feynman’s (Hamiltonian) computer



- The Hamiltonian

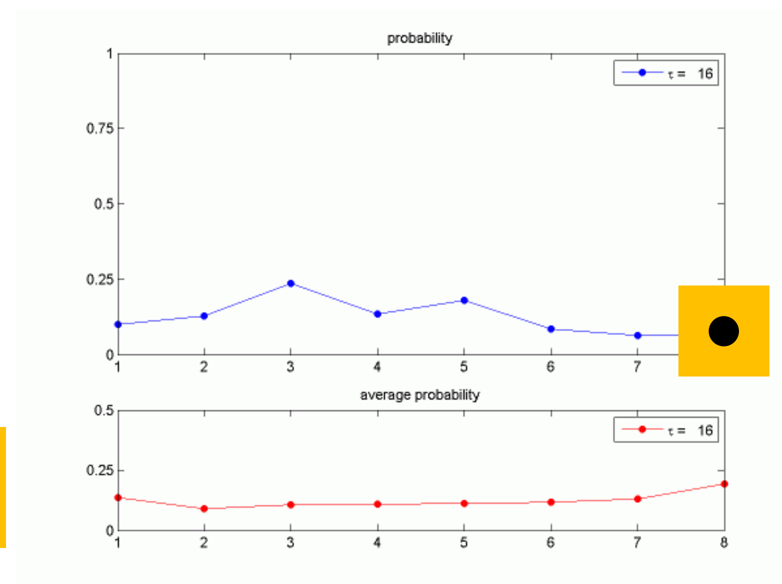
$$H_F = -\sum_{t=1}^{L} \left(U_t \otimes |t\rangle \langle t-1| + U_t^{\dagger} \otimes |t-1\rangle \langle t| \right)$$

- A quantum walk on a “line”

$$|\varphi_0\rangle \otimes |0\rangle_c$$

the output

$$U_L \cdots U_1 |\varphi_0\rangle \otimes |L\rangle$$



- Feynman's Hamiltonian

$$H_F = -\sum_{t=1}^{L}\left(U_t \otimes |t\rangle\langle t-1| + U_t^\dagger \otimes |t-1\rangle\langle t|\right)$$

- The "line" of states

$$\begin{aligned} |\varphi_0\rangle &\otimes |0\rangle_c \\ U_1|\varphi_0\rangle &\otimes |1\rangle_c \\ U_2U_1|\varphi_0\rangle &\otimes |2\rangle_c \\ U_3U_2U_1|\varphi_0\rangle &\otimes |3\rangle_c \\ U_4U_3U_2U_1|\varphi_0\rangle &\otimes |4\rangle_c \end{aligned}$$

$$H_F = -\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The eigenvectors: combinations of plane waves

- Feynman's Hamiltonian

$$H_F = -\sum_{t=1}^{L} \left(U_t \otimes |t\rangle \langle t-1| + U_t^\dagger \otimes |t-1\rangle \langle t| \right)$$

- The "line" of states

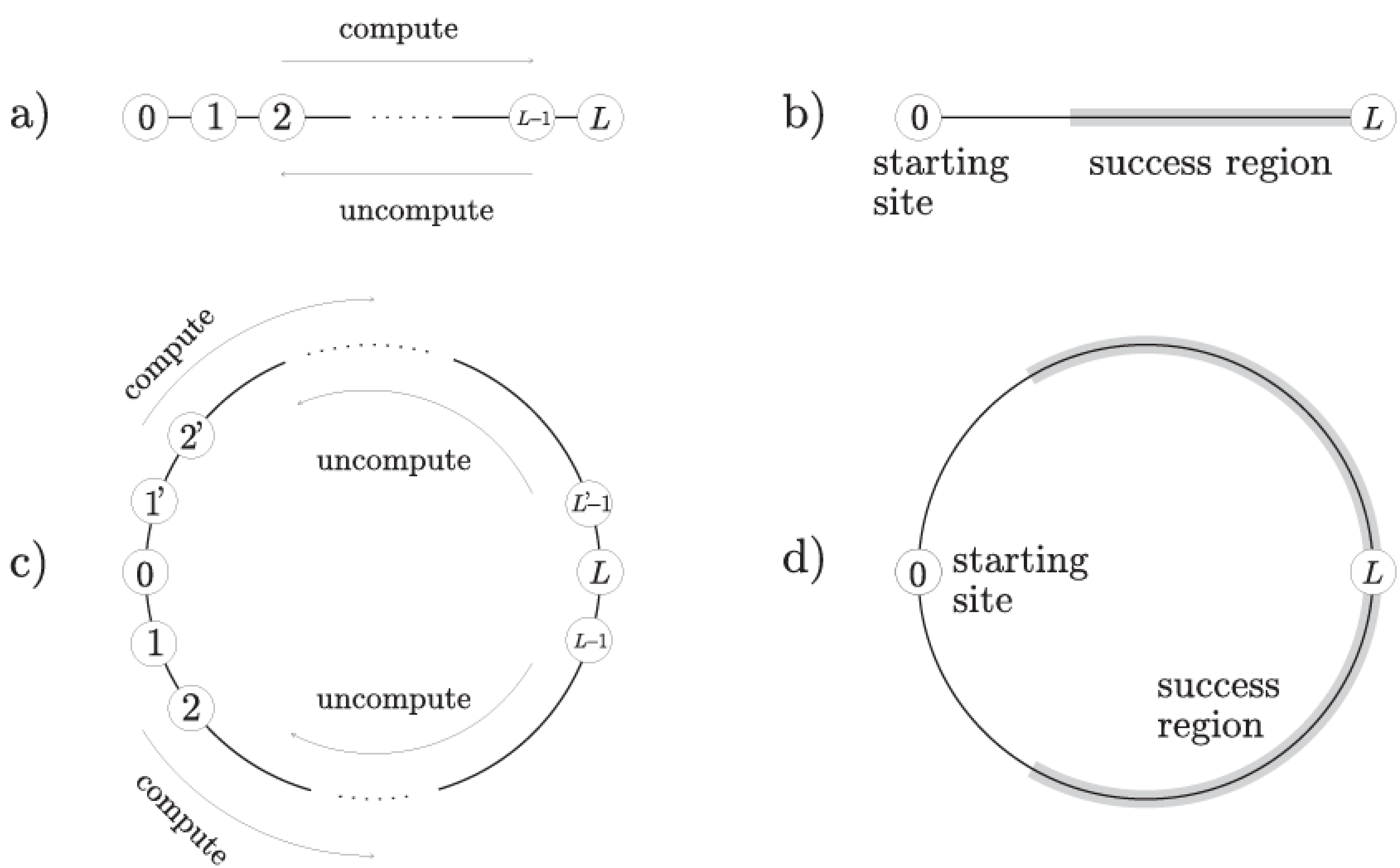
a possibility: wrap around a circle

$$\begin{aligned} &|\varphi_0\rangle \otimes |0\rangle_c \\ U_1 &|\varphi_0\rangle \otimes |1\rangle_c \\ U_2 U_1 &|\varphi_0\rangle \otimes |2\rangle_c \\ U_3 U_2 U_1 &|\varphi_0\rangle \otimes |3\rangle_c \\ U_4 U_3 U_2 U_1 &|\varphi_0\rangle \otimes |4\rangle_c \end{aligned}$$

$$H_F = - \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- The eigenvectors: combinations of plane waves

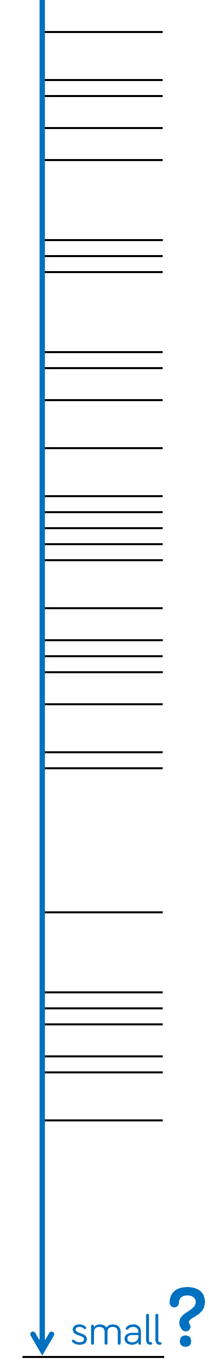
2 Boosting the success probability



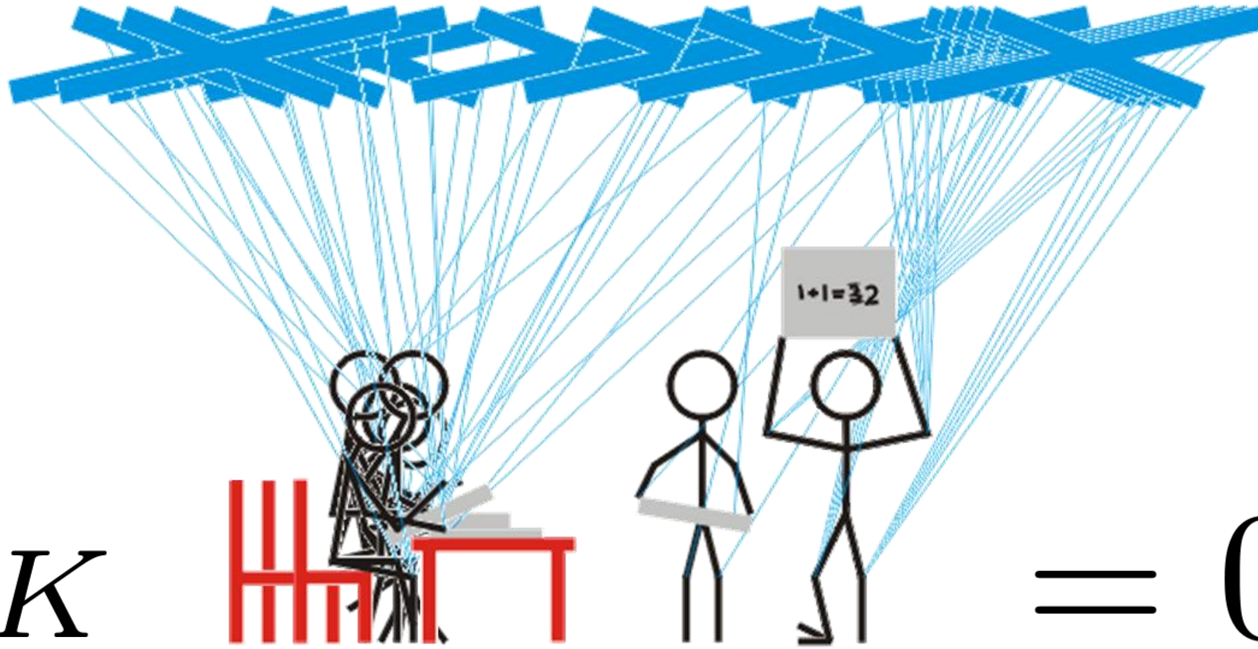
2 The Local Hamiltonian problem

Is
the
ground
state
energy
of a

LH



2 The history state: a ground state

H_K  $= 0$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} \underbrace{|\varphi_t\rangle}_{U_t \cdots U_1 |\varphi_0\rangle} \otimes |t\rangle$$

k-local

c-o-n-d-i-t-i-o-n-s

clock encoding

state progression

initialization

outpu

$$|\cdots 000 \cdots 0\rangle \otimes |0\rangle$$

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

$$\langle T| \otimes \langle 1 \cdots 1|$$

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

- uniform superpositions: zero-energy eigenstates

$$H_t = \tfrac{1}{2}\left(|t+1\rangle\langle t+1| + |t\rangle\langle t|\right) - \tfrac{1}{2}\left(U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^{\dagger} \otimes |t\rangle\langle t+1|\right) = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Feynman's Hamiltonian

a projector

$$|\varphi_t\rangle \otimes |t\rangle$$

$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

Antisymmetry checks.

- uniform superpositions: zero-energy eigenstates

$$H_t = \frac{1}{2}\Big(|\varphi_{t+1}\rangle|t+1\rangle - |\varphi_t\rangle|t\rangle\Big)\Big(\langle\varphi_{t+1}|\langle t+1| - \langle\varphi_t|\langle t|\Big)$$

- an energy penalty for antisymmetric combinations

$$|\varphi_t\rangle \otimes |t\rangle$$
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

Antisymmetry checks.

$$\sum_{t=1}^{L} H_t = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

positive semidefinite

$$|\varphi_t\rangle \otimes |t\rangle$$
$$|\varphi_{t+1}\rangle \otimes |t+1\rangle$$

a nice basis

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\varphi_t\rangle \otimes |t\rangle$$

2 Checking proper computation

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$$\sum_{t=1}^{L} H_t = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

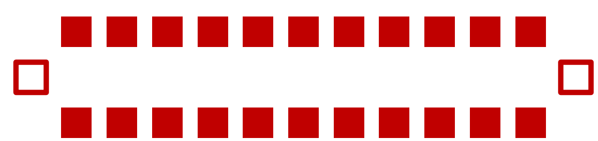
positive semidefinite

local?

$$\sum_t e^{-ipt} |\varphi_t\rangle \otimes |t\rangle$$

eigenvectors: combinations of plane waves

L



2(L+1)

an L^{-2} eigenvalue gap

a clock workshop

3 Constructing local clocks

- the pulse

transitions: 2-local

00100

↓

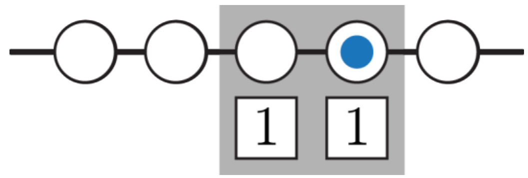
+00010

- joining the states by projectors

$$|01 - 10\rangle\langle 01 - 10|$$

3 Constructing local clocks

- the pulse



transitions: 2-local
2-qubit gates: 4-local

interaction
with the data

- joining the states by projectors

$$|01 - 10\rangle\langle 01 - 10|$$

3 Constructing local clocks

- the pulse

transitions: 2-local
2-qubit gates: 4-local

00000 a “dead” state

Initialization!

- joining the states by projectors

$$|01-10\rangle\langle 01-10|$$

3 Constructing local clocks

- the domain wall

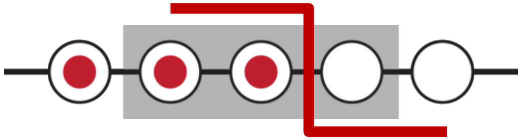
$$|t\rangle = |4\rangle$$
$$= |11110\rangle$$

- 2-local terms "compatible" with **11...1100...00**

?

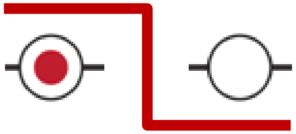
$$|01\rangle\langle 01|$$

3 Constructing local clocks

- the domain wall  transitions: 3-local

$$|t\rangle = |3\rangle = |11100\rangle$$

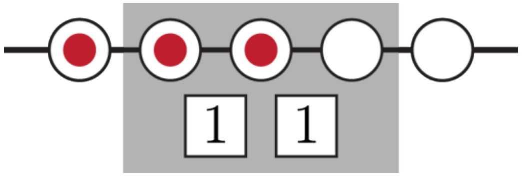
- joining states by transitions? $|100 - 110\rangle\langle 100 - 110|$

- enforce a domain wall: fix the ends 

- the ground state $\cdots + |2\rangle + |3\rangle + \ldots$

3 Constructing local clocks

- the domain wall



transitions: 3-local
2-qubit gates: 5-local

- interacting with work (data) qubits

$$H_t = \frac{1}{2}\left(|t+1\rangle\langle t+1| + |t\rangle\langle t|\right) - \frac{1}{2}\left(U_{t+1} \otimes |t+1\rangle\langle t| + U_{t+1}^\dagger \otimes |t\rangle\langle t+1|\right)$$

5-local

3 Local Hamiltonian: putting it all together

- punish bad ancilla initialization

$$\sum_{a=1}^{n_a} |1\rangle\langle 1|_a \otimes |10\rangle\langle 10|_{c_1,c_2} \qquad |\varphi_0\rangle \otimes |0\rangle_c$$

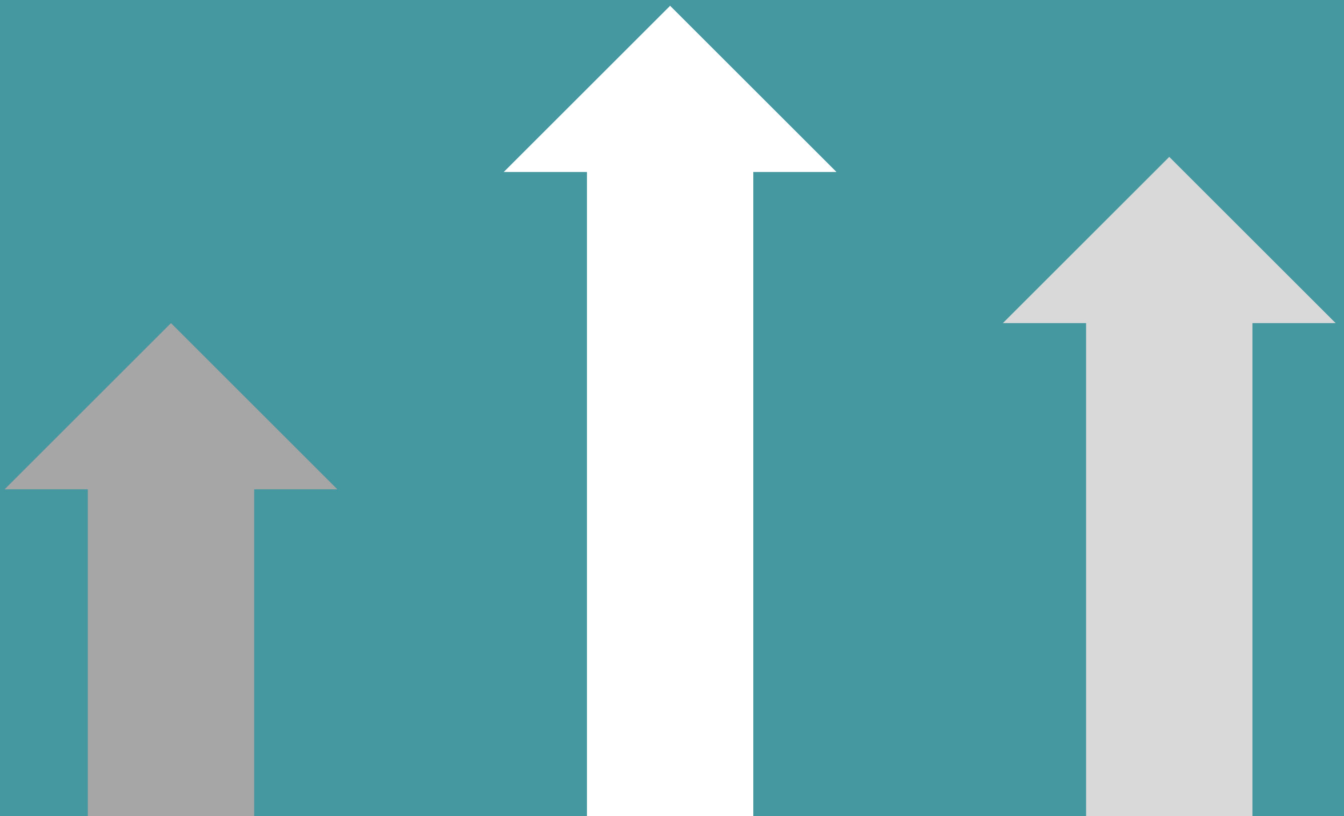
- check the computation

$$H_t = \tfrac{1}{2}\left(|t+1\rangle\langle t+1| + |t\rangle\langle t|\right) - \tfrac{1}{2}\left(U_{t+1}\otimes|t+1\rangle\langle t| + U_{t+1}^\dagger \otimes |t\rangle\langle t+1|\right)$$

- punish non-accepting computations

$$|0\rangle\langle 0|_{out} \otimes |1\rangle\langle 1|_{c_L} \qquad |\varphi_L\rangle \otimes |L\rangle_c$$

lower bounding the ground state **energy**



good
clock
states

... 01 ...
bad
clock
states

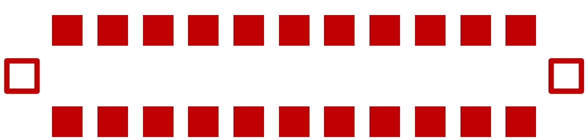
history states

non-uniform
superpositions

history states

a polynomially small gap

$\Delta = O\left(L^{-2}\right)$



well	badly
initialized history states	

well

initialized histories

accepted states

well

initialized histories

accepted states

$H_A + H_B$

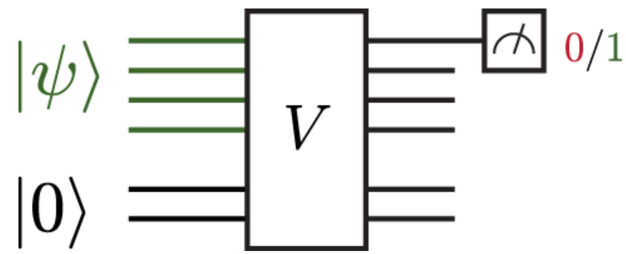
$$\lambda_0 \geq \sin^2 \frac{\vartheta}{2} \times \min(\Delta_A, \Delta_B)$$

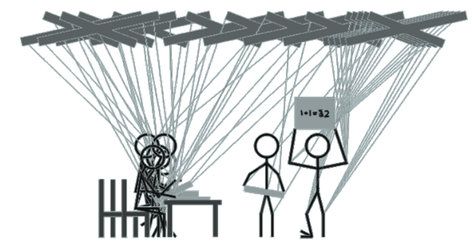
L^{-2}

L^{-1}

3 LH and QMA verification

$$H_{clock} + H_{init} + H_{prop} + H_{out}$$

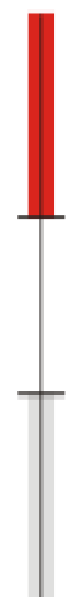




NO no witness accepted by V more likely than ε

any state has energy

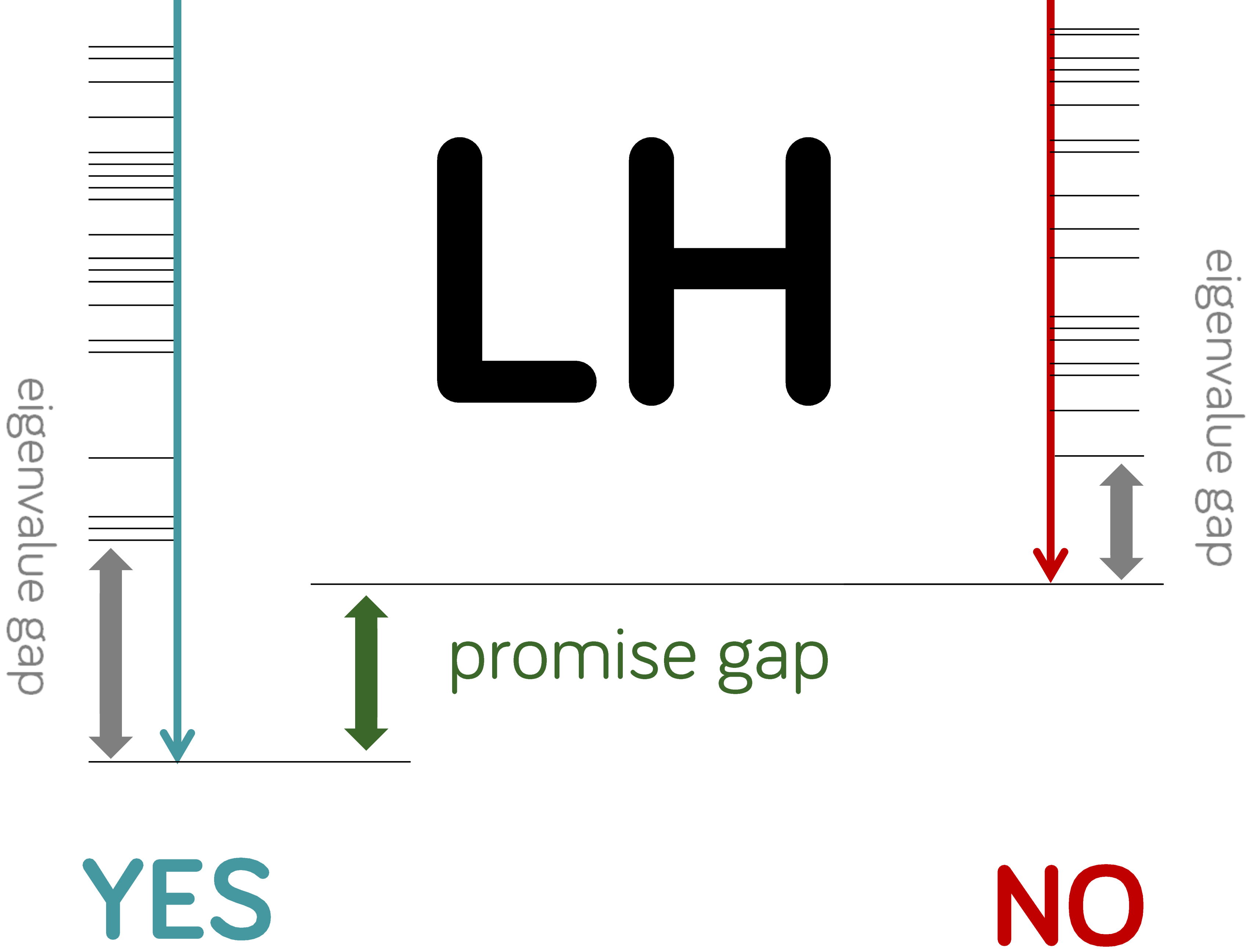
$$\langle\eta|\, H \,|\eta\rangle \geq \frac{c\left(1-\sqrt{\epsilon}\right)}{L^3}$$



YES there is a proof accepted by V with probability 1-ε

$$\langle\psi_{hist}|\, H \,|\psi_{hist}\rangle \leq 0 + 0 + \frac{\epsilon}{L+1}$$

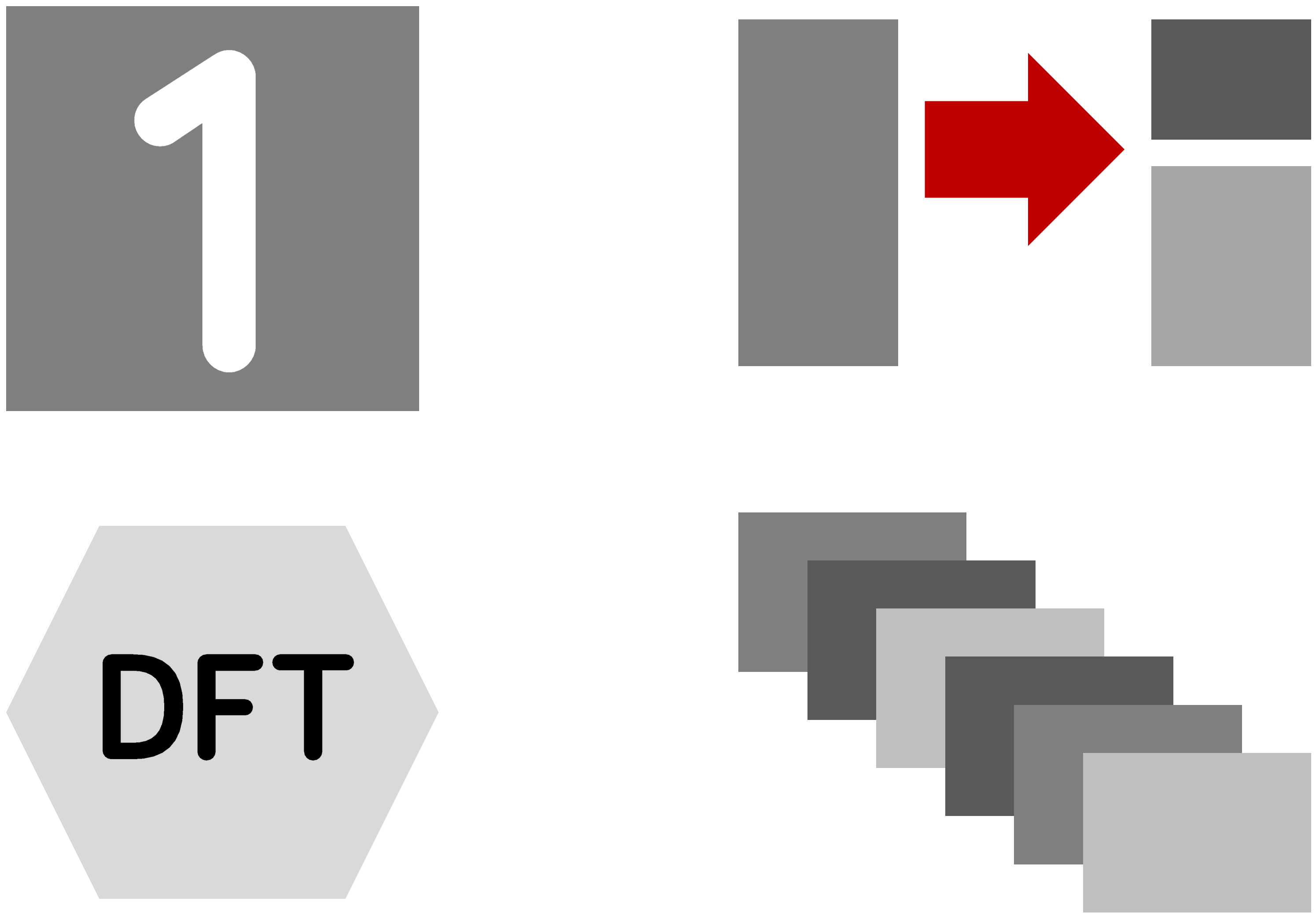
the history for the proof



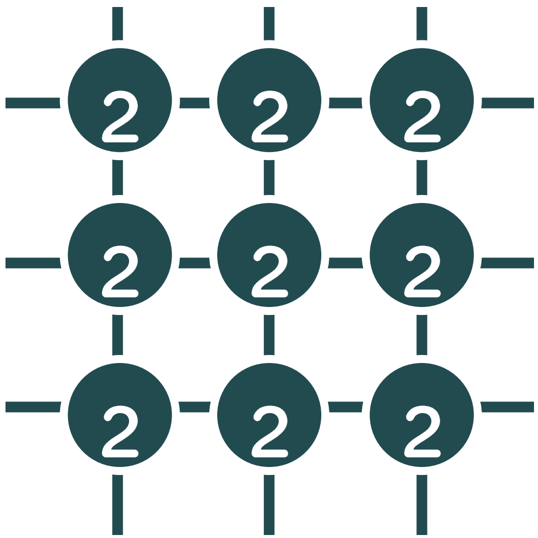
LH
eigenvalue gap
eigenvalue gap
promise gap
YES
NO

2 Other QMA-complete problems

[Bookatz '13]

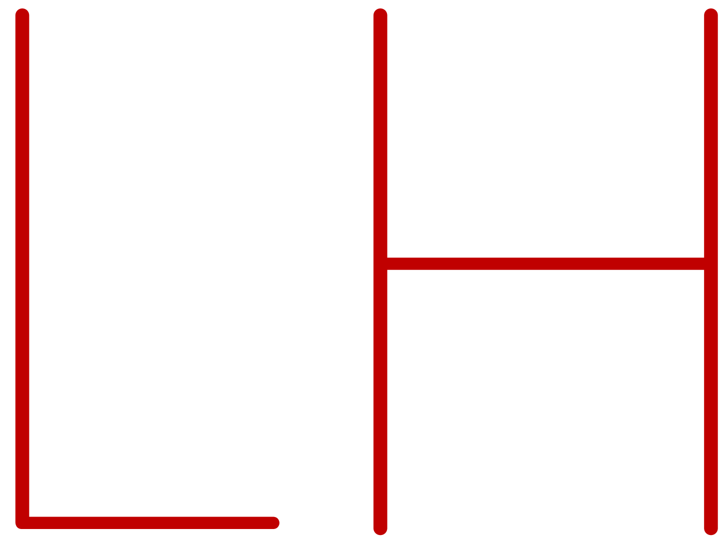


2 2-local Hamiltonian is QMA complete

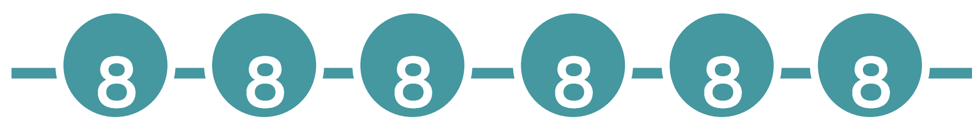


[Oliveira, Terhal ’04]

a global minimum

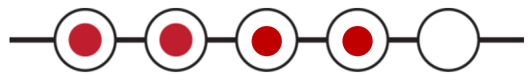


$$\sum H_{jk}$$



[Hallgren, Nagaj, Narayanaswami ’13]

3 From a 5-local to a 3-local clock [Kempe, Regev]

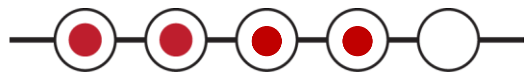
- the domain wall  transitions: 1-local

- 1-local transitions $\frac{1}{2}|1-0\rangle\langle 1-0|$

- punish mistimed transitions?

$$|01\rangle\langle 01|$$

3 From a 5-local to a 3-local clock [Kempe, Regev]

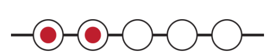
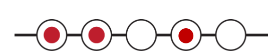
- the domain wall  transitions: 1-local

- 1-local transitions $\frac{1}{2}|1-0\rangle\langle 1-0|$

- punish mistimed transitions right away $|01\rangle\langle 01|$

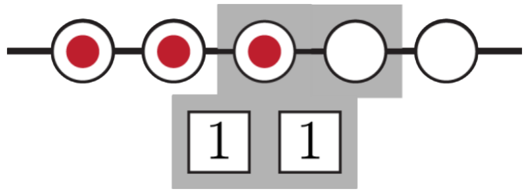
it was an eigenstate $H_{prop}^{5-loc}|\psi_{hist}\rangle = 0$

now: an E=0 state $\langle\psi_{hist}|H_{prop}^{3-loc}|\psi_{hist}\rangle = 0$

3 From a 5-local to a 3-local clock [Kempe, Regev]

- the domain wall



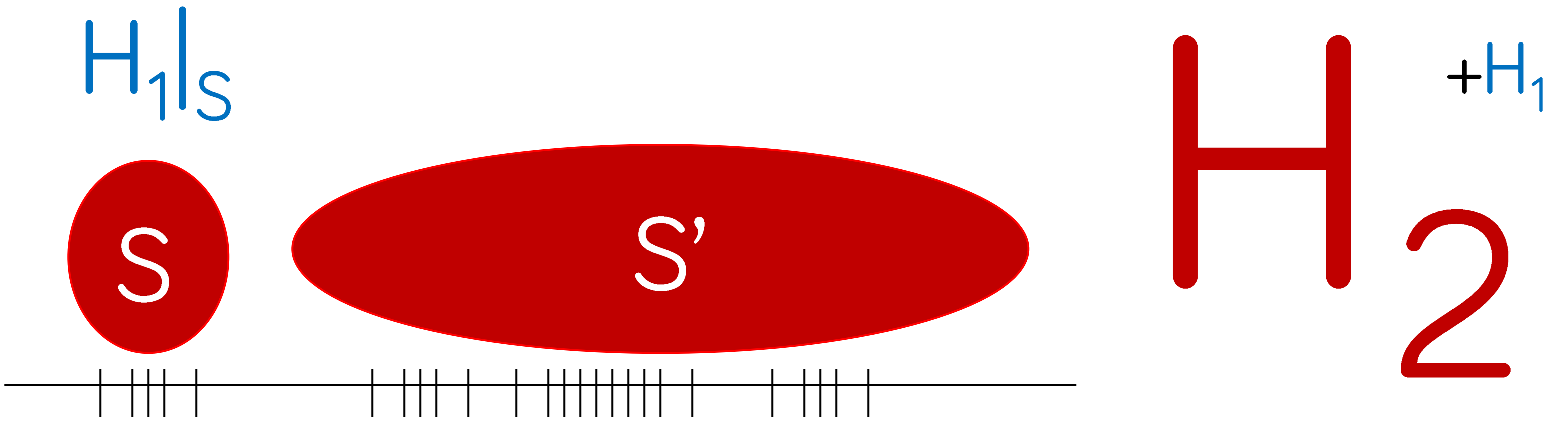
transitions: 1-local
2-qubit gates: 3-local

- 2-qubit gates $|1\rangle\langle 1| + |0\rangle\langle 0| - |1\rangle\langle 0| \otimes U - |0\rangle\langle 1| \otimes U^\dagger$

- punish mistimed transitions

$$|01\rangle\langle 01|$$

4 The projection lemma: a useful tool for proving gaps

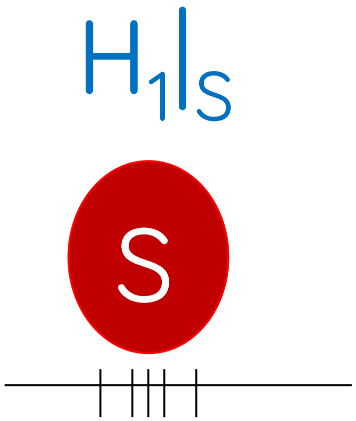


Lemma 1 *Let $H = H_1 + H_2$ be the sum of two Hamiltonians operating on some Hilbert space $\mathcal{H} = \mathcal{S} + \mathcal{S}^\perp$. The Hamiltonian H_2 is such that $\mathcal{S}$ is a zero eigenspace and the eigenvectors in $\mathcal{S}^\perp$ have eigenvalue at least $J > 2\|H_1\|$. Then,*

$$\lambda(H_1|_\mathcal{S}) - \frac{\|H_1\|^2}{J - 2\|H_1\|} \leq \lambda(H) \leq \lambda(H_1|_\mathcal{S}).$$

- a HIGH energy penalty for "illegal" states?
- the low energy states live near the "legal" subspace

4 The projection lemma: a useful tool for proving gaps



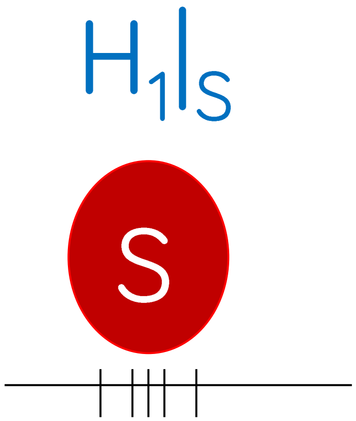
$$\frac{1}{2}|1-0\rangle\langle 1-0|$$

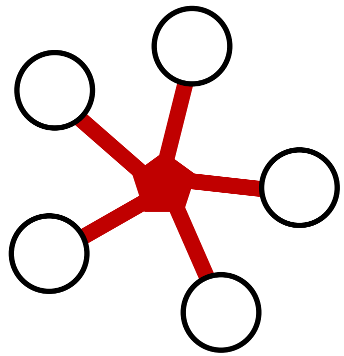
$$\frac{1}{2}|110-100\rangle\langle 110-100|$$

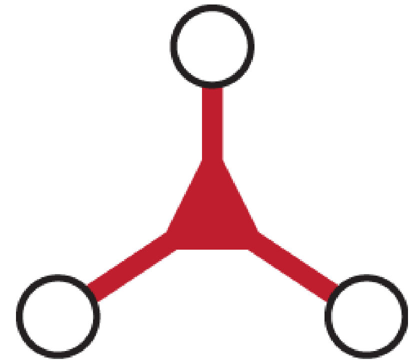
$$\frac{1}{2}\Big(|t+1\rangle-|t\rangle\Big)\Big(\langle t+1|-\langle t|\Big)$$

- a HIGH energy penalty for “illegal” states?
- the low energy states live near the “legal” subspace

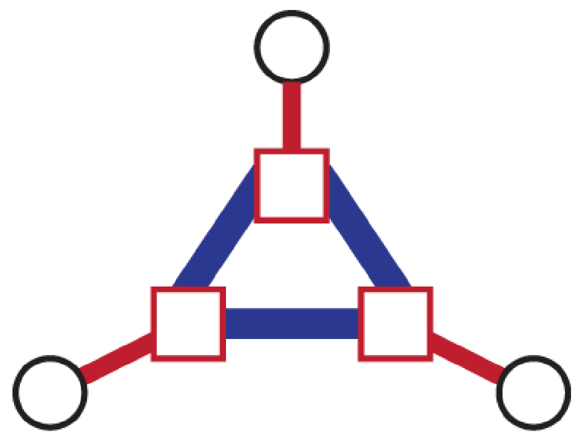
4 The projection lemma: a useful tool for proving gaps





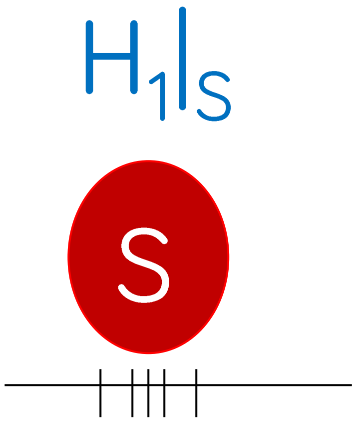


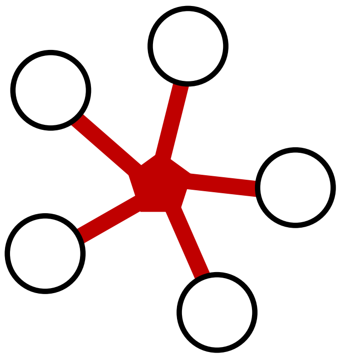
- 3-local H. that works "just as the 5-local one" in the "good clock subspace"
- 2-local H. from effective interactions

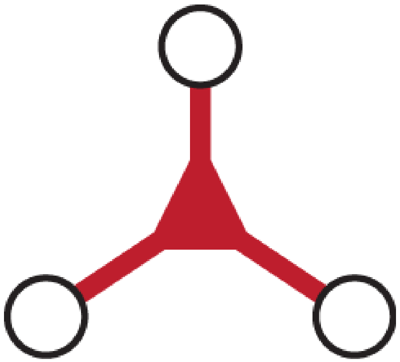


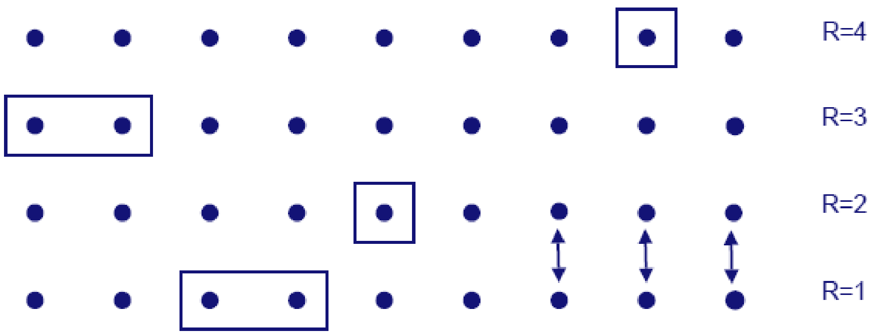
[Kempe, Kitaev, Regev '03]

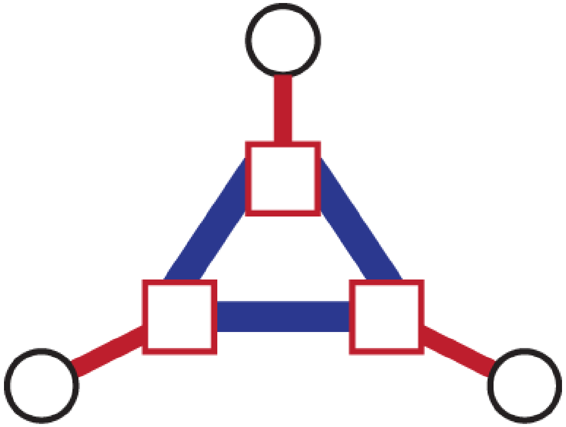
4 The projection lemma: a useful tool for proving gaps



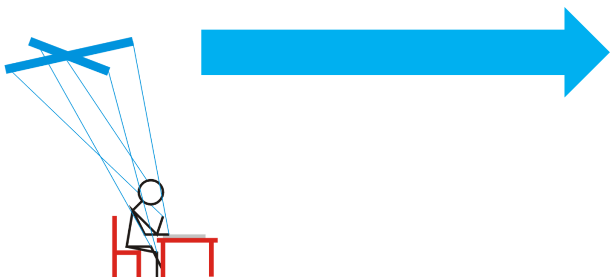




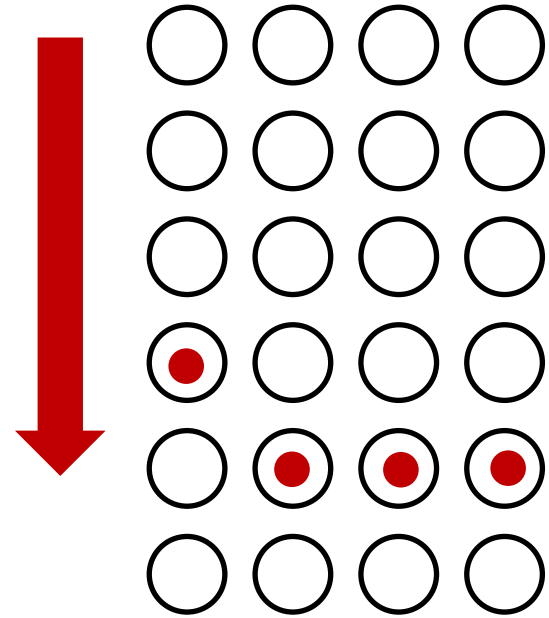


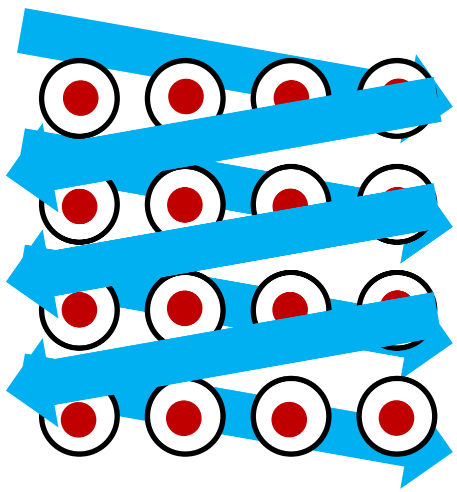


- 2-loc. H. in 2D [Oliveira, Terhal '05] clock/work registers

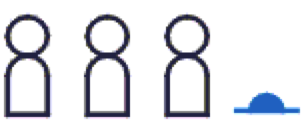


a geometric clock



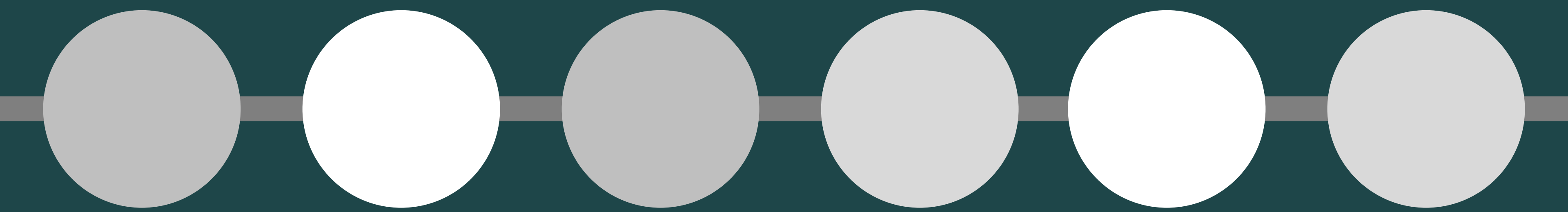


geometric locality

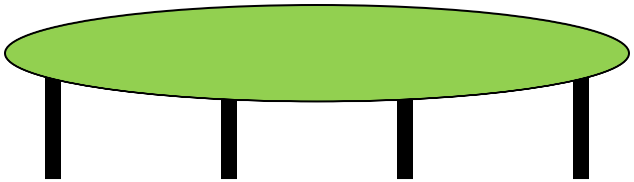


moving data on a line

l-H-a-m-i-l-t-o-n-i-a-n-s-i-n-1-D-L-o-c-a-l-H-a-m-i-l-t-o-n-i-a-n-s-i-n-1-D-L-o-c-a-l-H-a-m

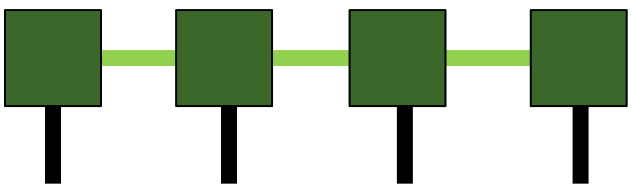


4 Matrix Product States



$$|\psi\rangle = \sum_{s,t,u,v=0}^{1} c_{stuv}|stuv\rangle$$

- Schmidt decompositions ➡ a local description



$$c_{stuv} = \sum_{a,b,c=1}^{\chi} A_a^s B_{ab}^t C_{bc}^u D_c^v$$

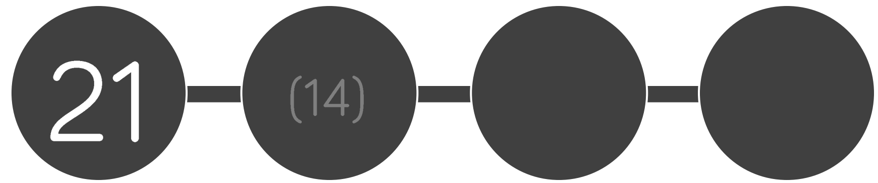
- Matrix Product States & DMRG

low entanglement
local optimization

4 1D ground states

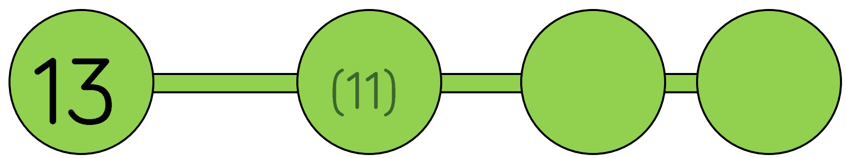
How hard is it to find/describe them?

entropy ~ L



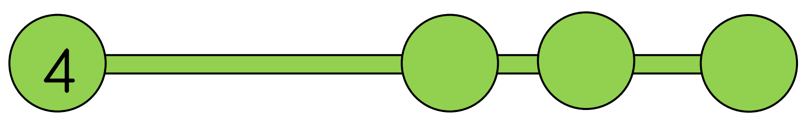
trans. invariant
[Irani]

QMA$_1$-comp.



history state
[AGIK ’06]

very entangled

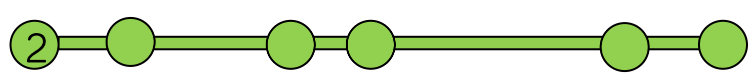


random projectors
[M+’10]

const. entropy

trans. invariant
AKLT model

product states



Q 2-SAT in P
[Bravyi, C+]

- Are they hard to find?

entanglement

- Can they be very entangled?

d=2: NO
[Chen+]

qubits

- Are they hard to find?
- Can they be very entangled?

entanglement

d=2: NO
[Chen+]

d=11: YES
[AGIK] [N]

qudits

- Are they hard to find?
- A quite entangled ground state.

entanglement

d=2: NO
[Chen+]

d=11: YES
[AGIK] [N]

qu3its

[Bravyi+ ’12]

- Are they hard to find?
- Very entangled ground states.

entanglement

d=2: NO

[Chen+]

d=11: YES

[AGIK] [N]

qu8its

[Hallgren+ '12]

4 Ground states in 1D

How hard is it to find/describe them?

entropy ~ L

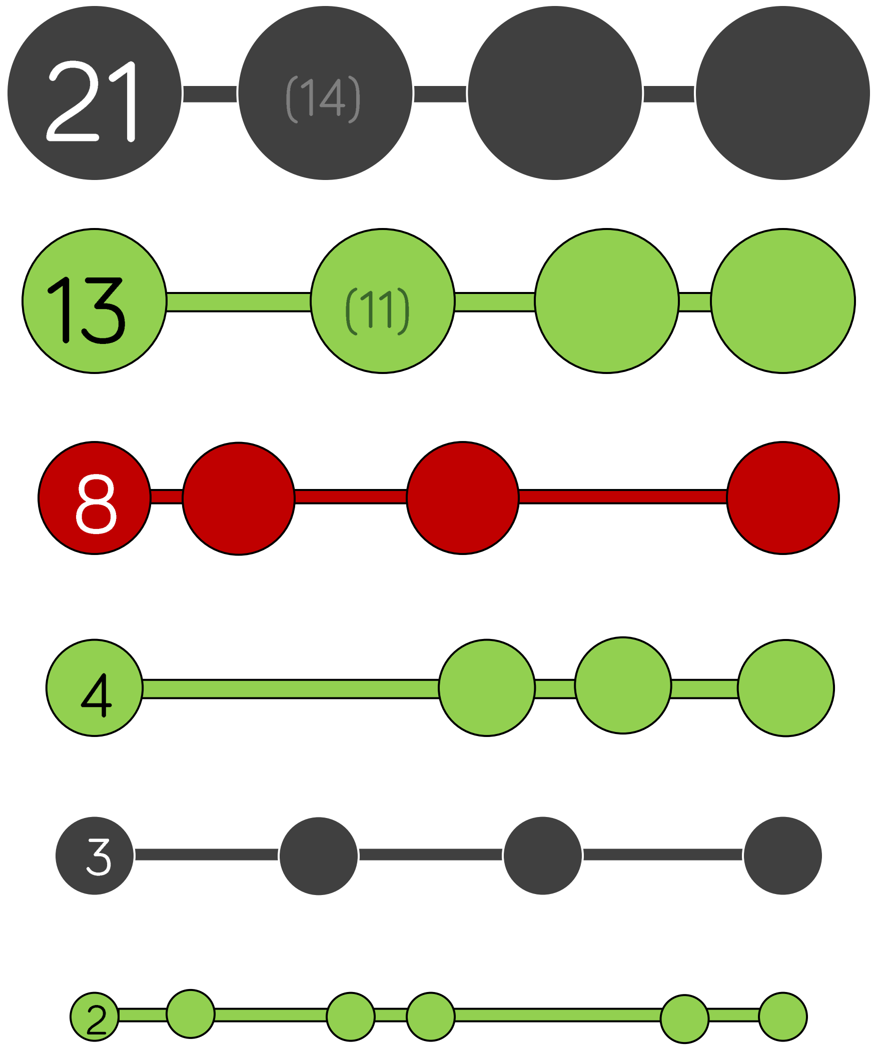
QMA_1-comp.

■ QMA-comp.

very entangled

■ entropy: log L

product states



trans. invariant
[Irani]

history state
[AGIK '06]

frustrated
[Hallgren+'13]

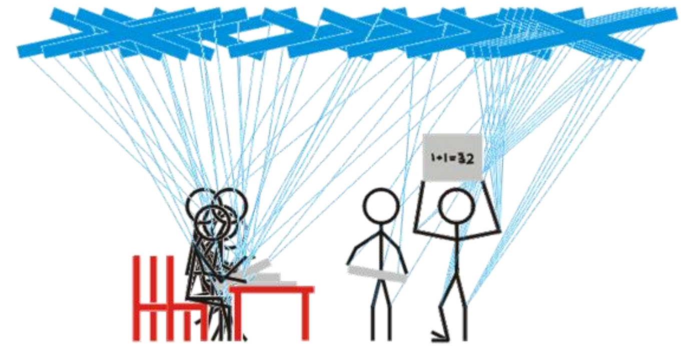
random projectors
[M+'10]

trans. invariant
[B+'12]

Q 2-SAT in P
[Bravyi, C+]

4 Computational histories in 1D

d = 8, 11, 13, ~~12~~: QMA-comp.



- the history state

$$|\psi_{hist}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\psi_t\rangle$$

a global & local ground state

- ground state of the propagation Hamiltonian

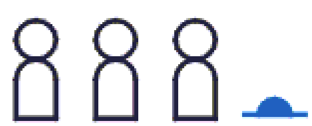
$$H_{prop} = \frac{1}{2} \sum_{t=1}^{T} \left(|\psi_t\rangle - |\psi_{t-1}\rangle\right)\left(\langle\psi_t| - \langle\psi_{t-1}|\right)$$

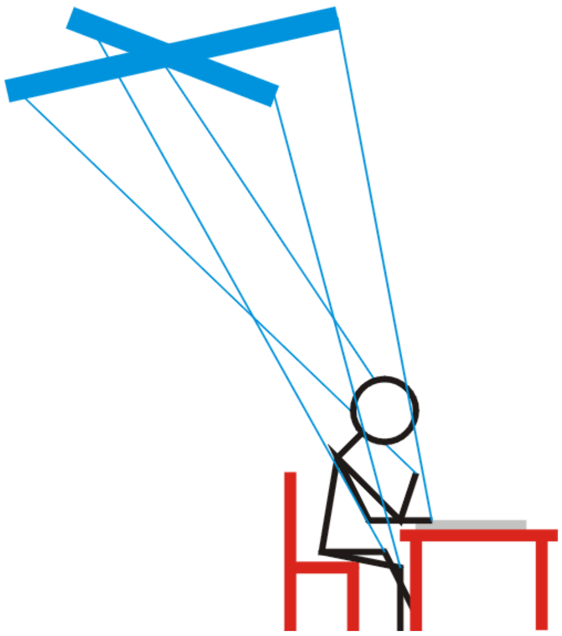
- using 2-locally checkable transitions

$$\frac{1}{2}\left(|ab\rangle - |cd\rangle\right)\left(\langle ab| - \langle cd|\right)$$

0-energy states

$$\cdots |ab\rangle \cdots$$
$$+ \cdots |cd\rangle \cdots$$





An introduction to

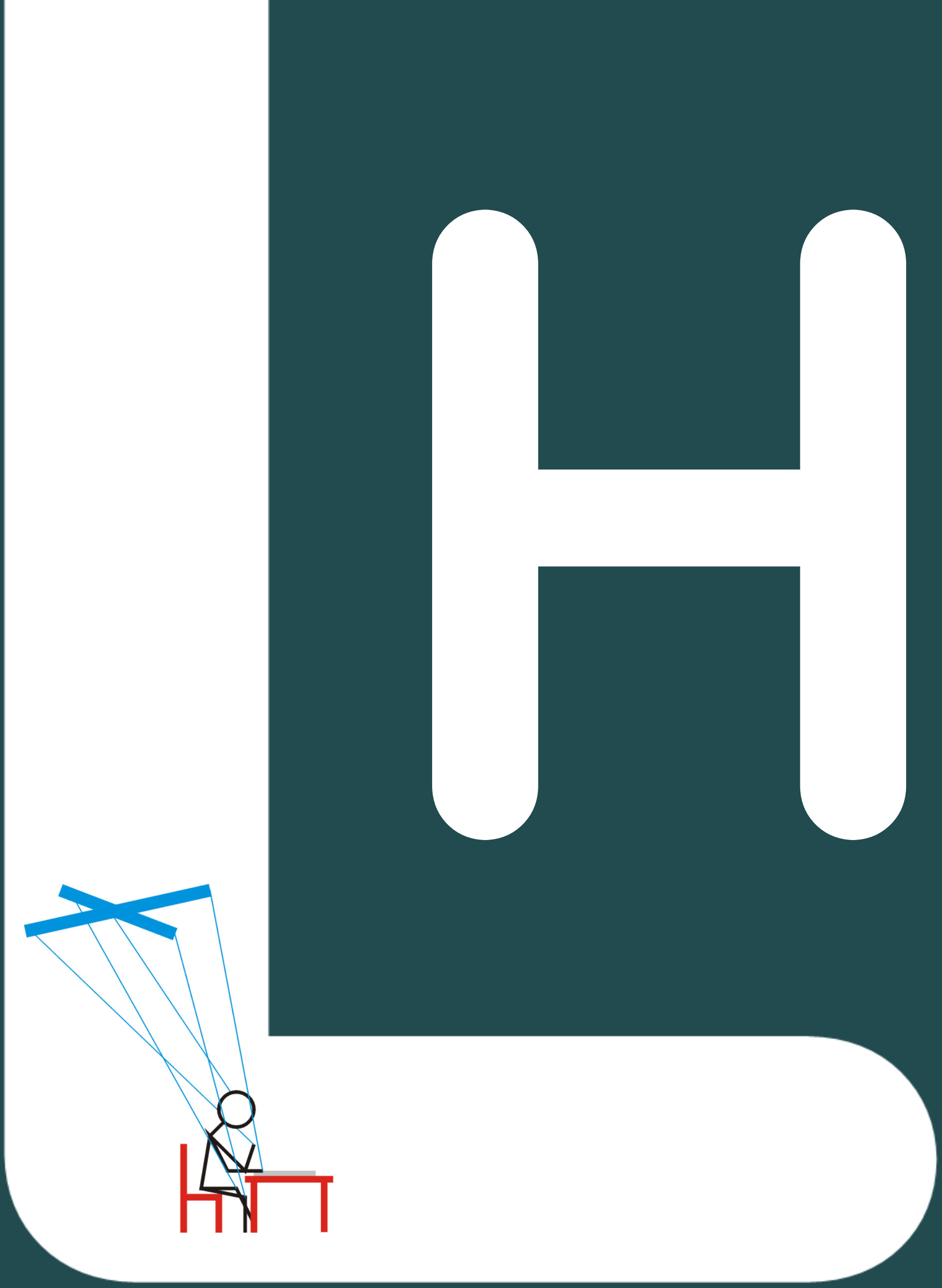
Local Hamiltonians & Quantum Complexity

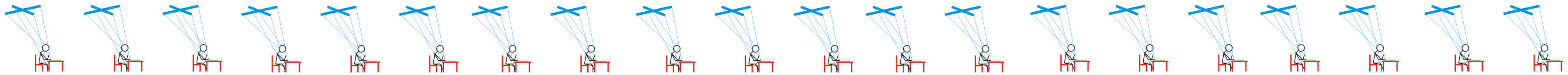
QMA
complete

history state

clock

constructions


LH



LH in 1D

Quantum SAT

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