QUANTUM WALKS

in continuous time on graphs

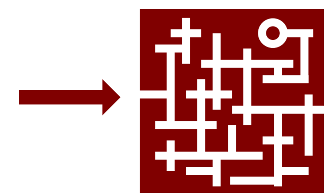
March 18, 2013
CoQuS colloquium

Based on work by E. Farhi, S. Gutmann, J. Goldstone, A. Childs, D. Gosset, Z. Webb, M. Kieferová, R. Somma, ...

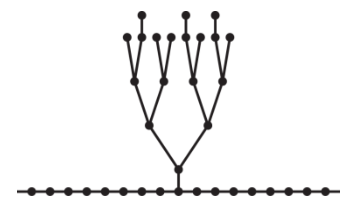
Daniel Nagaj
University of Vienna

1 CTQW overview
XX model, 1D dynamics

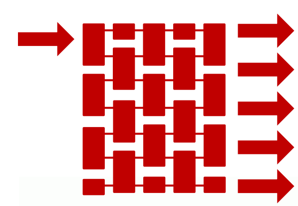
2 algorithmic applications
traversing, searching



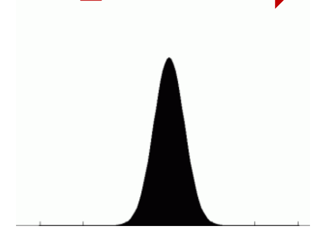
3 scattering and exploring
reflection, transmission & games



4 universal computation
single- and multi-particle



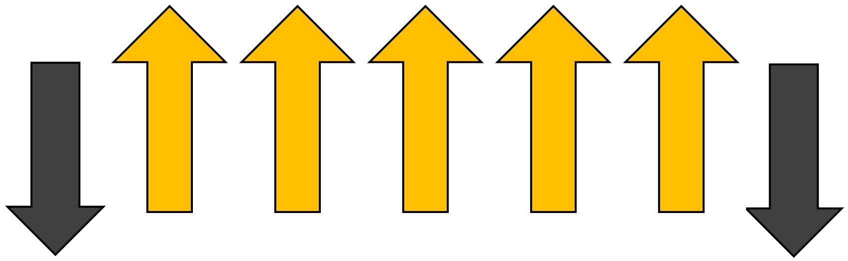
5 battling dispersion
keeping packets alive



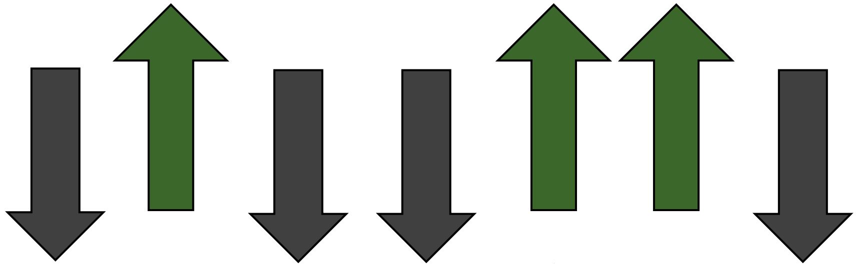
the basics

1 Dynamics of excitations

Spin-½ systems.

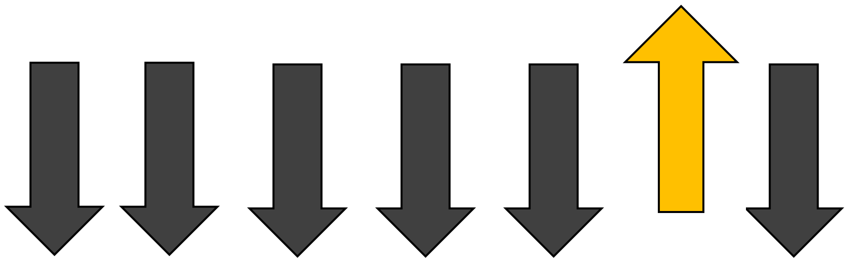


a single excitation hopping



multiple excitations

1 Dynamics of excitations



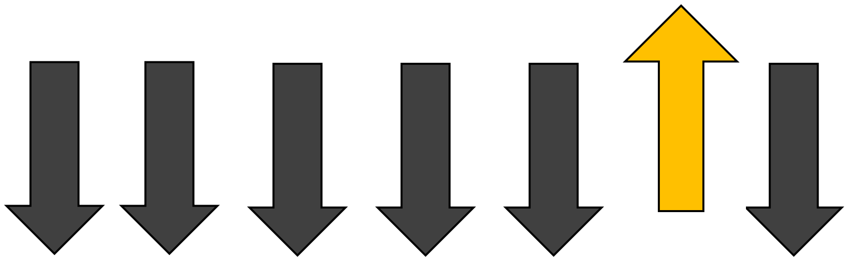
allowed transitions $01 \leftrightarrow 10$

translating to spins $\frac{1}{2}X_jX_k$ $\frac{1}{2}(1 - Z_jZ_k)$

the XX model $\frac{1}{2}(X_jX_k + Y_jY_k)$

1 Continuous-time quantum walks

A small basis.



the subspace with a single excitation

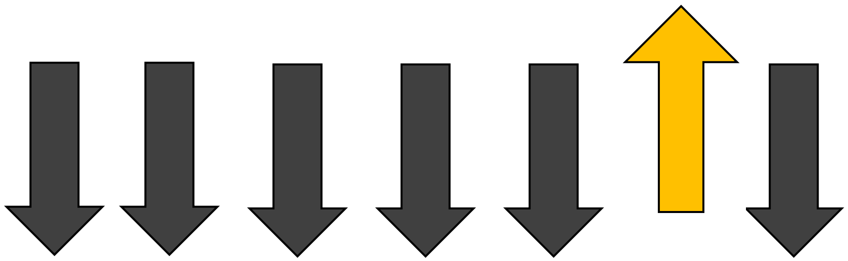
allowed transitions $01 \leftrightarrow 10$

a discrete basis: excitation location $|x\rangle$

the XX model $\frac{1}{2}\left(X_j X_k + Y_j Y_k\right)$

1 Continuous-time quantum walks

A small basis.



the subspace with
a single excitation

allowed transitions $01 \leftrightarrow 10$

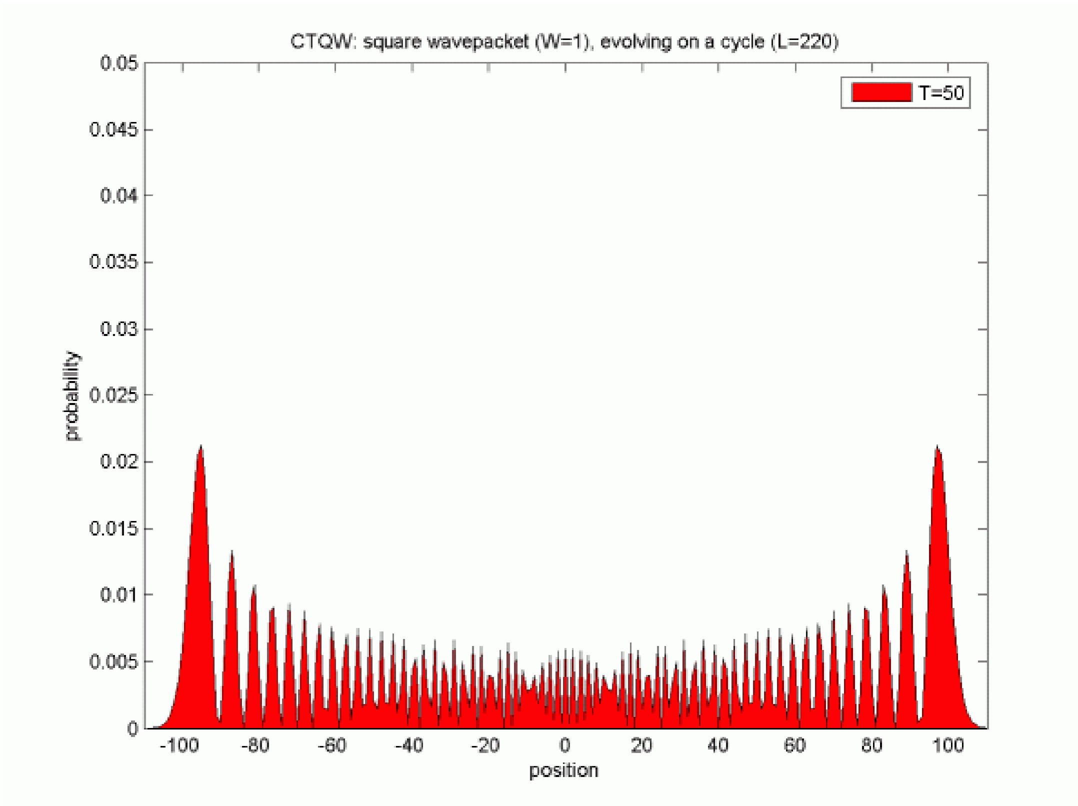
a discrete basis: excitation location $|x\rangle$

the Hamiltonian: (minus) the adjacency matrix

$$H_{1D} = -\sum_x \left(|x\rangle\langle x+1| + |x+1\rangle\langle x|\right)$$

1 A quantum walk on a line

How does it move?



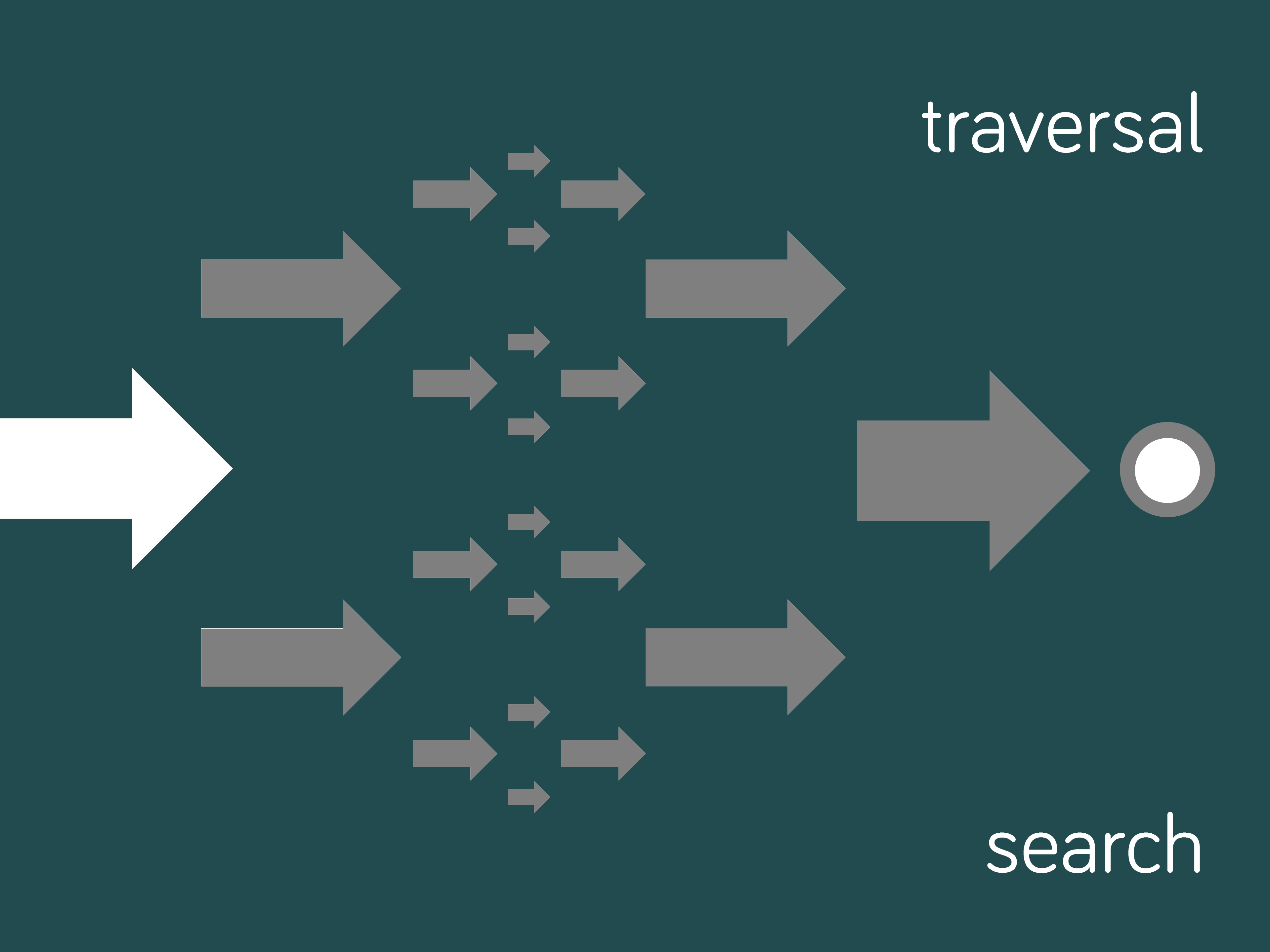
start: localized

plot: probability

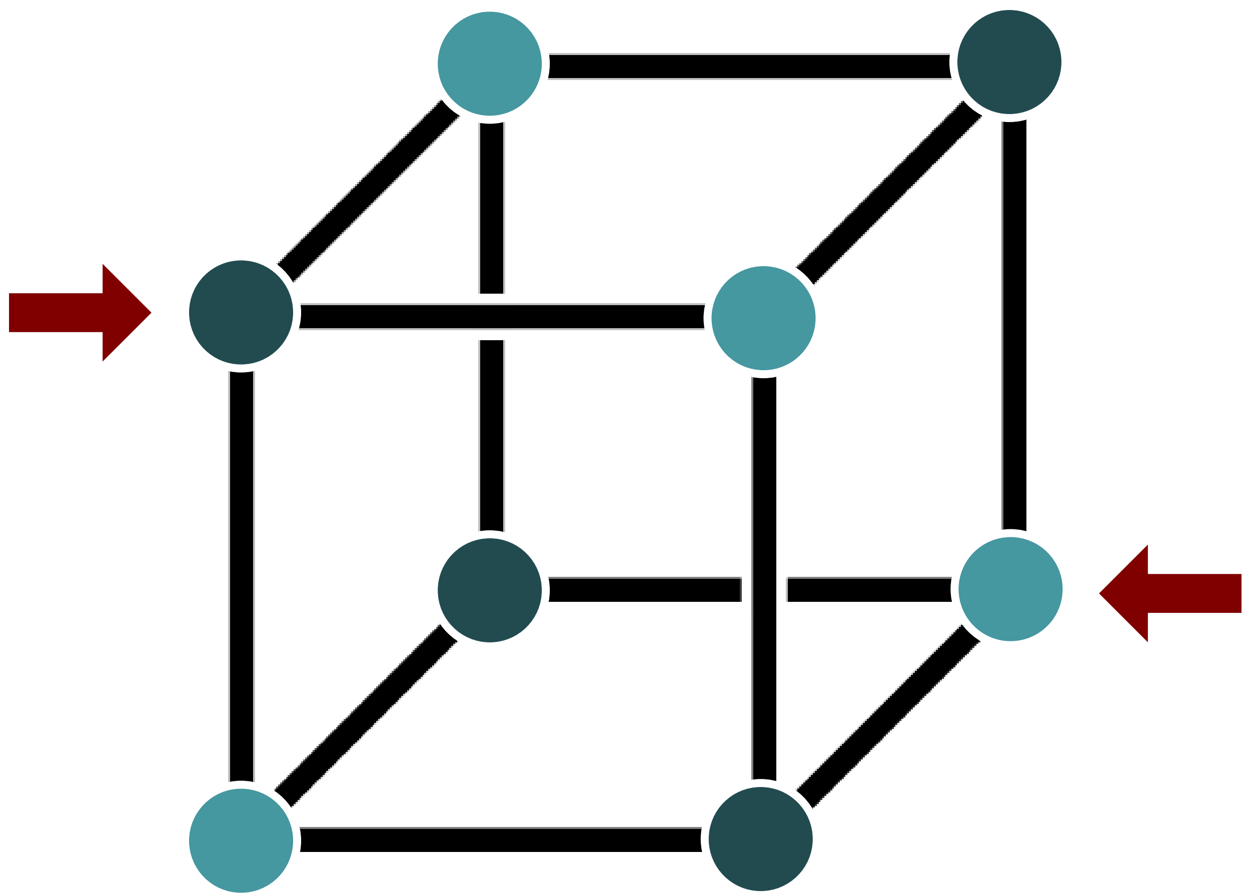
the mean distance grows linearly with time

the Hamiltonian: (minus) the adjacency matrix

$$H_{1D} = -\sum_x \left(|x\rangle\langle x+1| + |x+1\rangle\langle x| \right)$$


traversal
search

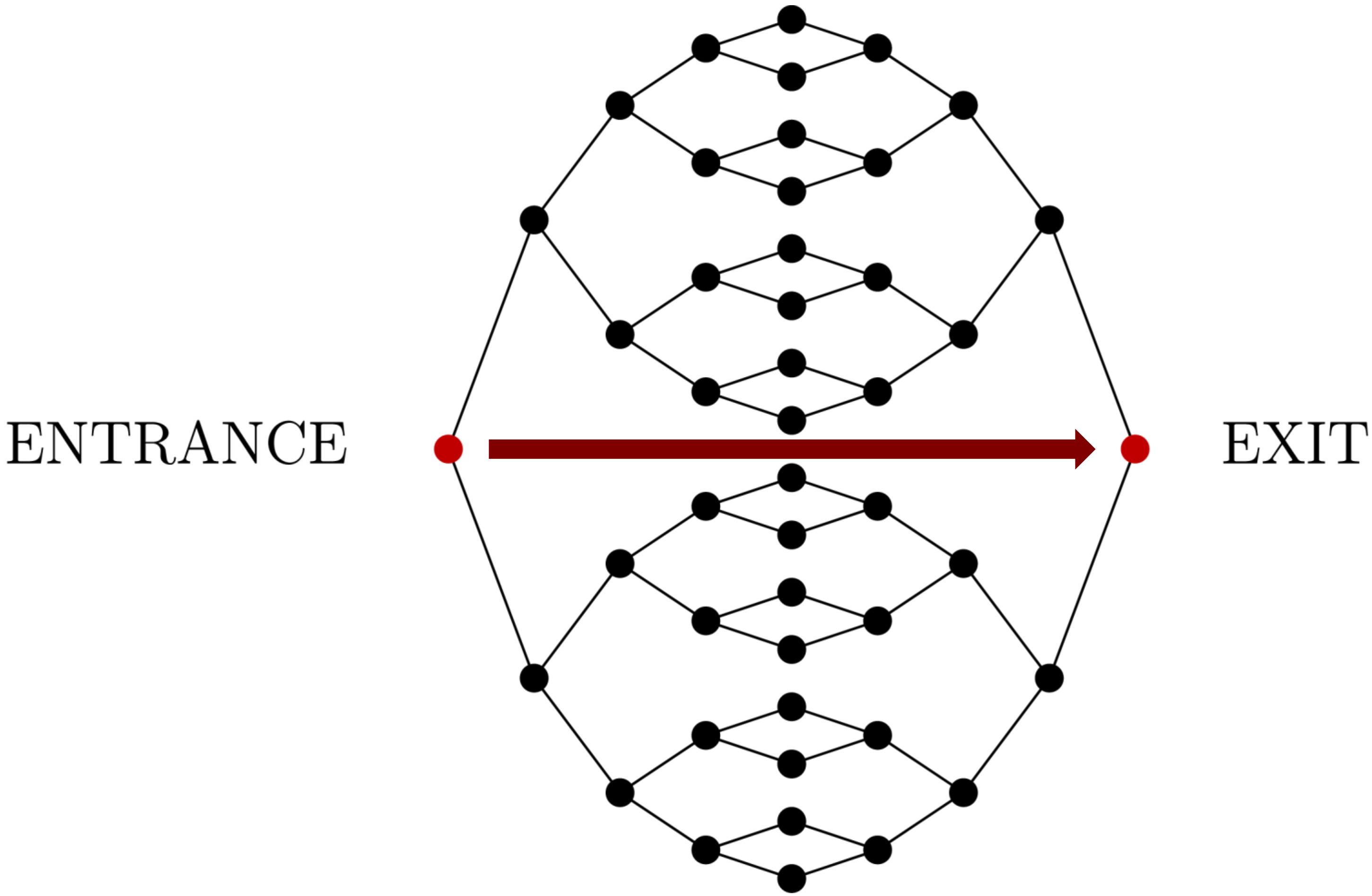
2 Traversing a (*d*-dimensional) hypercube



- a classical random walk would be stuck in the “middle”

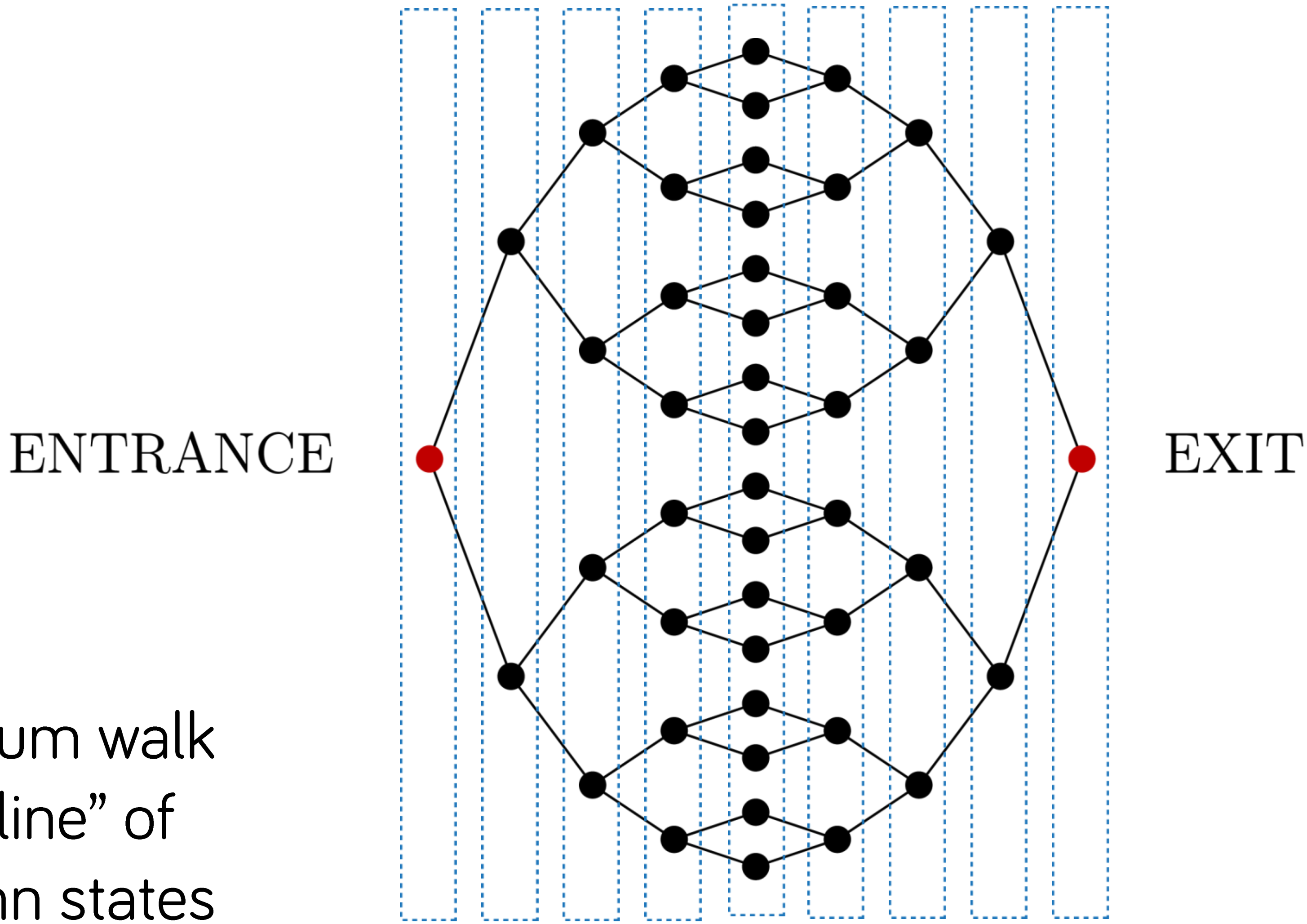
2 Traversing glued trees

Find the "name" of the EXIT vertex.



2 Traversing glued trees

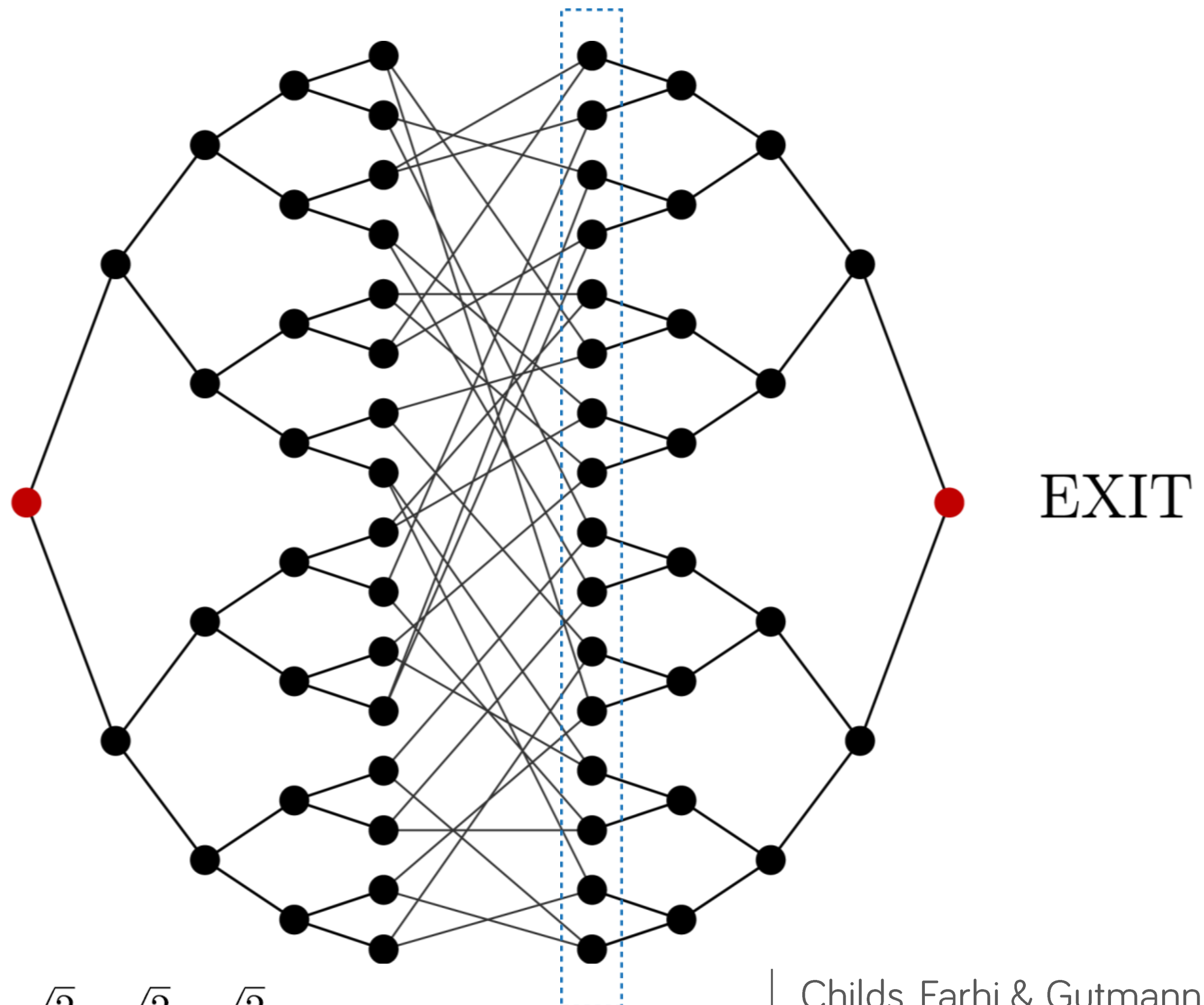
Symmetry for the win.



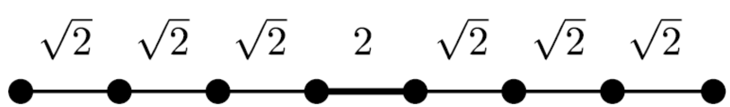
- quantum walk on a "line" of column states

2 Traversing randomly glued trees

An exponential speedup.



- a weighted quantum walk on a line

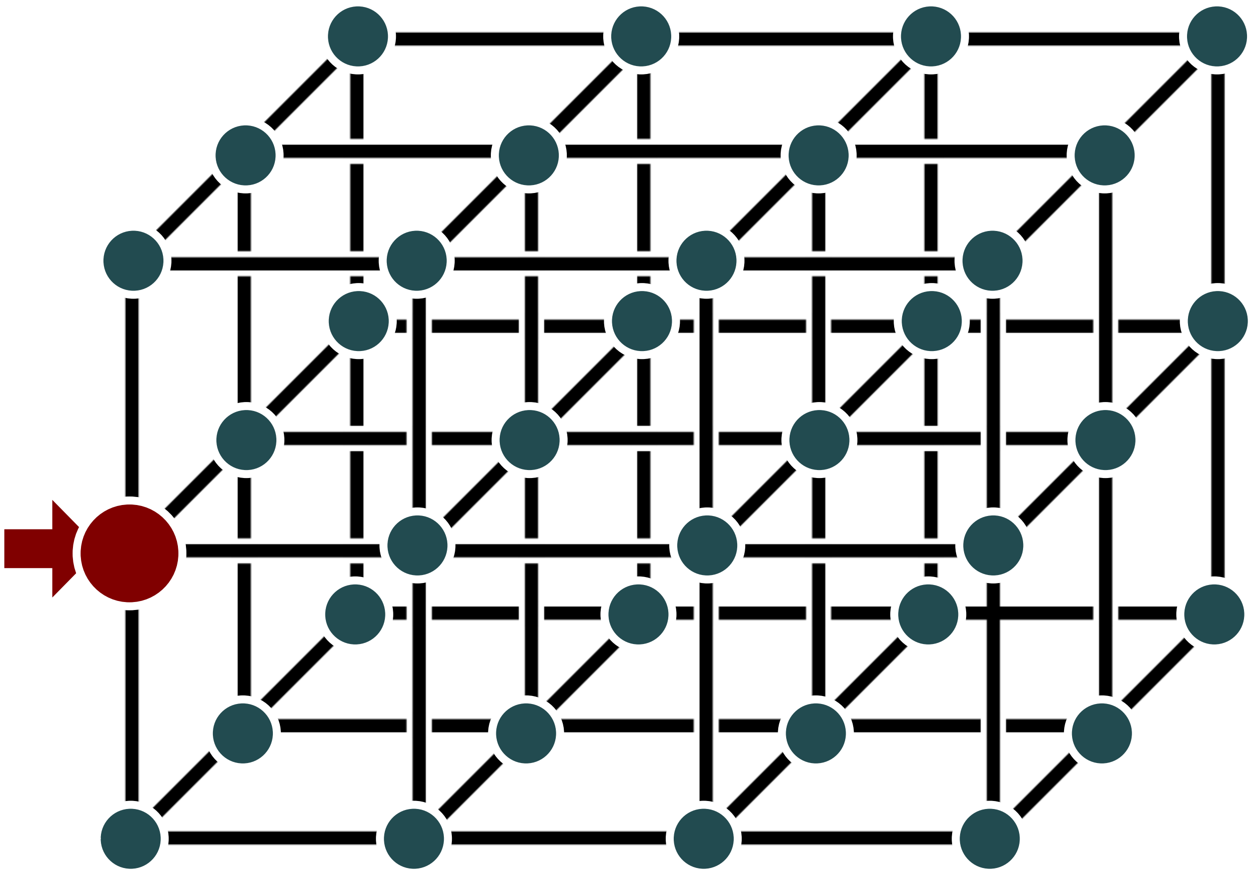


Childs, Farhi & Gutmann
QIP 1, 3543 (2002)

2 Searching on a *d*-dimensional lattice

One special vertex.

$$H = -A - |w\rangle\langle w|$$



- runtime: $O\left(\sqrt{N}\right)^{\text{in 2D}}_{\times polylog(N)}$, needs spin in 2D & 3D

Childs, Goldstone PRA 70, 022314 & 042312 (2004)

... scattering

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eringscatter ...

scatteringscatter ...

atteringscatter ...

3 Plane waves

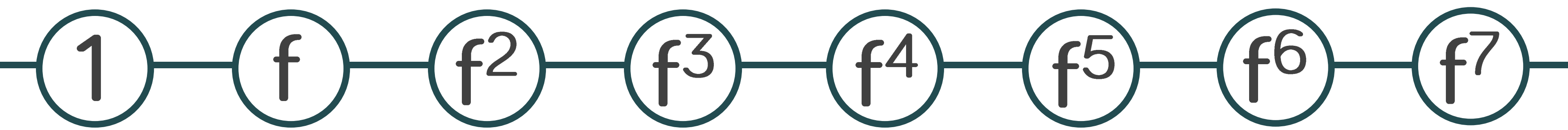
H: minus the adjacency.

- eigenstates

$$\langle \phi_p | x \rangle = e^{-ipx} = f^x$$

$$E_p = -2\cos p$$

$$-\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



3 A plane wave with p=π/2

H: minus the adjacency.

- eigenstates

$$\langle \phi_{\frac{\pi}{2}} | x \rangle = e^{-i\frac{\pi}{2}x}$$

$$E_p = -2\cos p$$

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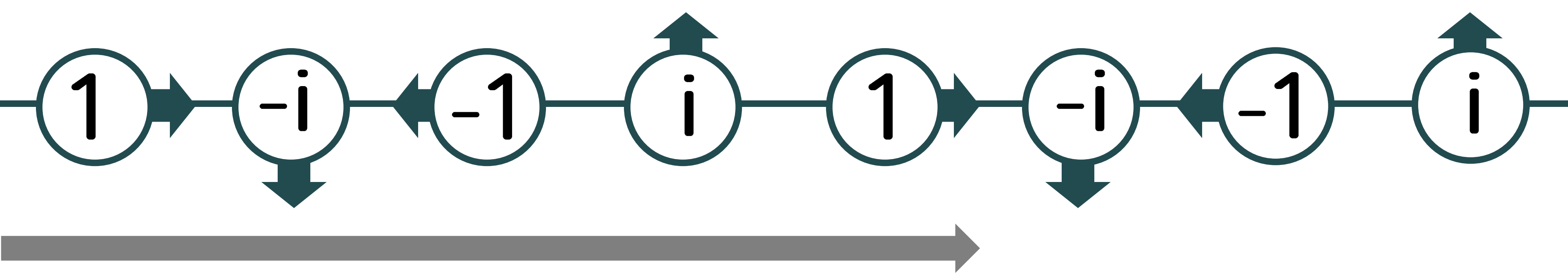


a right-moving plane wave e^{-ipx}

3 A plane wave with p=π/2

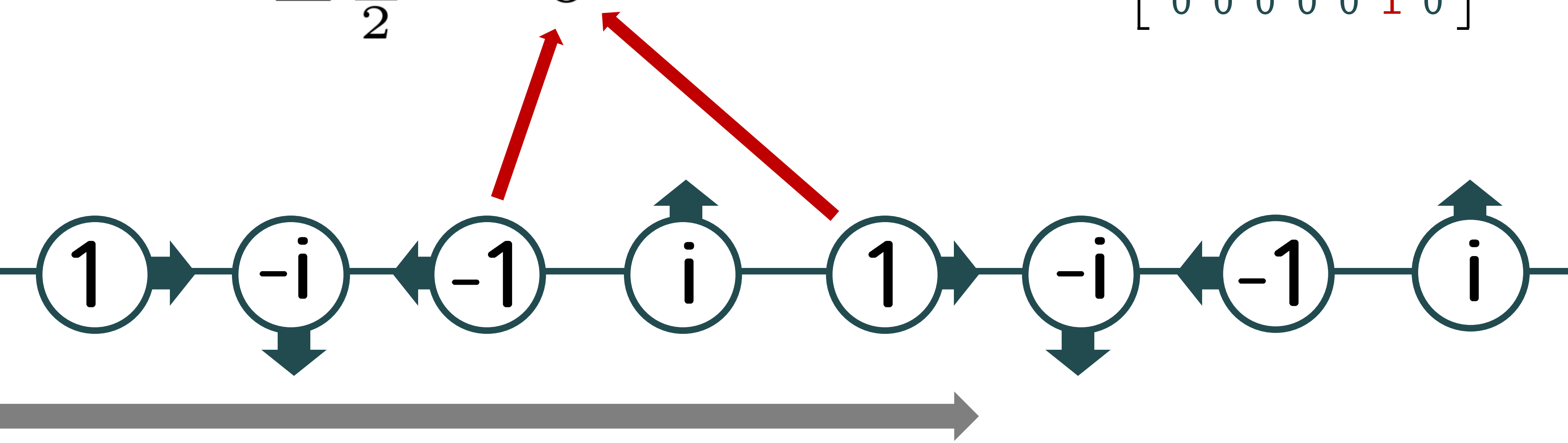
H: minus the adjacency.

- eigenstates

$$\langle \phi_{\frac{\pi}{2}} | x \rangle = e^{-i\frac{\pi}{2}x}$$

$$E_{\frac{\pi}{2}} = 0$$

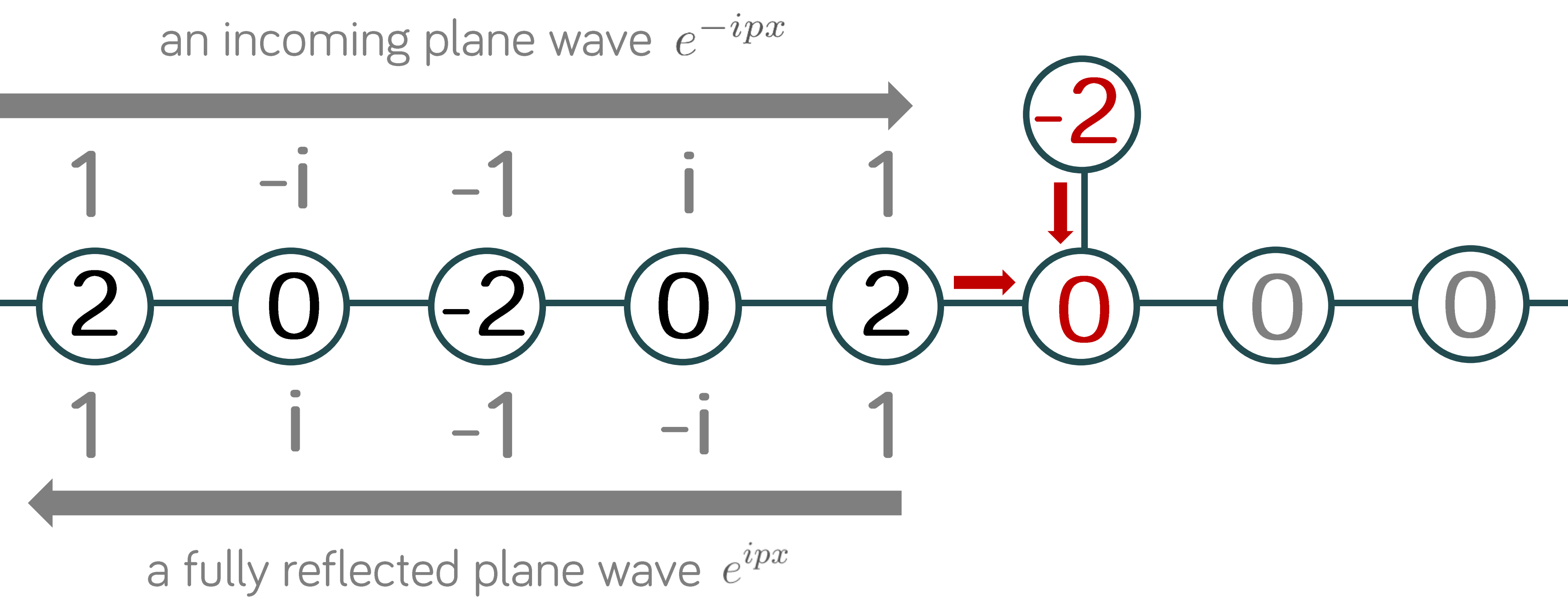
$$-\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



a right-moving plane wave e^{-ipx}

3 A plane wave with p=π/2

- look for an eigenstate with $E = 0$



3 A plane wave with p=π/2

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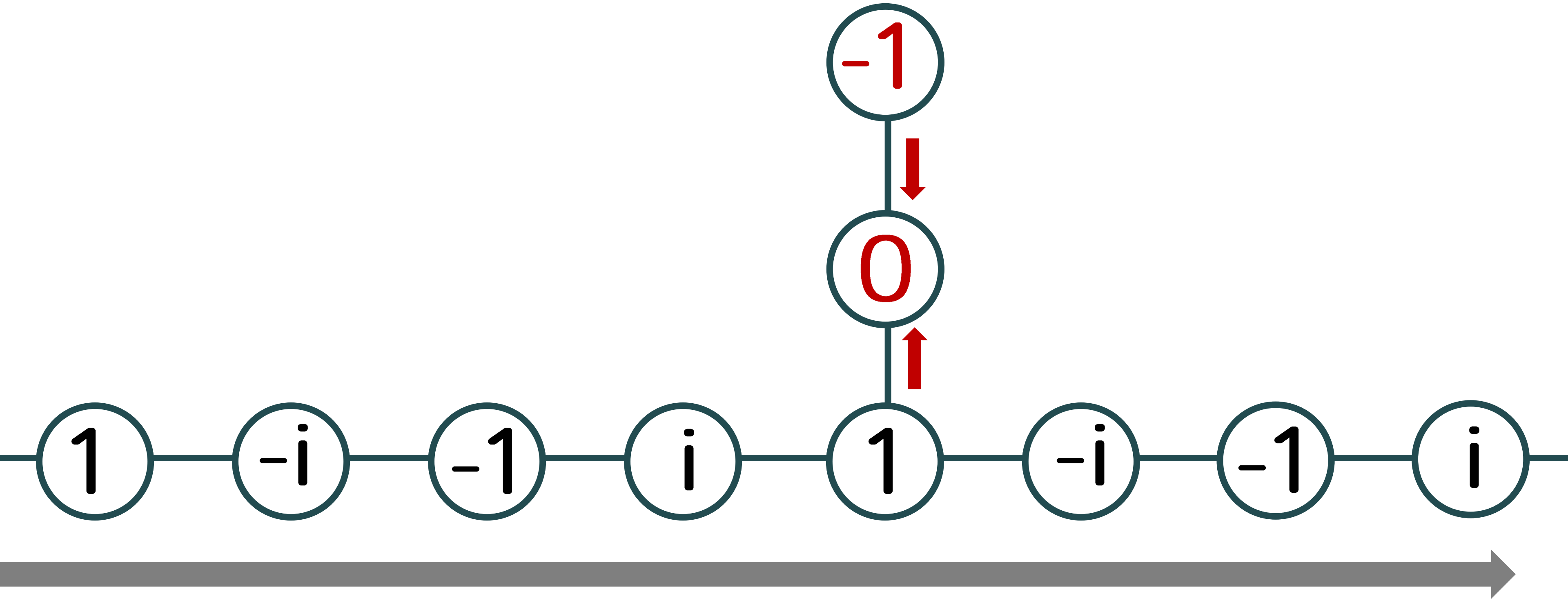
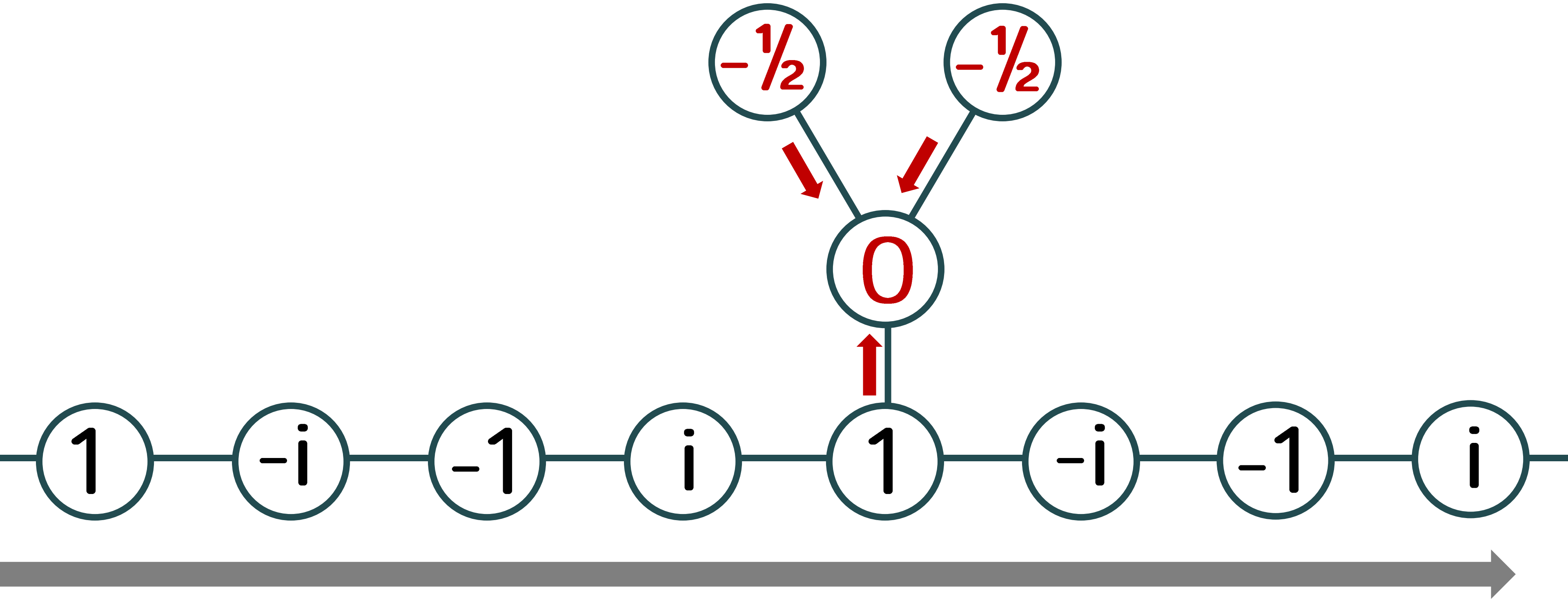


a fully transmitted plane wave e^{-ipx}

3 A plane wave with p=π/2

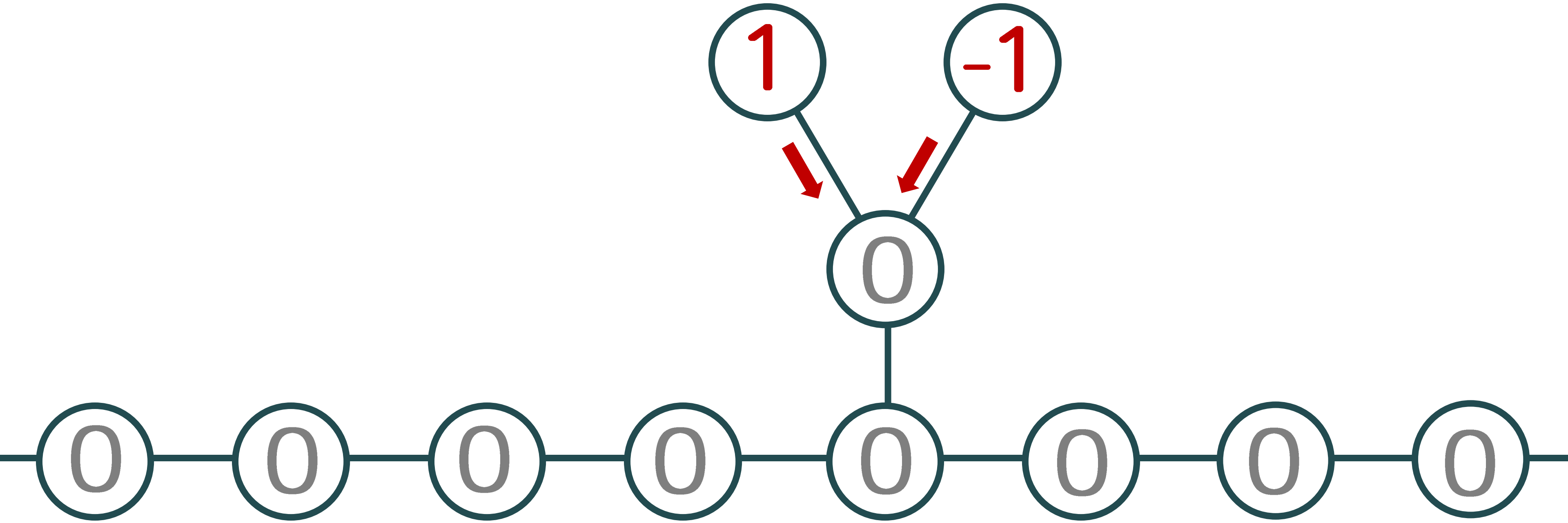
- look for an eigenstate with $E = 0$



a fully transmitted plane wave

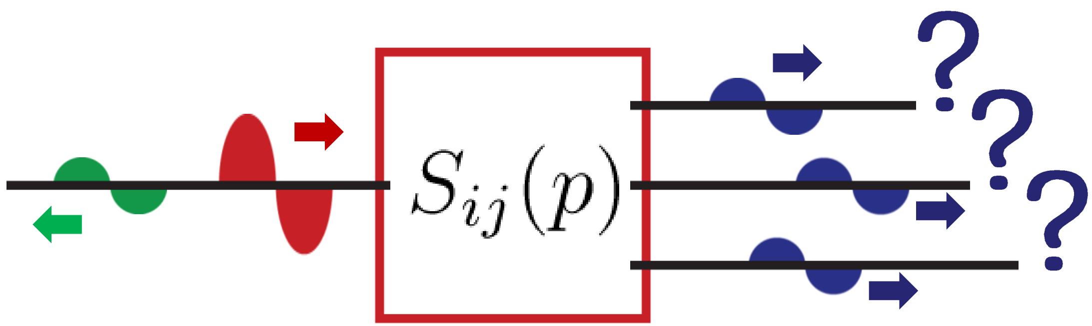
3 Bound states

- another eigenstate with $E = 0$

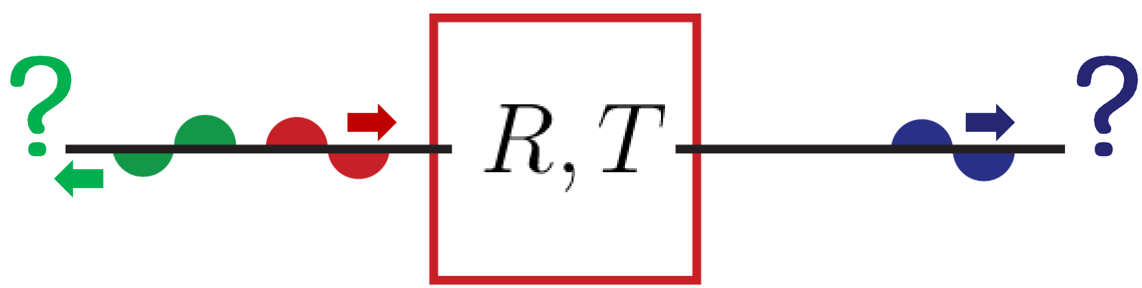


3 Quantum walks and scattering

- infinite runways
- far away: plane waves (with momentum p)



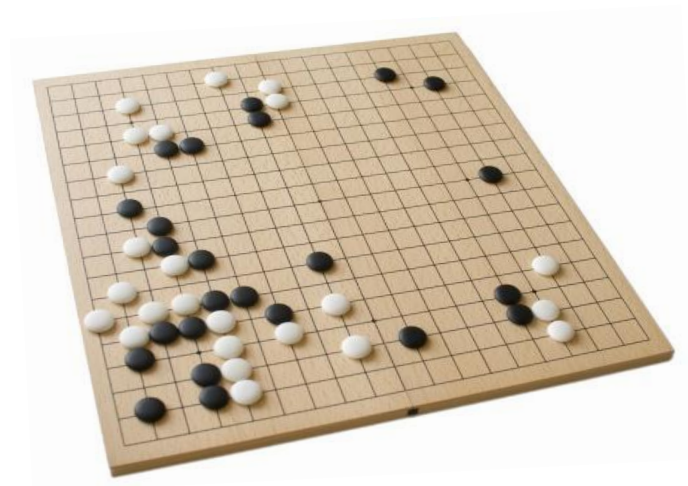
scattering matrix



reflection/transmission coefficient

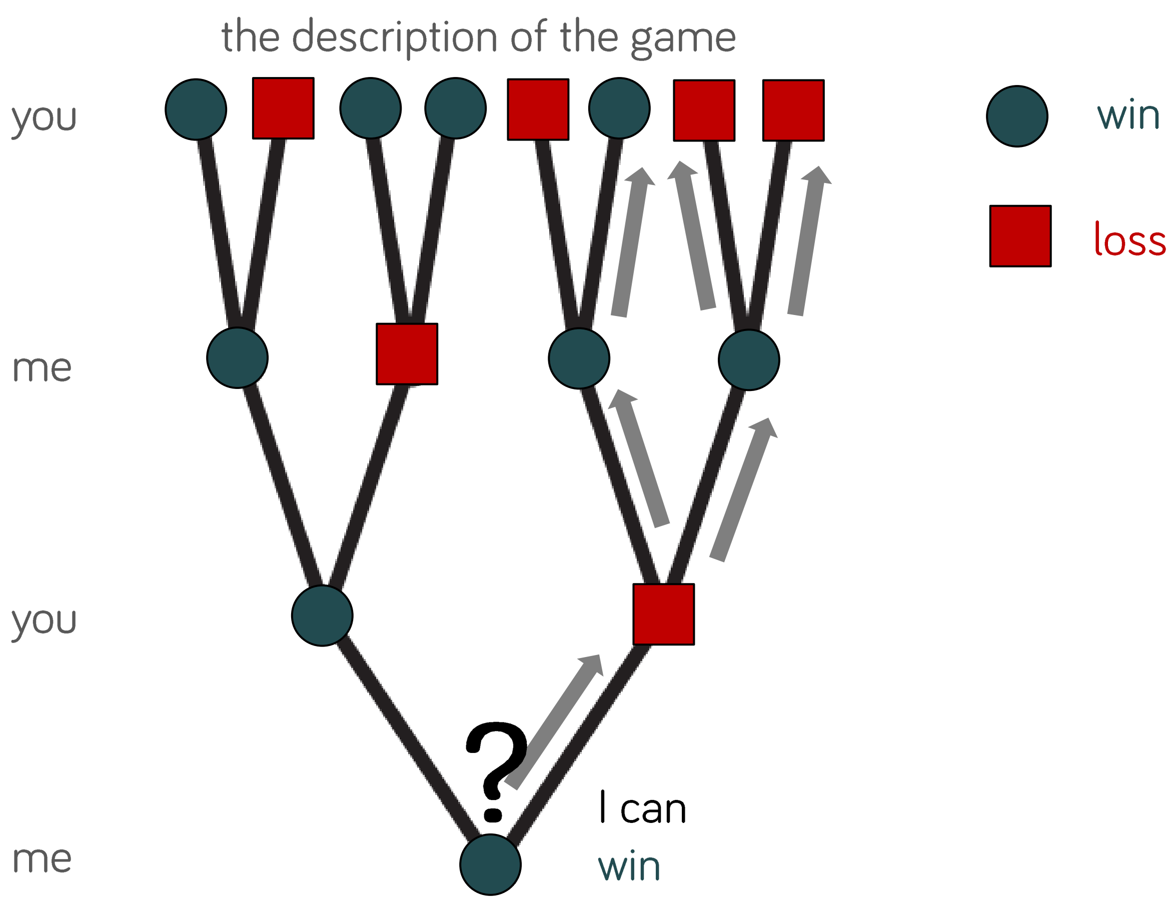
3 Playing games

- my move… your move…



3 Evaluating game trees

- my move... your move... can I (the first player) win?



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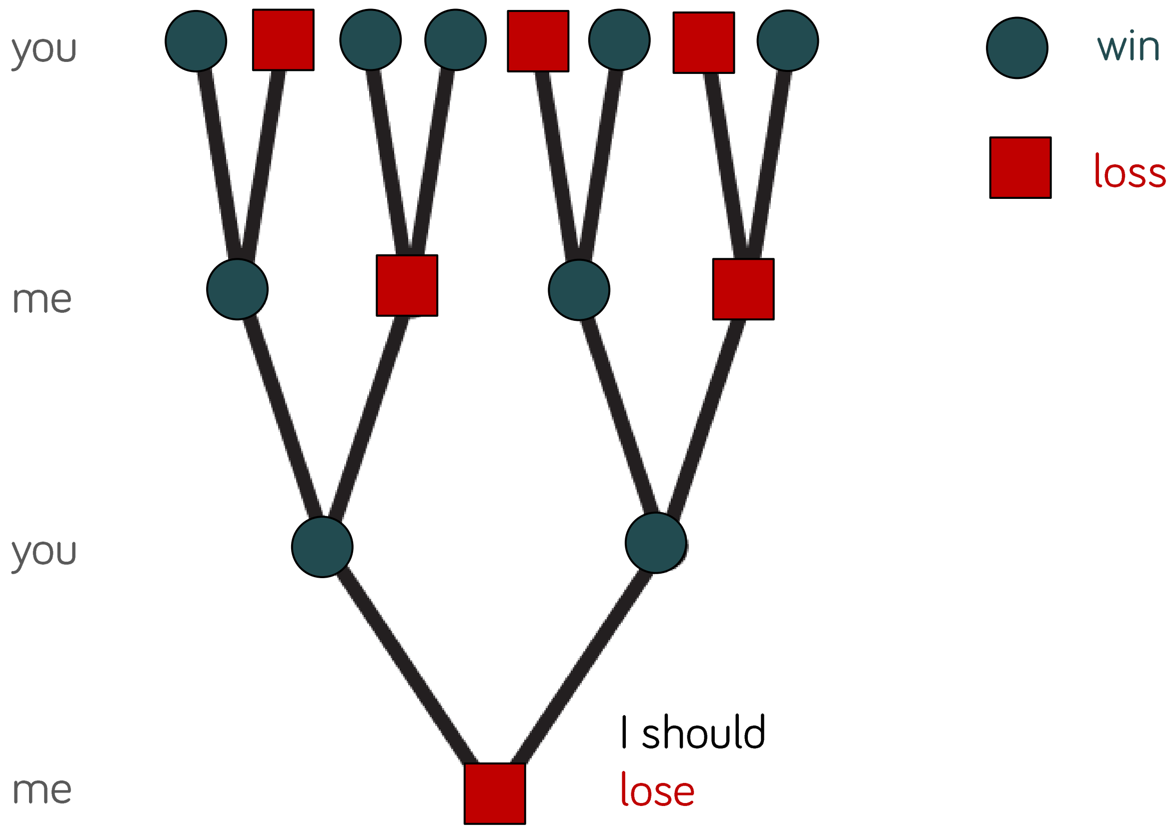
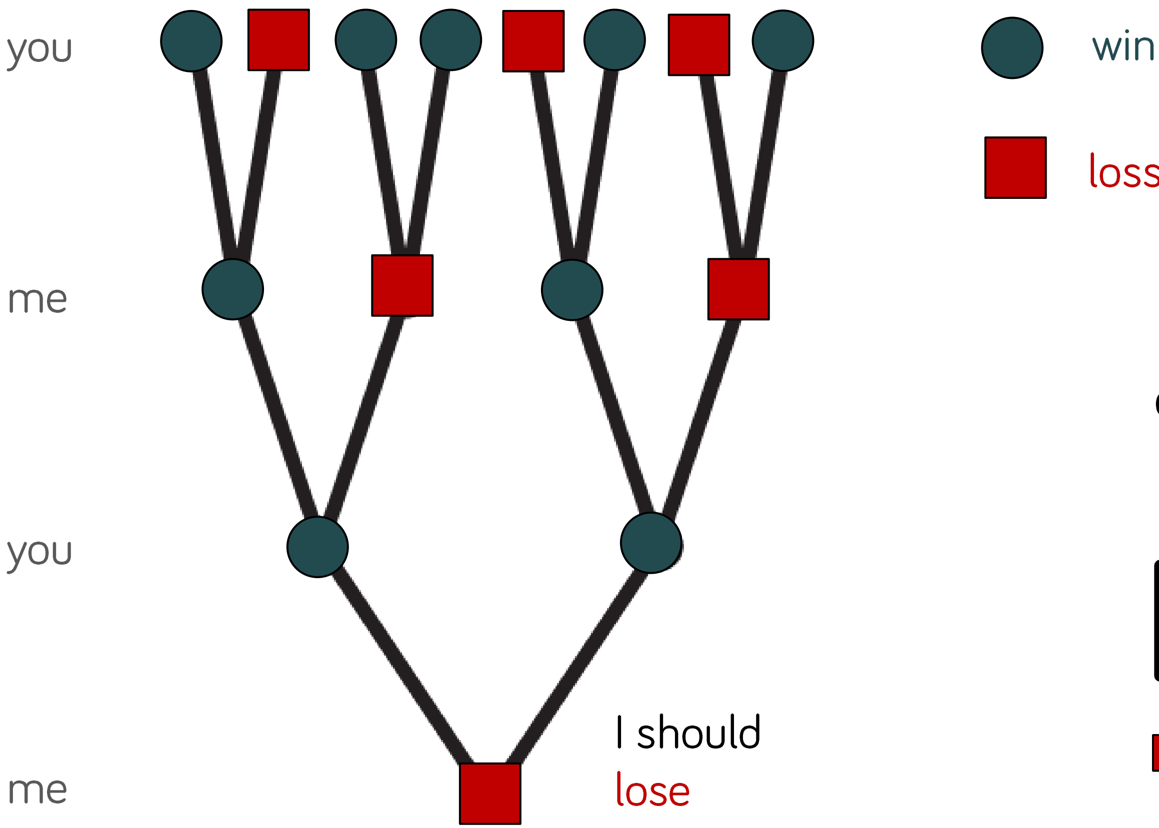


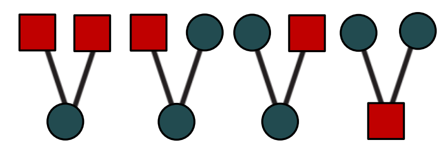
3 Evaluating NAND trees

- my move… your move… can I (the first player) win?

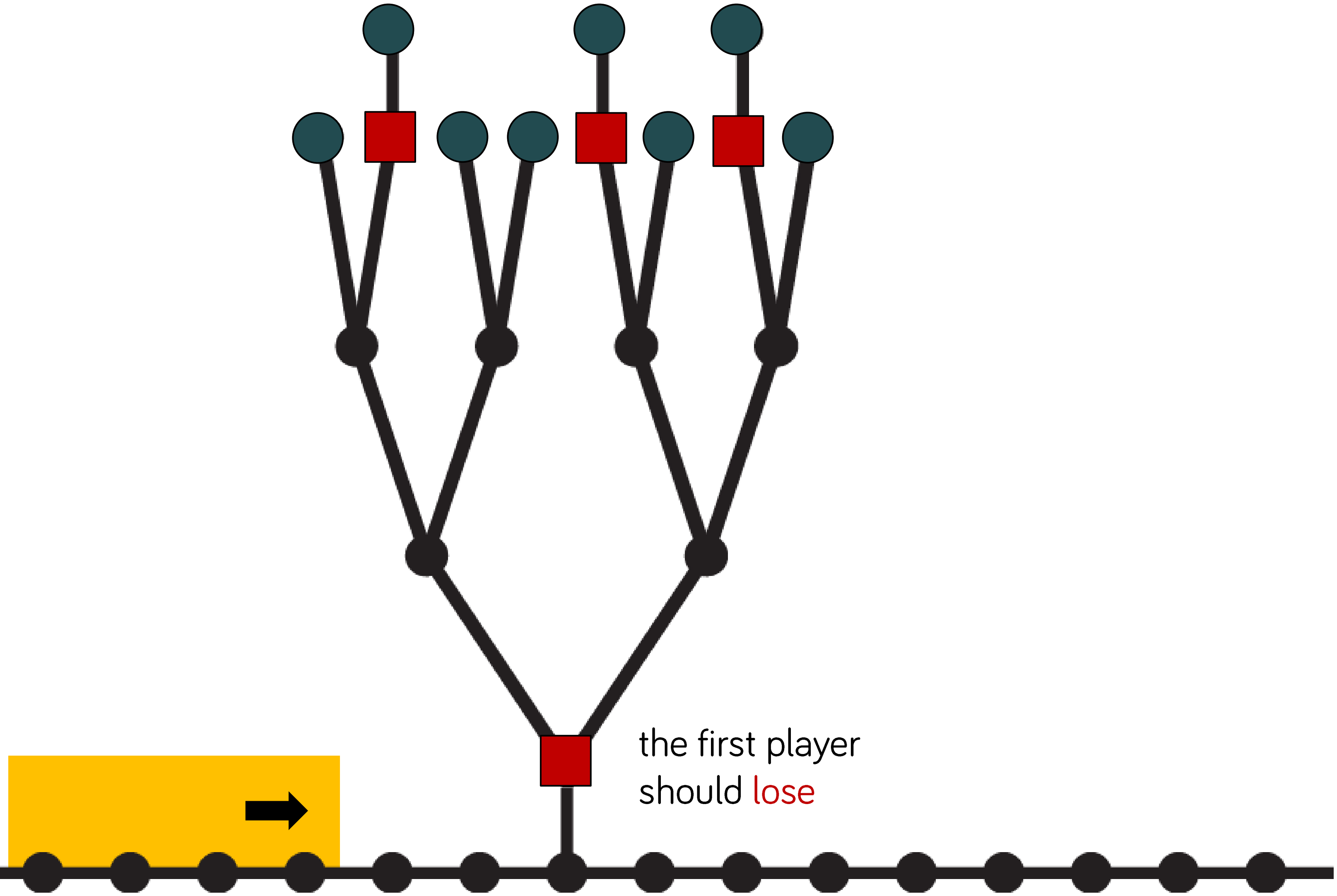


classically: $N^{0.753}$

NAND

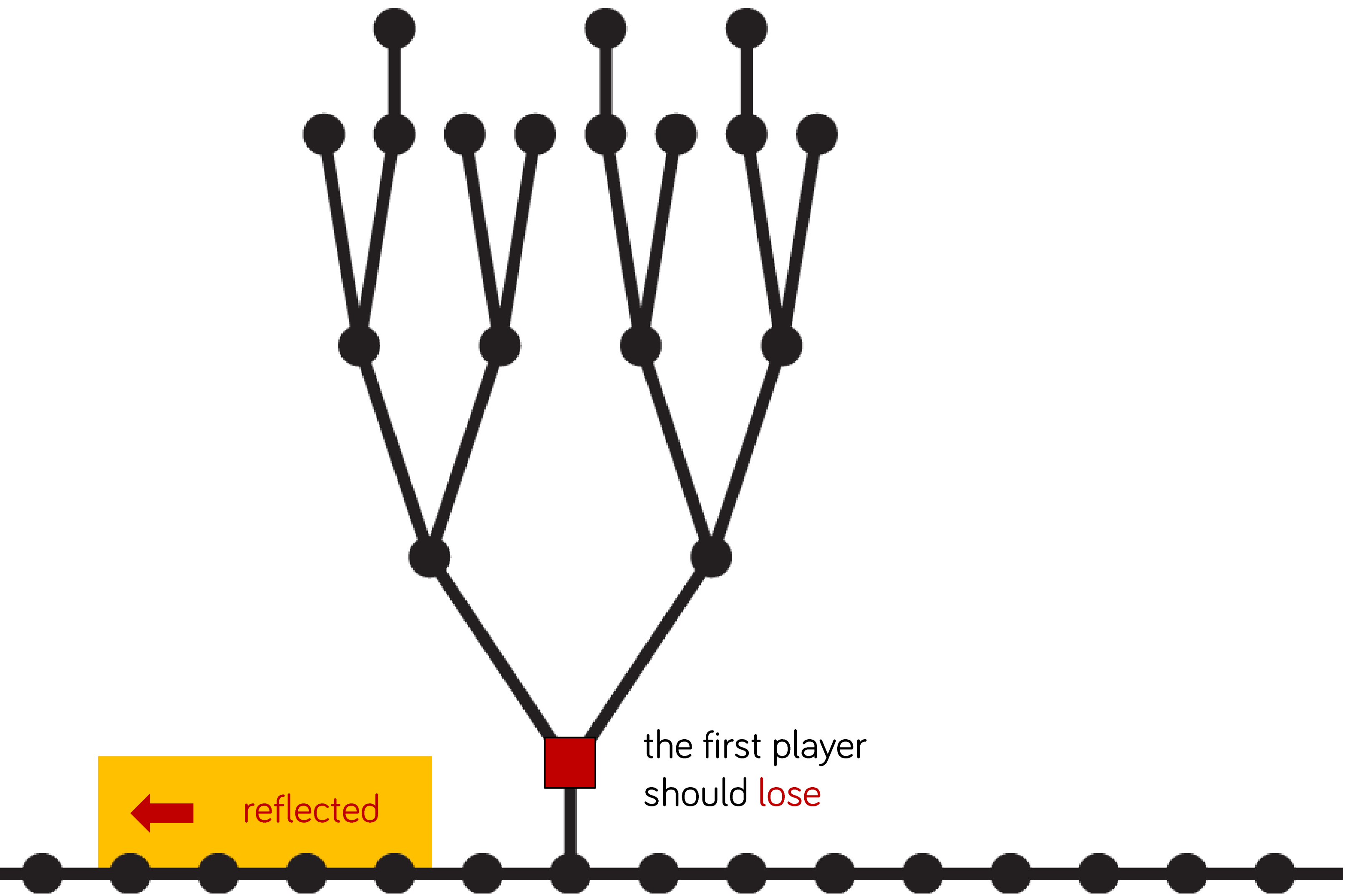


3 Evaluating NAND trees by scattering



the first player
should lose

3 Evaluating NAND trees by scattering



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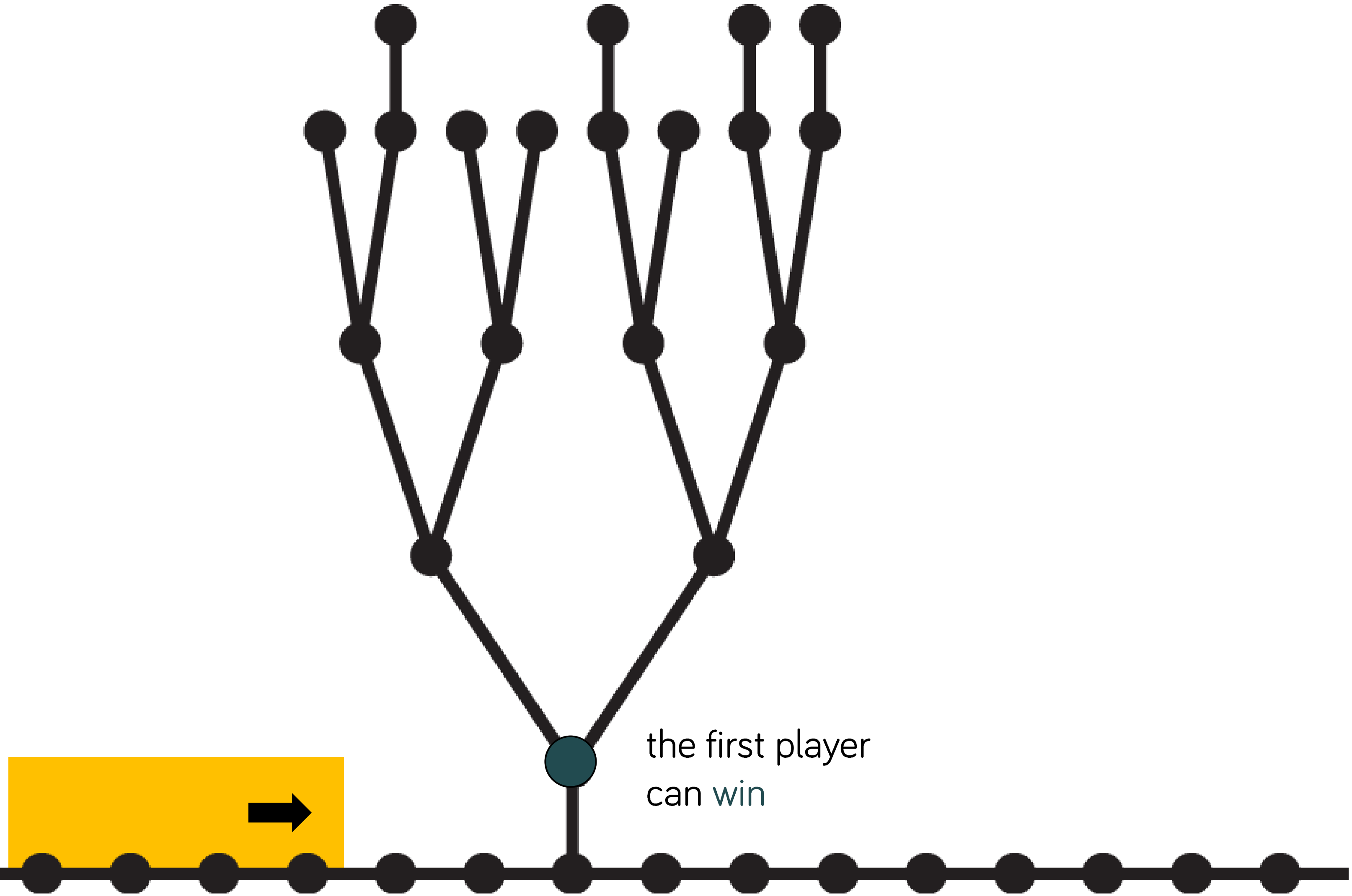
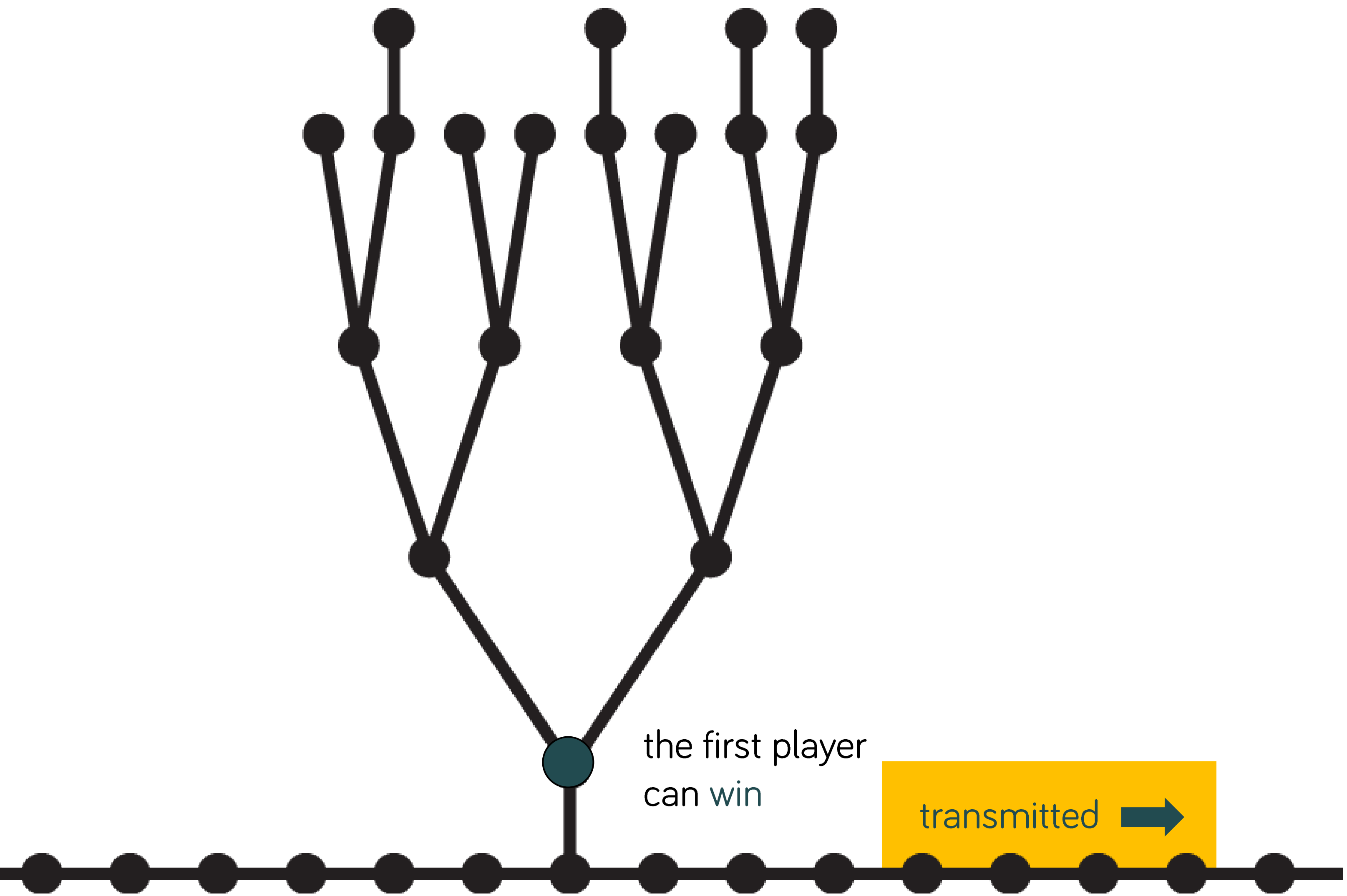


the first player
can win

3 Evaluating NAND trees by scattering



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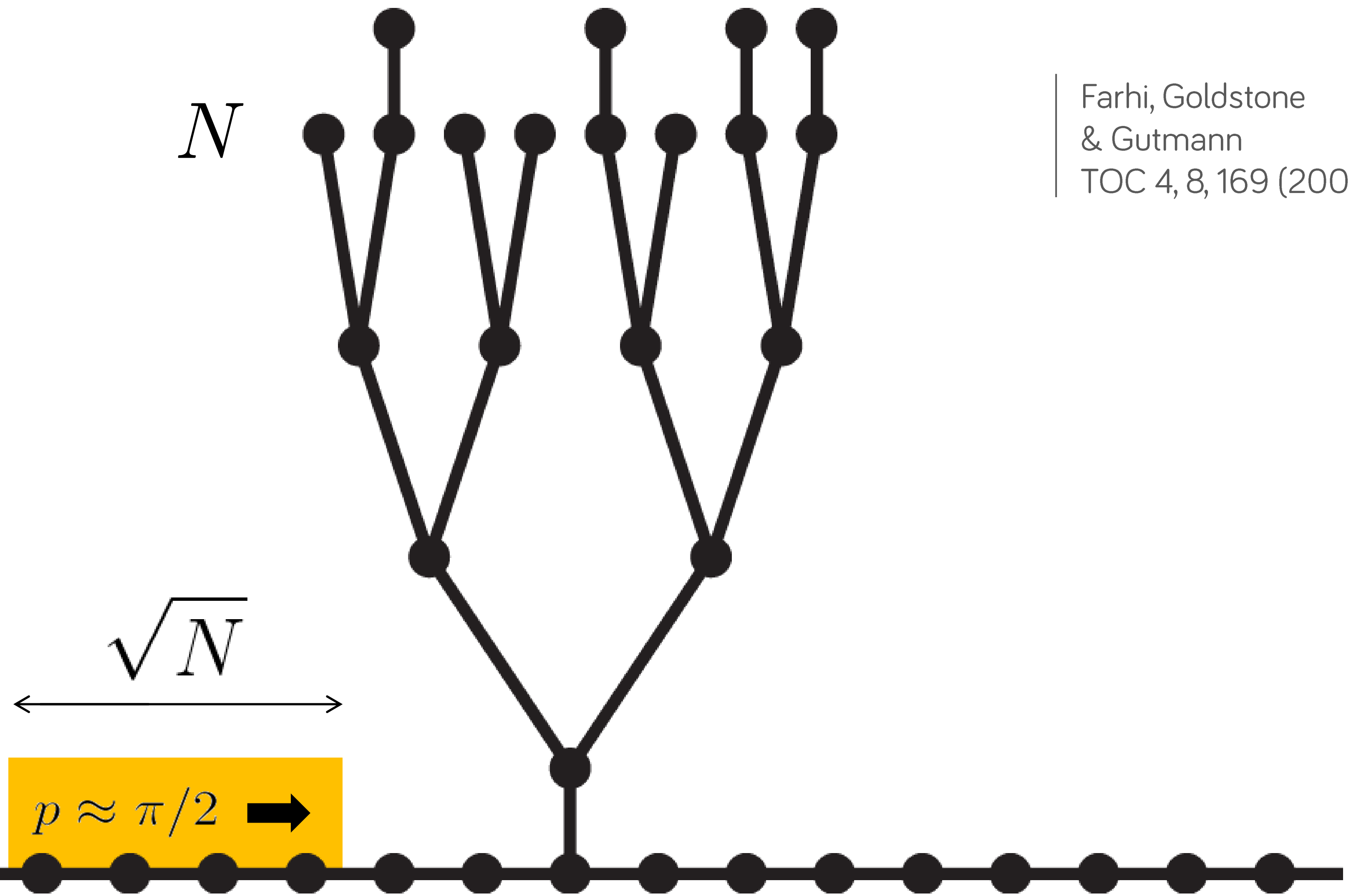
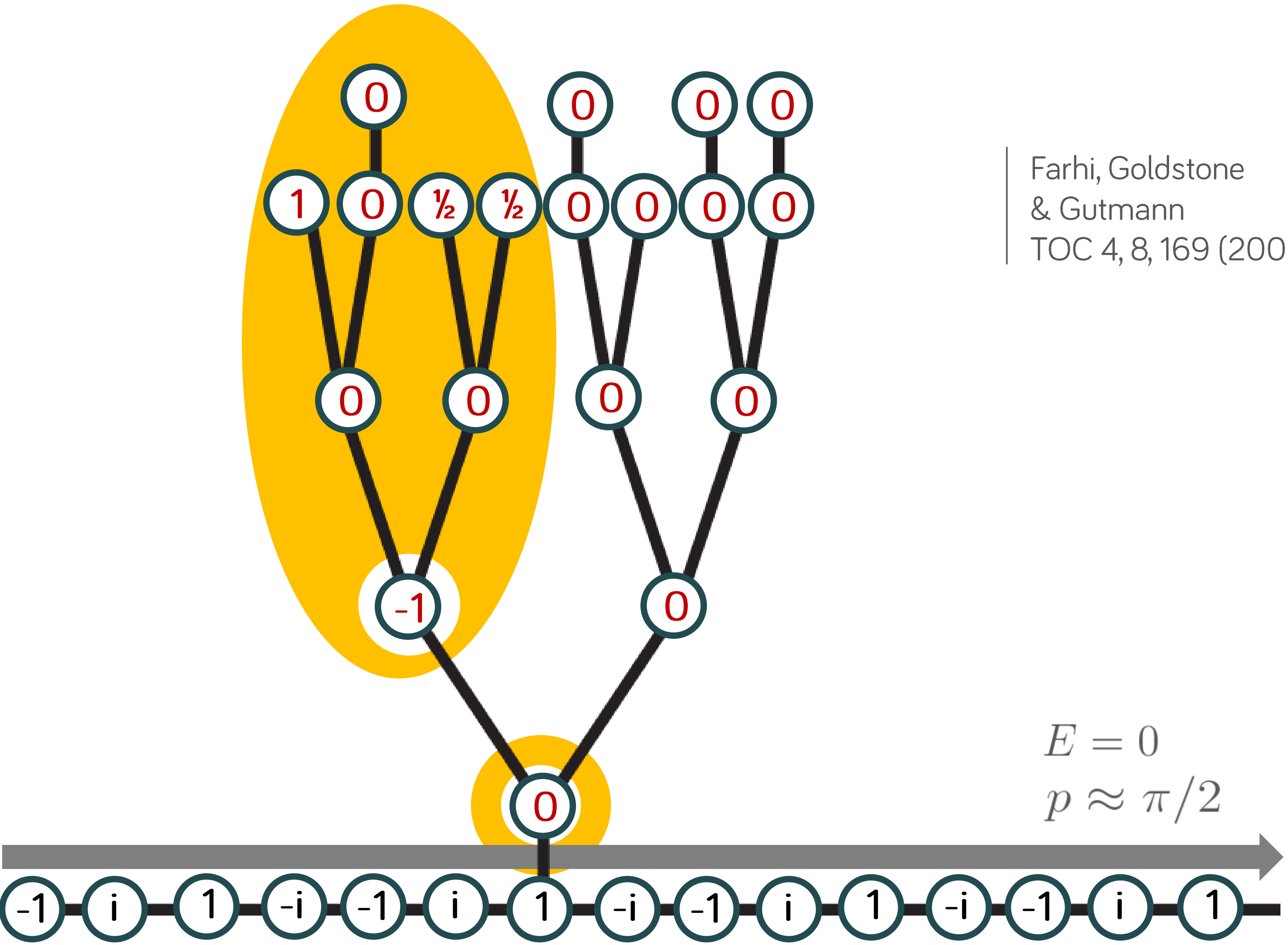


Farhi, Goldstone & Gutmann
TOC 4, 8, 169 (2008)

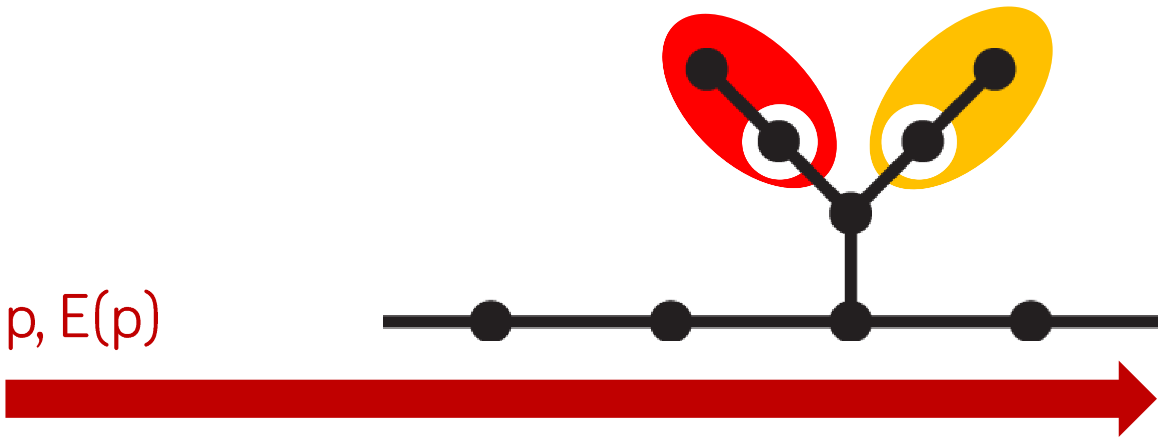
3 Evaluating NAND trees by scattering



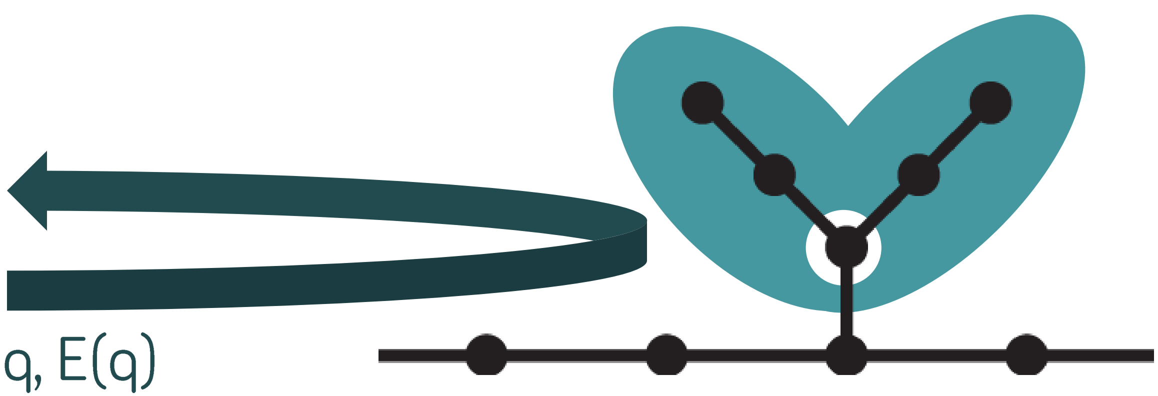
Farhi, Goldstone & Gutmann TOC 4, 8, 169 (2008)

$E = 0$
$p \approx \pi/2$

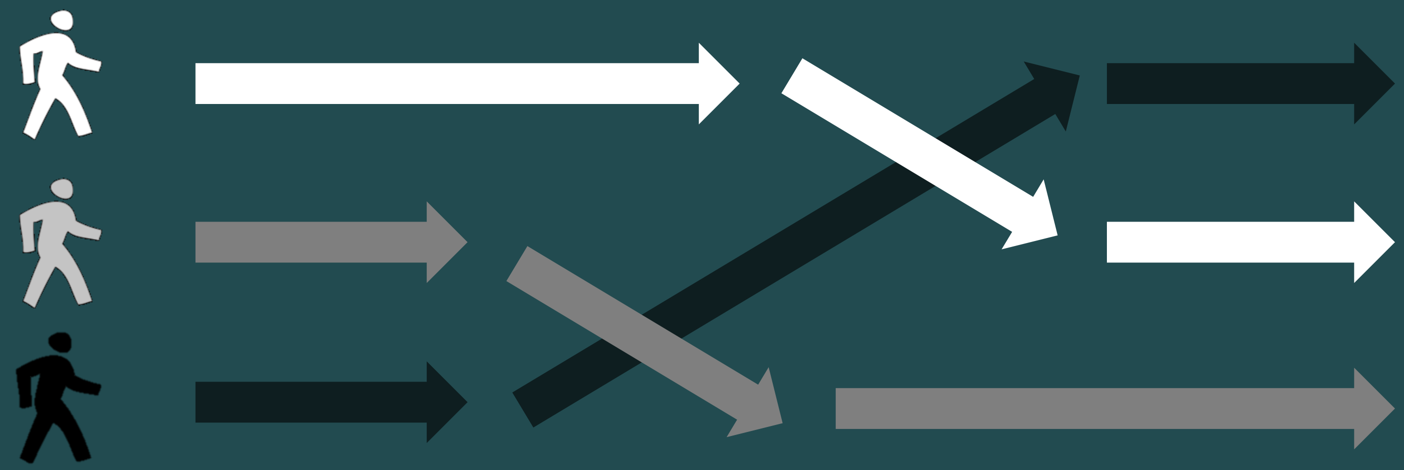
3 Does a tree a transmit or reflect?



at least one eigenvector with a nonzero amplitude



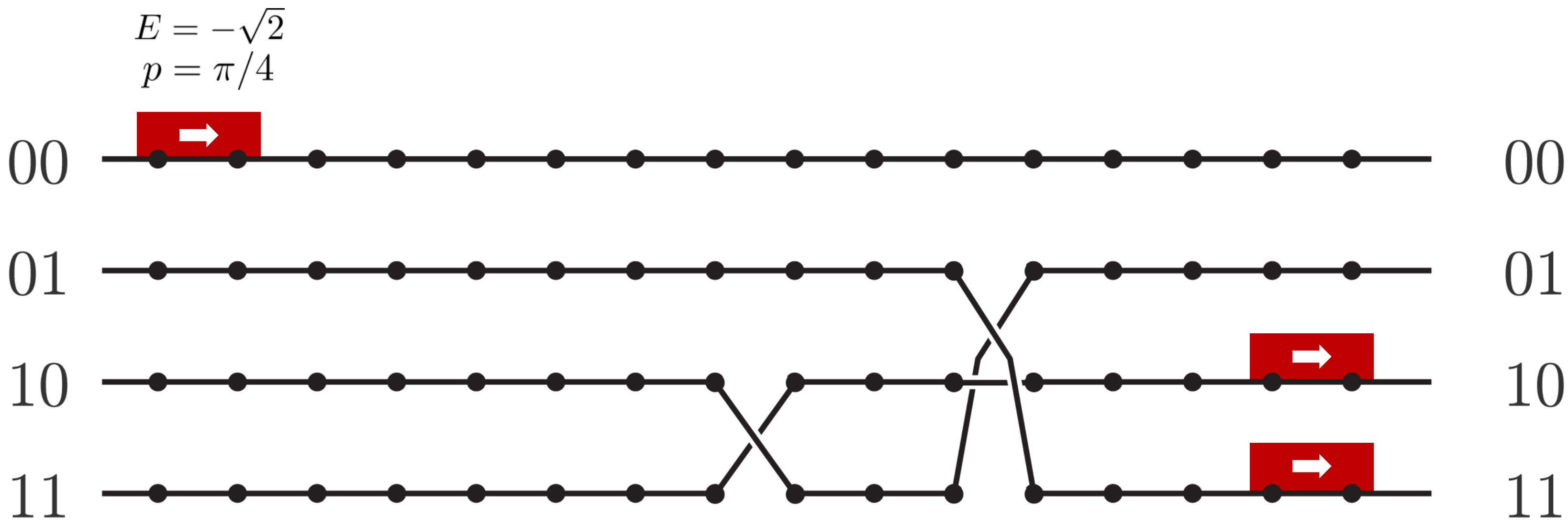
an eigenvector with a nonzero amplitude

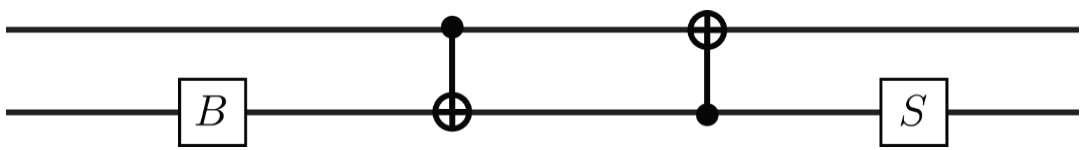


com + put = ation

Universal computation by QW

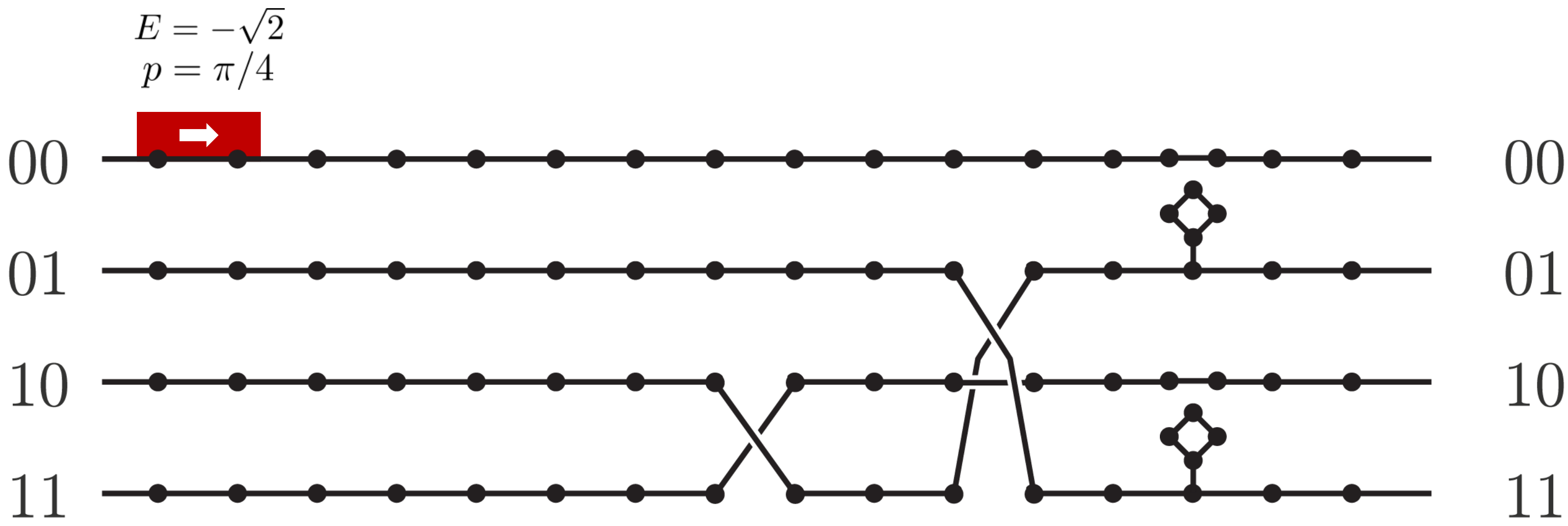
CNOTs.

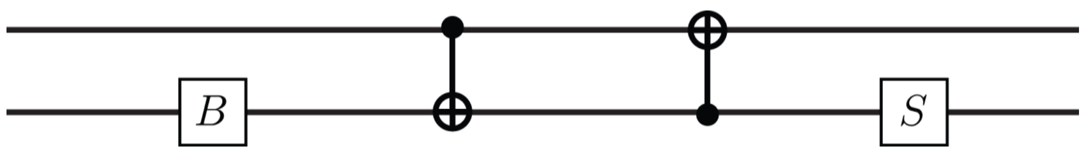




Universal computation by QW

Phase gate.





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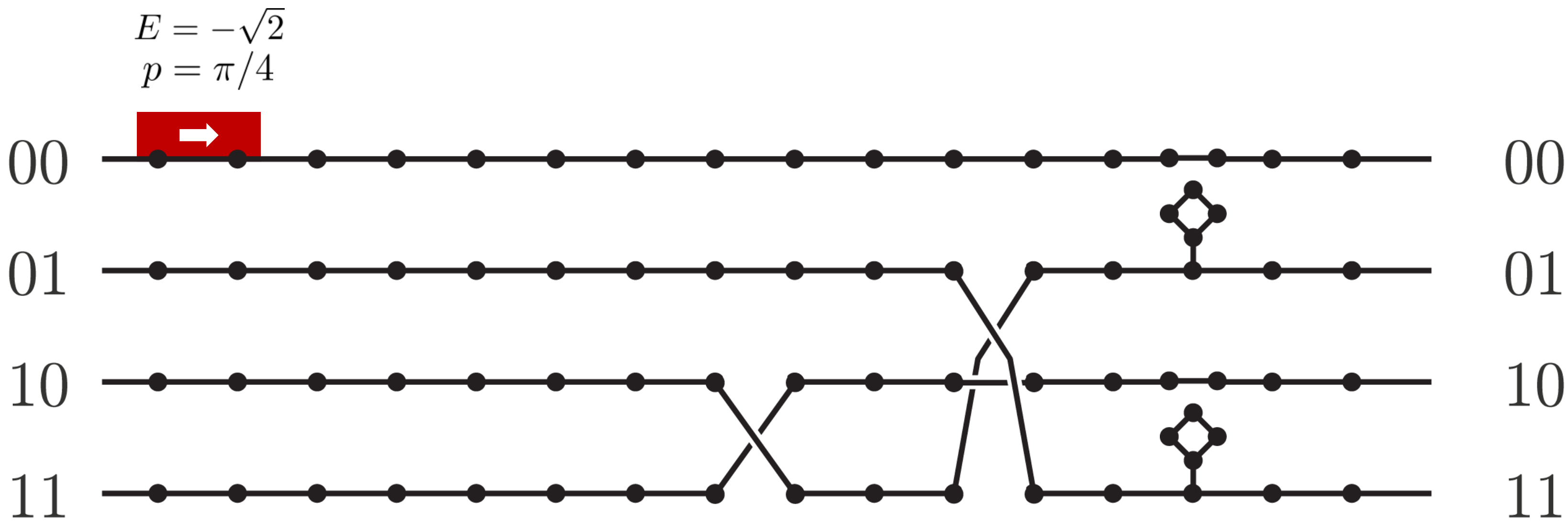
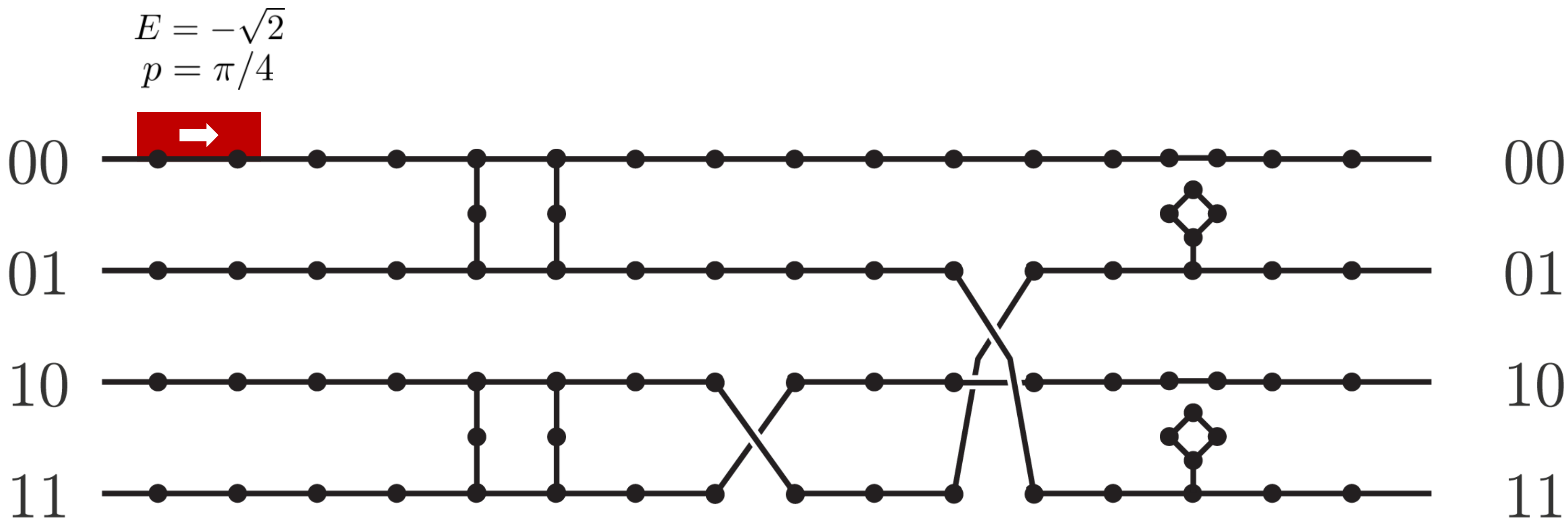
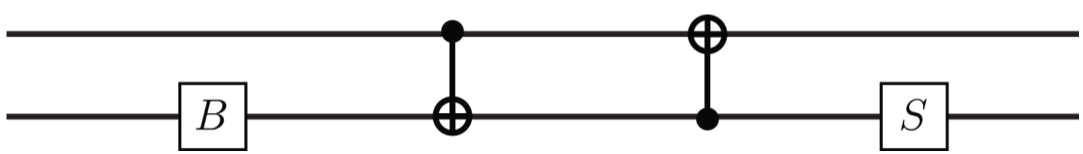


fully transmited, but delayed...

4 Universal computation by QW

Basis-changing.

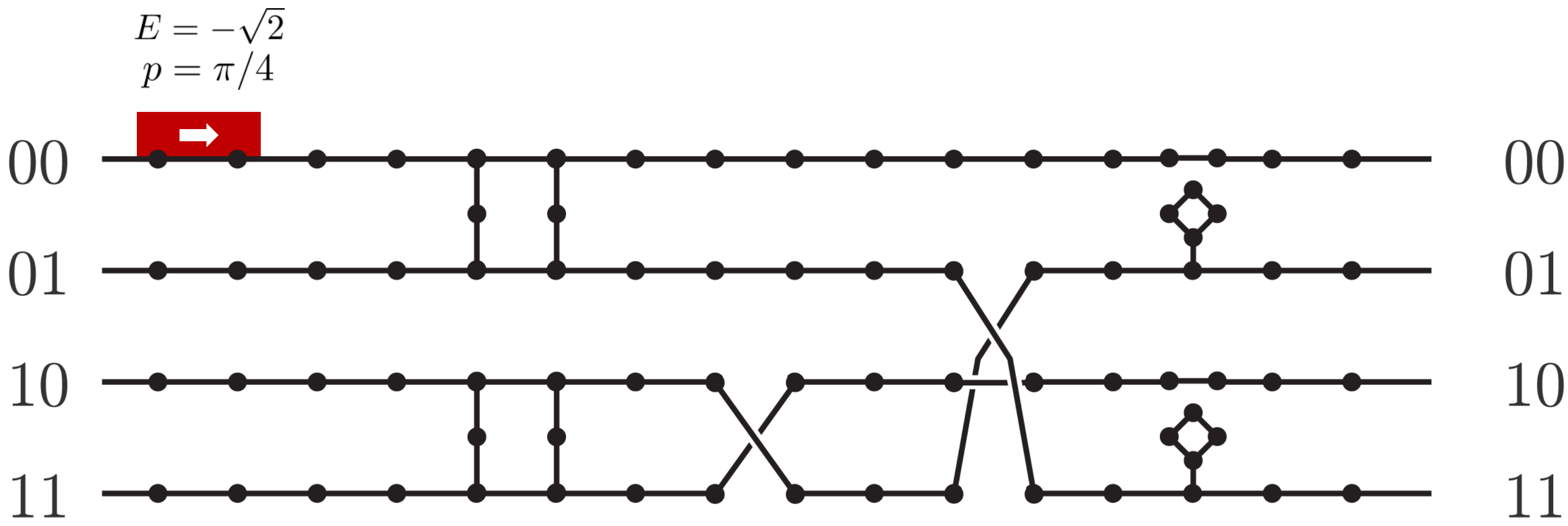


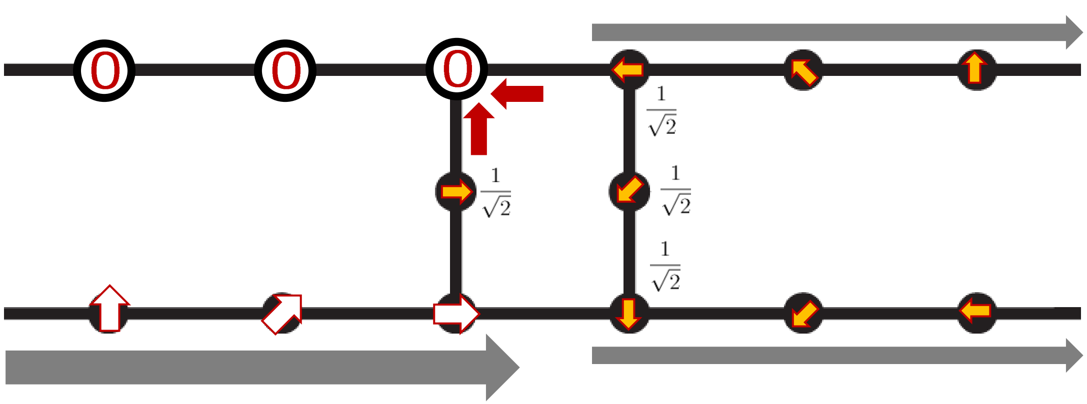


- B transforms Z & Y bases (similar to Hadamard)

4 Universal computation by QW

Basis-changing.

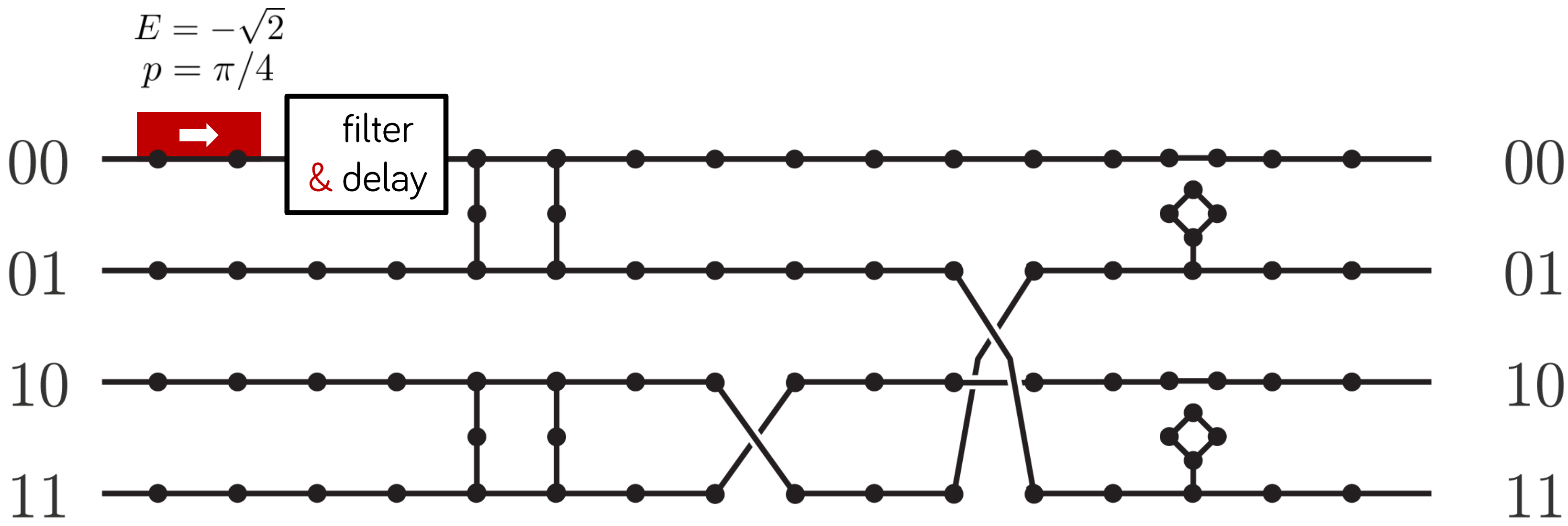


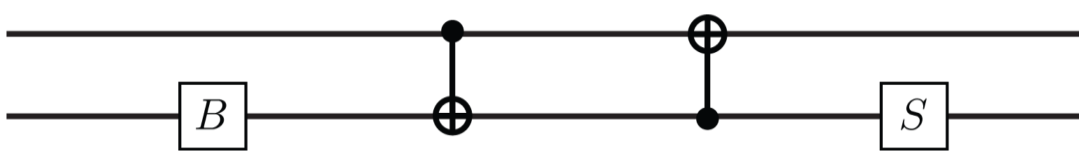


an equal magnitude
& a phase difference

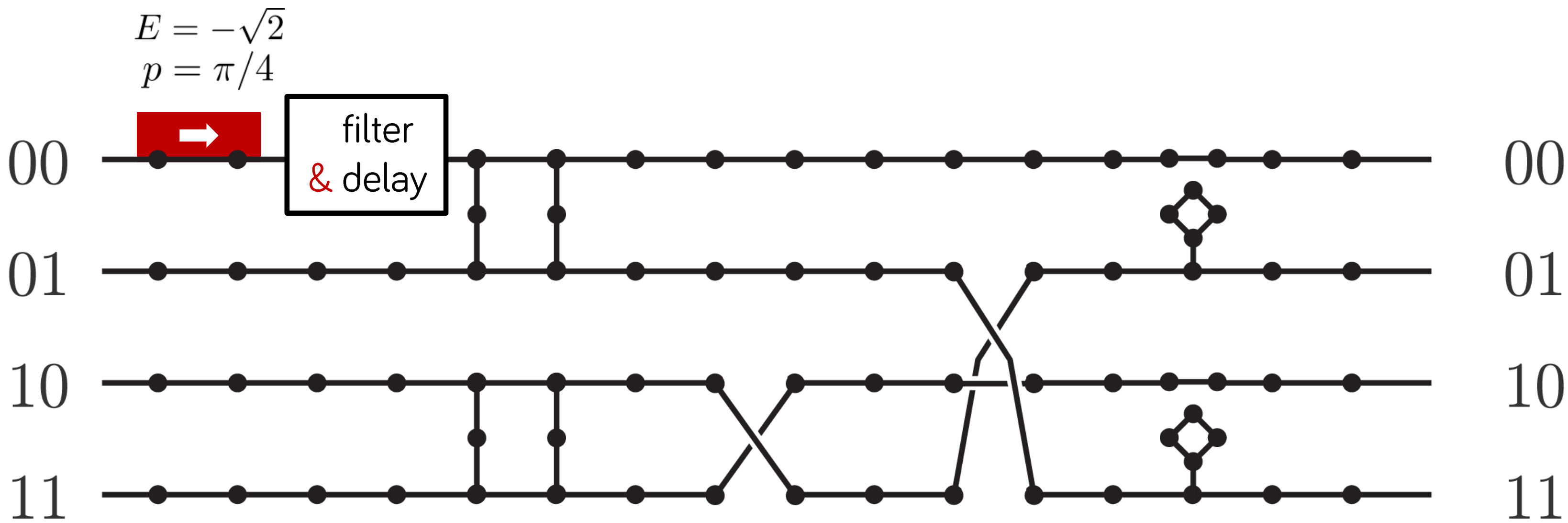
Universal computation by QW

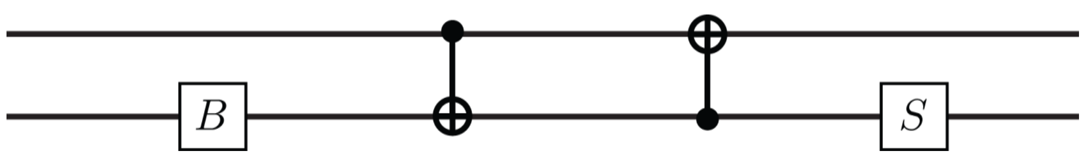
Picking the momentum.



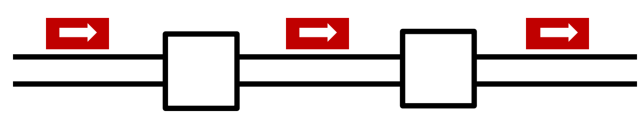


Universal computation by QW

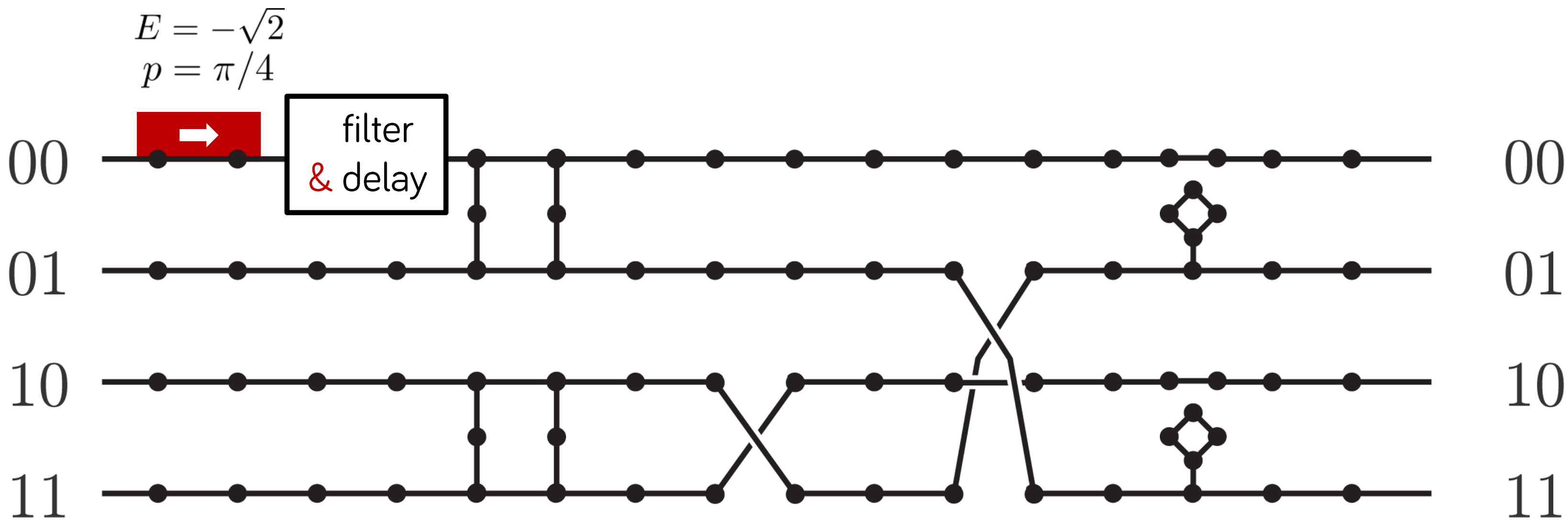


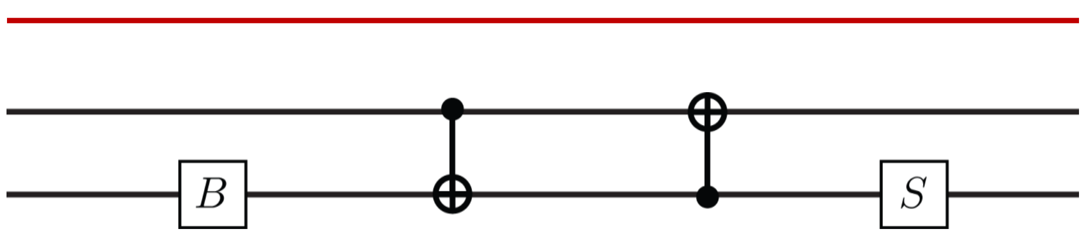


- composing gates?
  wide packet/long lines



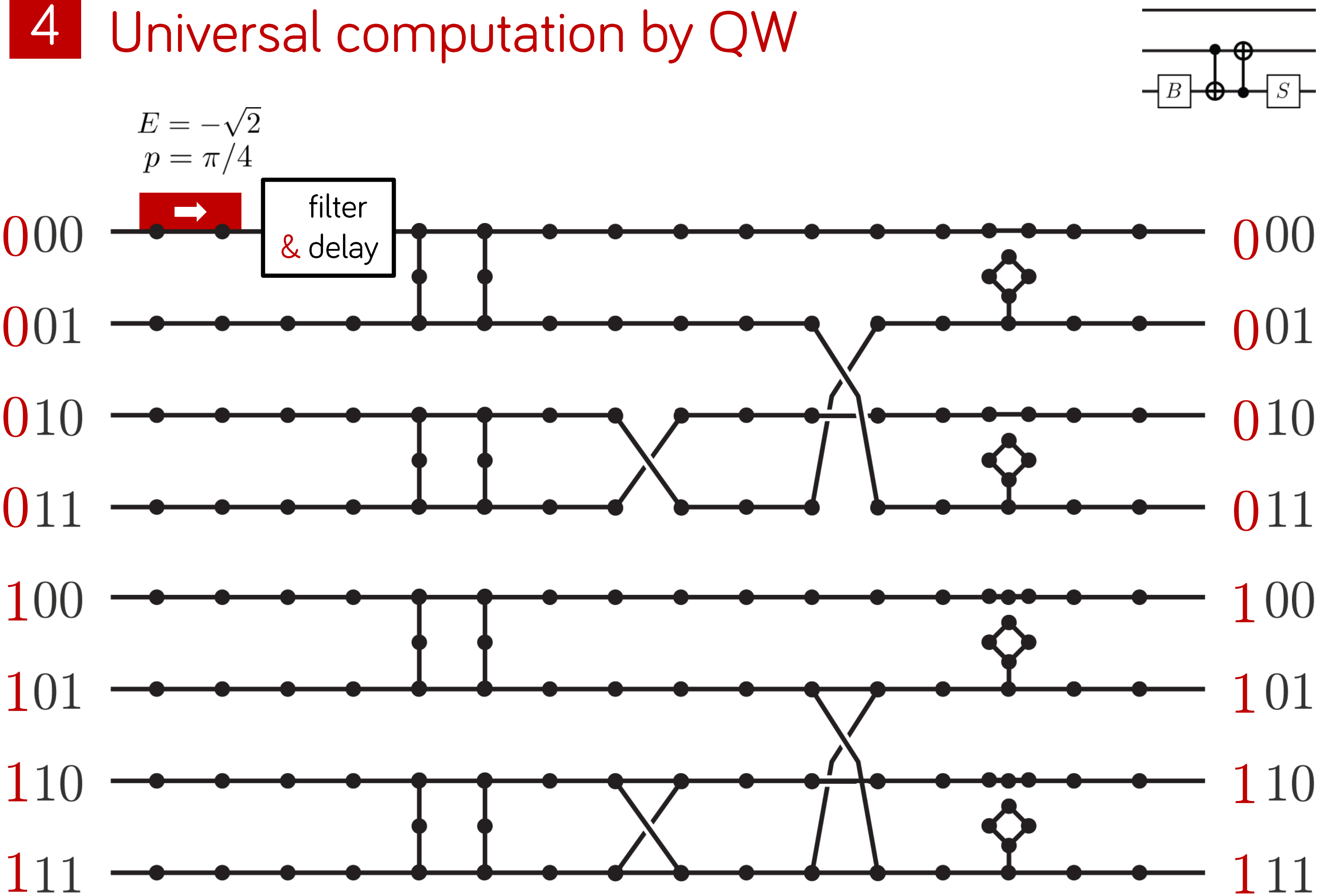
Universal computation by QW





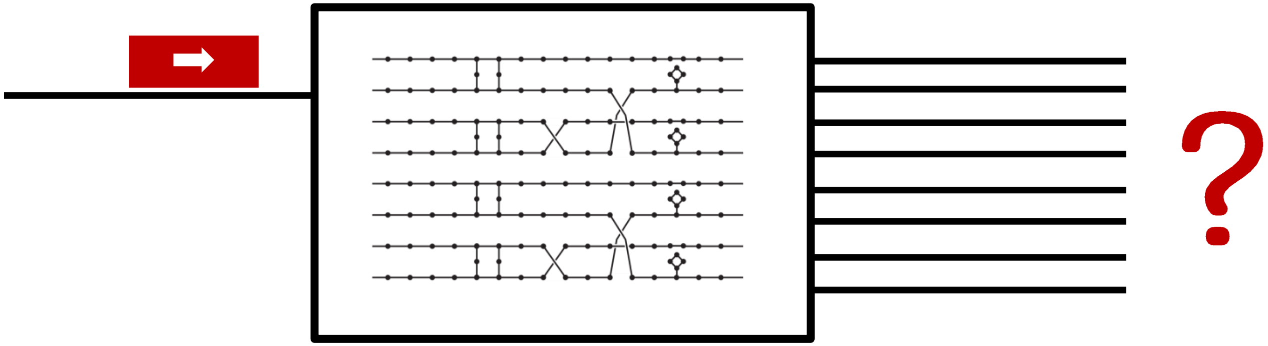
- adding more qubits?

4 Universal computation by QW



4 Universal computation by QW

an exponentially
large graph



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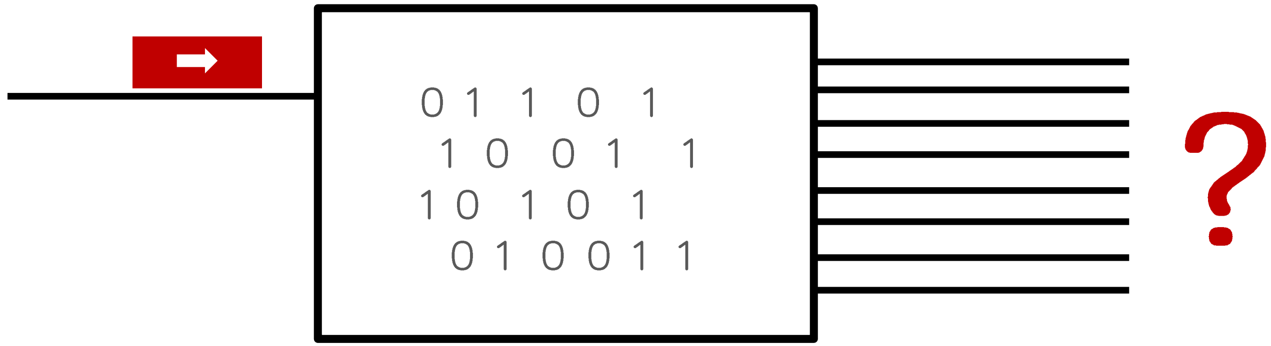


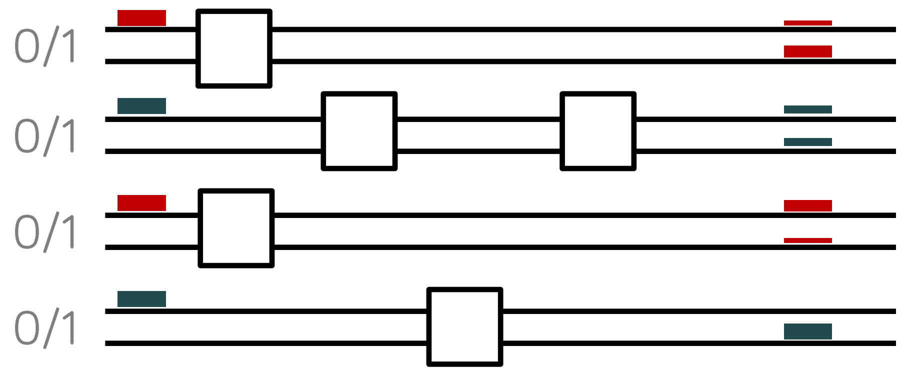
a sparse, 0/1 matrix
with a simple description

- Can you solve such scattering problems?
  You can do quantum computation (BQP).

Childs
PRL 102 (18), 180501 (2009)

4 Universal computation by multi-particle quantum walk

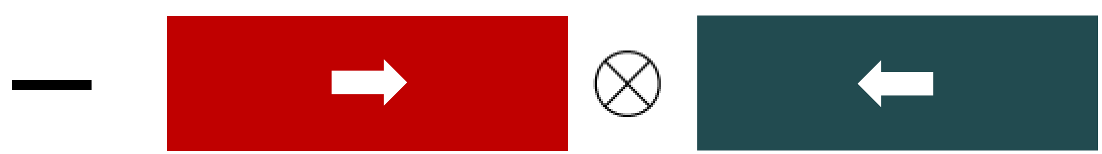
- dual-rail encoding with N wavepackets



$$a_j^\dagger a_k + a_k^\dagger a_j$$

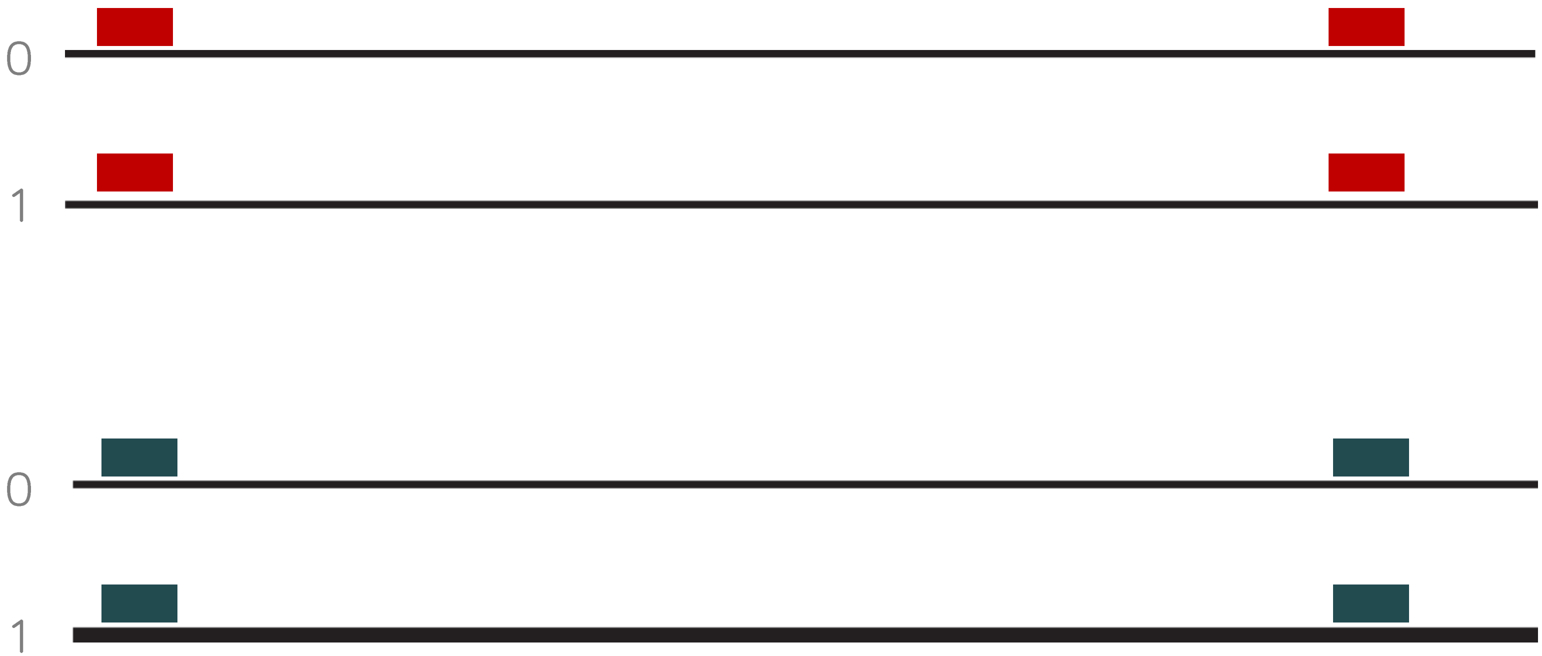
- single-qubit gates? we already have them
- 2-qubit (CPHASE)? requires interaction

$$a_j^\dagger a_k^\dagger a_j a_k$$



2 packets with different momenta meet on a line

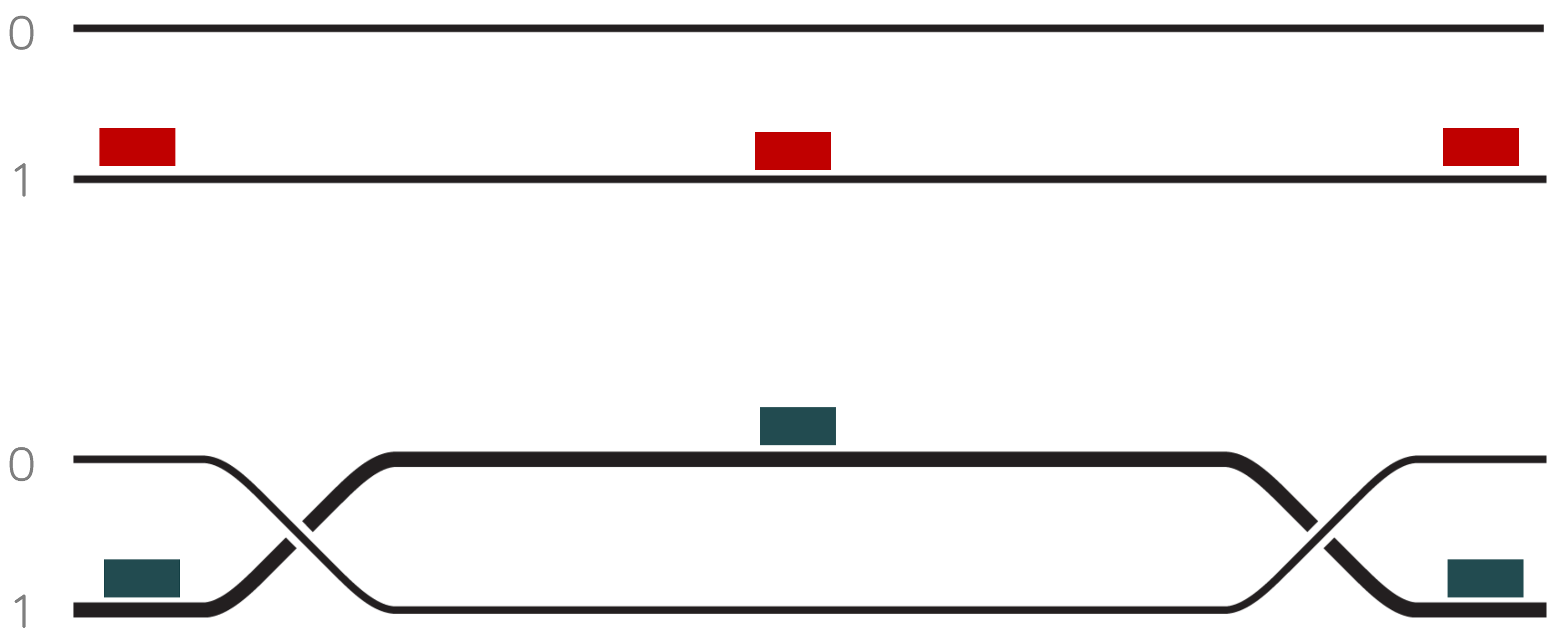
4 A meeting of two walkers



- how to get a minus sign in the “11” case?

4

A meeting of two walkers



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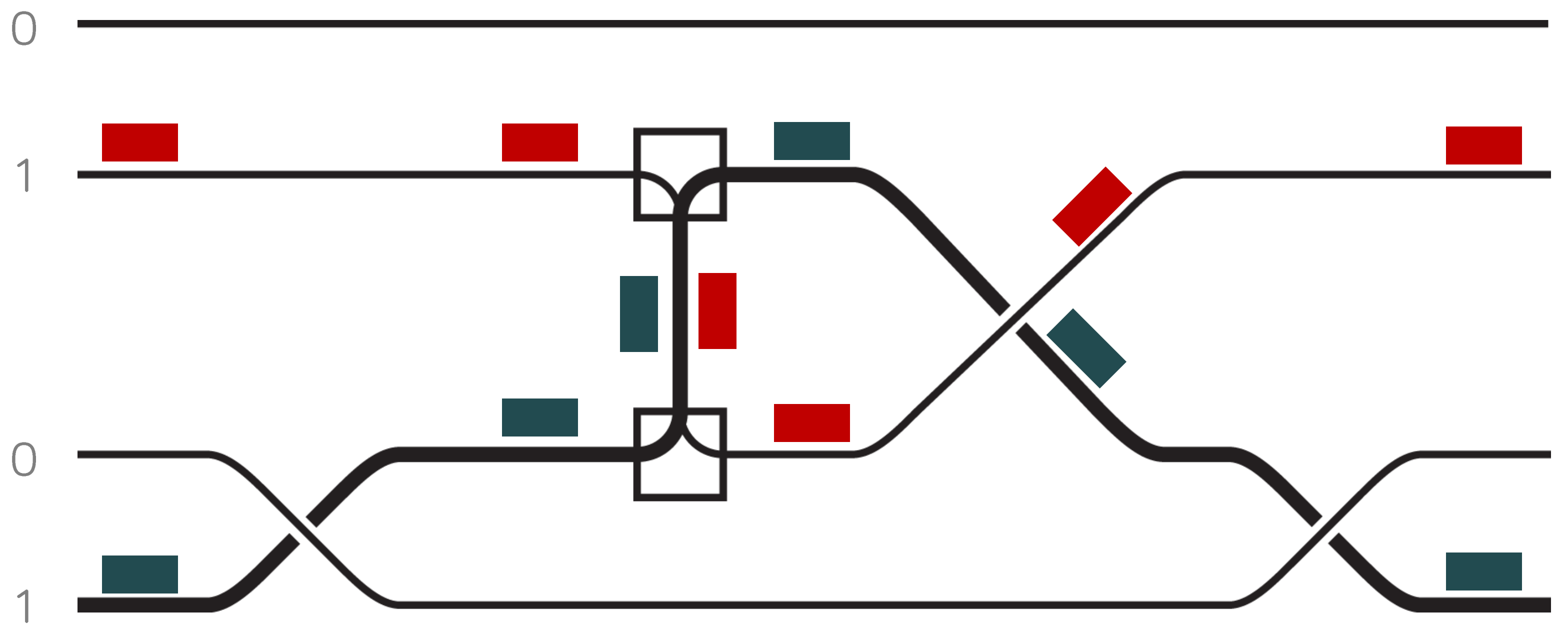
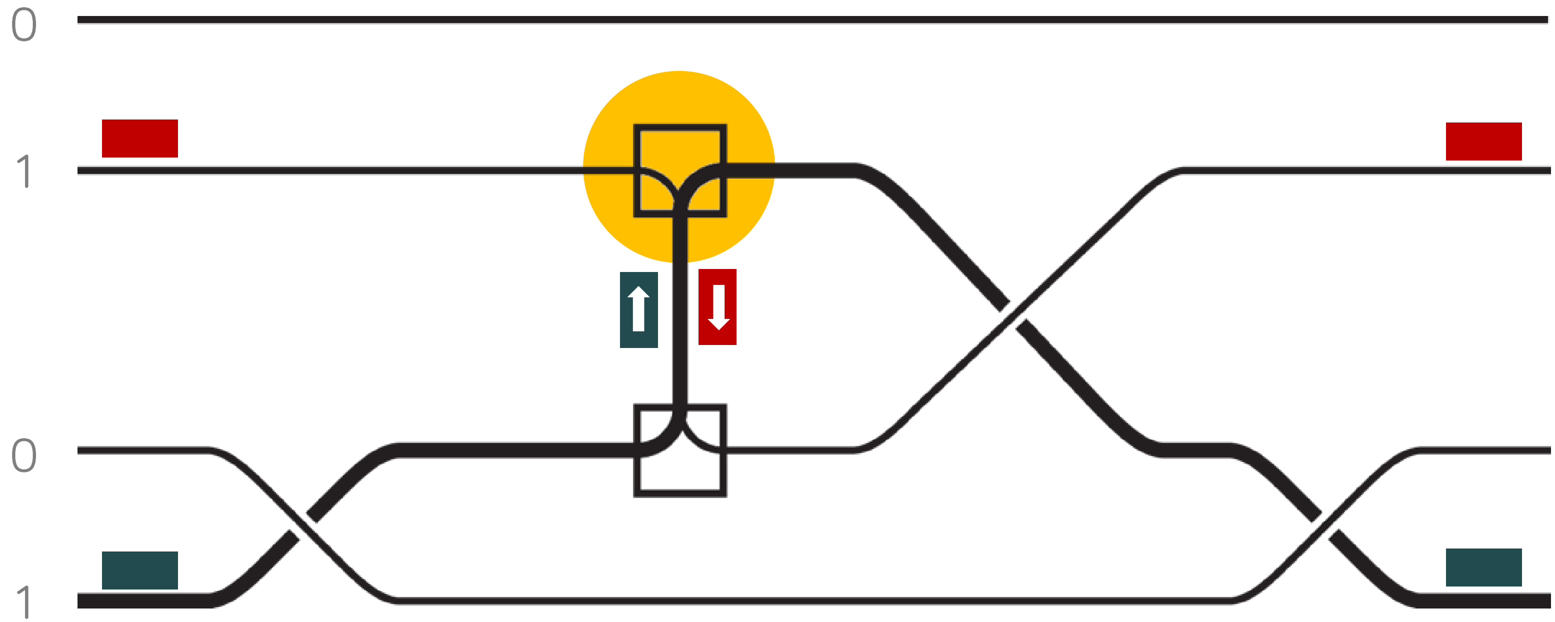
4 A meeting of two walkers

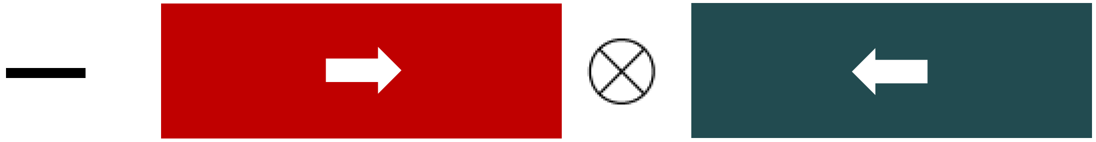


- how to get a minus sign in the “11” case?

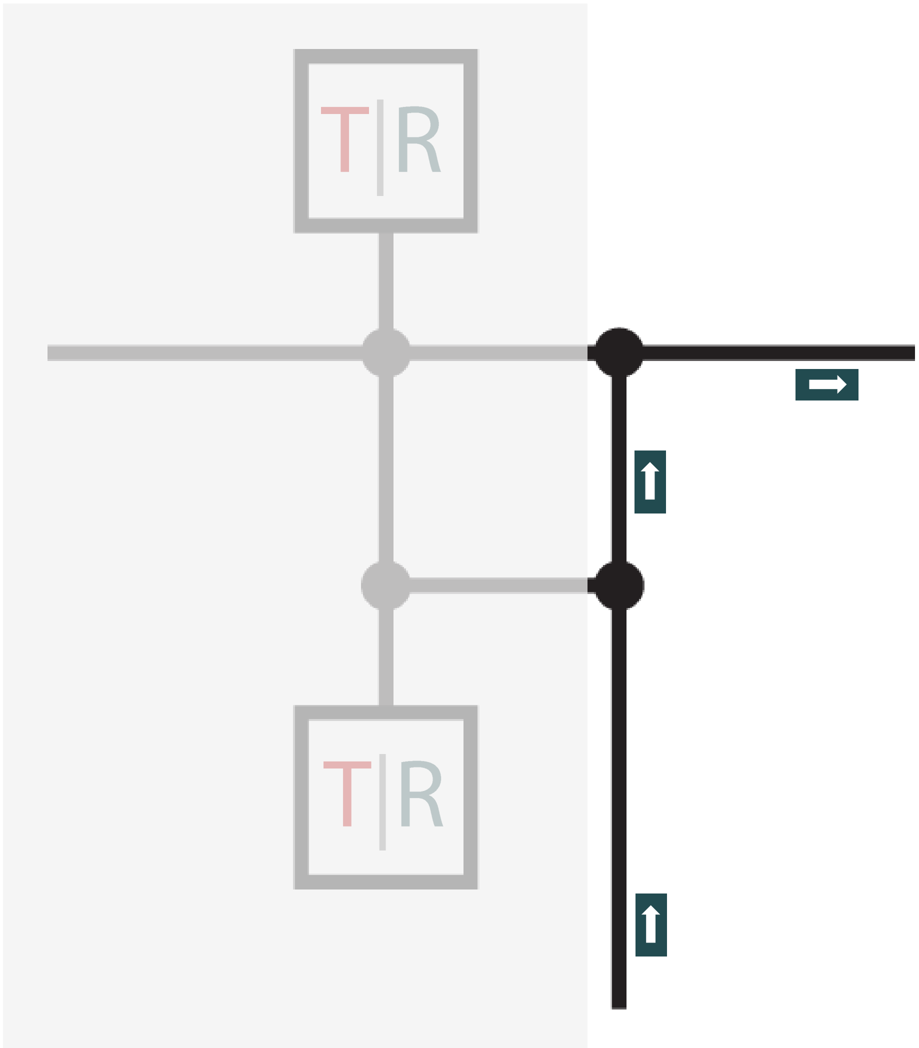
4 A meeting of two walkers

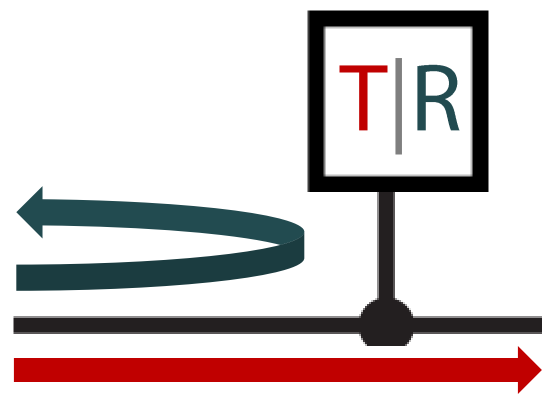
C-PHASE





4 The momentum switch





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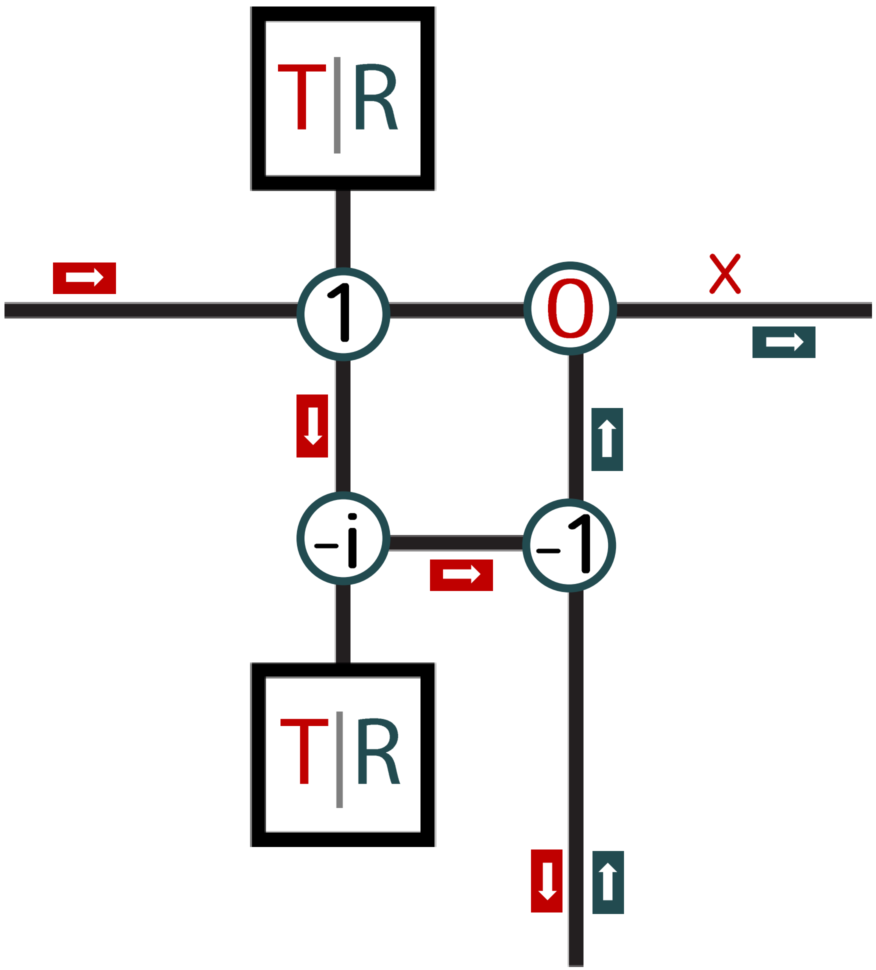
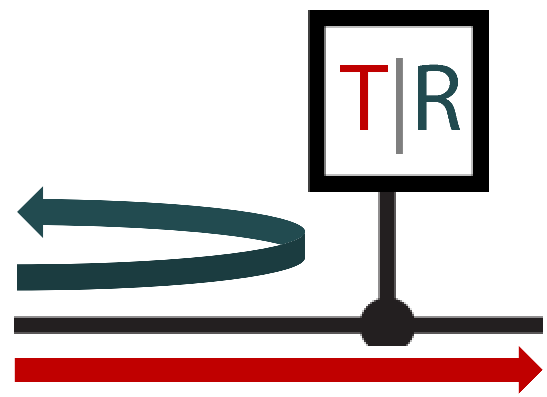
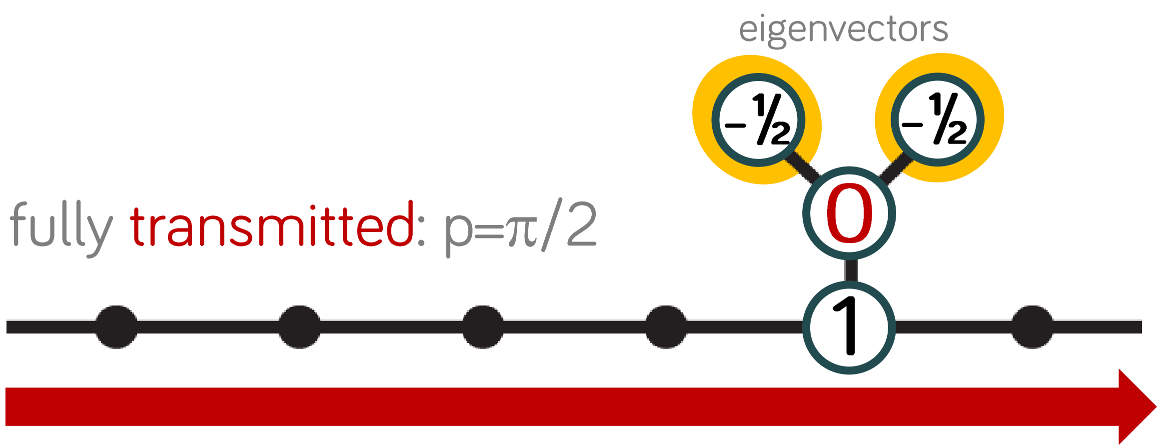




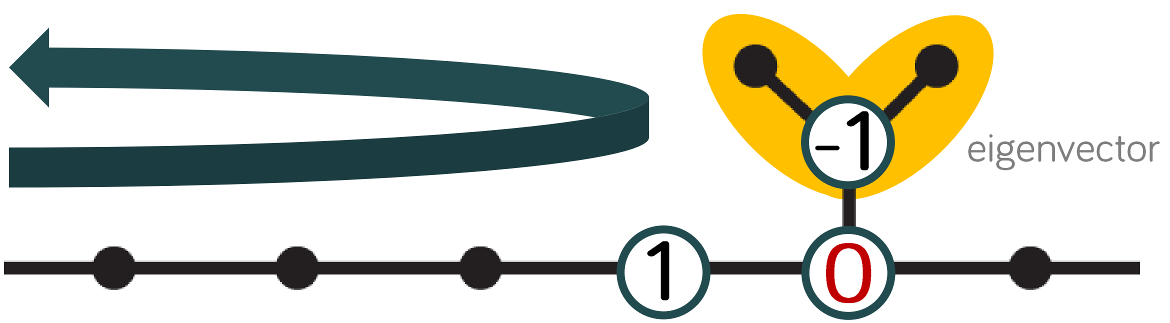
4 The momentum switch

- a transmitter/reflector for 2 momenta

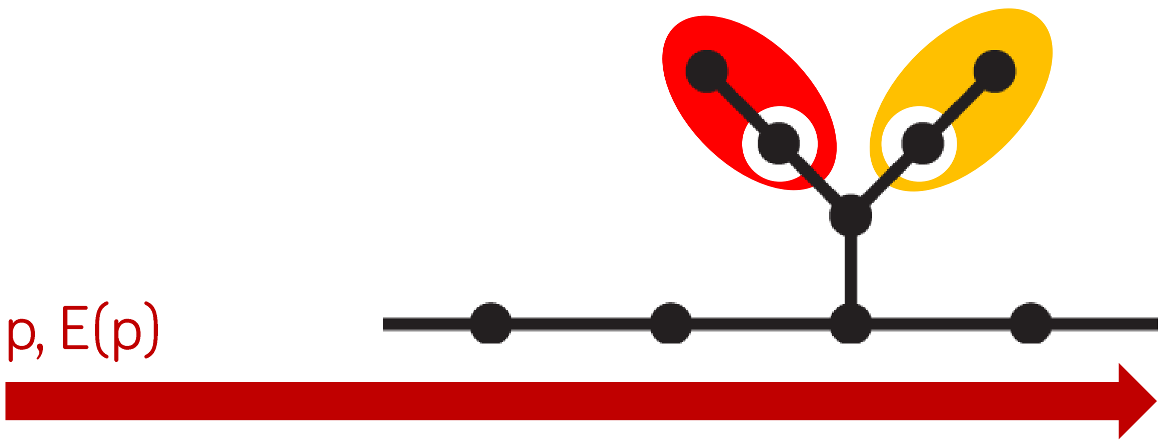
fully **transmitted**: $p=\pi/2$



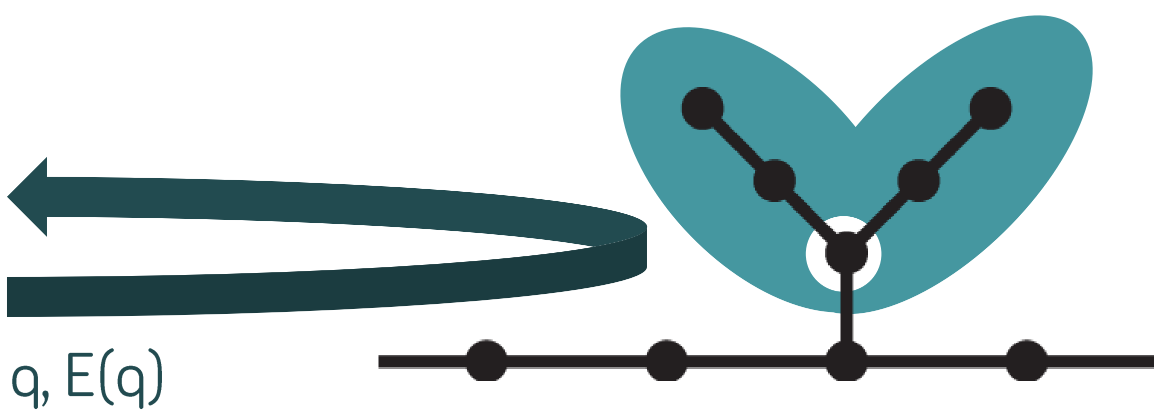
fully **reflected**: $p=\pi/4$



4 General transmitter/reflector design



at least one eigenvector with a nonzero amplitude

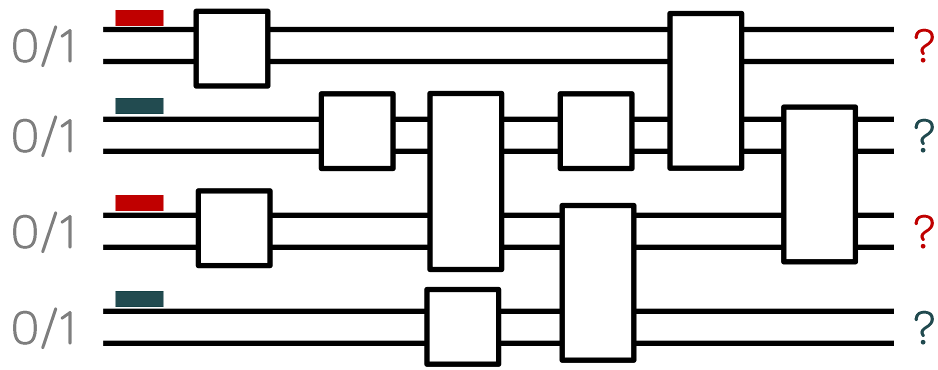


an eigenvector with a nonzero amplitude

4 Universal computation by multi-particle quantum walk

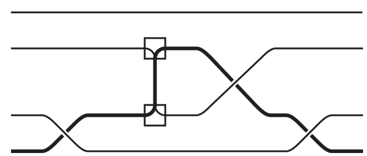
- dual-rail encoding
  with N wavepackets

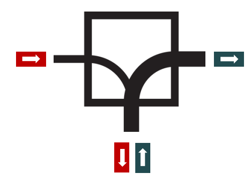
$$a_j^\dagger a_k + a_k^\dagger a_j$$

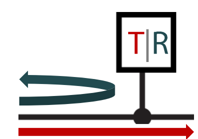


- CPHASE: interaction

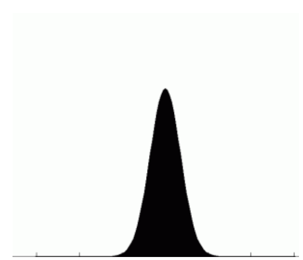
$$a_j^\dagger a_k^\dagger a_j a_k$$



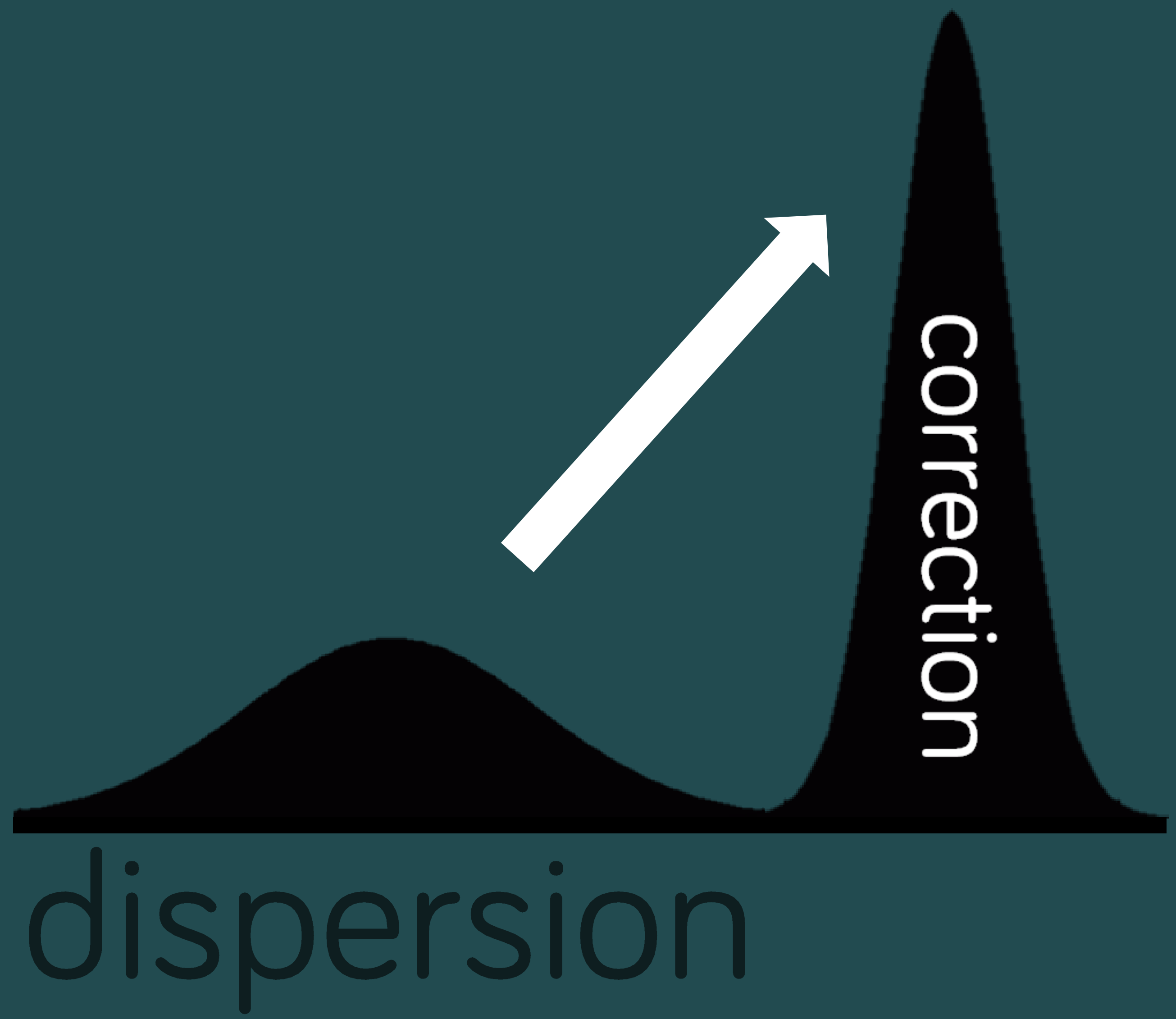




- very wide packets (& a big graph)
  sharp momentum, low error



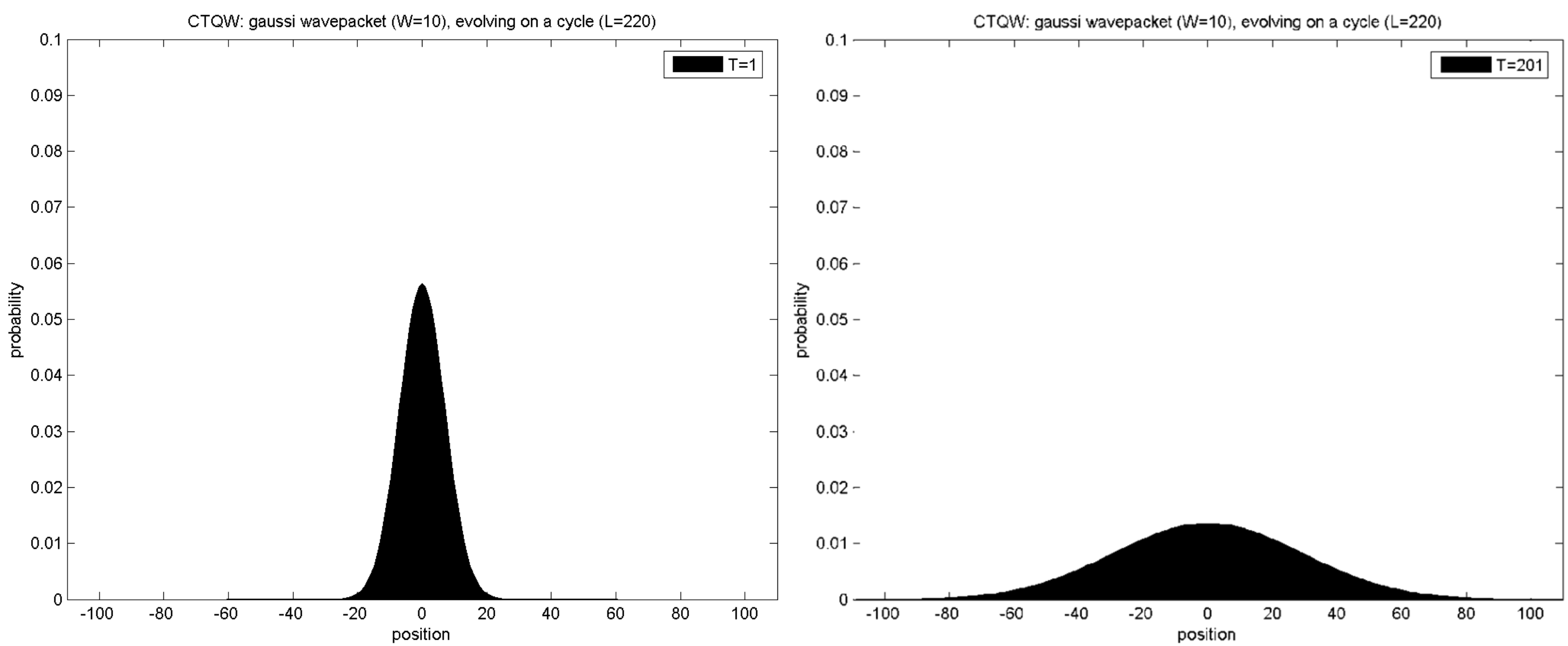
Childs, Gosset, Webb
Science 339 (6121), 791 (2013)


correction
dispersion

5 Dispersing wavepackets on a line

- a Gaussian's width grows

$$M(t) = L\sqrt{1 + \left(\frac{2(\cos p)t}{L^2}\right)^2}$$



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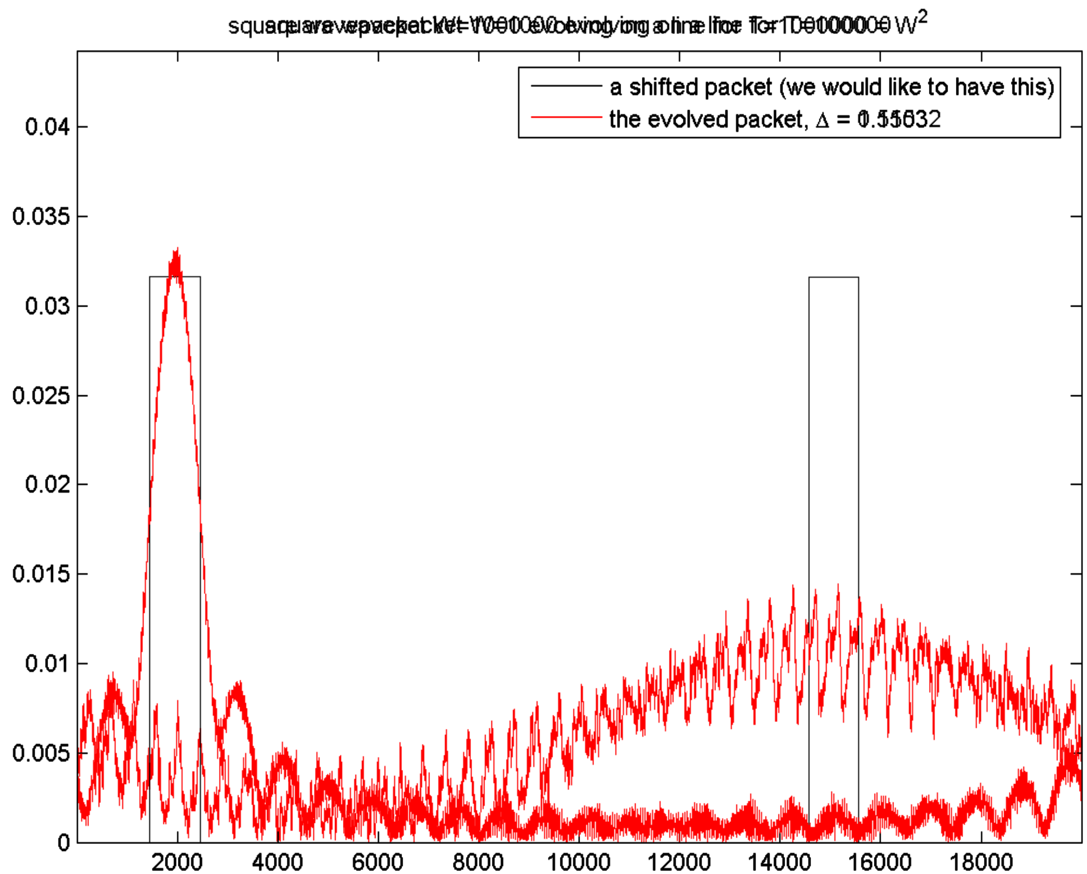
- a rectangular packet behaves even worse



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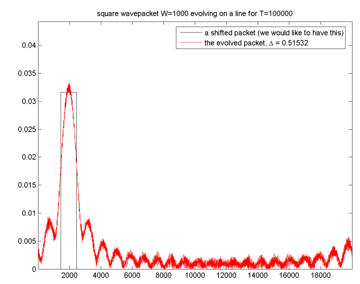

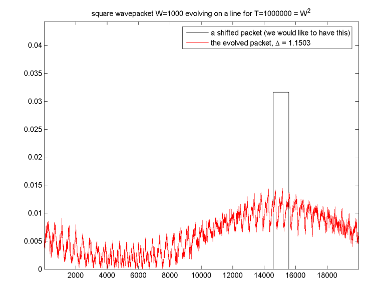

- propagation speed

$$v(p) = 2\sin p$$

5 Dispersing wavepackets on a line

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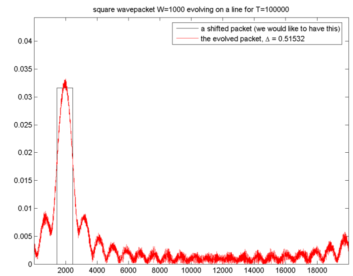
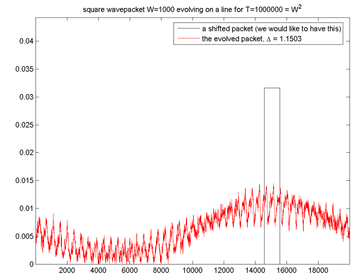
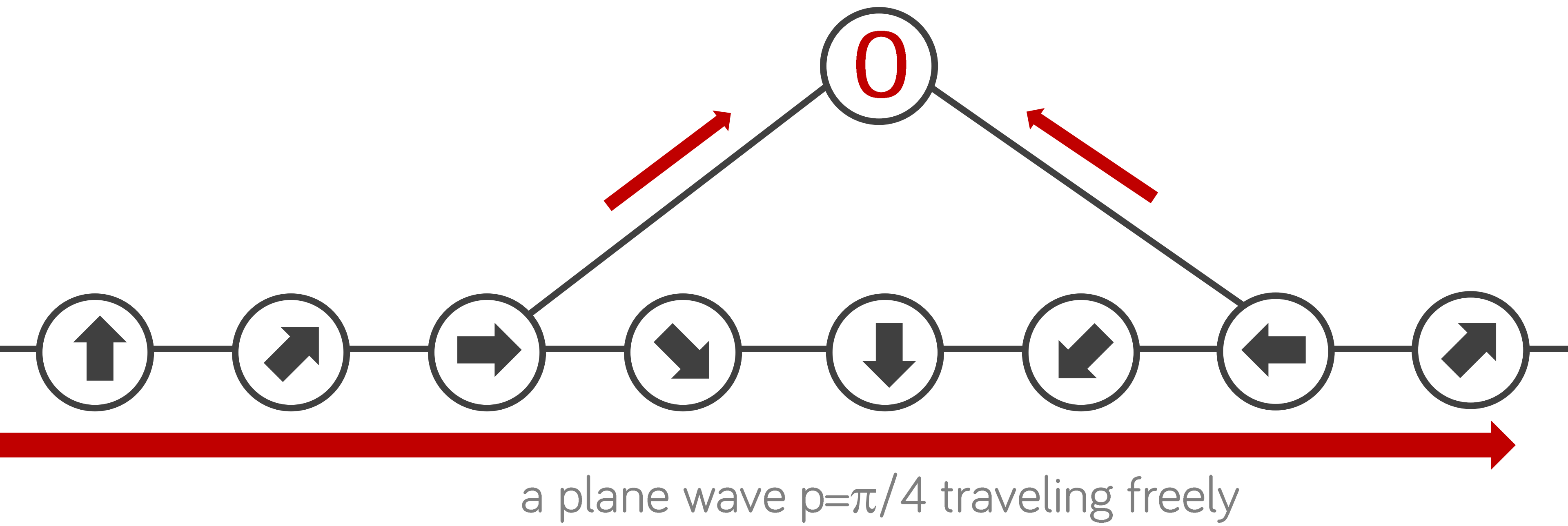




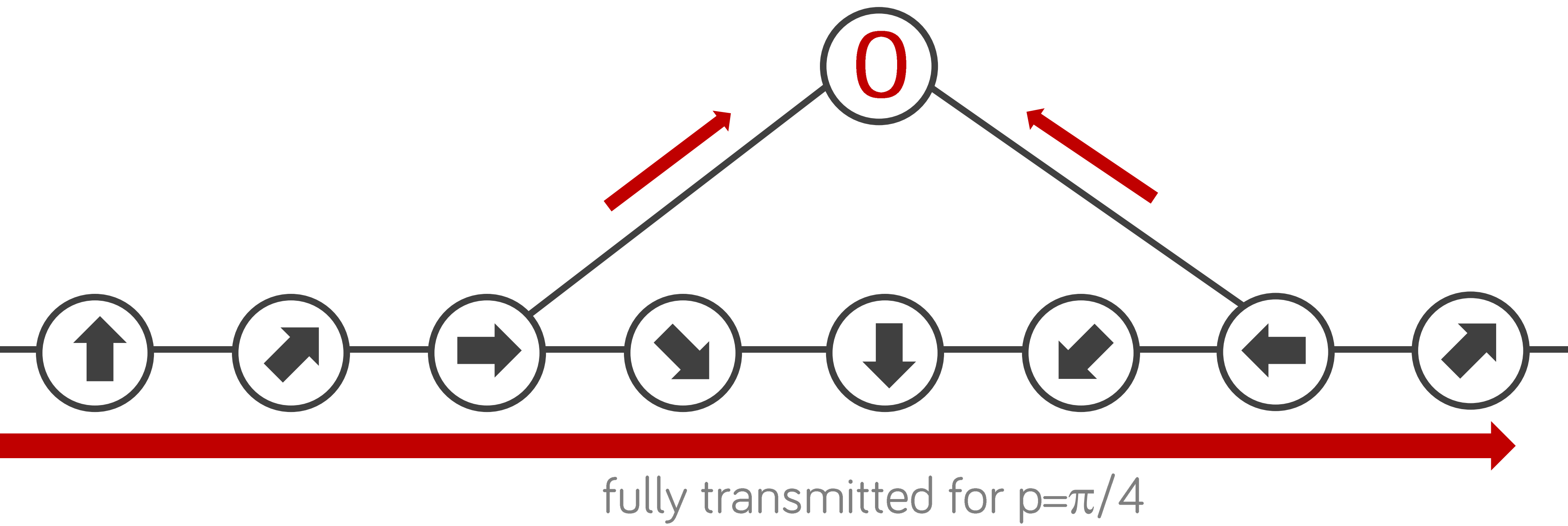
- could we develop an anti-dispersion gadget to "repair" dispersed packets?

5 An anti-dispersion gadget for p=π/4



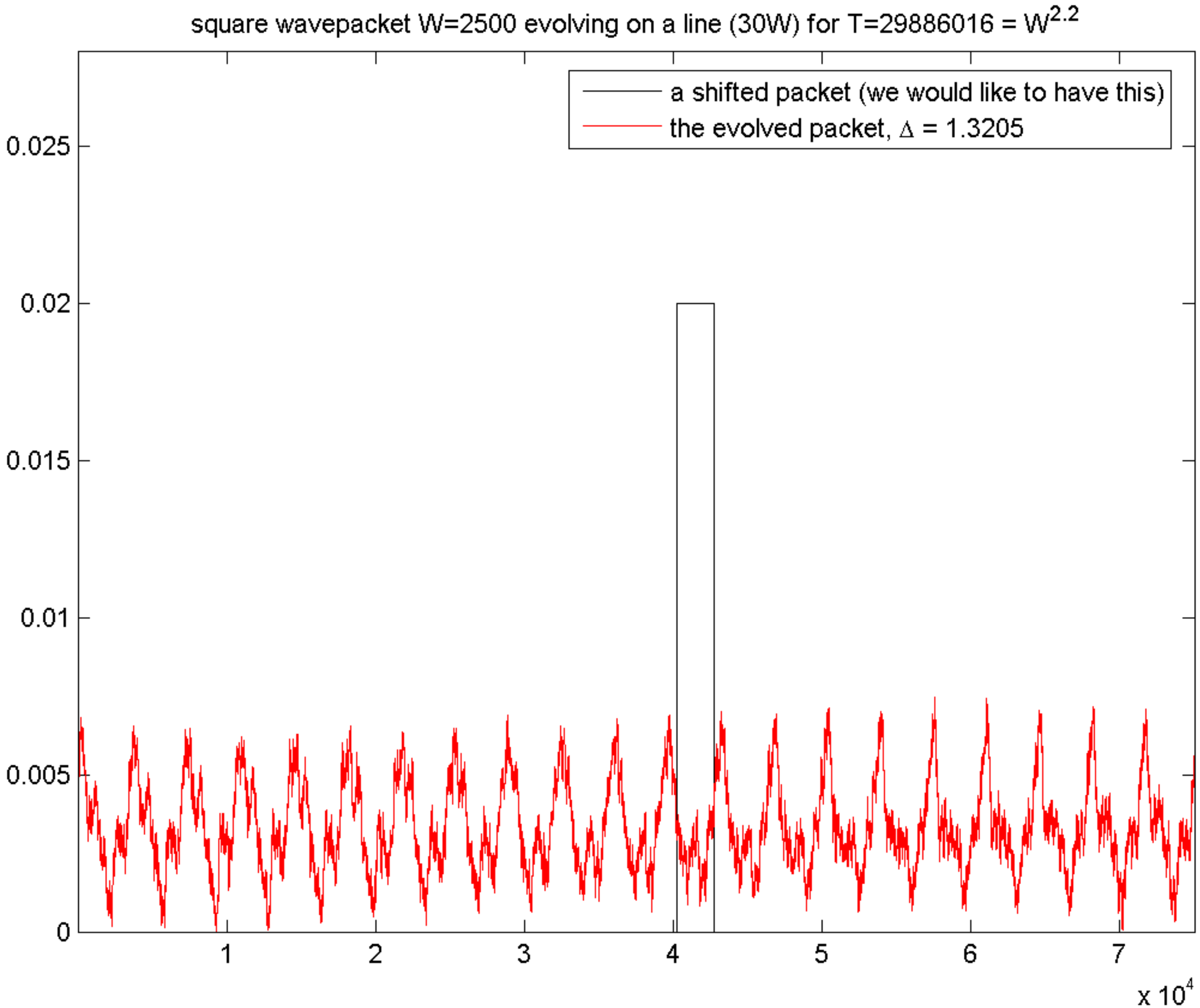
5 An anti-dispersion gadget for p=π/4

- a “reverse” travel time/momentum dependence near π/4

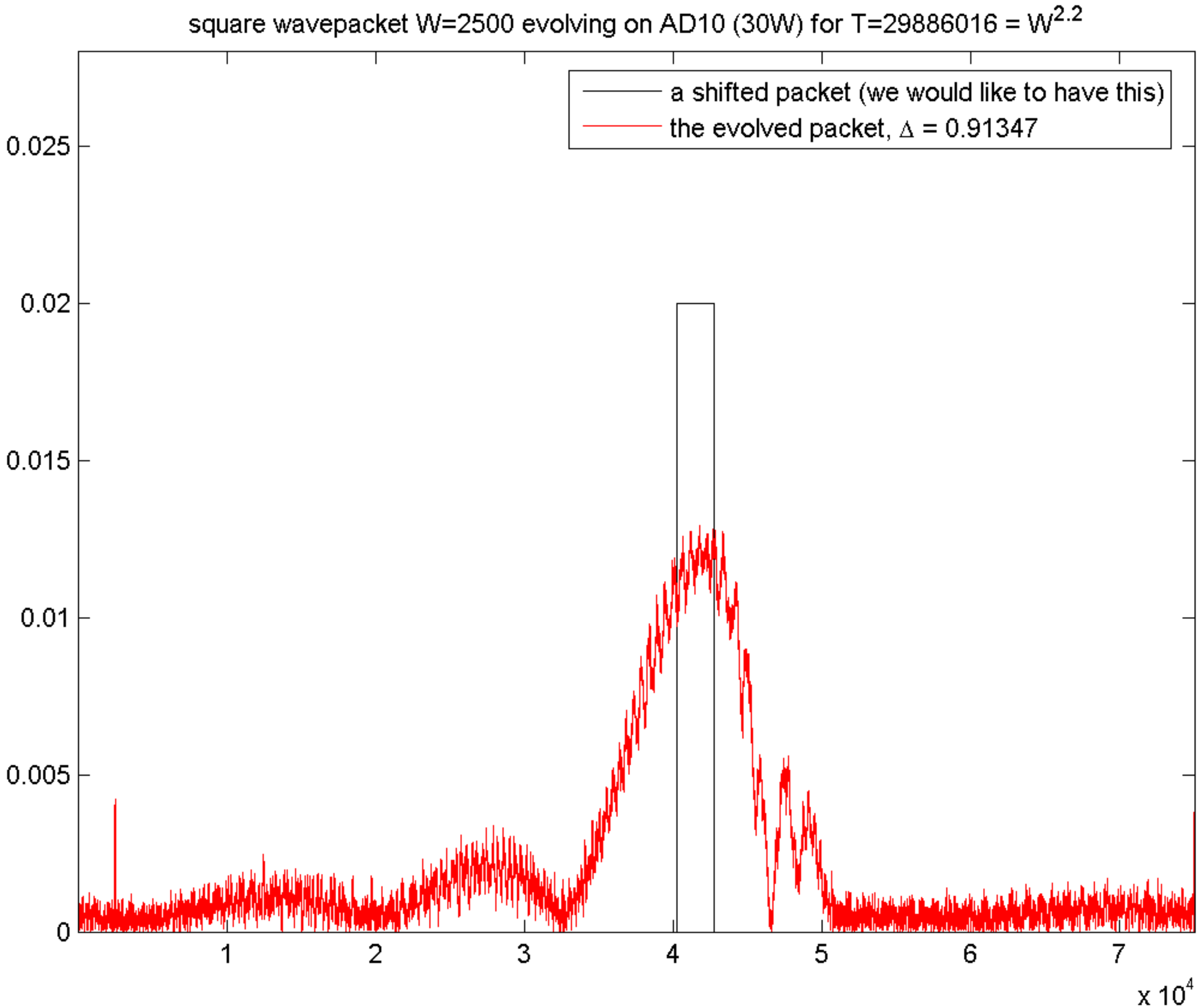


- “repairs” 10 vertices’ worth of dispersion

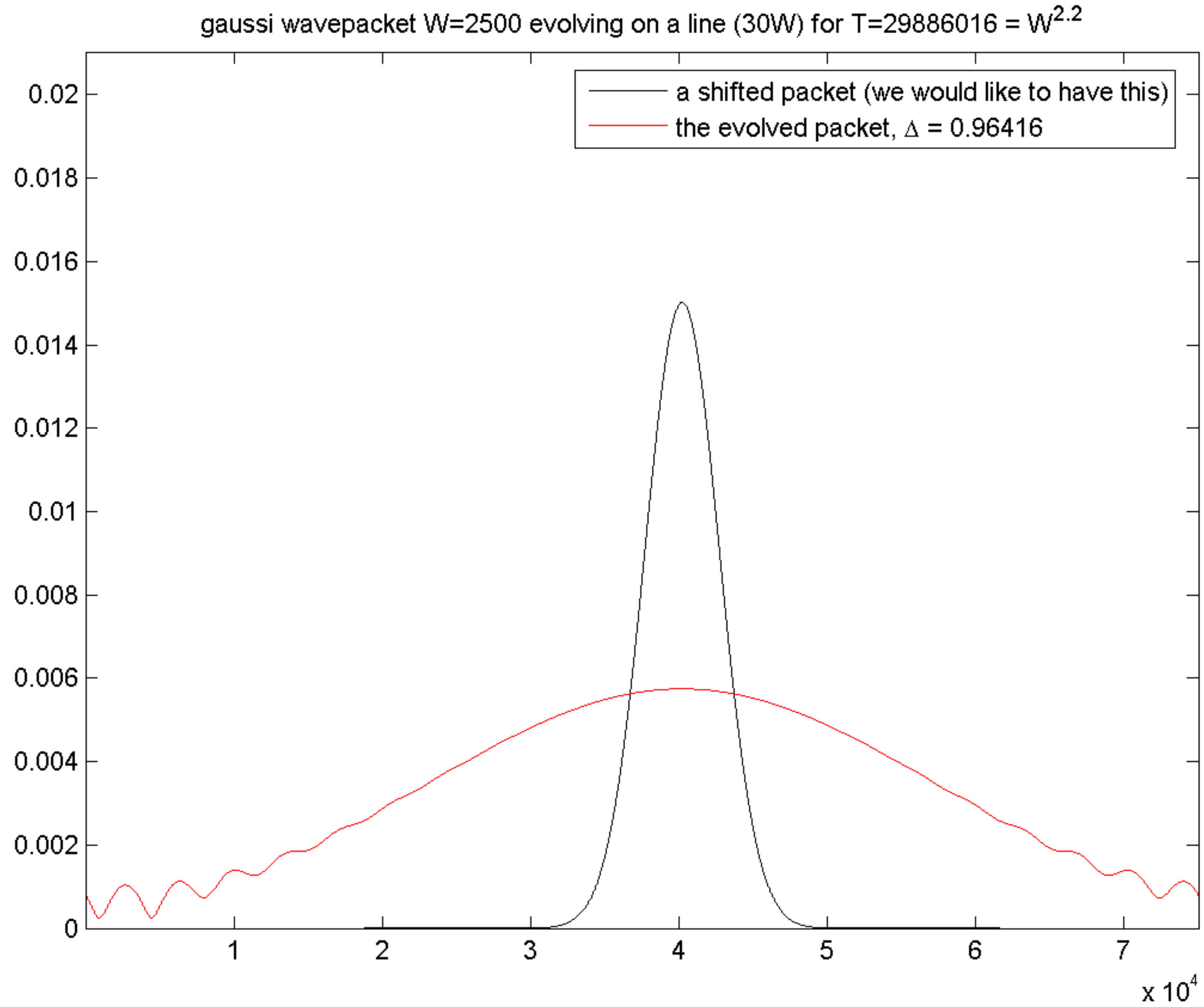
5 Rectangular packet, evolving for $T=W^{2.2}$ on a line



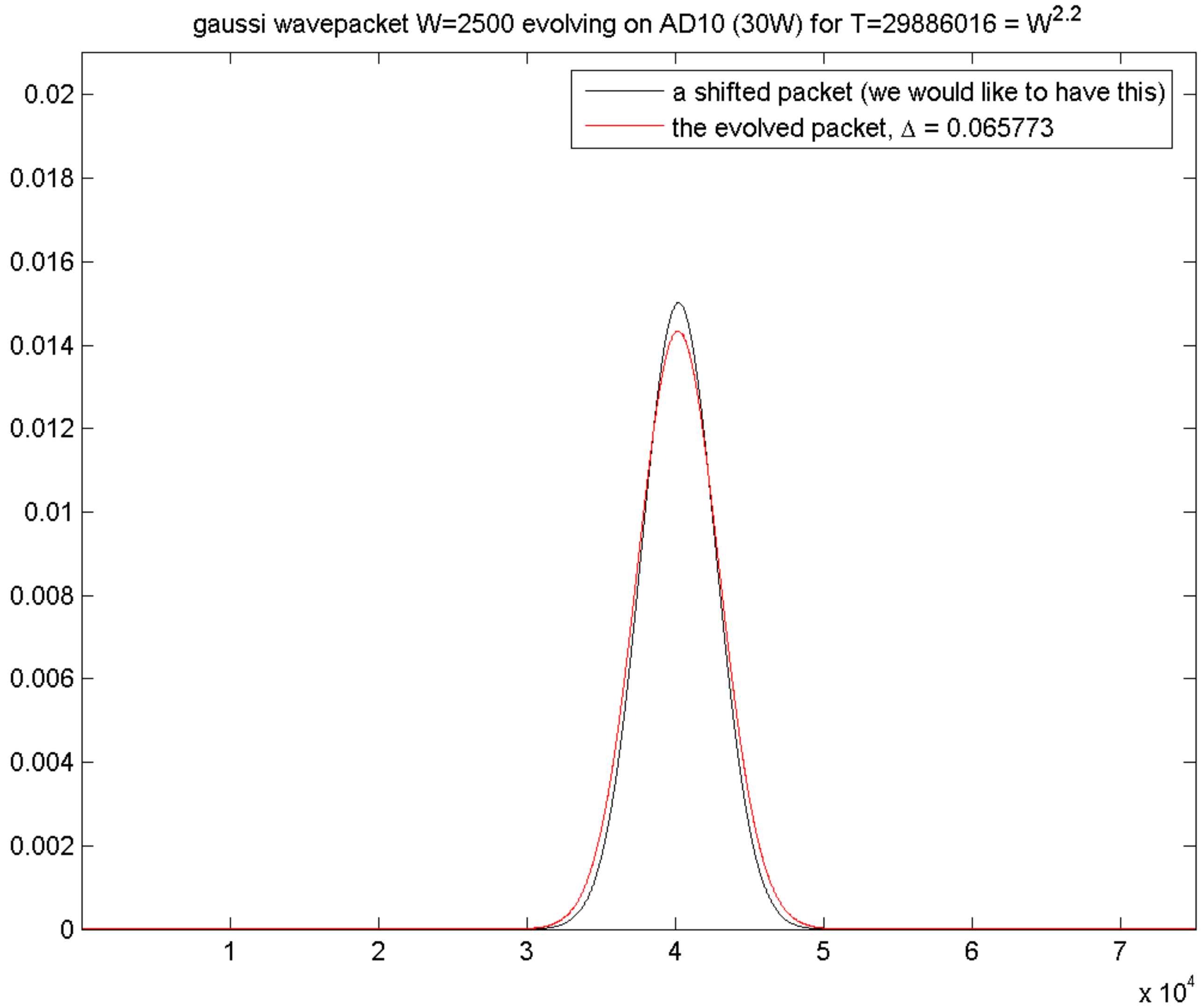
5 Rectangular packet, evolving for $T=W^{2.2}$ with correction



5 Gaussian packet, evolving for $T=W^{2.2}$ on a line



5 Gaussian packet, evolving for $T=W^{2.2}$ with correction



a powerful tool

many algorithms
computational universality

there's much we don't know

interesting graph properties
that we could determine?

practical applications?

let's research & experiment!

QUANTUM WALKS
Daniel Nagaj