QIP 2013
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towards perfect completeness in QMA

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arXiv: 1210.1290

towards perfect completeness in QMA

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1 what it means to be really convincing

2 classical witnesses $QCMA_1$

3 a few EPR's

$QMA_1^{c.EPR}$

$QIP_1(2)$

1 The MA protocol

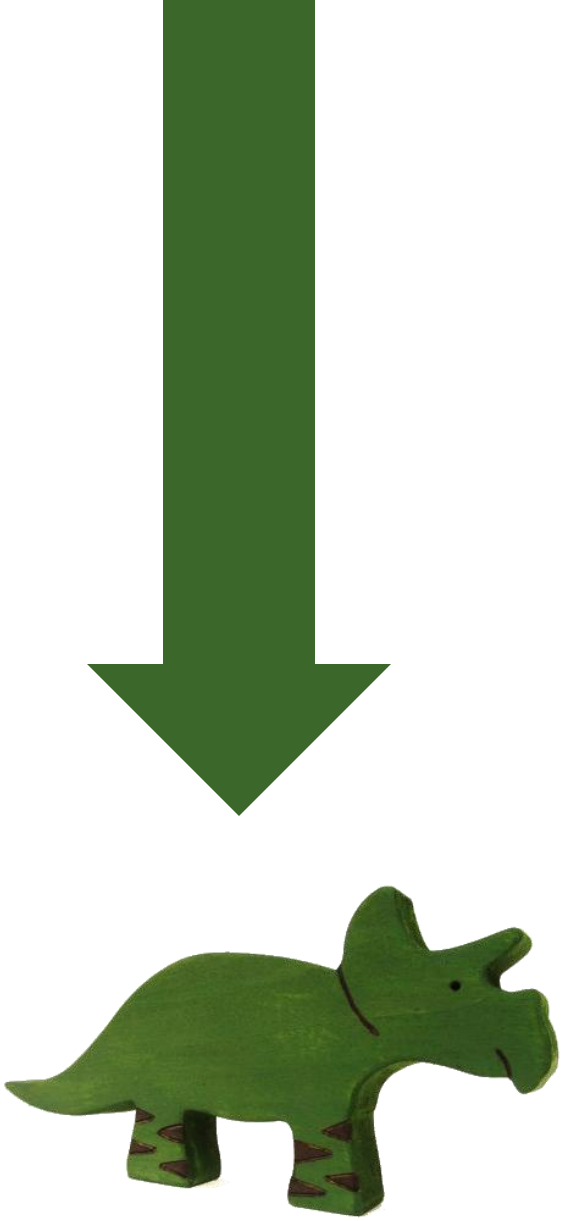
Did dinosaurs exist?

triceratops

?

1 The MA protocol

Did dinosaurs exist?

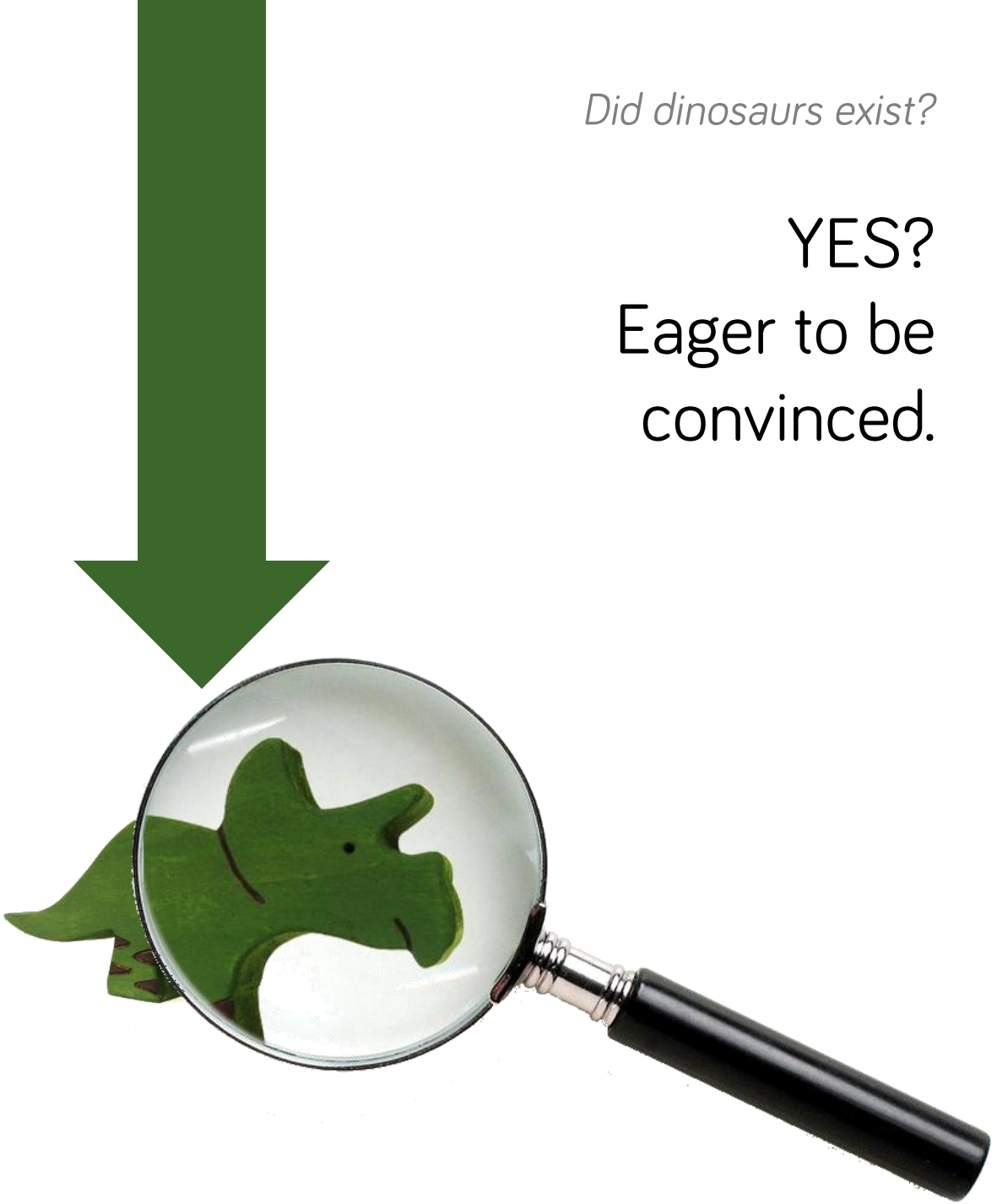


[wooden animals: Imagination Kids Toys]

1 The MA protocol

Did dinosaurs exist?

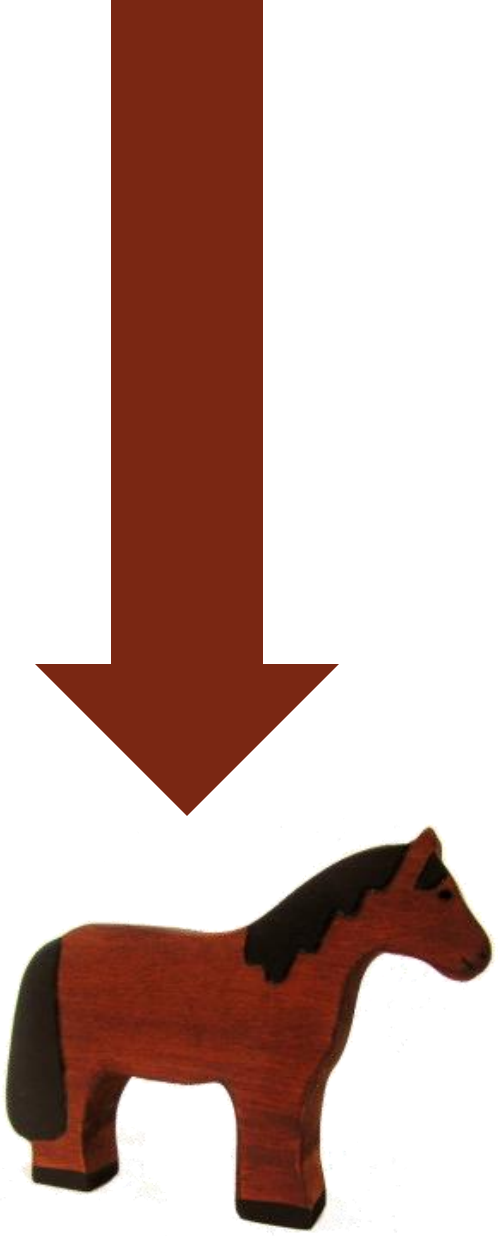
YES?
Eager to be convinced.



[magnifying glass: hillllllall]

1 The MA protocol

Recognizing fakes?



1 The MA protocol

Recognizing fakes?

NO?
Don't be
fooled
easily.

1 Probabilistic checks

Sometimes reject a genuine proof?

Accept a fake?

1 Perfect completeness

Never reject
a genuine proof?

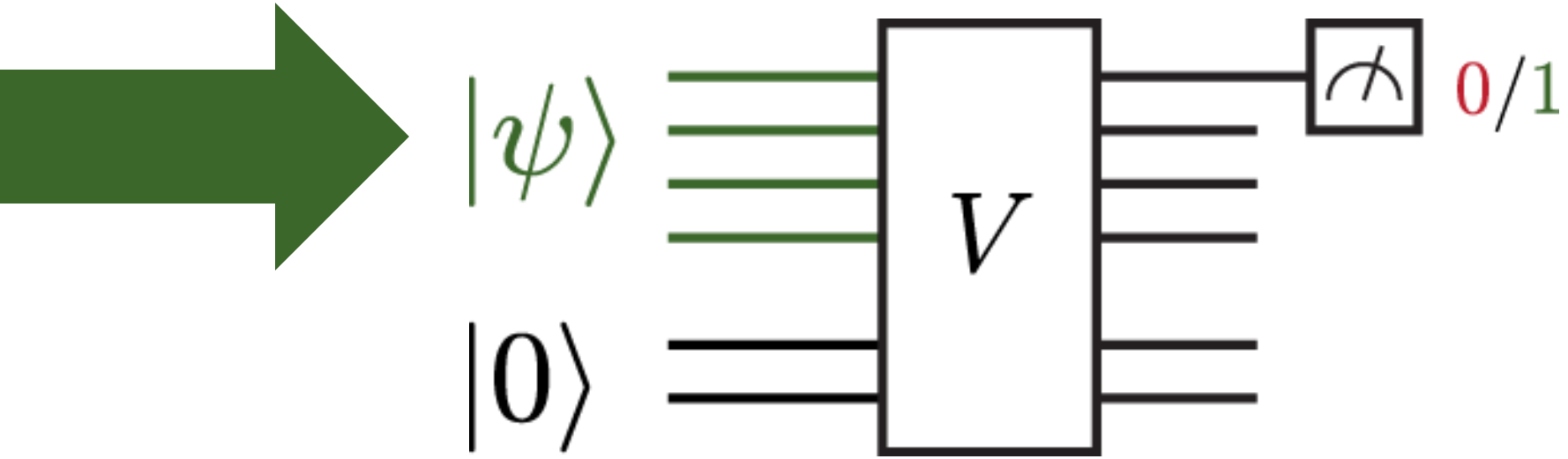
YES?
Accept some
proof without
any doubt.

NO?
Still don't get fooled easily.

YES?
Accept some proof without any doubt.

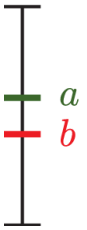
perfect completeness

1 The QMA protocol

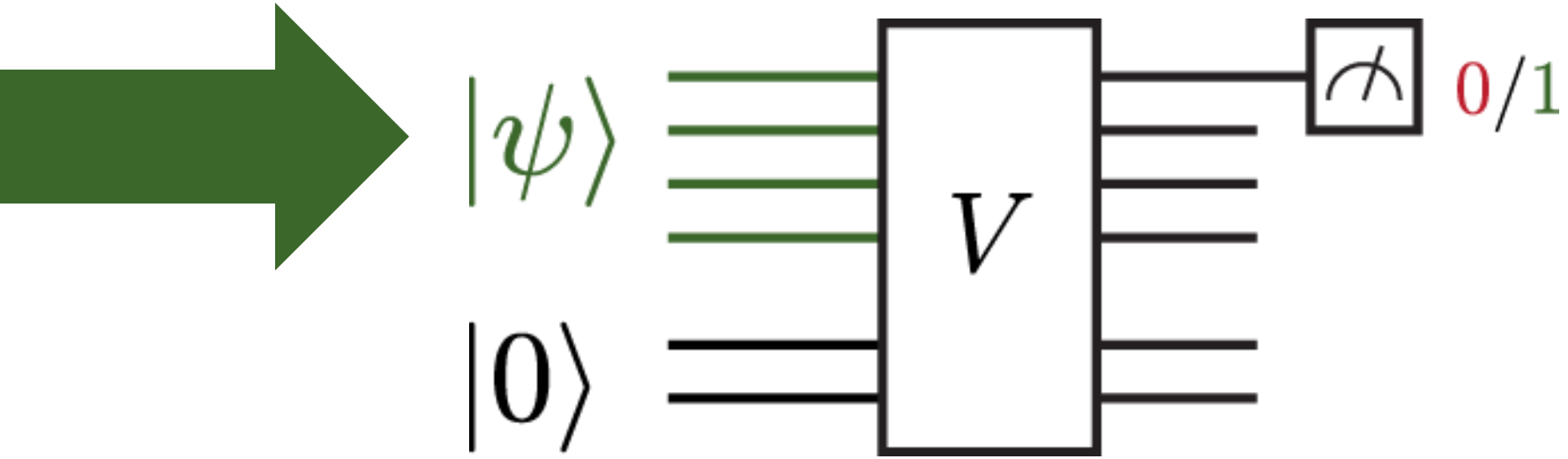


YES? Accept a good proof with $p > a$.

NO? Probability of accepting $p < b$.



1 The QMA protocol: amplification



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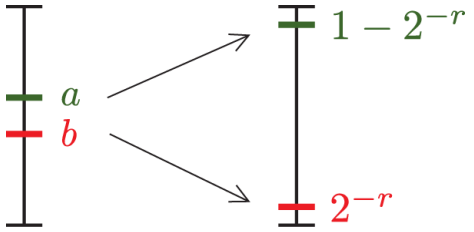
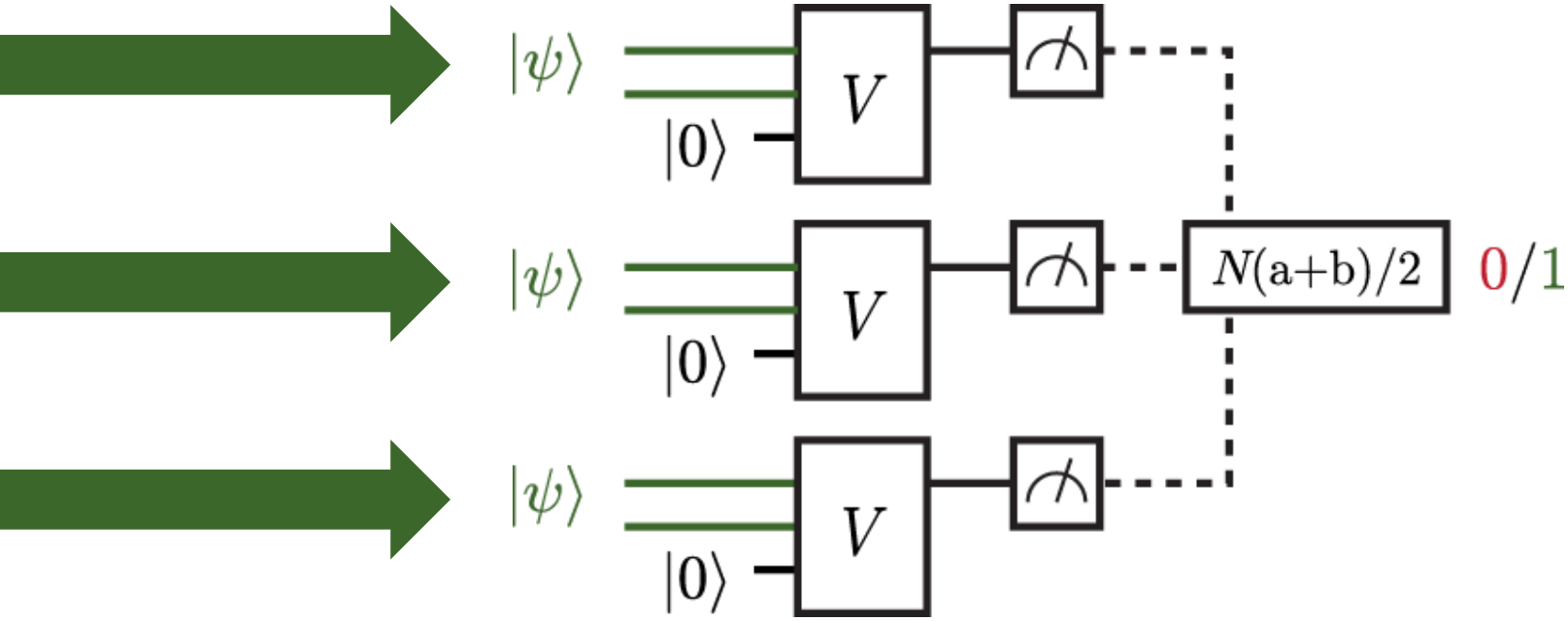


1 The QMA protocol: amplification [Kitaev]



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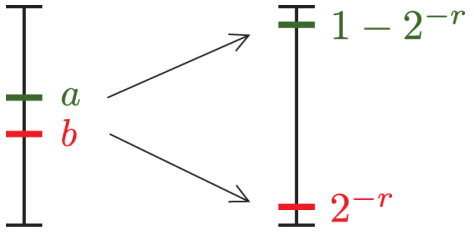
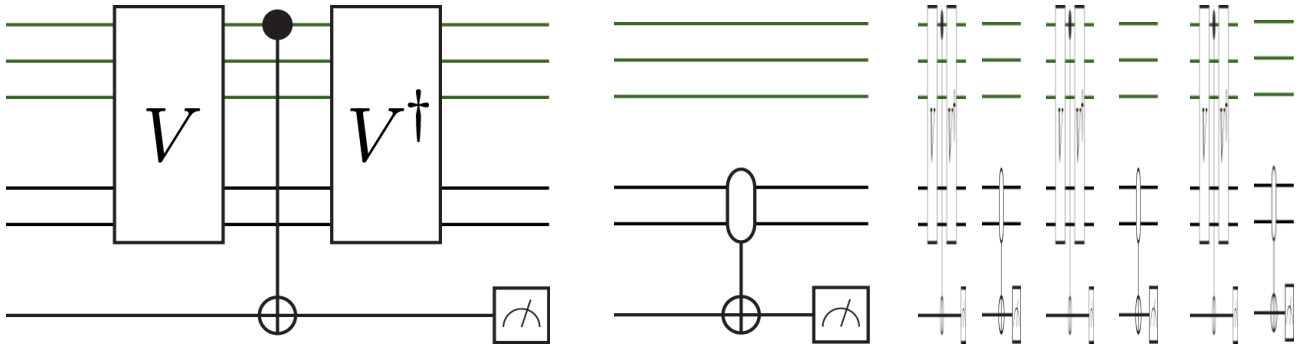


1 The QMA protocol: amplification [Mariott-Watrous]

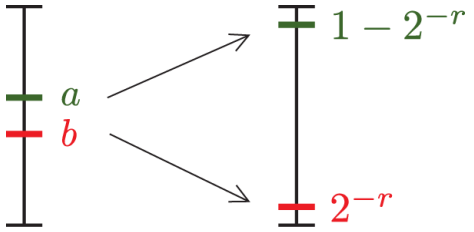
alternating projections P, Q



1, 0, 0, 0, 1, 1, 0, 1 …

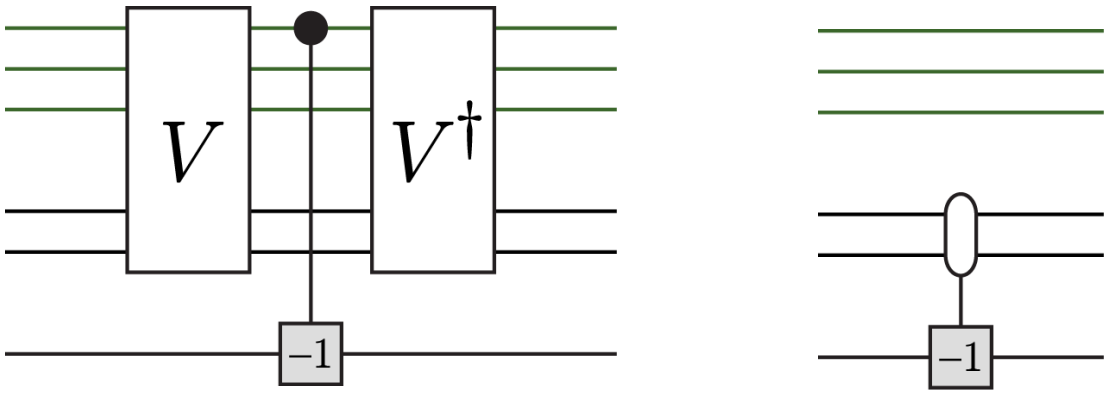
Life in a 2D subspace. [Jordan]

How many 00's and 11's?

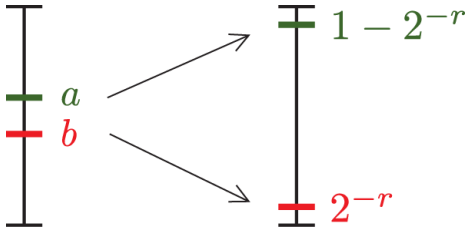


1 The QMA protocol: fast amplification [N.-Wocjan-Zhang]

alternating reflections R, S

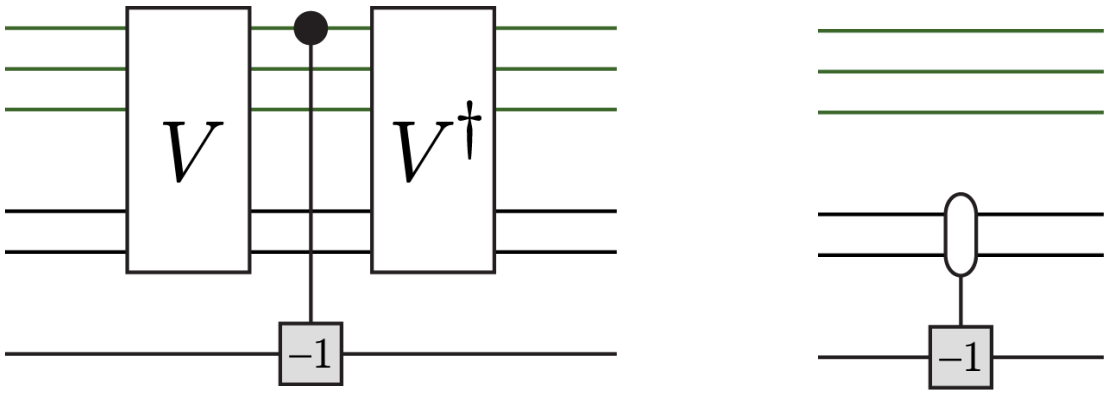


Together: a rotation.
Phase estimation of RS.



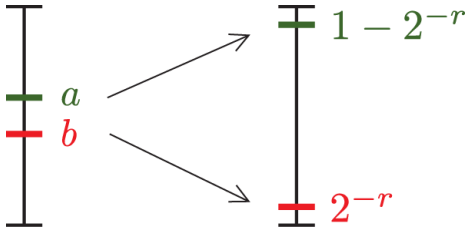
1 The QMA protocol: fast amplification [N.-Wocjan-Zhang]

alternating reflections *R*, *S*



Together: a rotation.

Perfect phase estimation of *RS*?

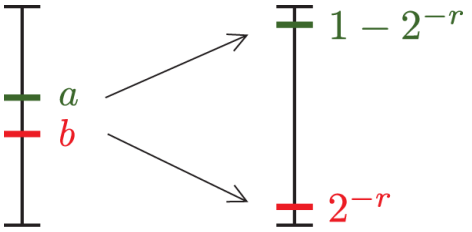


1 Amplification for MA & QMA.

amplification

YES? Accept with p almost 1.

NO? Get fooled with small p.

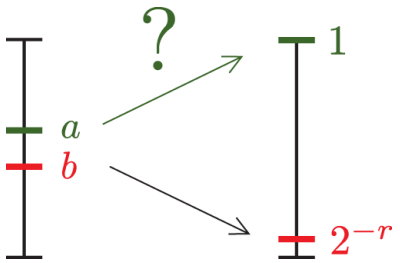


1 Perfect amplification for MA & QMA?

perfect
amplification

YES? Accept a good proof.

NO? Get fooled with small p.



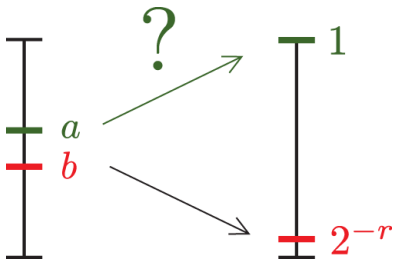
1 Perfect amplification for MA.

perfect classical amplification

$$\mathrm{MA} = \mathrm{MA}_1$$ [Zachos & Fürer]

YES? Accept a good proof.

NO? Get fooled with small *p*.



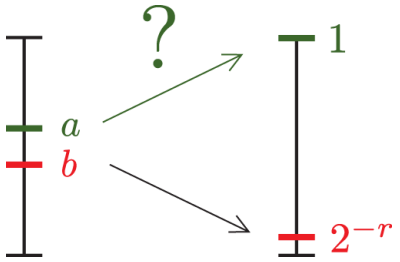
1 Perfect amplification for QMA?

perfect quantum amplification

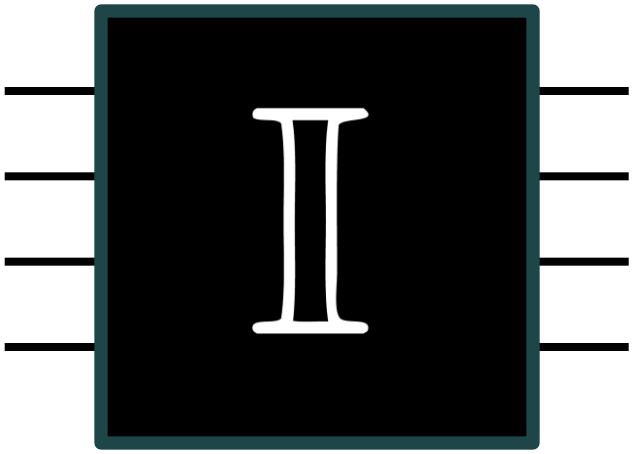
$$\text{QMA} \overset{?}{\to} \text{QMA}_1$$

YES? Accept a good proof.

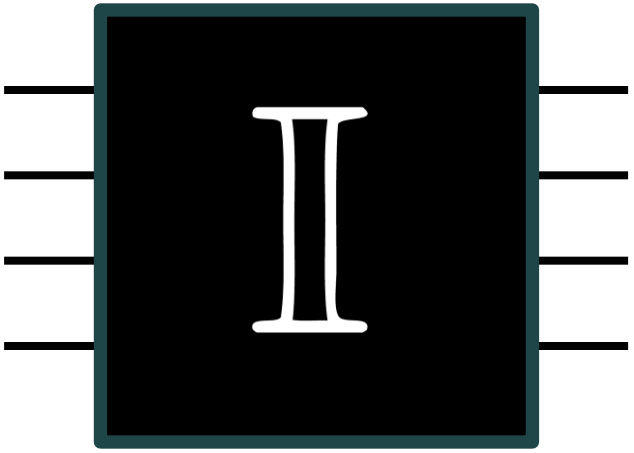
NO? Get fooled with small *p*.



An oracle separation of QMA & QMA_1



An oracle separation of QMA & QMA_1



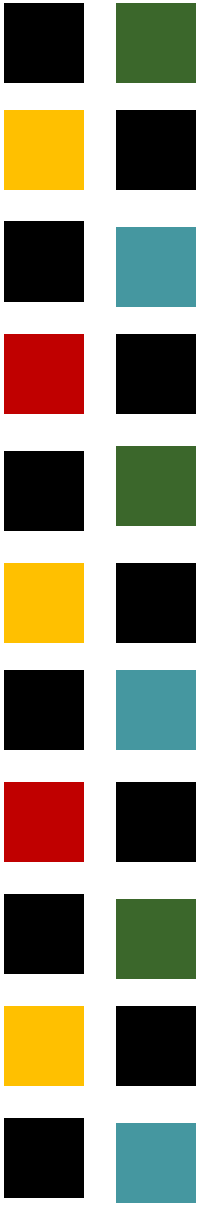
a continuous range of angles

Accept something without a doubt?

=

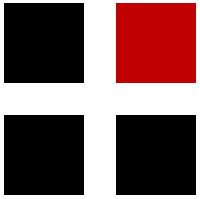
Accept everything…
[Aaronson '08]

Quantum Computing



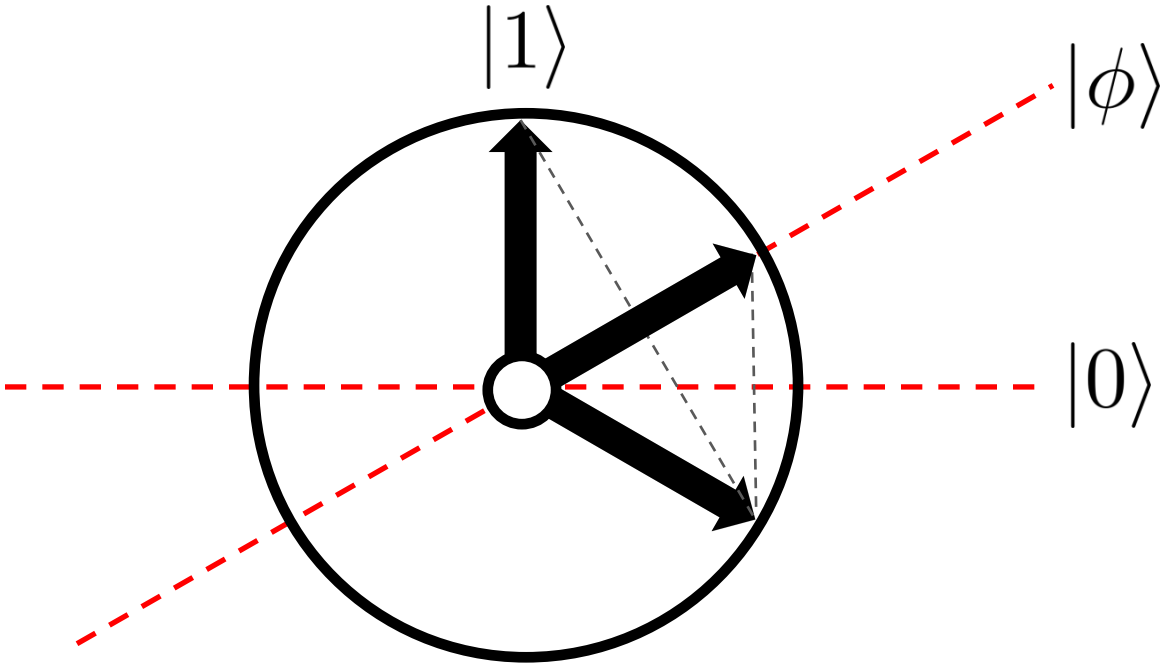
Exact

Quantum Computing



Exact Grover's search [L. Grover]

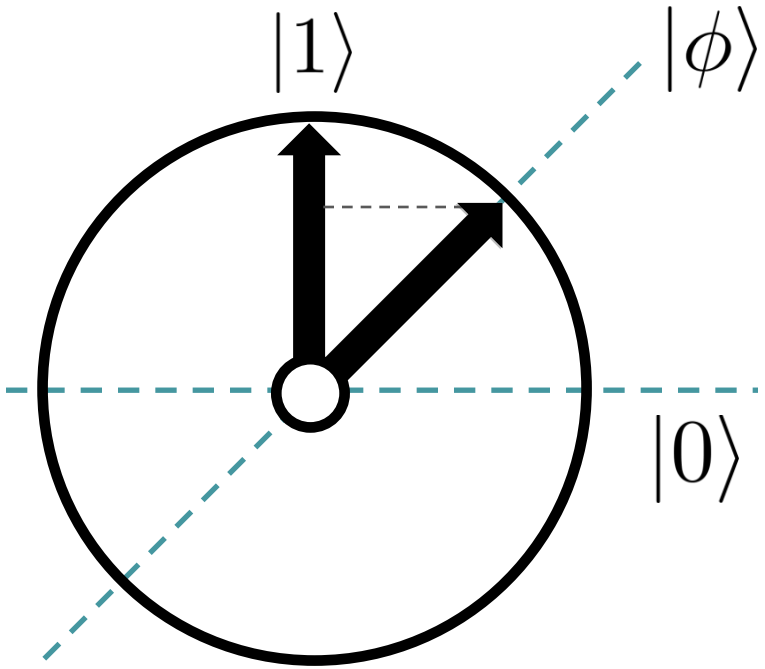
$$|\phi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

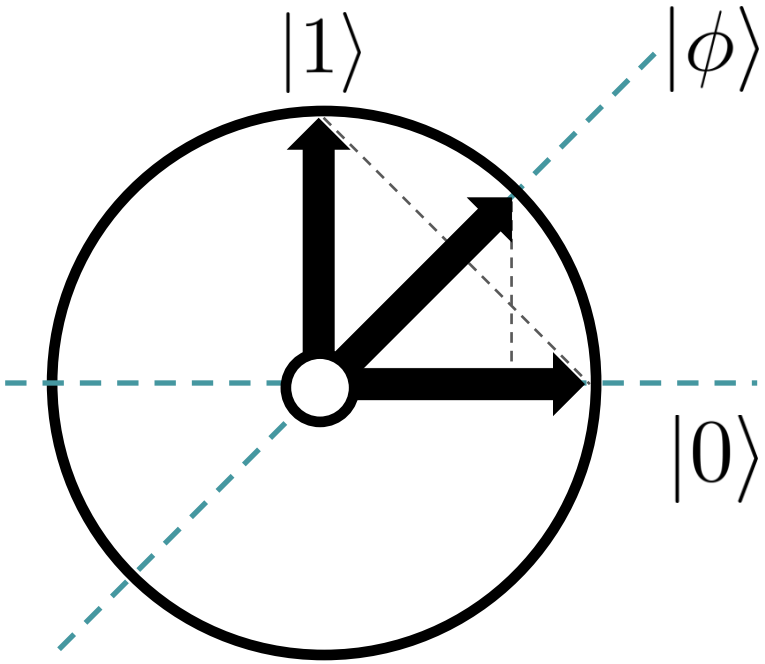


Exact quantum rewinding

[J. Watrous]

$$|\phi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

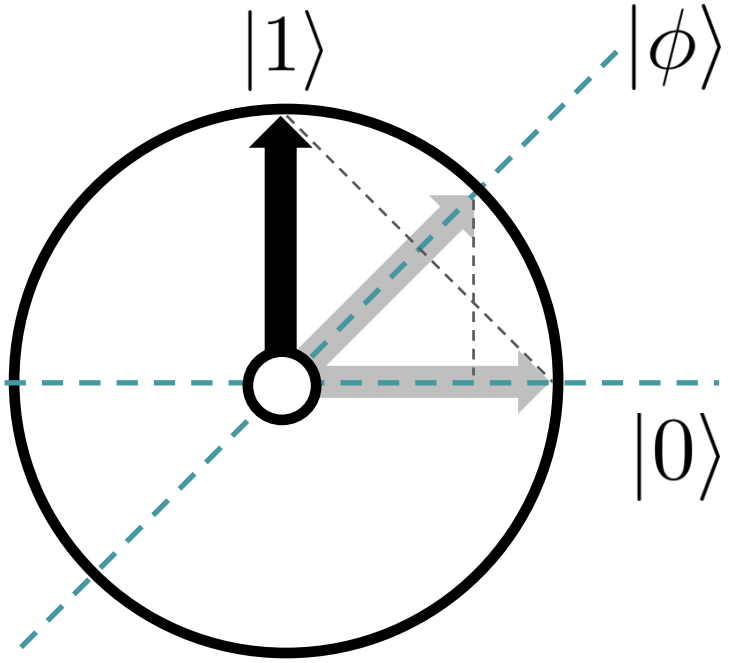




Exact quantum rewinding

$$|\phi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

- test acceptance
- reflect about $|\phi\rangle$
- test acceptance



Exact quantum rewinding

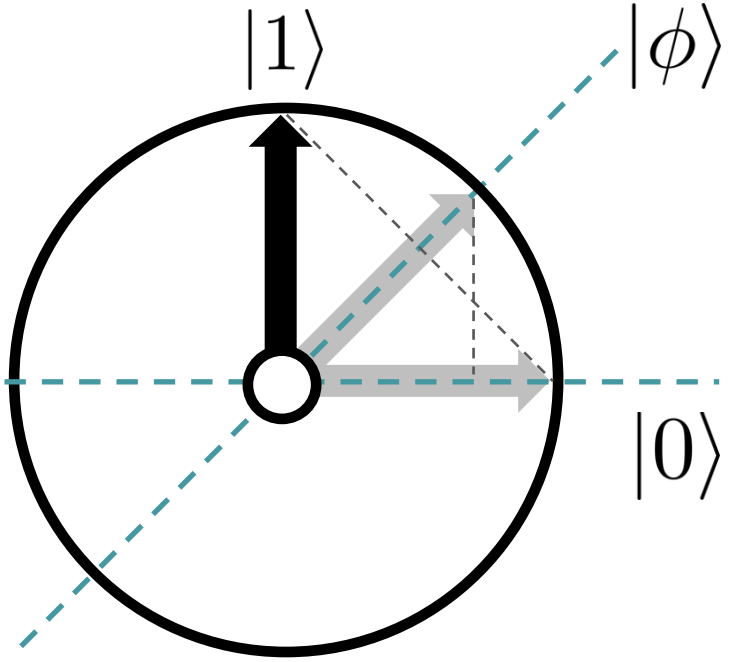
$$|\phi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

a state with a "nice" *p*

test acceptance

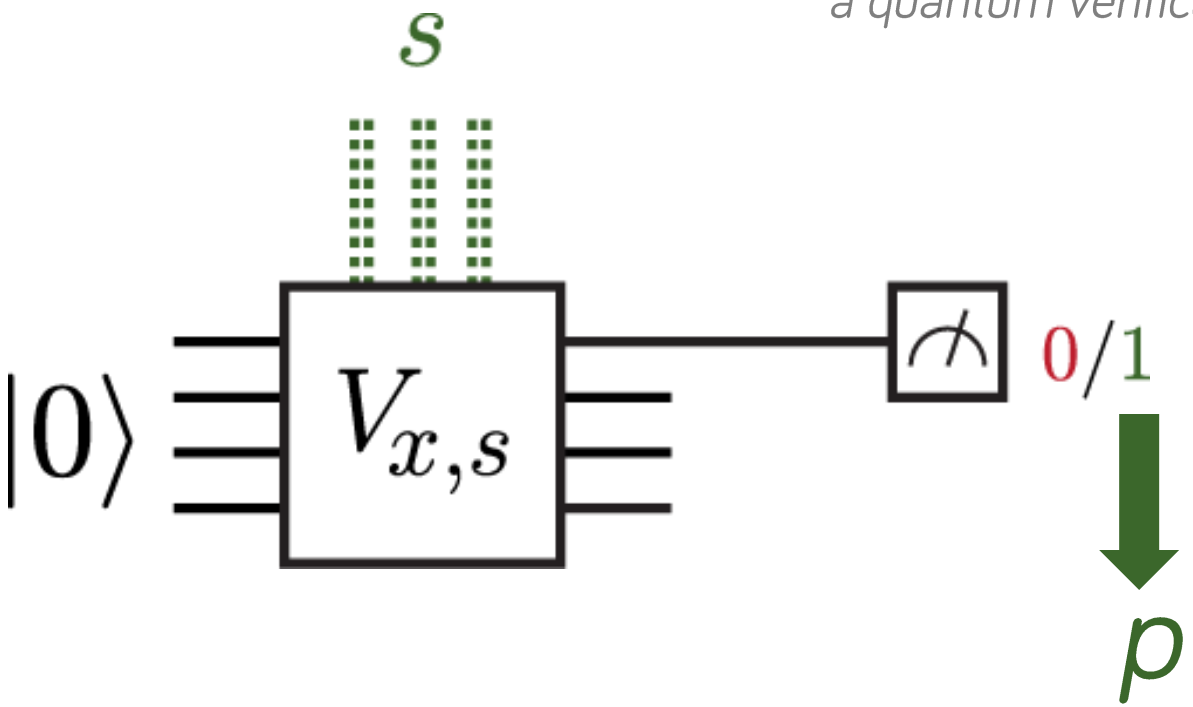
reflect about $|\phi\rangle$

test acceptance



2 QCMA (MQA)

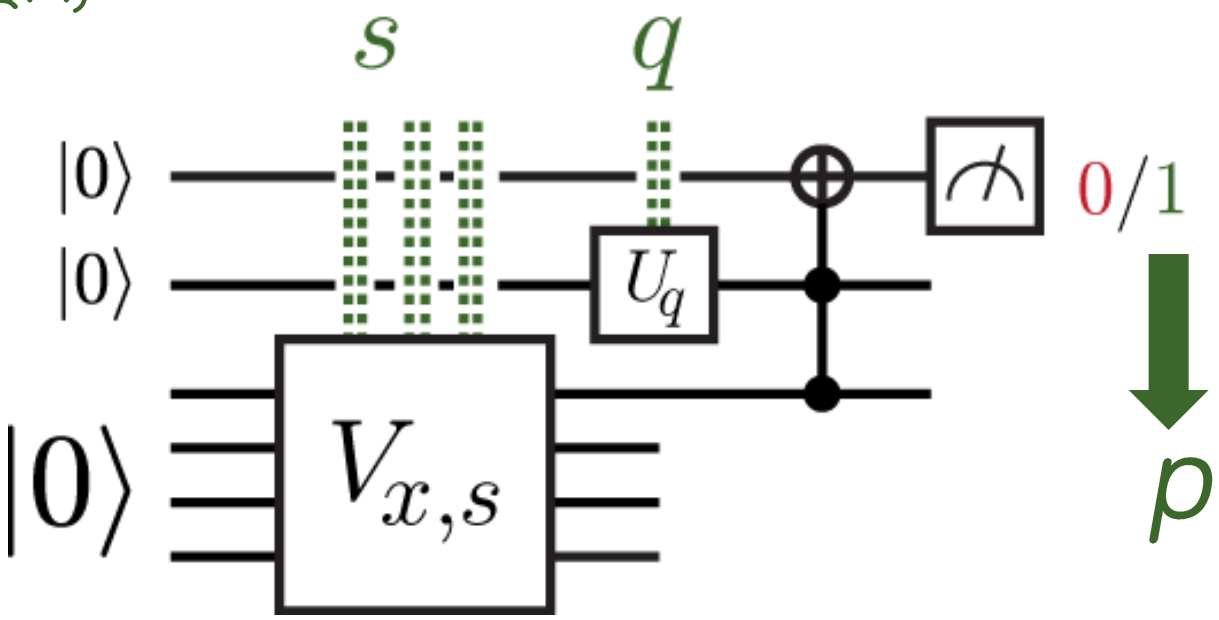
A classical witness,
a quantum verification.



Knowing how to prepare the witness...
we can reflect about it.

$$\sqrt{1-p}|\cdots 0\rangle + \sqrt{p}|\cdots 1\rangle$$

2 QCMA (MQA)

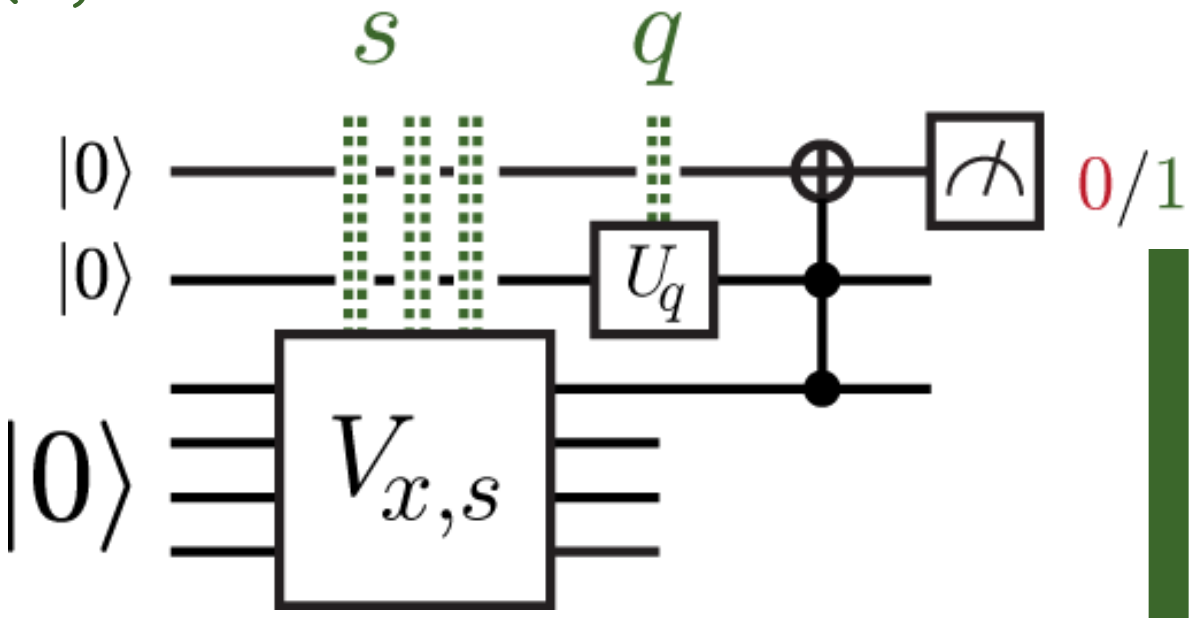


Knowing the acceptance probability...
add a rotated ancilla.

$$\sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$$

$$\sqrt{1-p}|\cdots 0\rangle + \sqrt{p}|\cdots 1\rangle$$

2 QCMA (MQA)



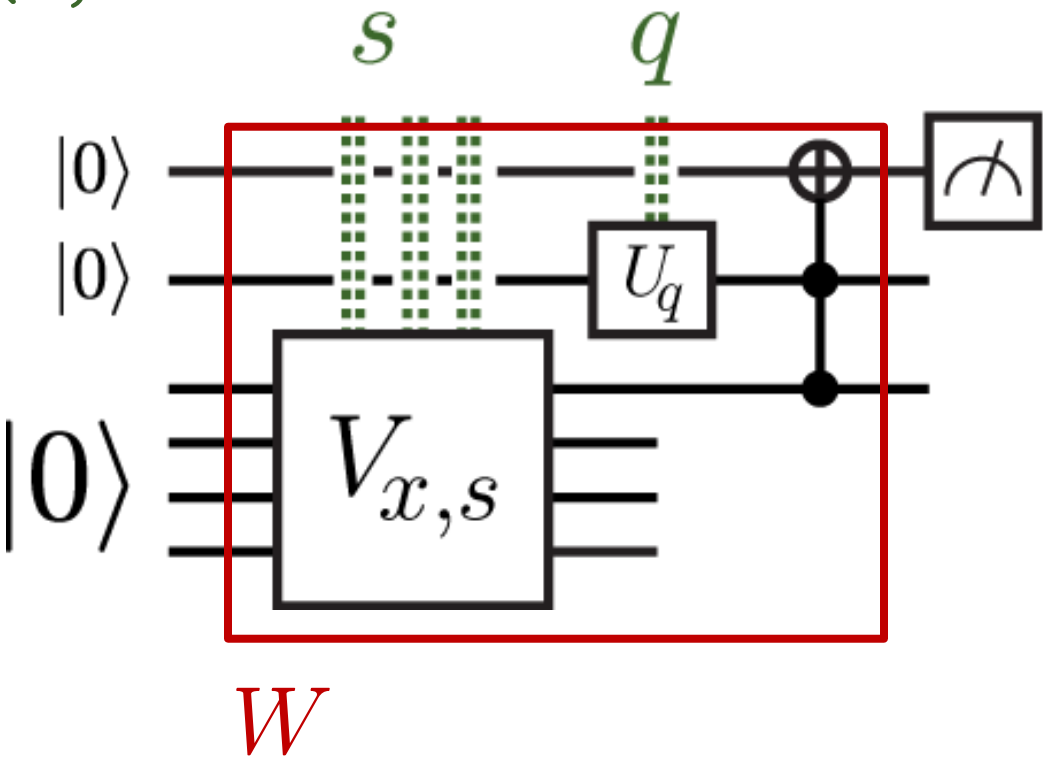
Knowing the acceptance probability...
add a rotated ancilla, get ¼ or ½.

$p'=pq=¼$

$$\sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$$

$$\sqrt{1-p}|\cdots 0\rangle + \sqrt{p}|\cdots 1\rangle$$

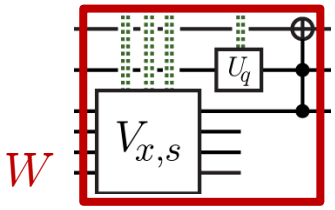
2 QCMA (MQA)

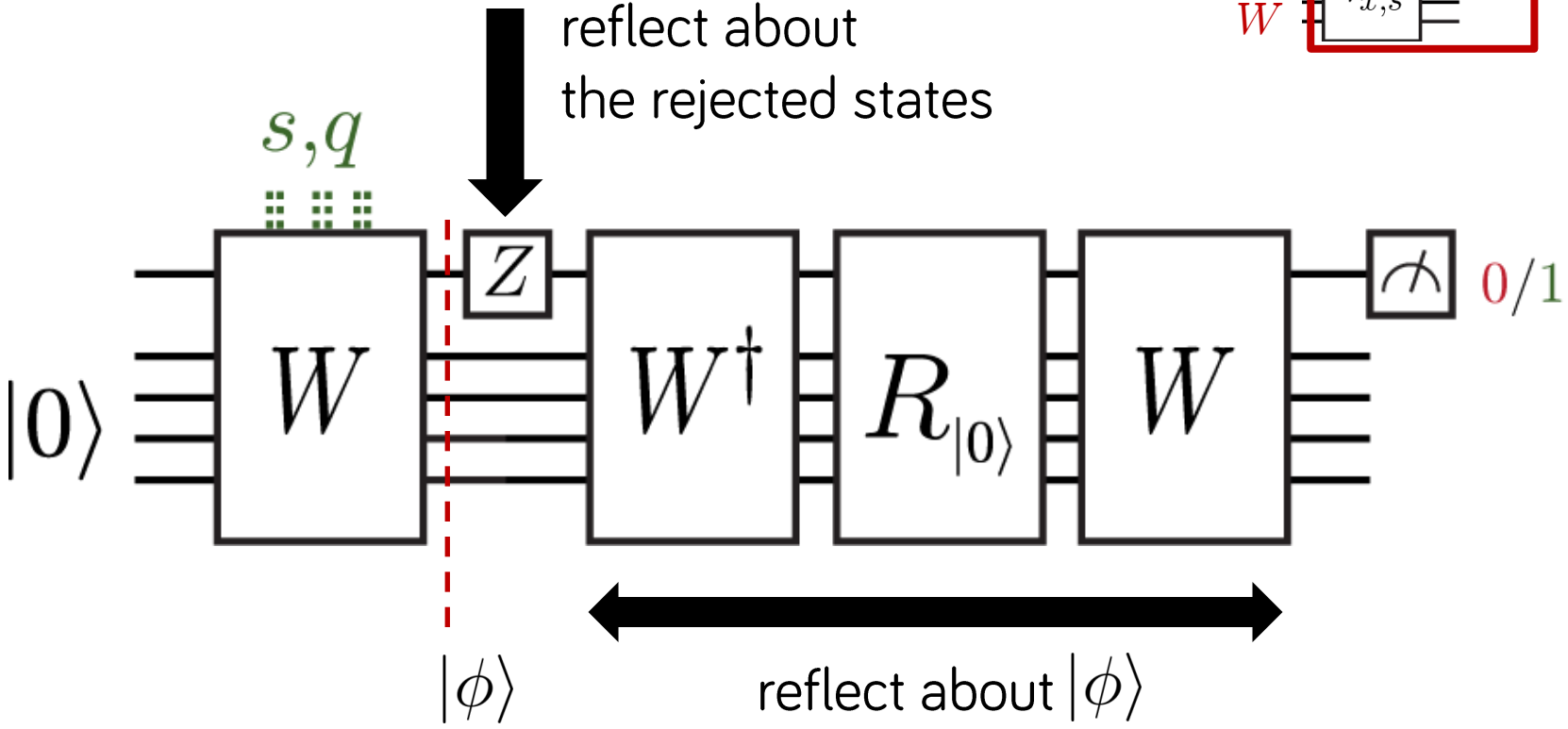


$p'=pq=¼$

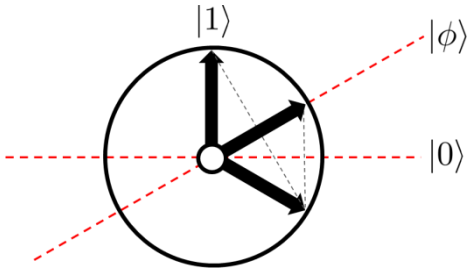
Gates with rational-number elements are universal.
Both p and q are rational.
It's doable.

2 QCMA (MQA) with perfect completeness

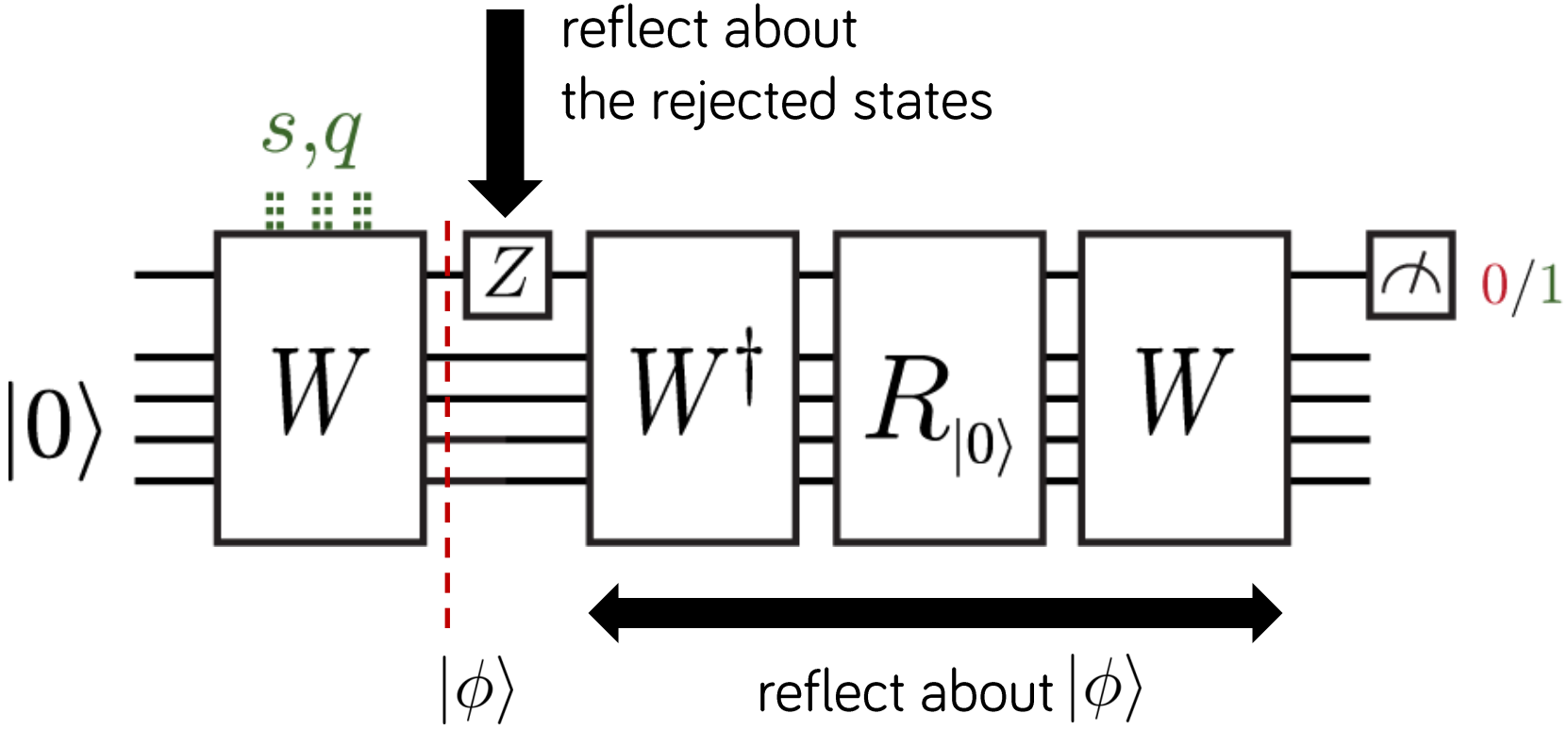




Perfectly accepts solid proofs.

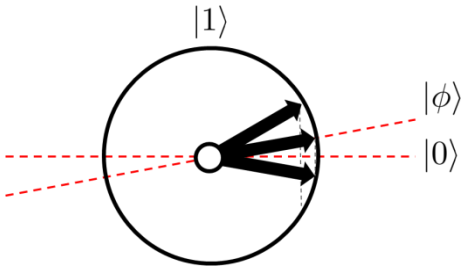


2 QCMA (MQA) with perfect completeness

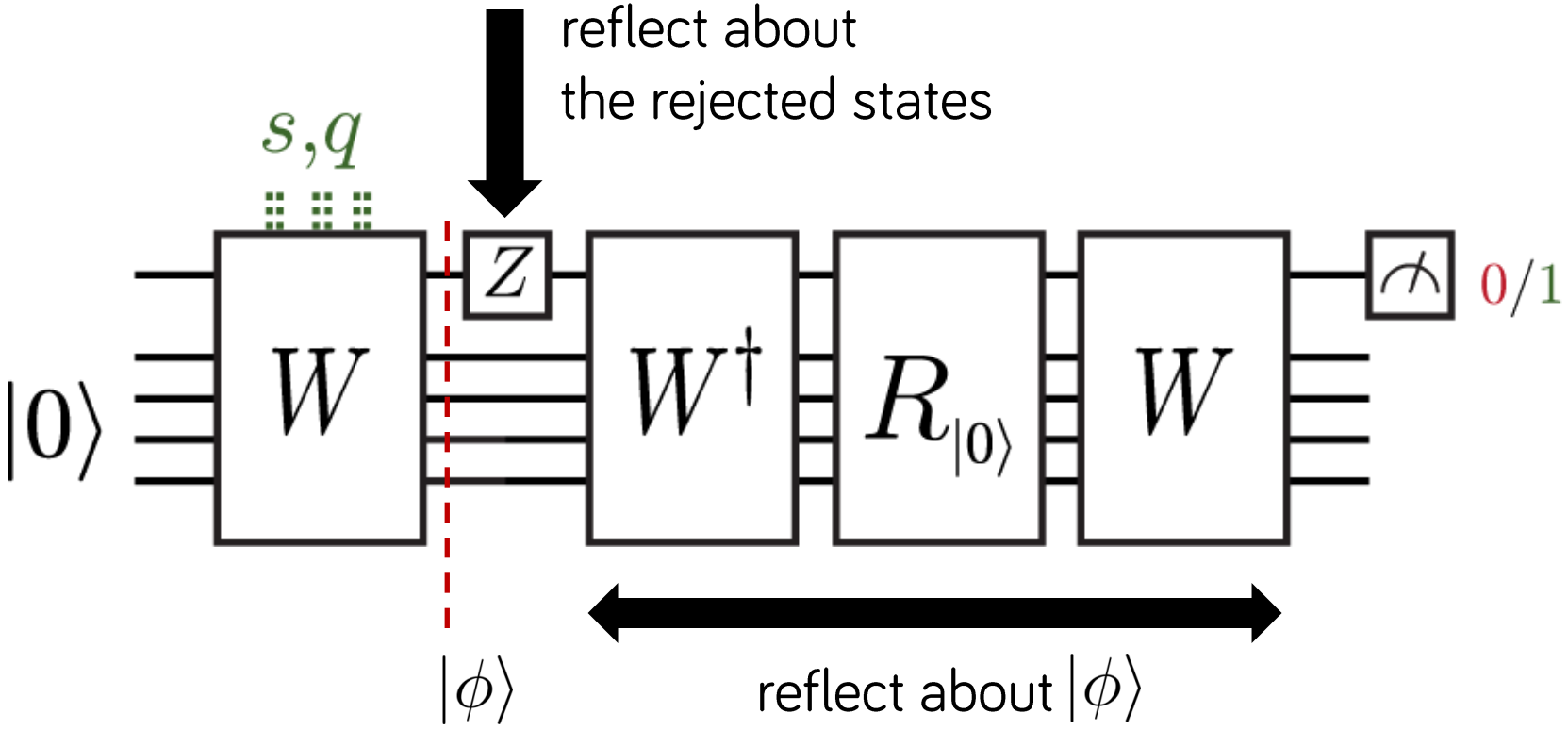


Perfectly accepts solid proofs.

The soundness doesn't break.



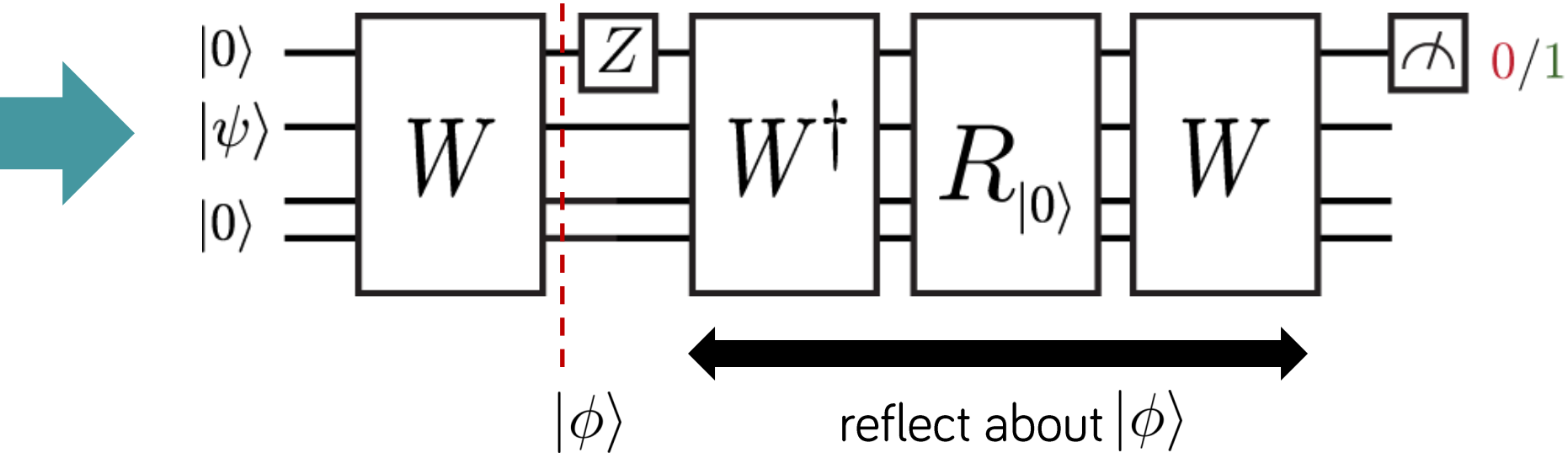
2 QCMA (MQA) with perfect completeness



$$\mathrm{QCMA}_1 = \mathrm{QCMA}$$

3 Towards perfect completeness in QMA...

Let's try the same with a quantum witness.



How to

correct *p* to something nice?

reflect about the unknown witness?

3 Towards perfect completeness in QMA...

Send us the witness.

Send us its acceptance probability p?

a correction q?

How to correct p to something nice?
reflect about the unknown witness?

3 Towards perfect completeness in QMA...

Send us the witness.

Send us its acceptance probability p?

a correction q?

a trustworthy encoding of q?

$$\sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$$

We'll give you some EPR pairs first.

How to

correct p to something nice?

reflect about the unknown witness?

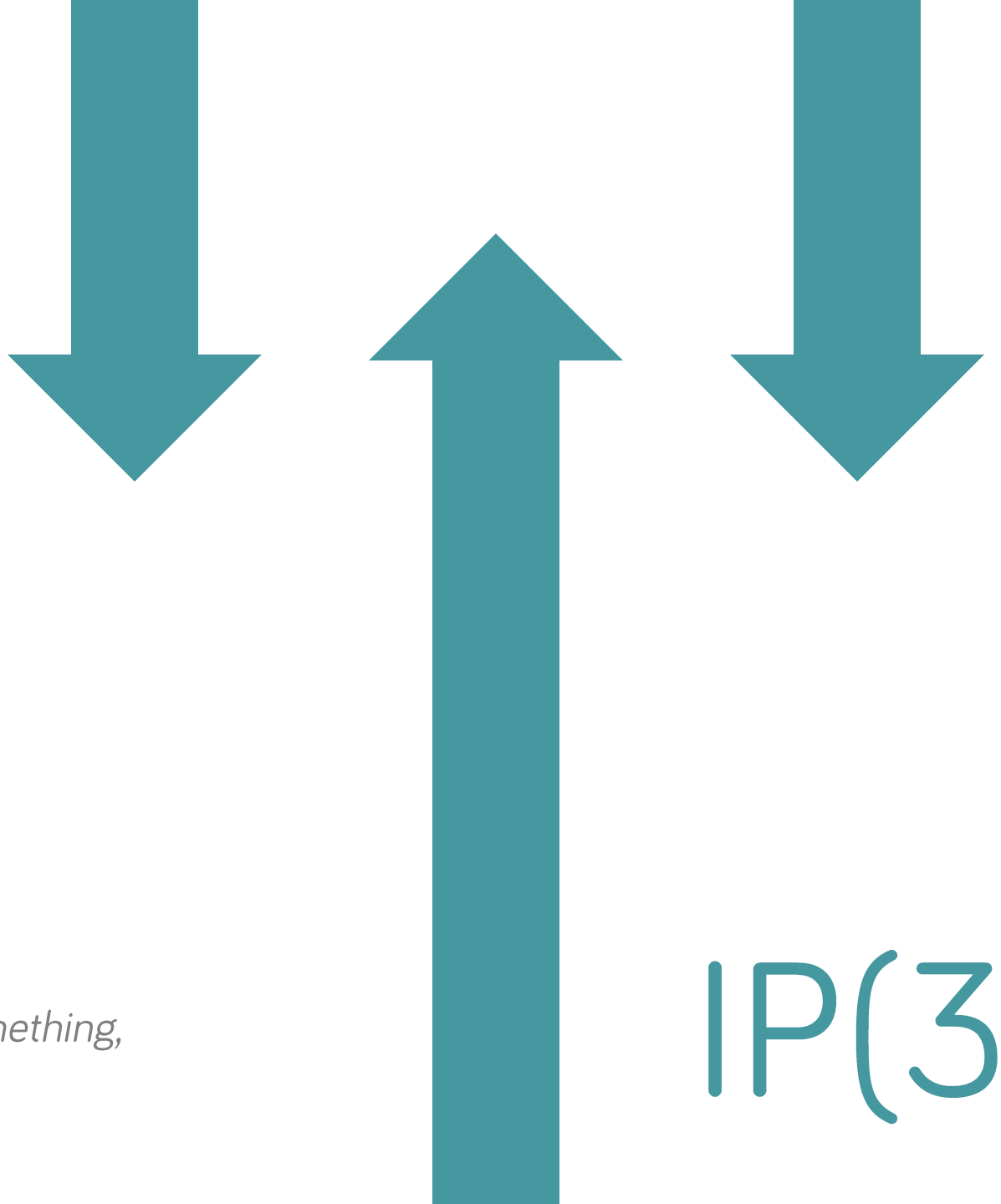
3 Interactive Proofs

Hey, Merlin, could you carve something from this material?

3 Interactive Proofs

Hey, Merlin, could you carve something from this material?

IP(2)

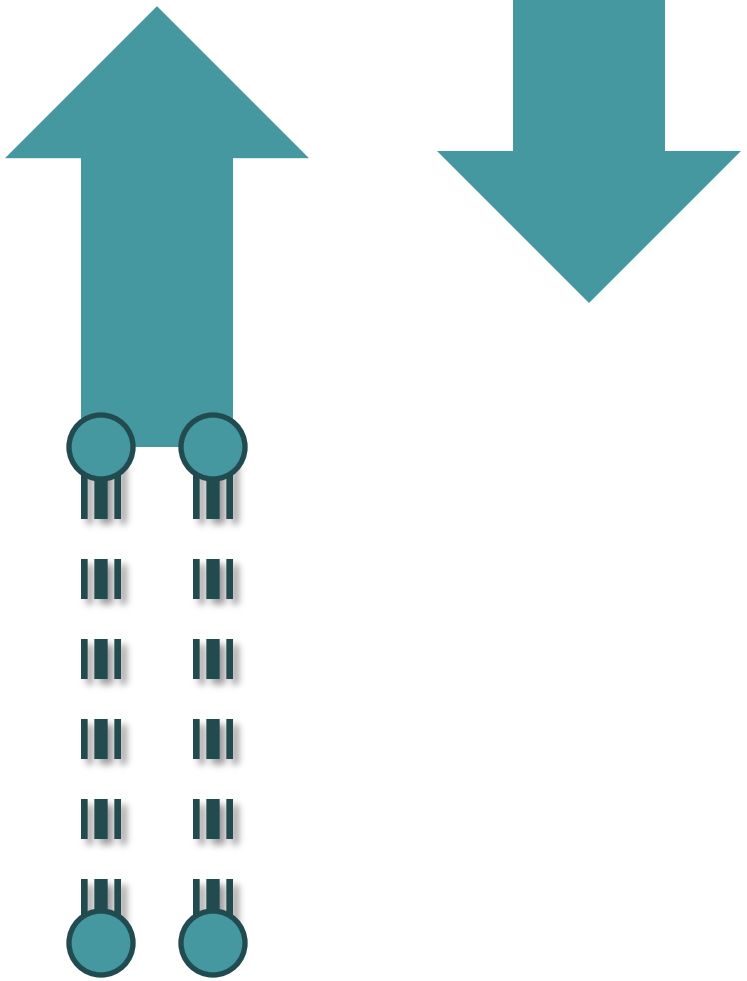


Receive,
compute, ask something,
receive, conclude.

IP(3)


IP(4)

$QIP_1(2)$



QMA_1 const. EPR

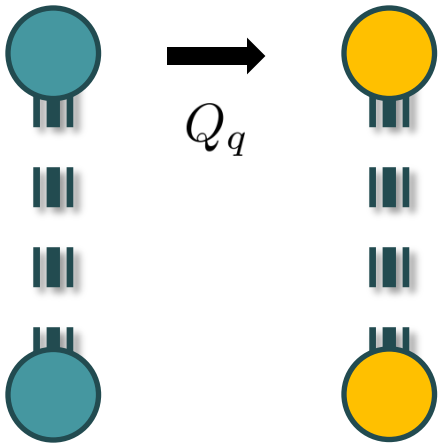
3 Correcting *p* to something "nice".

A "correcting" state $\sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$ with $pq = \frac{1}{2}$

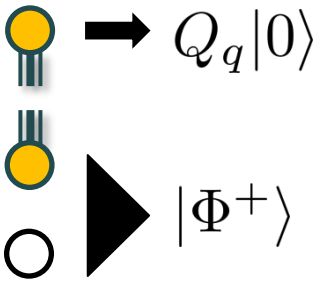
Prepared by Merlin using

$$Q_q = \begin{bmatrix} \sqrt{1-q} & \sqrt{q} \\ \sqrt{q} & -\sqrt{1-q} \end{bmatrix}$$

on a half of an EPR pair



$$\sqrt{1-q}|\Phi^-\rangle + \sqrt{q}|\Psi^+\rangle$$

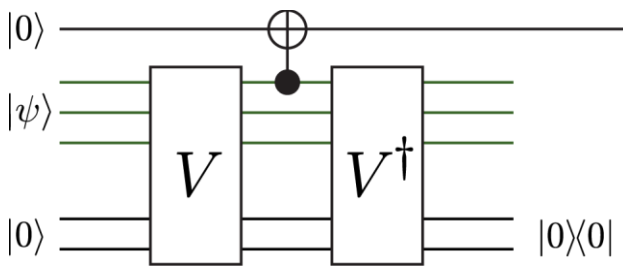


A Choi-Jamiołkowski state… it allows probabilistic (heralded) simulation of Q_q.

3 The soundness is much easier to prove with distillation

Instead of using $\sqrt{1-p}|\cdots 0\rangle + \sqrt{p}|\cdots 1\rangle$

"distill" the state



$\sqrt{1-r}|0\rangle + \sqrt{r}|1\rangle$

with *r* related to *p*

Use it to apply V_r probabilistically

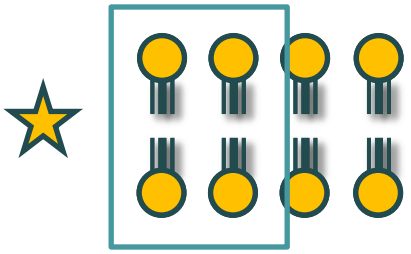
$$V_r = \begin{bmatrix} \sqrt{1-r} & \sqrt{r} \\ \sqrt{r} & -\sqrt{1-r} \end{bmatrix}$$

We can simulate the reflection about $|\phi\rangle = W(|\psi\rangle \otimes |0\rangle)$

3 The combined SOUND protocol

Send Merlin *N* halves of EPR pairs.

He applies Q_q, returns them & a witness.

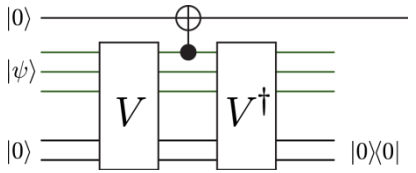


Permute the "EPR pairs".

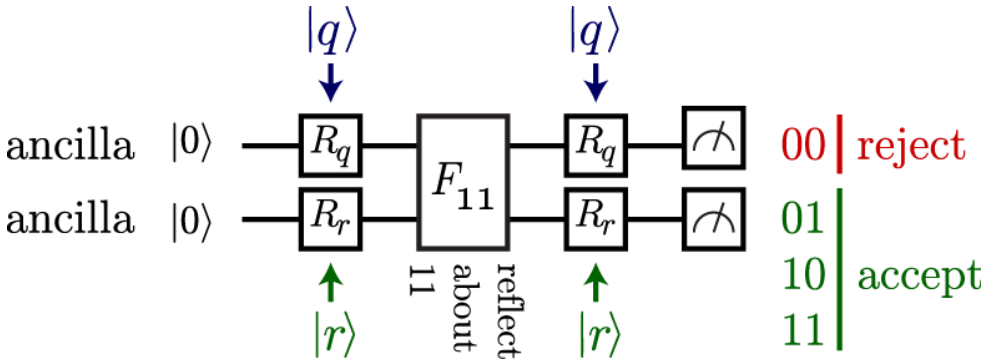
Pick the first two.

SWAP test & Subspace test.

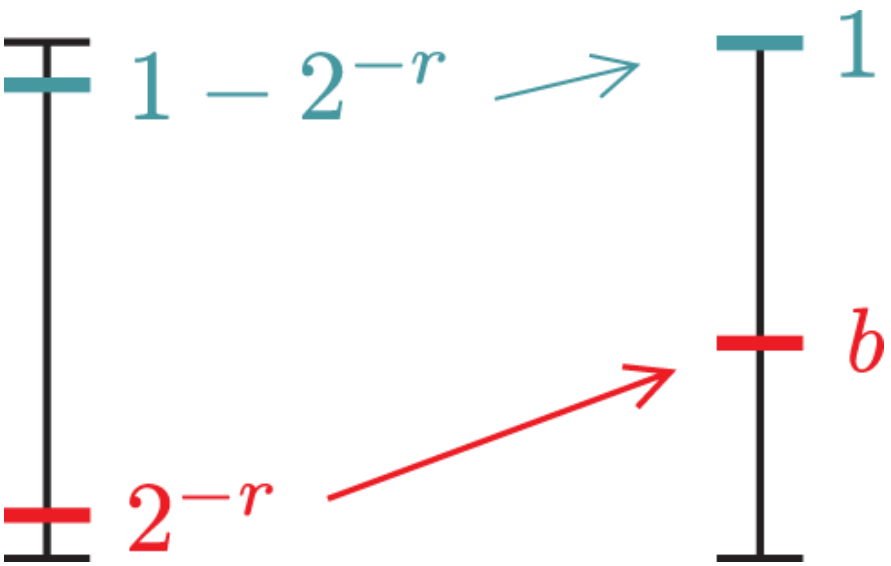
Distill 2 copies of $\sqrt{1-r}|0\rangle + \sqrt{r}|1\rangle$



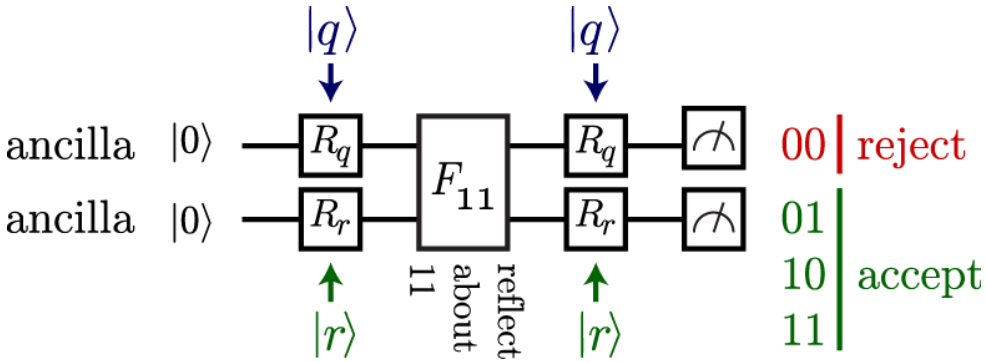
Simulate a modified verification. If the simulation fails, accept.



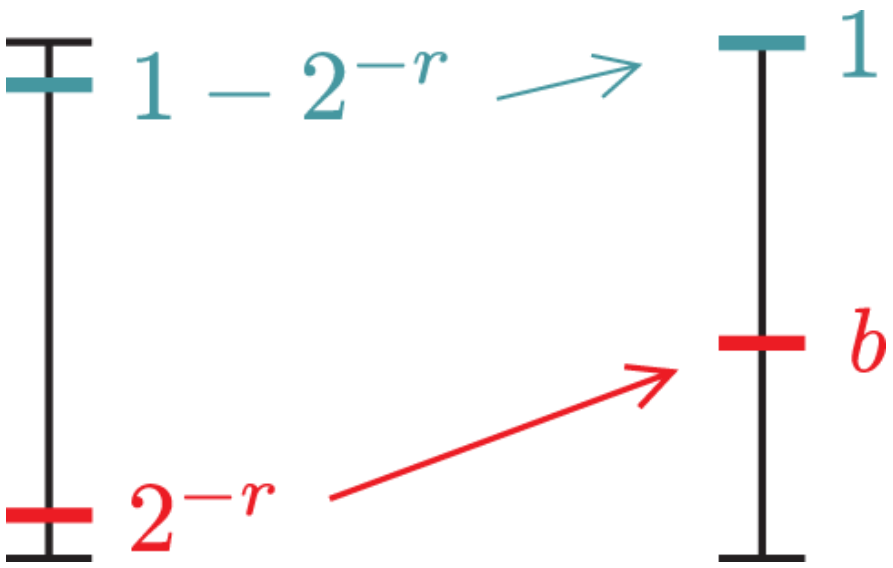
3 The second result



Simulate a modified verification. If the simulation fails, accept.



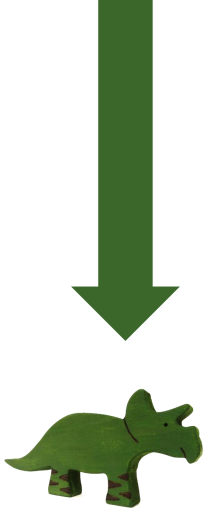
3 The second result



Simulate a modified verification. If the simulation fails, accept.

$$\mathrm{QMA} \subseteq \mathrm{QMA}_1^{\text{const. EPR}}$$

4 Towards perfect completeness for QMA



- It is quite difficult.

  The last, tiny but annoying step.
  An oracle separation to tackle.

- Classical & “nice” witnesses.

  Perfect quantum rewinding.
  Reflection about a known initial state.

$$\mathrm{QCMA} = \mathrm{QCMA}_1$$

- A constant # of EPR pairs.

  Simulating reflections probabilistically.
  Shared EPR pairs give us soundness.

$$\mathrm{QMA} \subseteq \mathrm{QMA}_1^{\mathrm{c.EPR}}$$

QMA

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QMA_1

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