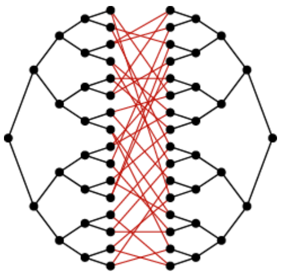


Quantum speedup by quantum annealing

continuous-time, adiabatic, annealing, stoquastic, exponential

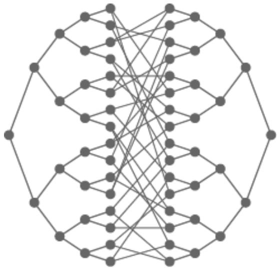
Rolando Somma LANL

Daniel Nagaj IPSAS Bratislava

Mária Kieferová IPSAS Bratislava

arXiv: 1202.6257 (soon in PRL)

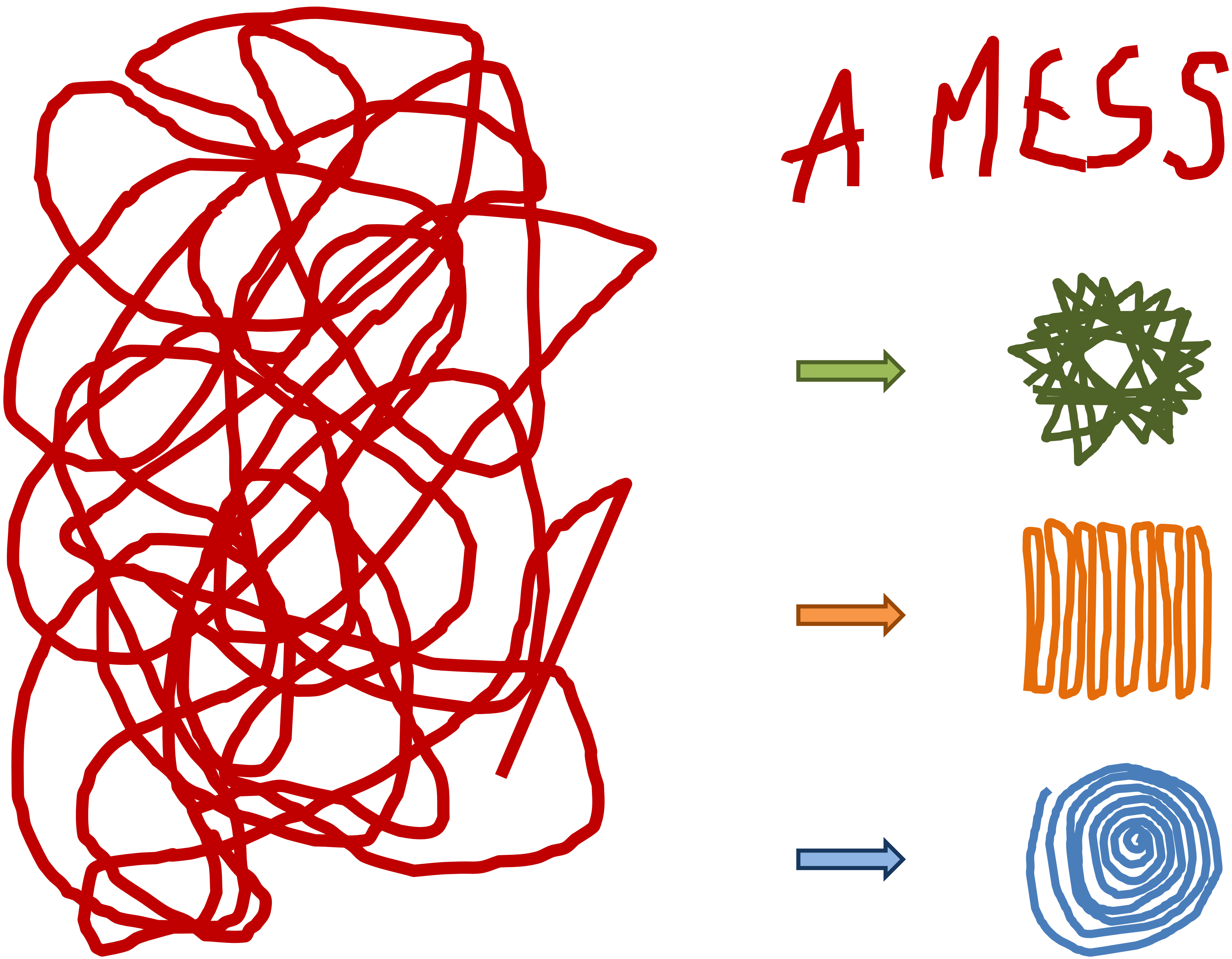
thanks: LPP.QWAC, APVV.COQI, meta-QUTE, Q-ESSENCE

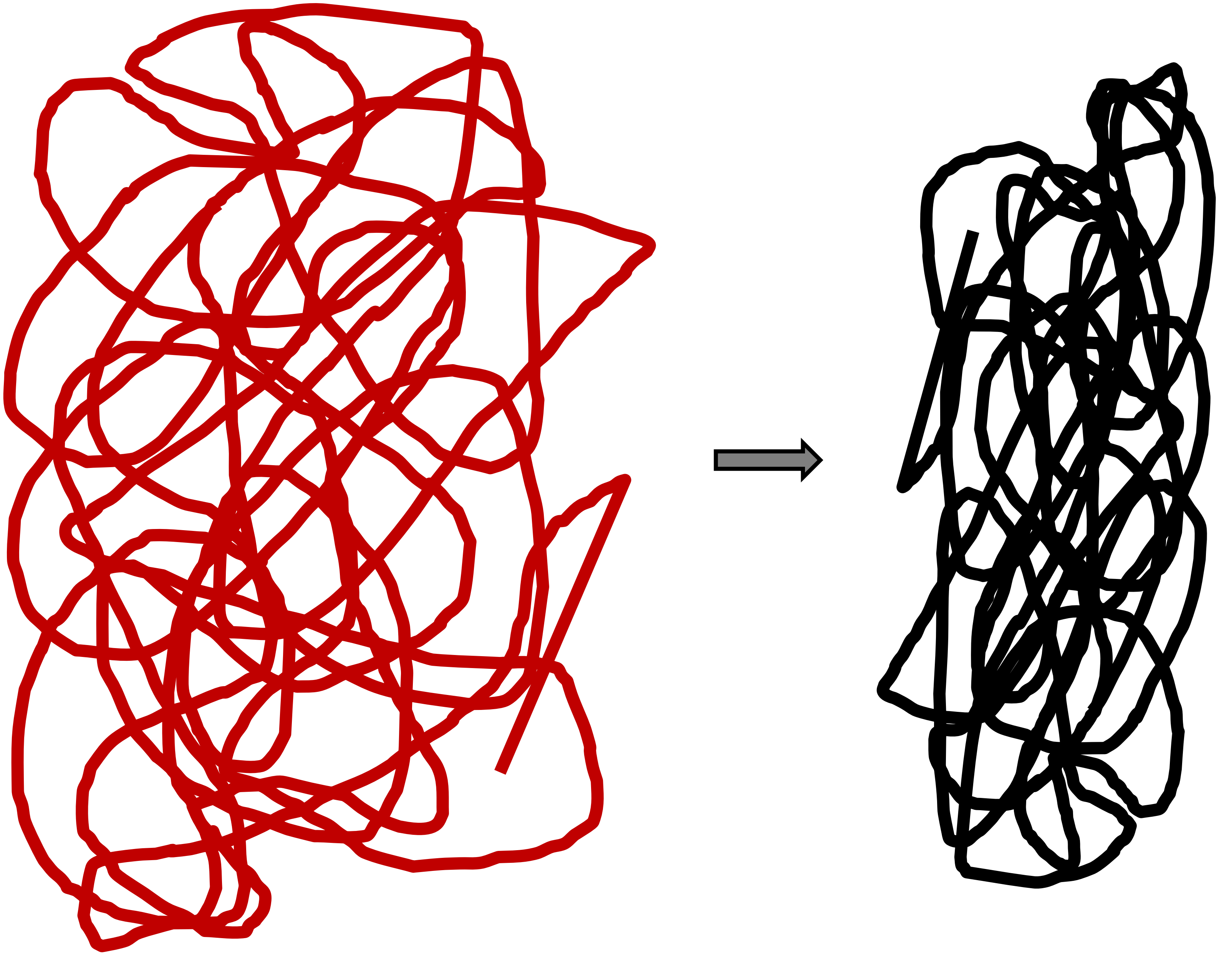


Quantum speedup by quantum annealing

1. Continuous-time quantum walks
2. Walking through glued trees efficiently
3. Adiabatic optimization & stoquastic H’s
4. Traversing trees by quantum annealing

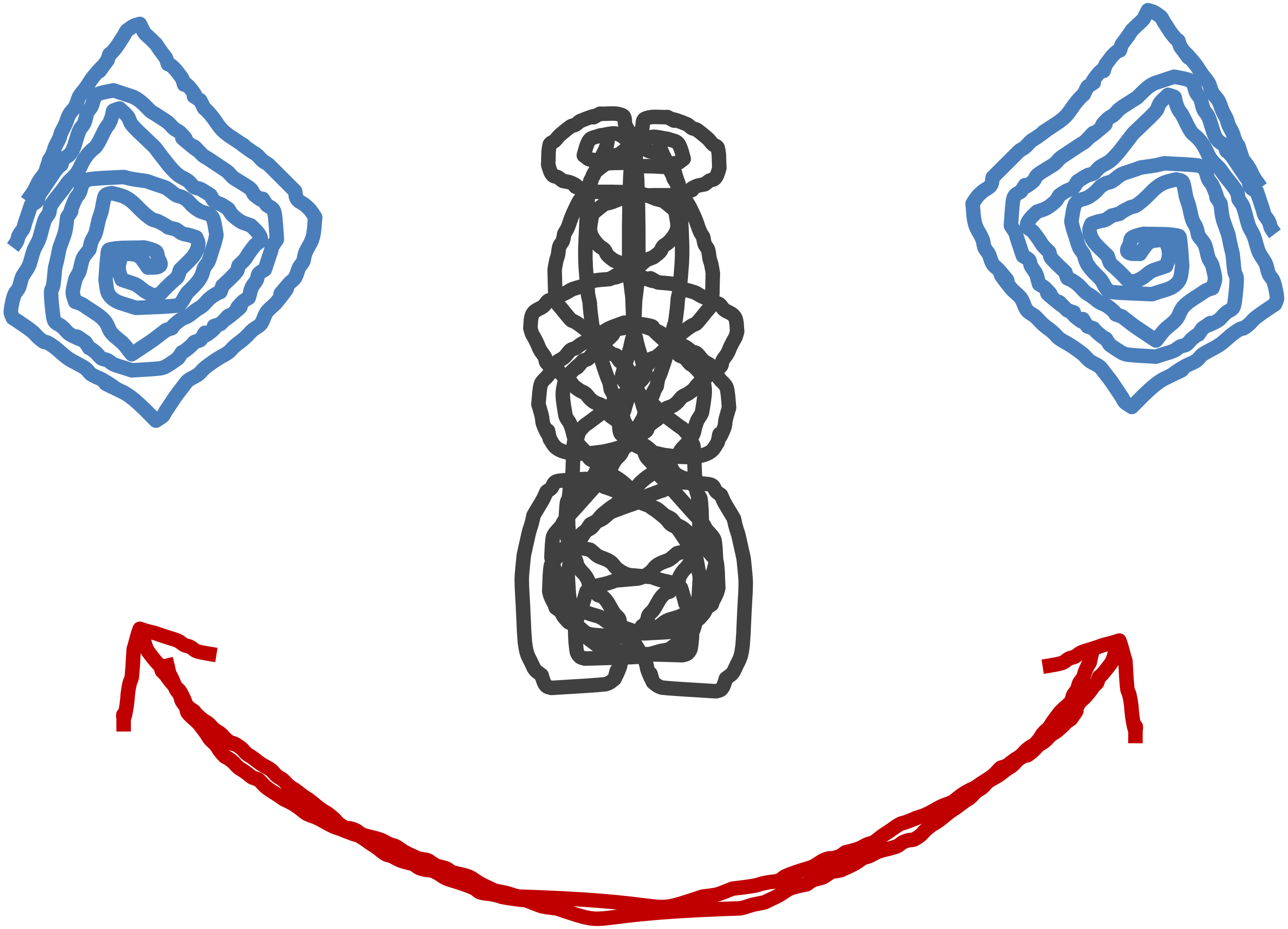
NICE BUT SLOW


A MESS



arXiv: 1202.6257, PRL (soon)

Quantum Speedup by Quantum Annealing



1 Continuous-time quantum walks

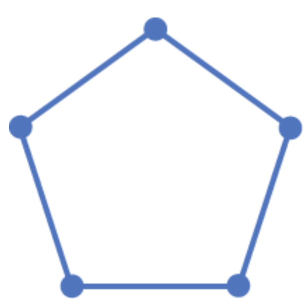
trying something beyond diffusion

classical diffusion: evolve probabilities

$$\frac{\mathrm{d}p_j(t)}{\mathrm{d}t} = \sum_{k\in V} L_{j,k}\, p_k(t)$$

Laplacian

$$L_{j,k} = \begin{cases} -\deg(j) & j = k, \\ 1 & (j,k) \in E, \\ 0 & \text{otherwise.} \end{cases}$$



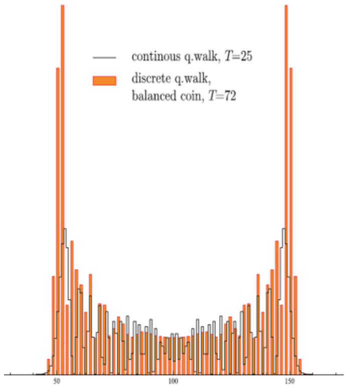
$$L_{\circ_5} = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}$$

1 Continuous-time quantum walks

trying something beyond diffusion

classical diffusion: evolve probabilities

$$\frac{\mathrm{d}p_j(t)}{\mathrm{d}t} = \sum_{k\in V} L_{j,k} p_k(t)$$



Schrödinger equation: evolve amplitudes

$$i\frac{\mathrm{d}}{\mathrm{d}t}\left|\psi(t)\right\rangle = L\left|\psi(t)\right\rangle$$

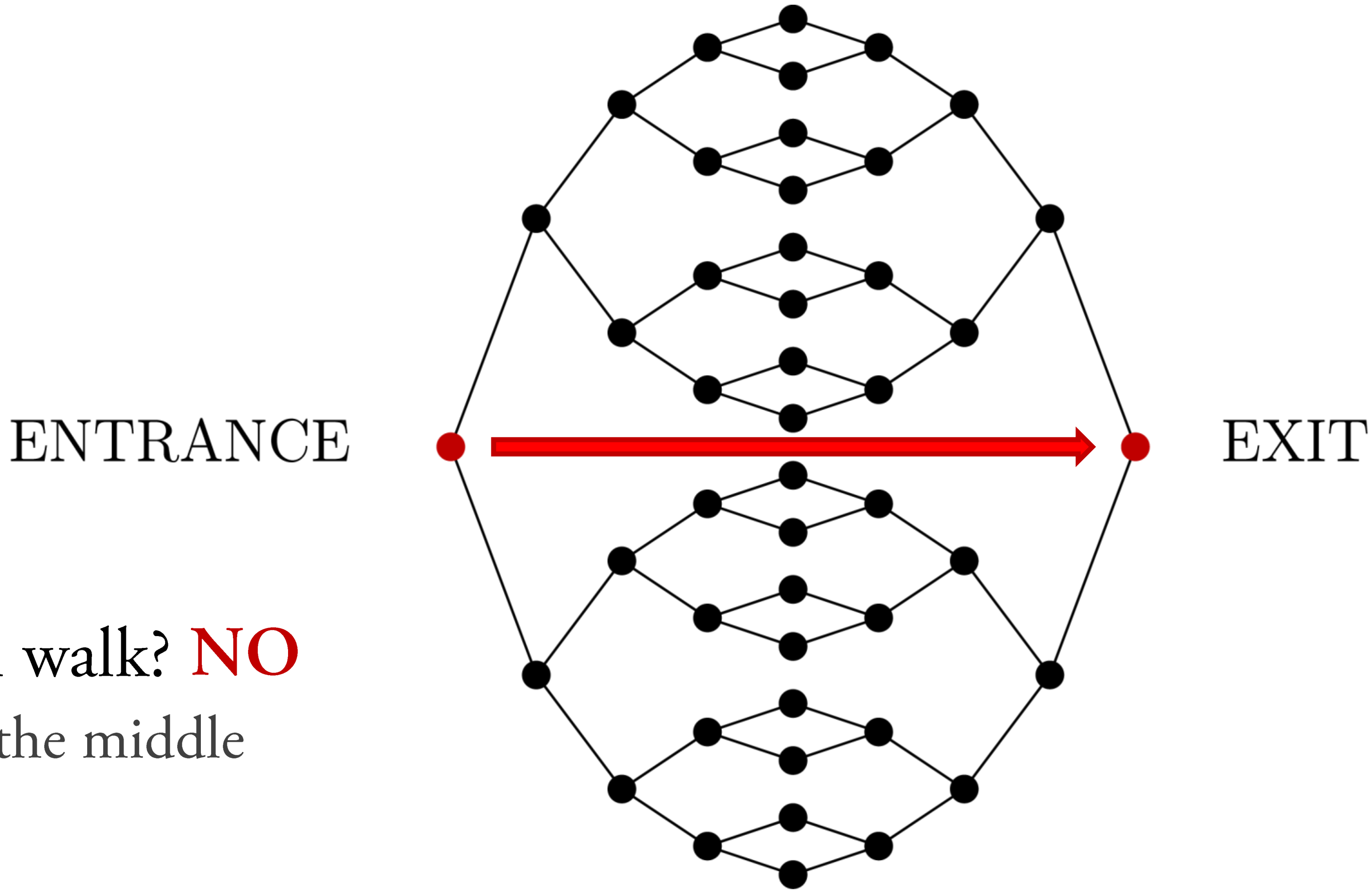
Farhi & Gutmann, Quantum computation and decision trees (1998)

- graph structure: Laplacian, adjacency matrix, …
- strong decoherence (measure often): back to diffusion
- quantum algorithms, universality

Childs, Gosset, Webb, Universal comp… (2012)

2 Efficiently walking through glued trees

finding an exit from a maze



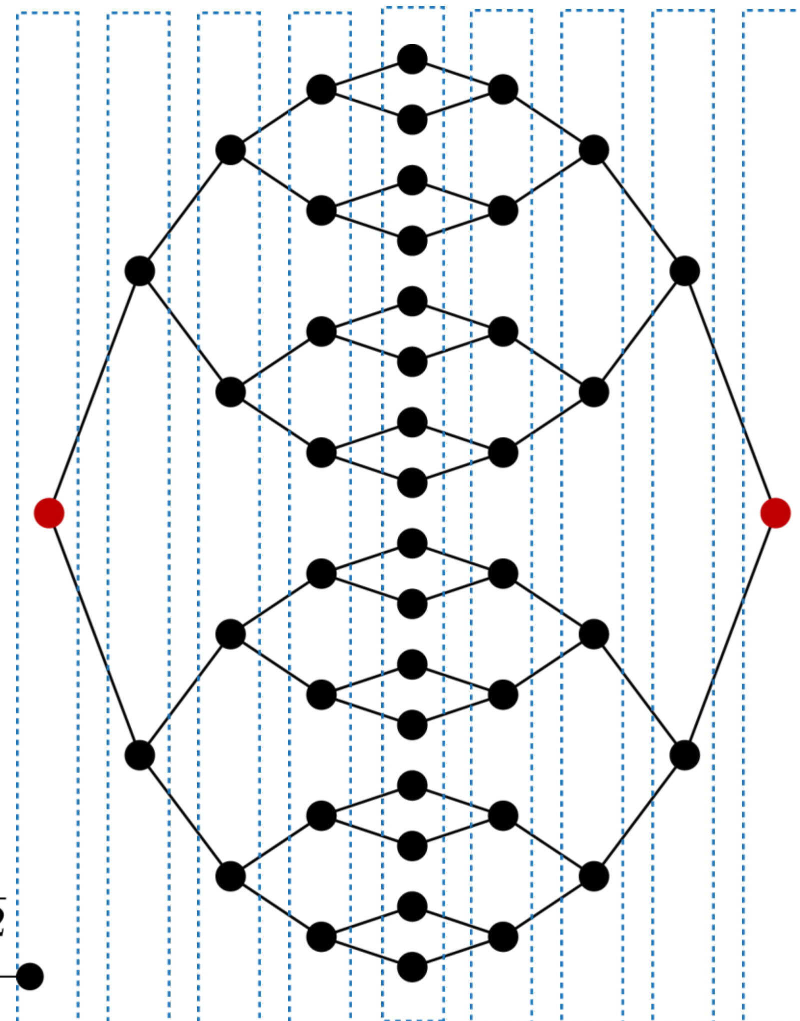
- random walk? **NO**

  stuck in the middle

2 Efficiently walking through glued trees

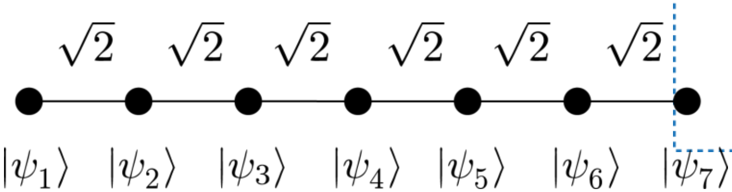
walking in superposition over columns

ENTRANCE



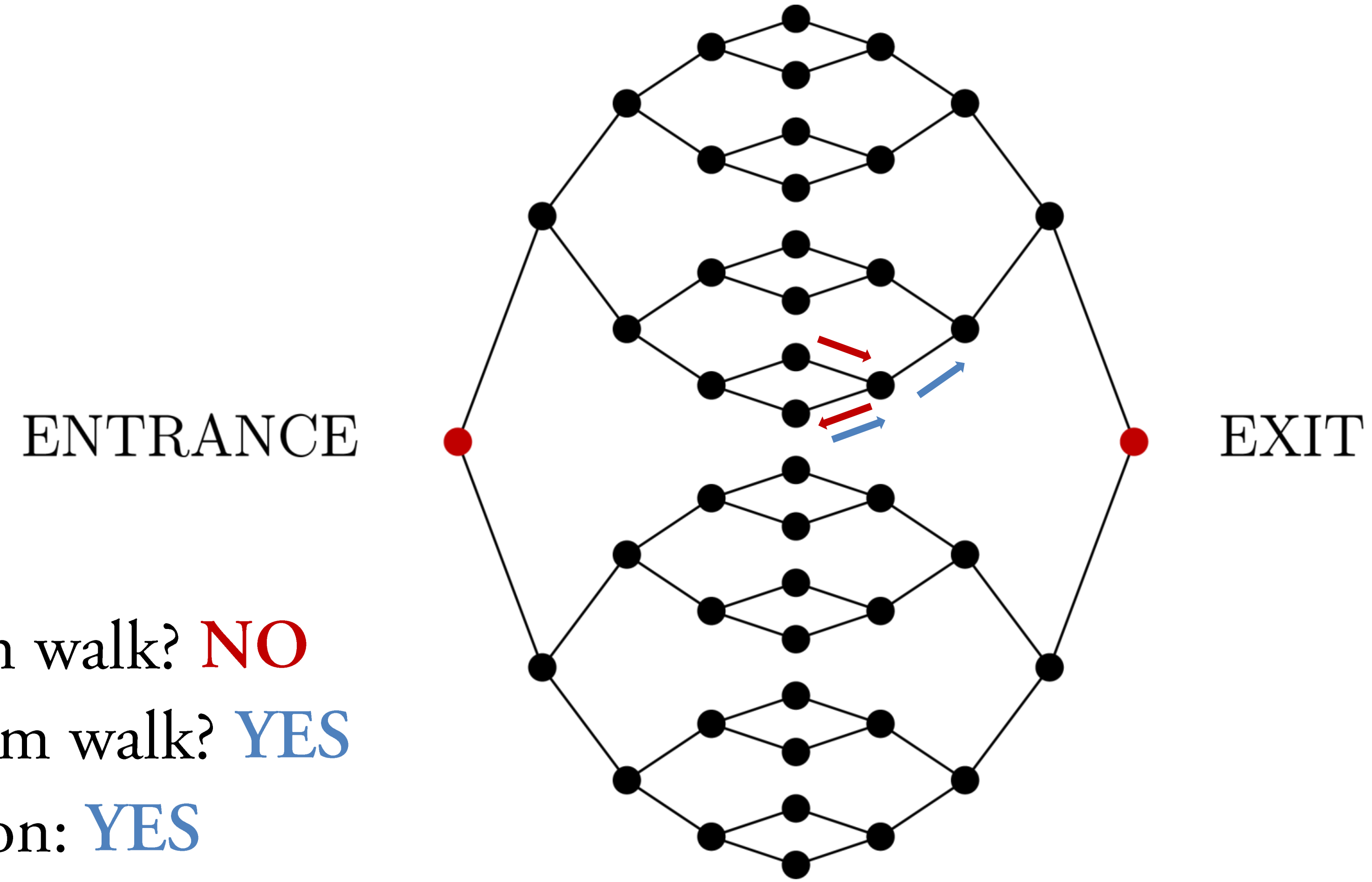
EXIT

- random walk? **NO**
- quantum walk? **YES**



2 Efficiently walking through glued trees

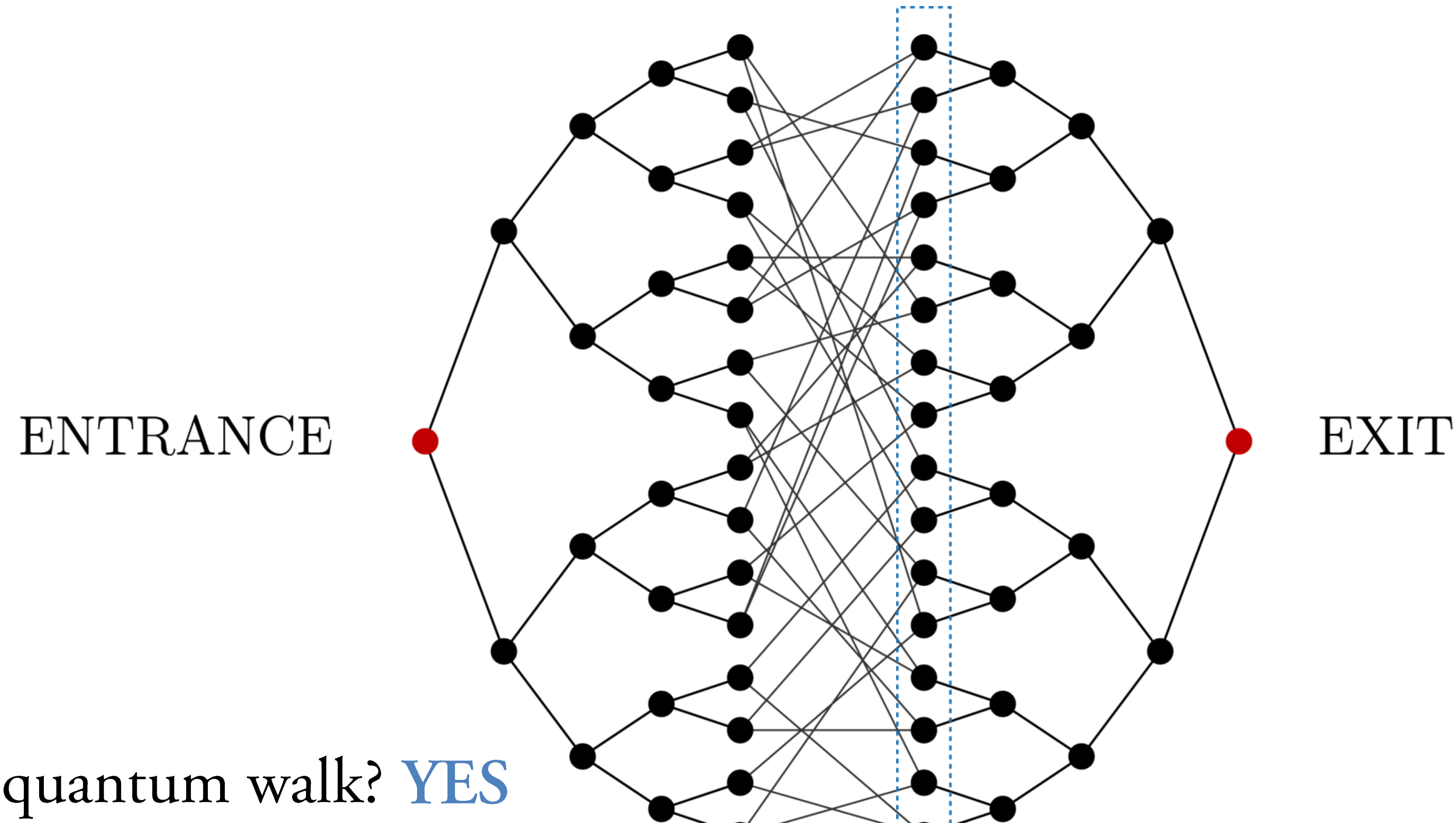
walking in superposition over columns



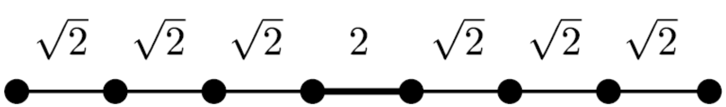
- random walk? NO
- quantum walk? YES
- recursion: YES

2 Efficiently walking through **randomly** glued trees

exponentially hard classically, polynomial for quantum walks



- quantum walk? YES



2 Efficiently walking through **randomly** glued trees

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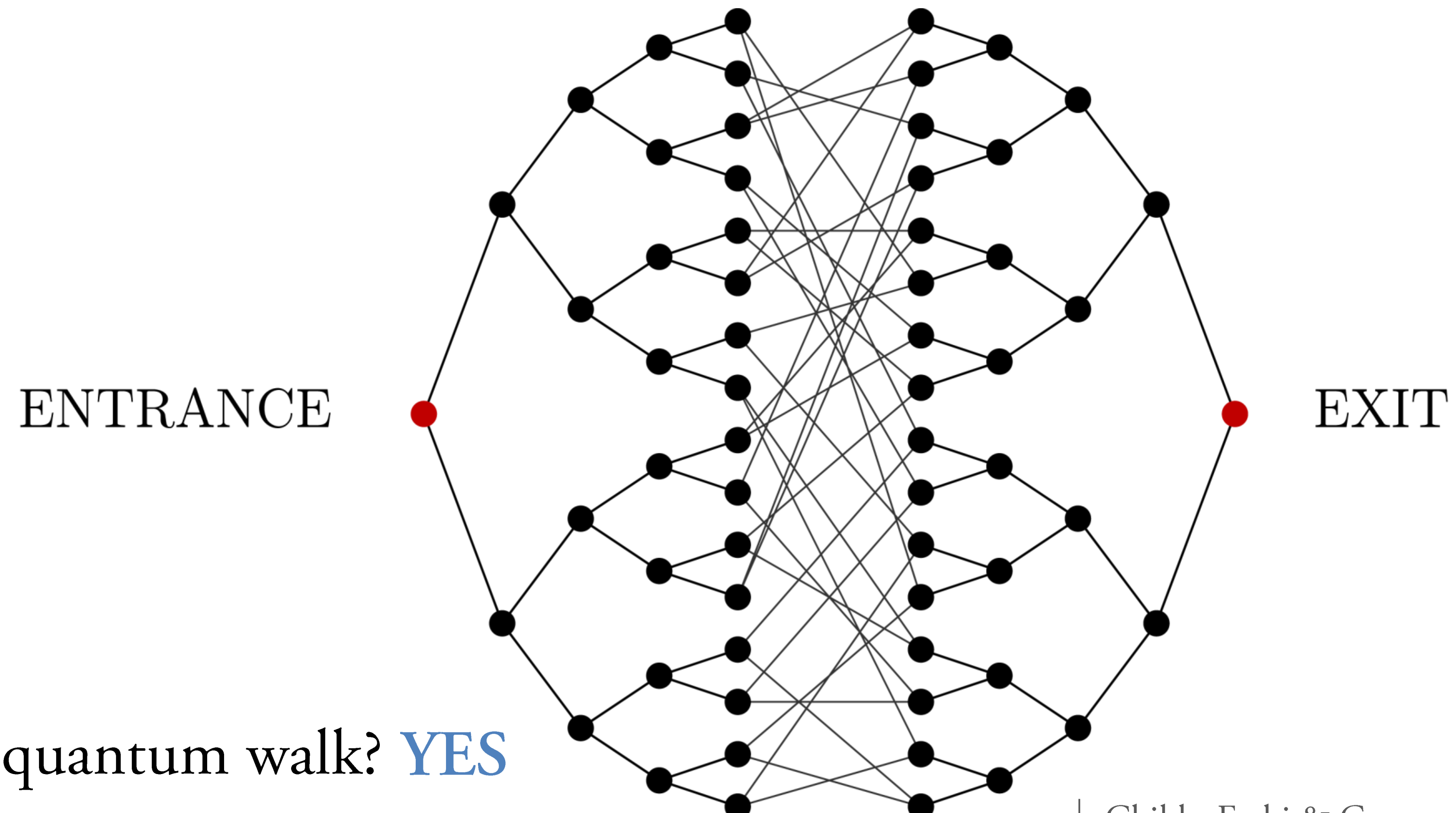


- quantum walk? YES
- classical algorithms? NO

Childs, Farhi & Gutmann
QIP 1, 3543 (2002)

3 Adiabatic Quantum Computation

evolving with a time-dependent Hamiltonian

$$i\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H(t)|\psi(t)\rangle$$

- fast: sudden (diabatic) approximation

  almost no change in the state

$$H_1 \rightarrow H_2$$

$$|\psi\rangle \rightarrow |\psi\rangle + \epsilon|\psi'\rangle$$

3 Adiabatic Quantum Computation

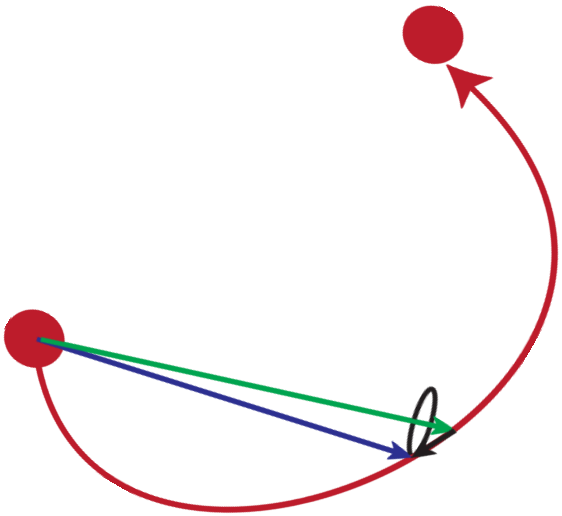
evolving with a time-dependent Hamiltonian

$$i\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H(t)|\psi(t)\rangle$$

- slow: adiabatic approximation

$$H(0) \longrightarrow H\left(s = \frac{t}{T}\right)$$

$$|\phi_i(0)\rangle \longrightarrow$$



3 Adiabatic Quantum Computation

evolving with a time-dependent Hamiltonian

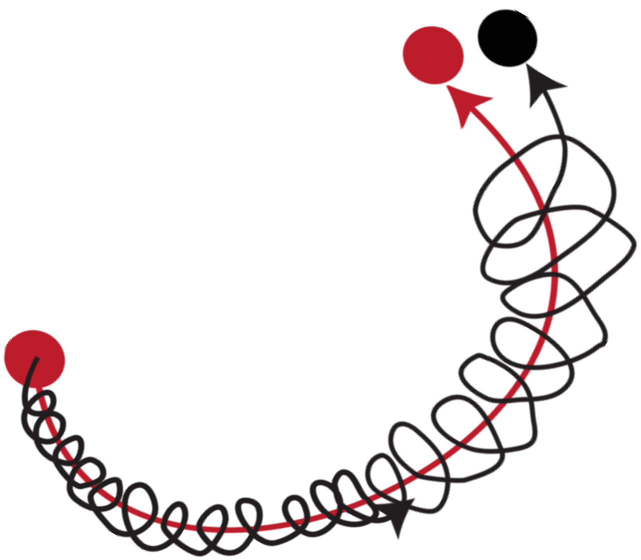
$$i\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H(t)|\psi(t)\rangle$$

- slow: adiabatic approximation

  an evolved eigenstate remains close to an eigenstate

$$H(0) \longrightarrow H\left(s = \frac{t}{T}\right)$$

$$|\phi_i(0)\rangle \longrightarrow |\phi_i(s)\rangle + \epsilon|\phi'\rangle$$



3 Adiabatic Quantum Computation

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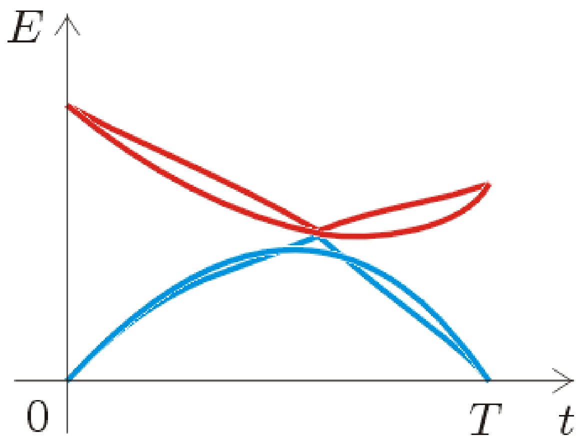


- which T makes it "slow" enough?

  norm of $||H(s)||, ||\dot{H}(s)||, \ldots$

  small gaps = easy excitations)-:

3 Adiabatic Quantum Computation

preparing states utilizing the adiabatic theorem

- adiabatic approximation

  an evolved eigenstate stays
  close to an eigenstate

3 Adiabatic Quantum Optimization

preparing ground states utilizing the adiabatic theorem

- **adiabatic approximation**
  an evolved ground state stays close to the ground state

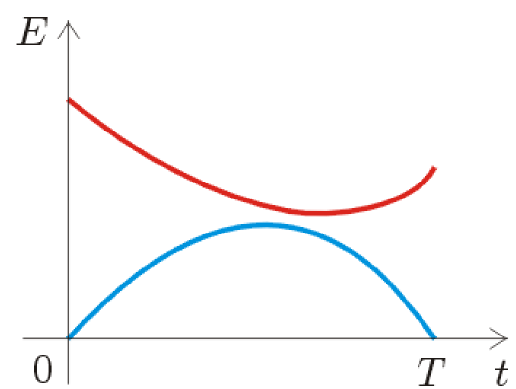
- **goal: the ground state of H_P**
  start in a simple ground state of some H_B

Farhi +, Q. comp. by adiabat. evolut. (2000)

$$H_B \rightarrow H_P$$

$$H(s) = (1-s)H_B + sH_P$$

small gaps are bad (avoided crossings)



3 Adiabatic Quantum Computation

why do we like this approach to quantum computing

- provably good for a few easy problems
- "not discouraging" numerics for interesting ones

It doesn't work!

It doesn't work!

3 Adiabatic Quantum Computation

why do we like this approach to quantum computing

- provably good for a few easy problems
- "not discouraging" numerics for interesting ones

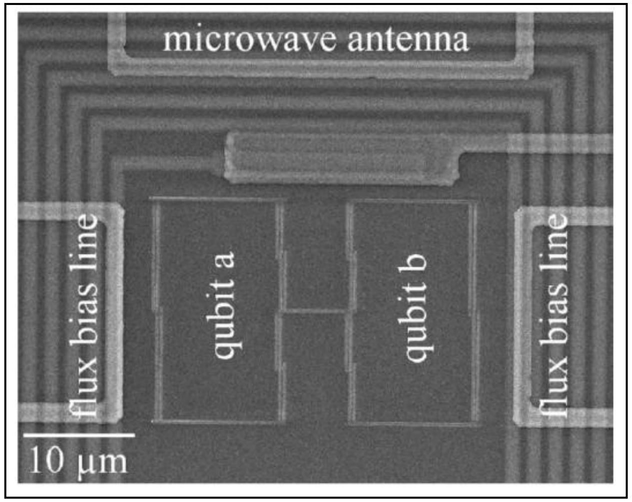
Choose a different H_B.

Add a little noise.

3 Adiabatic Quantum Computation

why do we like this approach to quantum computing

- provably good for a few easy problems
- "not discouraging" numerics for interesting ones
- universal for QC | Aharonov +, AQC is equiv. (2004)
- realizable: superconducting qubits



Izmalkov et al., PRL 101, 017003 (2008)

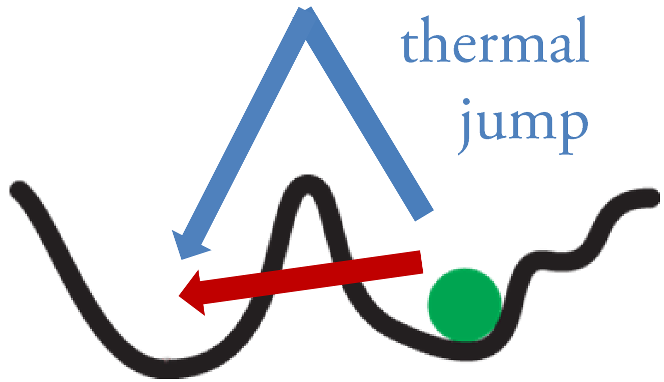
D-wave (2011)

3 Quantum Annealing with Stoquastic Hamiltonians

a classical & a quantum method

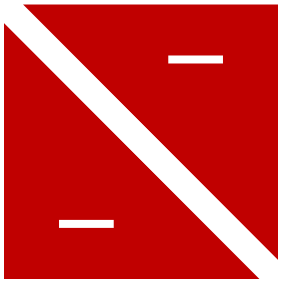
- QA: an optimization heuristic

  pick a path (Hamiltonian)
  a classical simulation
  a quantum algorithm



- stoquastic: negative off-diagonal elements

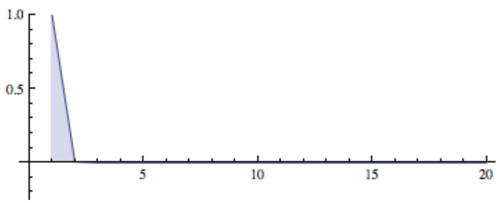
  not universal for quantum computing
  used in adiabatic algorithms
  no sign problem: Monte Carlo? **NO!**

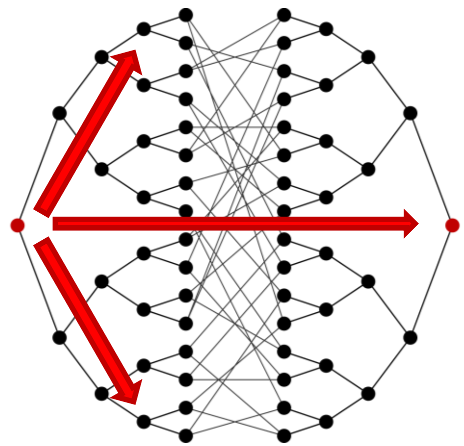


4 Traversing trees adiabatically

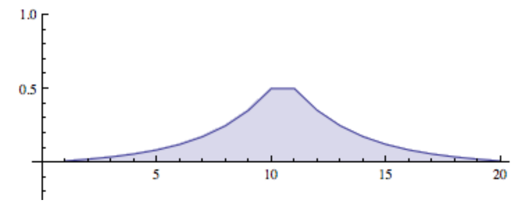
it doesn't really work with simple AQC

$H_B = -|0\rangle\langle 0|$





$H_P = -A$



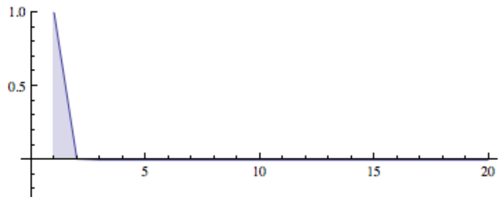
- start at the left endpoint
  go to a superposition over columns

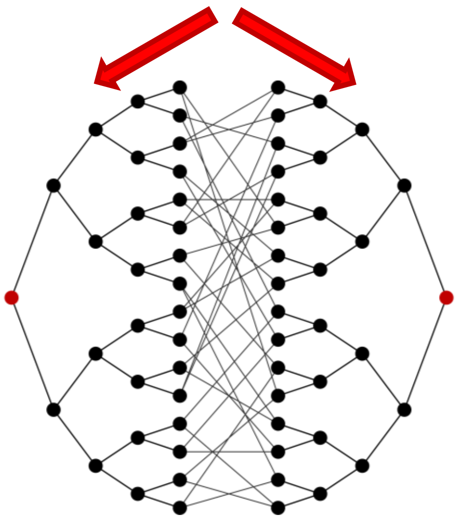
  the gap is exponentially small
  the final state has exp-small overlap with the EXIT

4 Traversing trees adiabatically

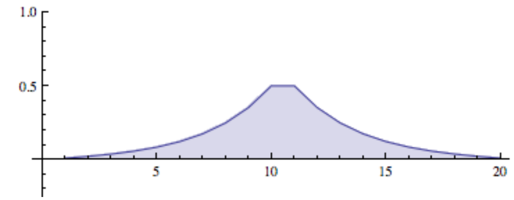
it doesn't really work with simple AQC

$$H_B = -|0\rangle\langle 0|$$





$$H_P = -A$$

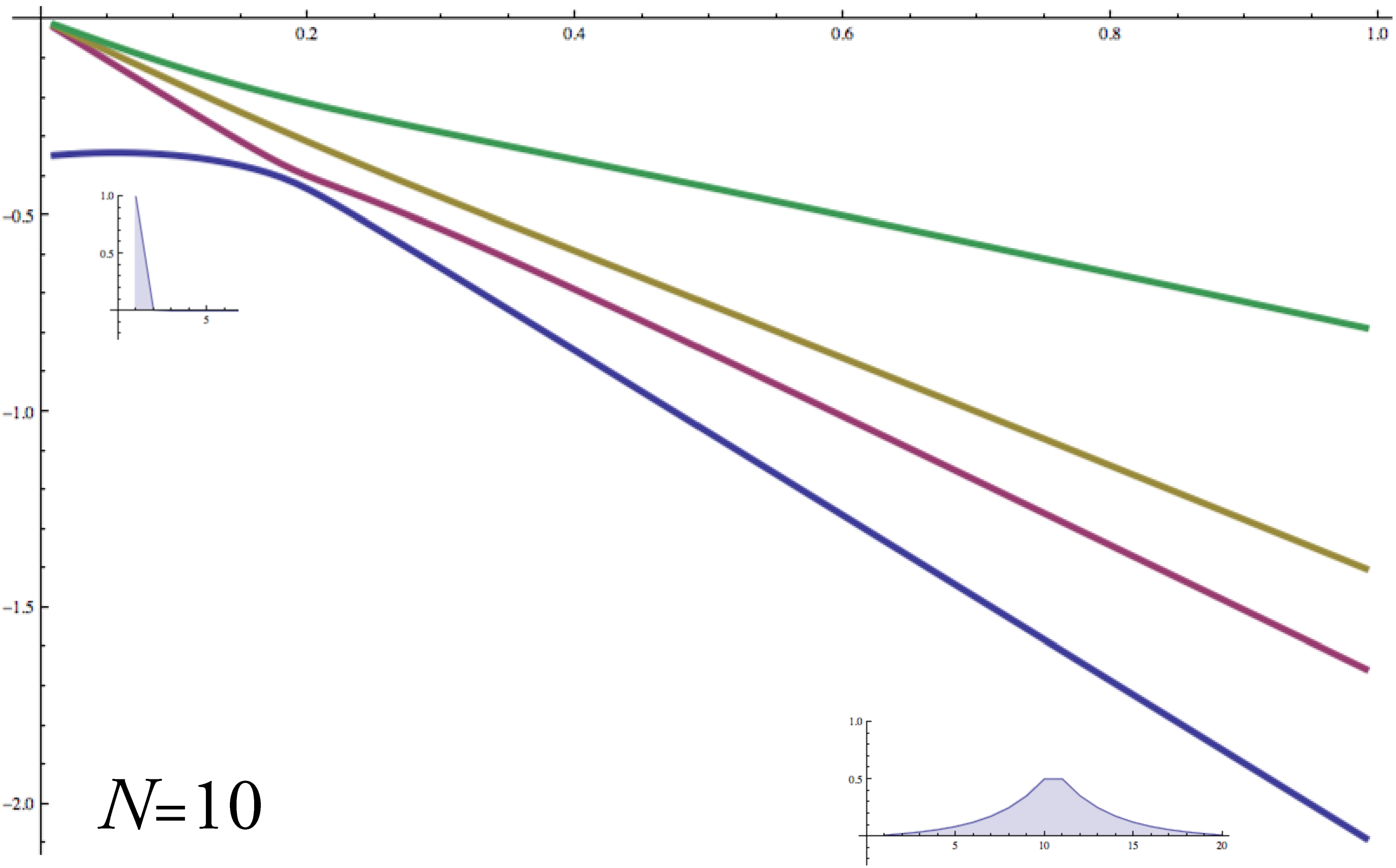


- start in a superposition over columns
  go to an endpoint

  the gap is exponentially small

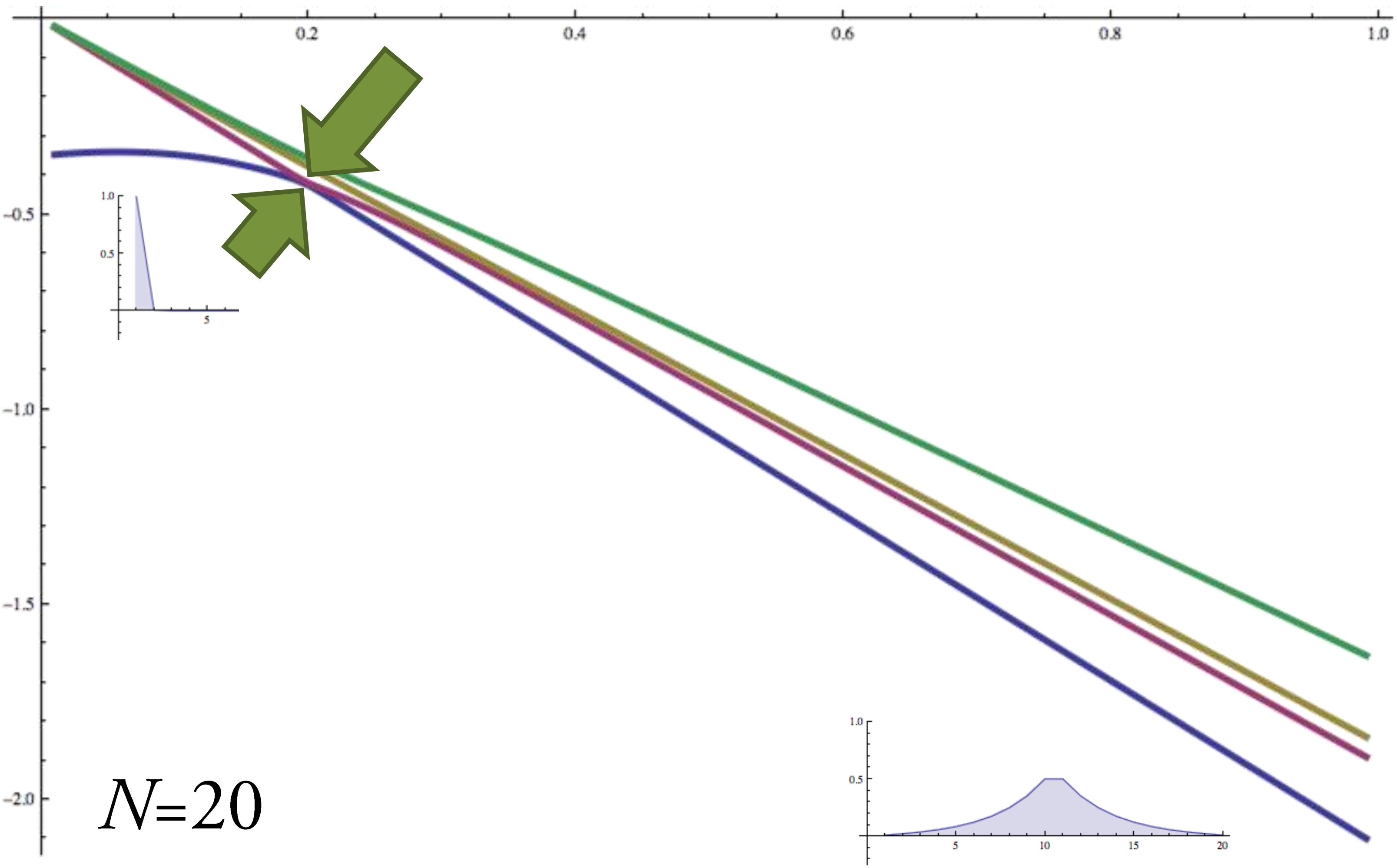
4 Traversing trees adiabatically

it doesn't really work with simple AQC



4 Traversing trees adiabatically

it doesn't really work with simple AQC – the gap is exp-small!

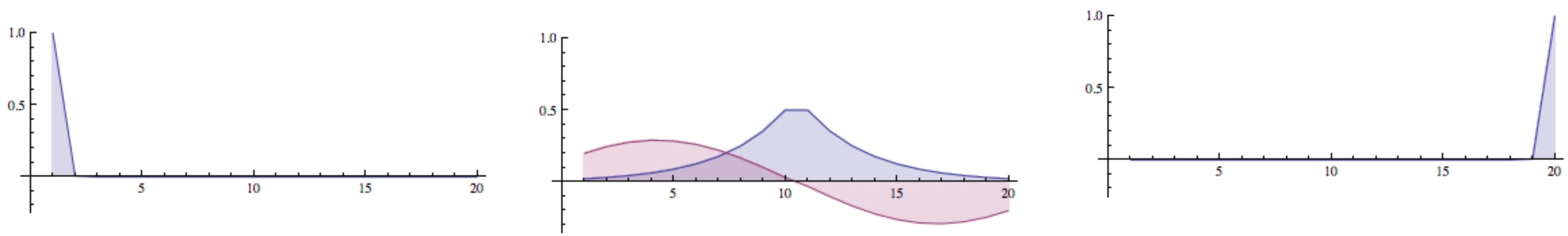


4 Traversing trees by annealing

a story of two gaps – exponential and polynomial

- a symmetric (3-component) algorithm

$$H_B = -|0\rangle\langle 0| \quad \rightarrow \quad H_M = -A \quad \rightarrow \quad H_P = -|E\rangle\langle E|$$

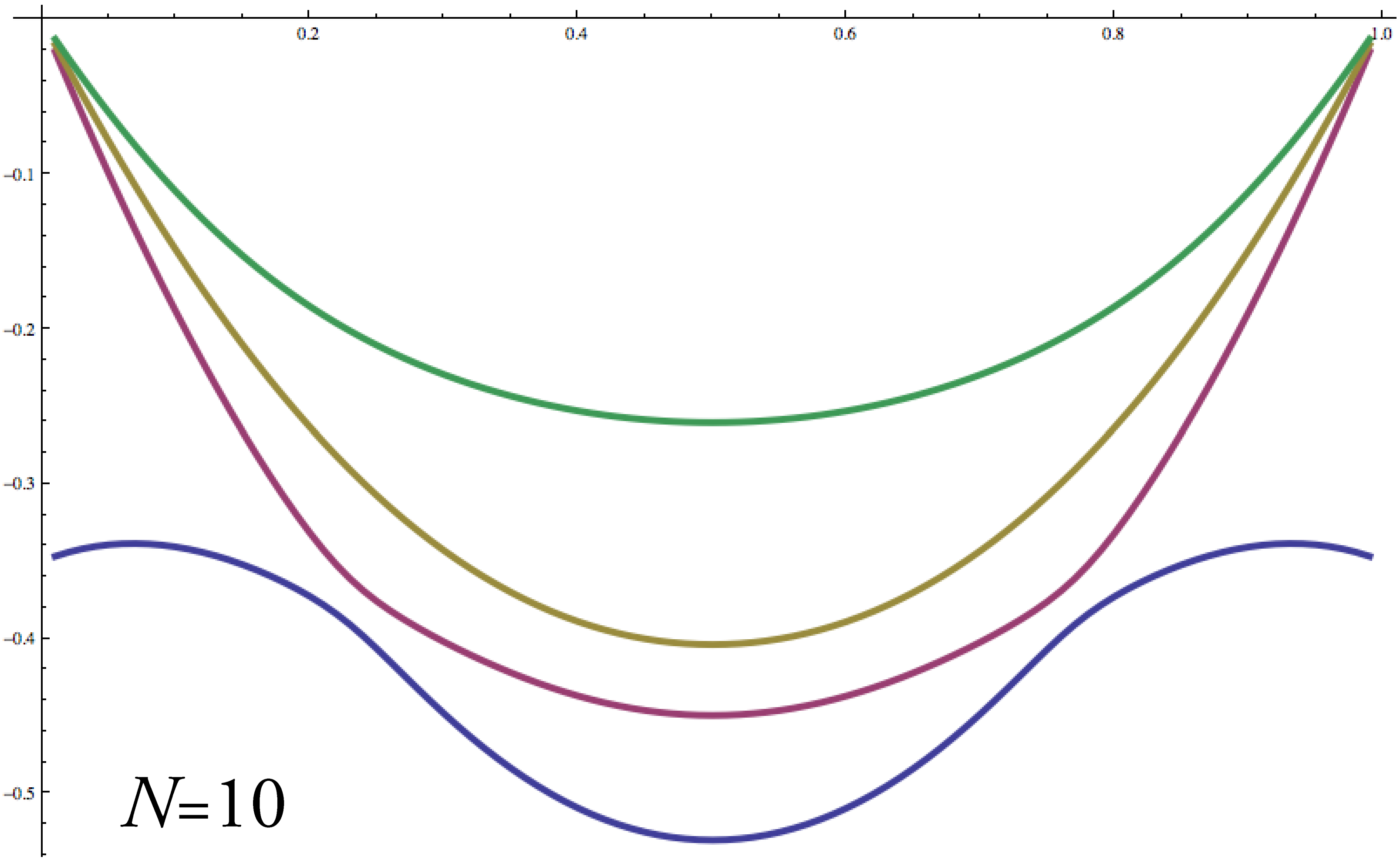


$$H(s) = (1-s)H_B + s(1-s)H_M + sH_P$$

The gaps still seem small… Will it necessarily jump?
Could we use the first excited state?

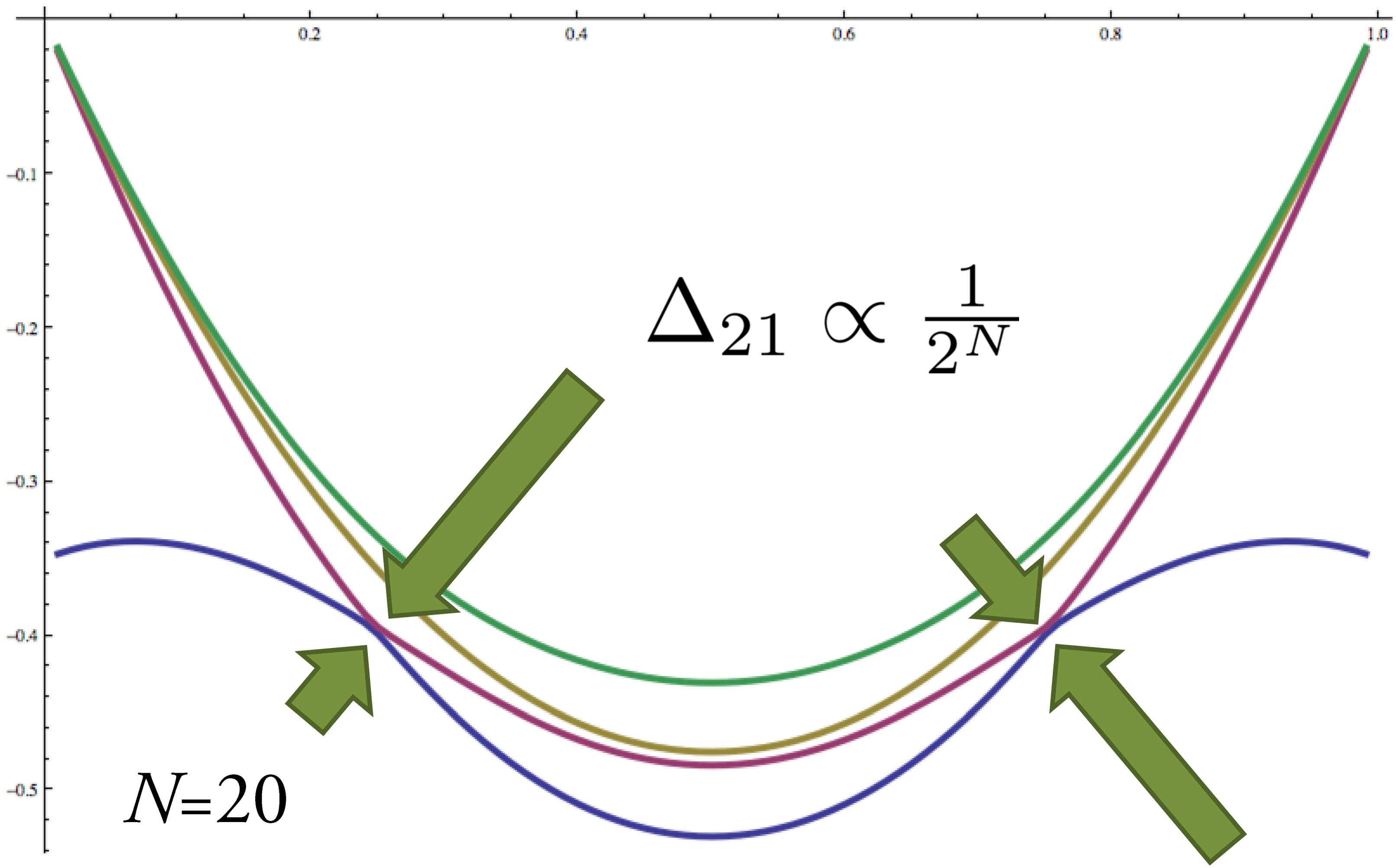
4 Traversing trees by annealing

a story of two gaps – exponential



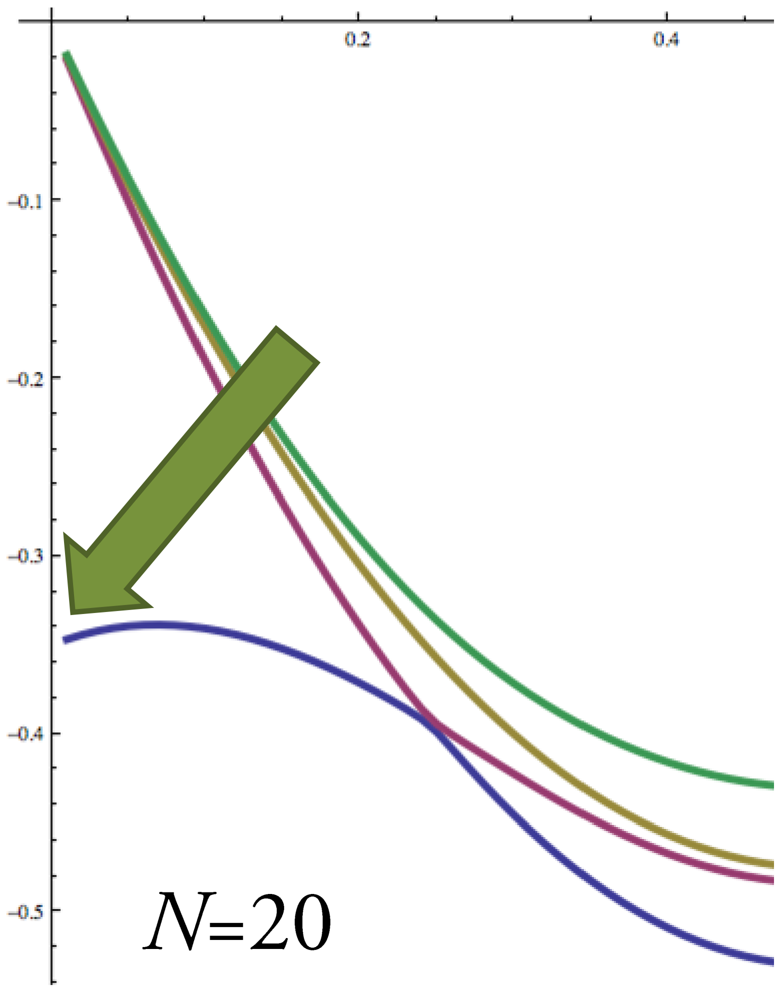
4 Traversing trees by annealing

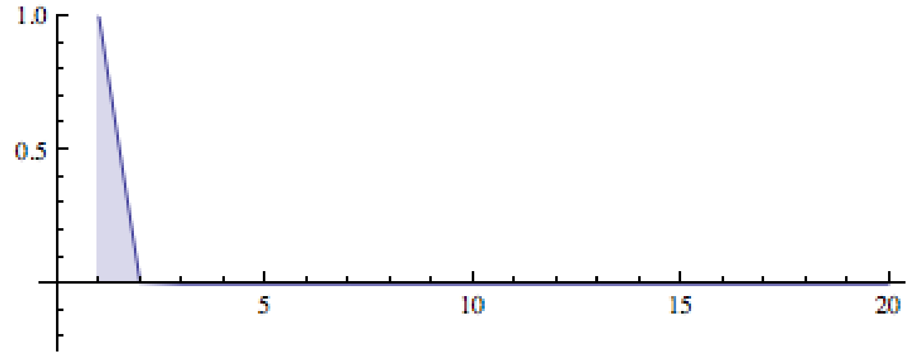
a story of two gaps – exponential



4 Traversing trees by annealing

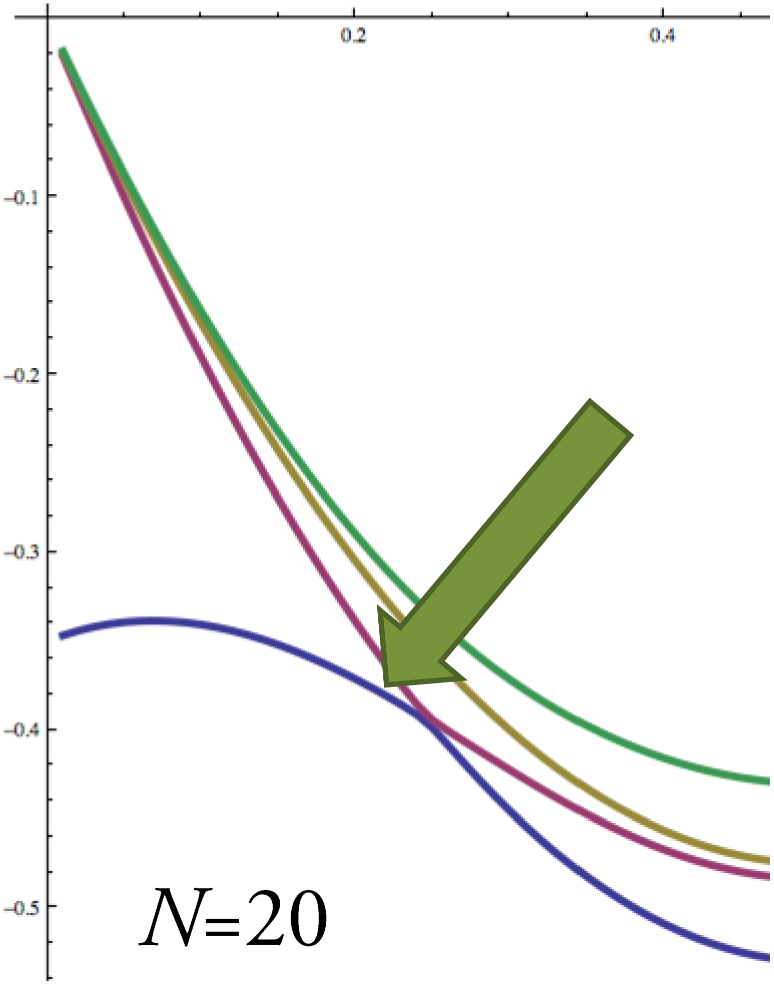
a story of two gaps – exponential

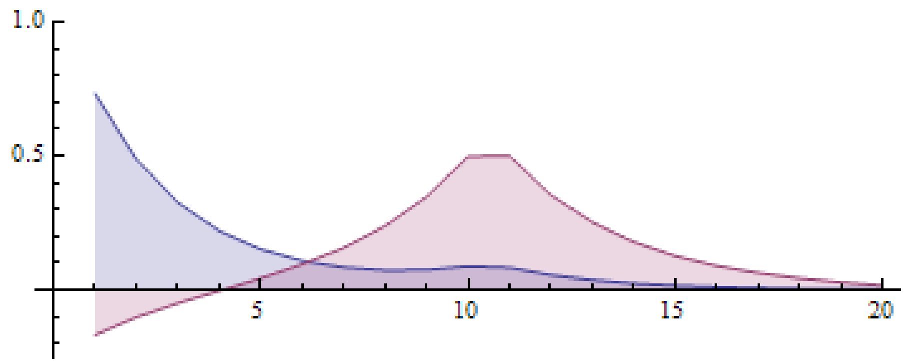




4 Traversing trees by annealing

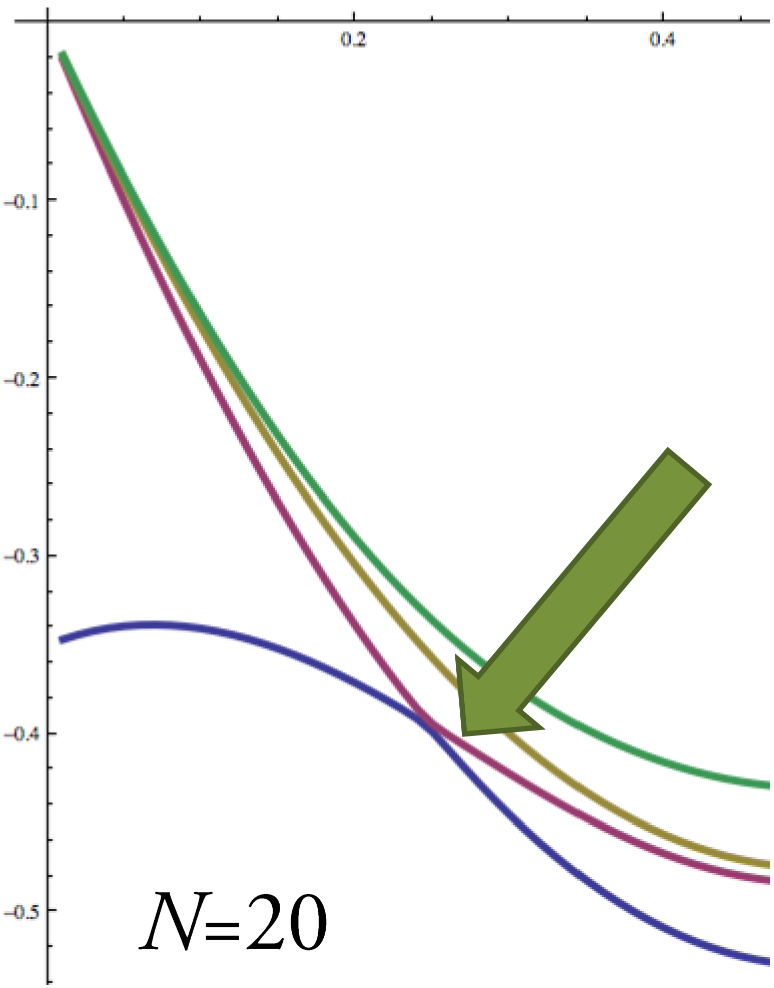
a story of two gaps – exponential



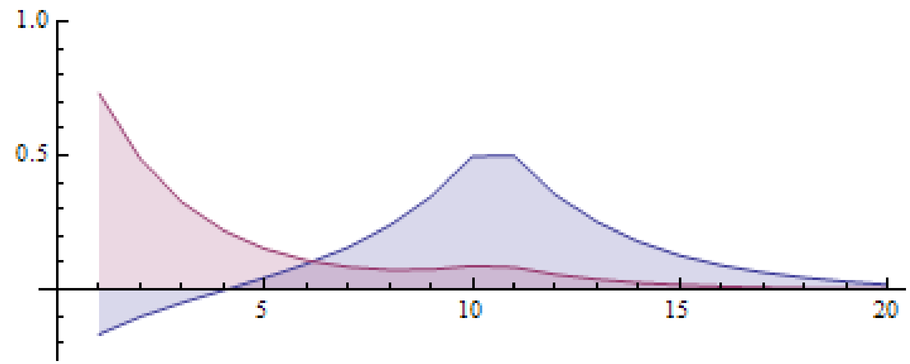


Traversing trees by annealing

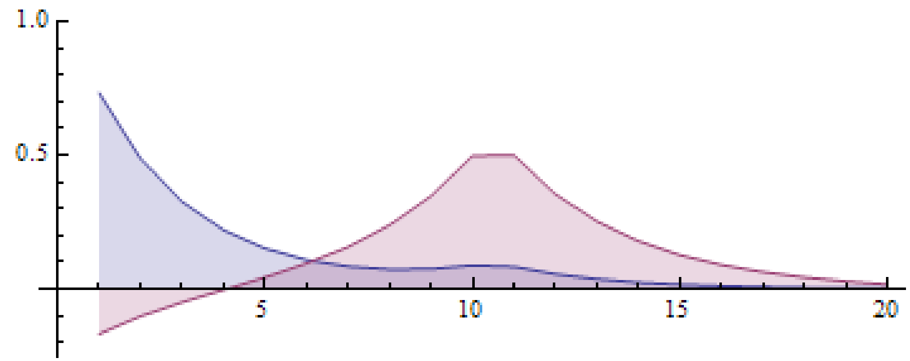
a story of two gaps – exponential



the energies (almost) cross

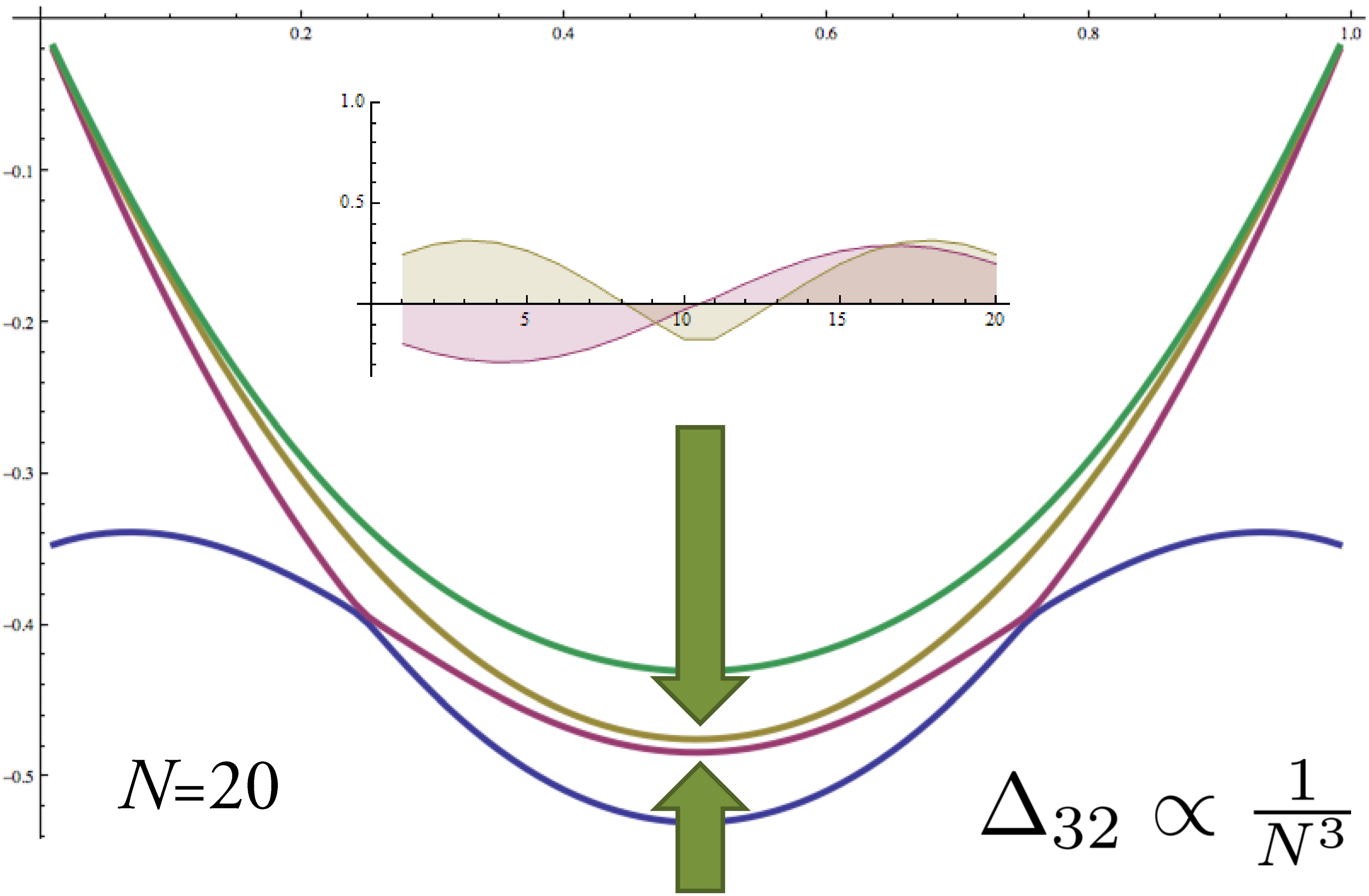


ground ↔ excited state exchange



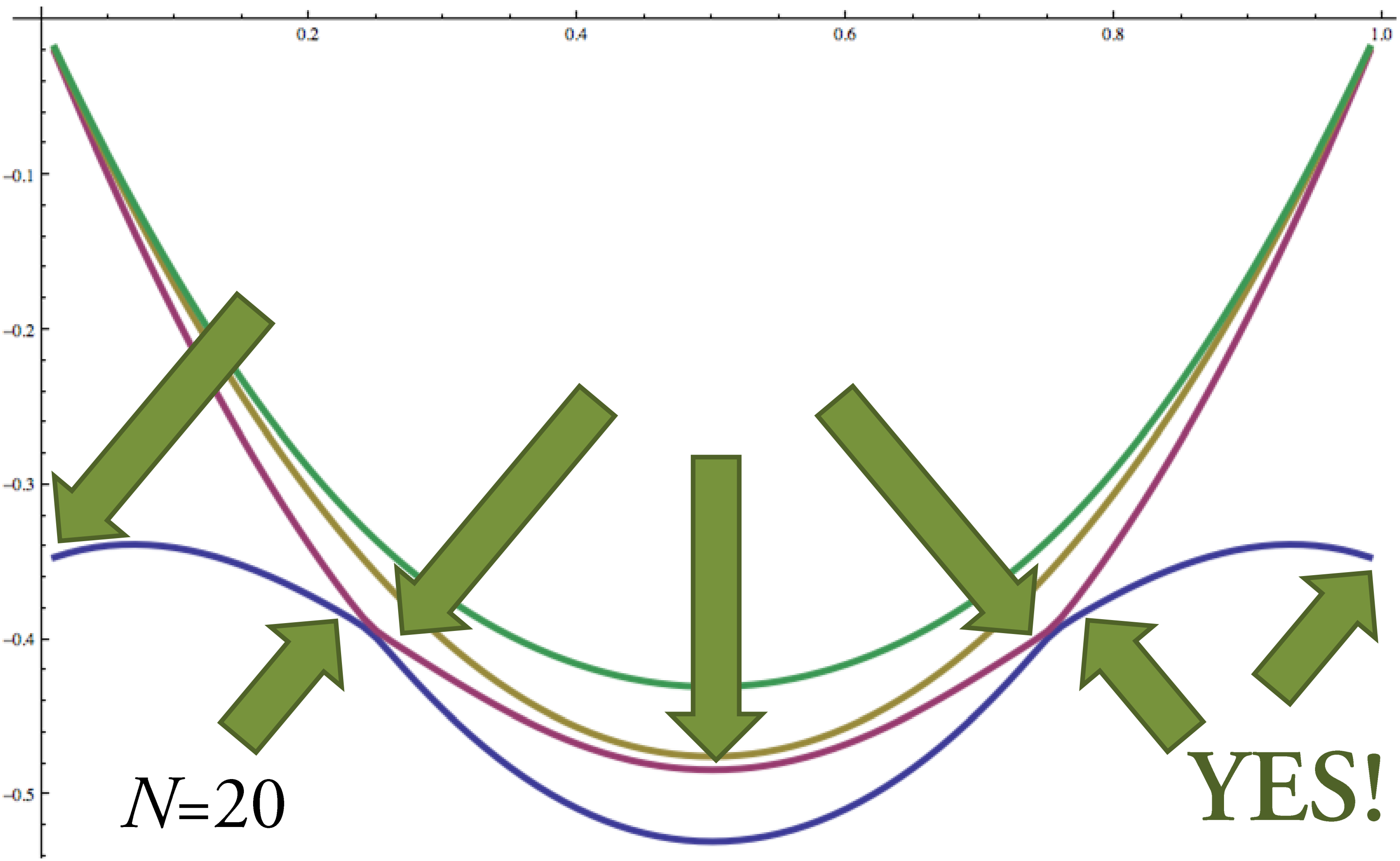
4 Traversing trees by annealing

a story of two gaps – exponential and polynomial



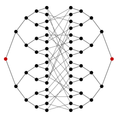
4 Traversing trees by annealing

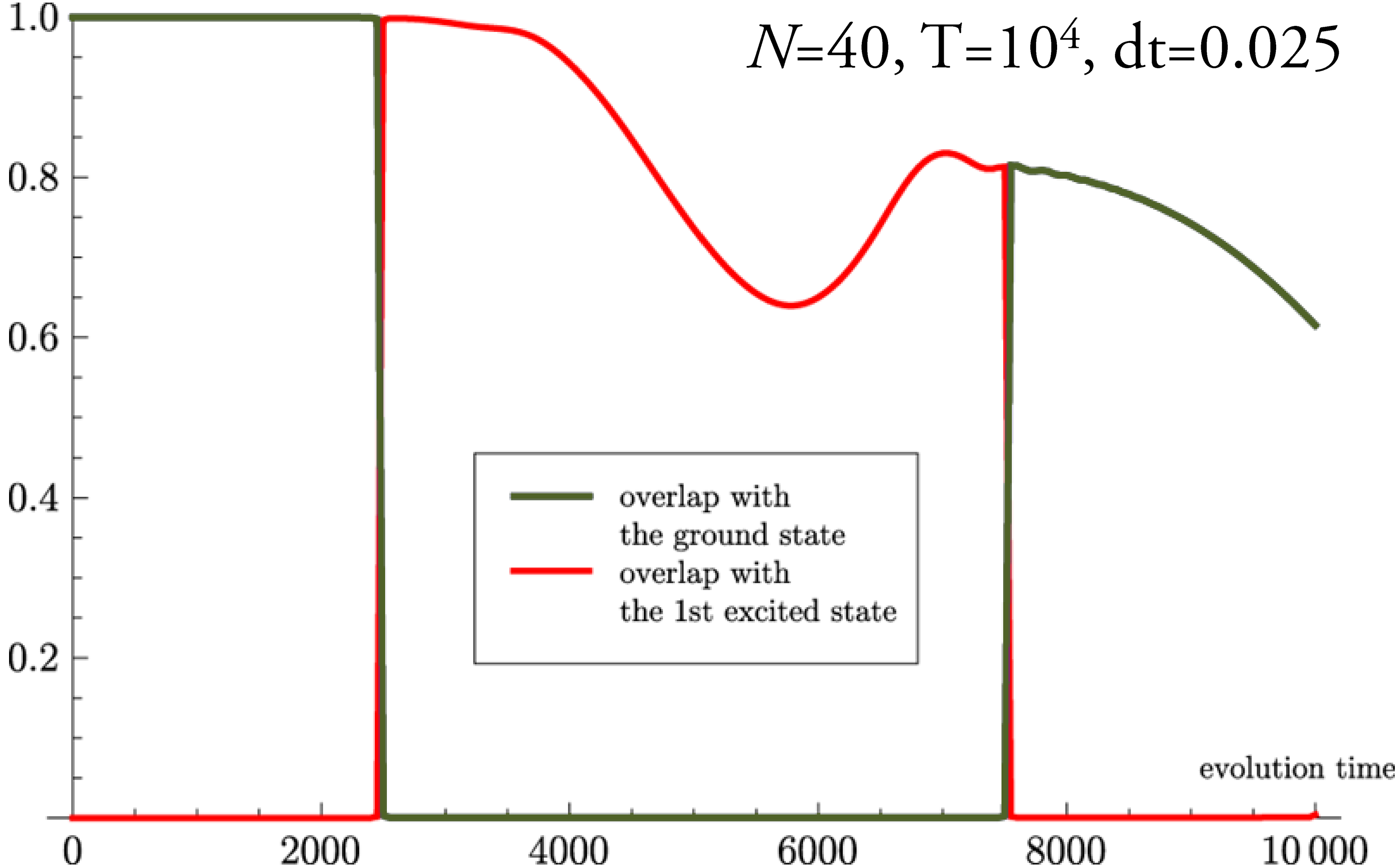
a story of two gaps – exponential and polynomial



4 Traversing trees by annealing

jumping through the exponential and not the polynomial gaps

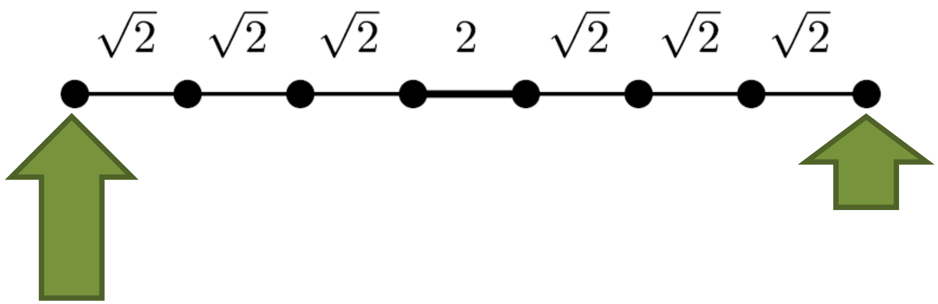




4 Traversing trees by annealing

a story of two gaps – exponential and polynomial

- a quantum walk solvable in the $N \to \infty$ limit



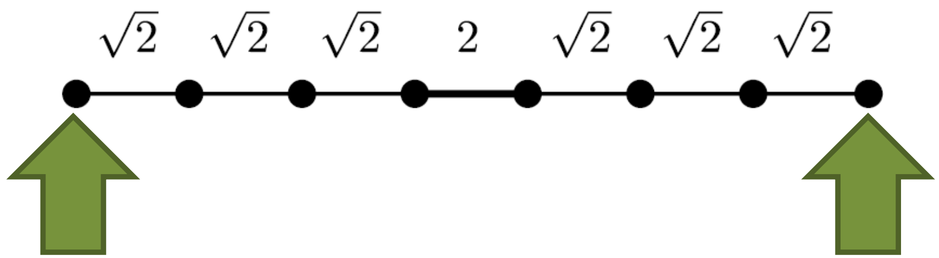
$$H(s) = (1-s)H_B + s(1-s)H_M + sH_P$$

$$H'(s) = \frac{1}{s}H_B + H_M + \frac{1}{1-s}H_P$$

4 Traversing trees by annealing

a story of two gaps – exponential and polynomial

- a quantum walk solvable in the N→∞ limit



$$H(s) = (1-s)H_B + s(1-s)H_M + sH_P$$

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a story of two gaps – exponential and polynomial

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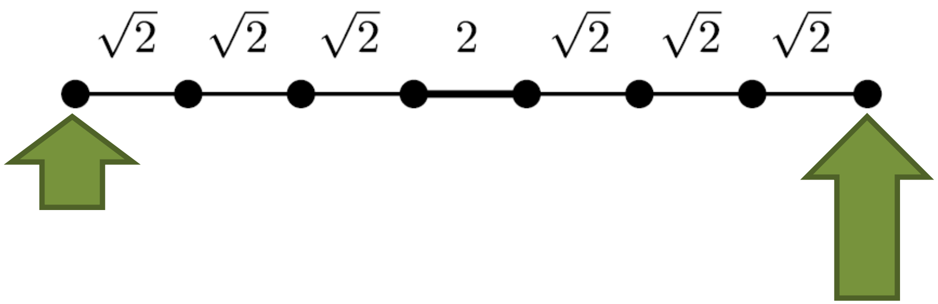

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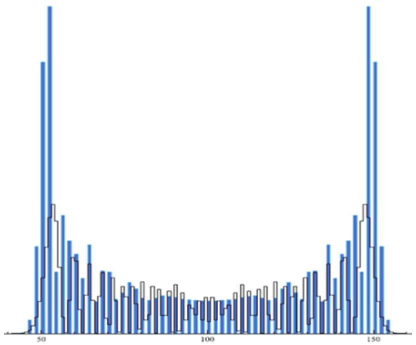
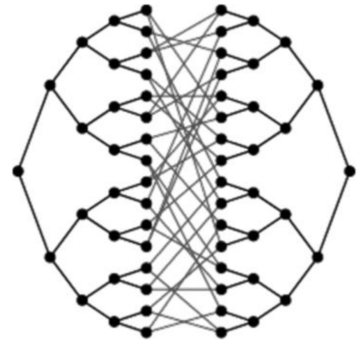
Rolando Somma at the Quantum Cryptography workshop in Dagsthul, 9/2012

Mária Kieferová at the Research Center for Quantum Information, Bratislava, 6/2012

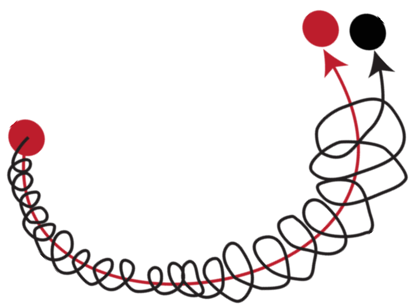
- c.t. quantum walks



- traversing glued trees



- adiabatic q. computing



- a new QA algorithm

arXiv: 1202.6257